MATHS

BOOKS - ARIHANT MATHS (HINGLISH)

APPLICATIONS OF DERIVATIVES

ILLUSTRATIVE EXAMPLES

1. A particle moves along the curve $x^2 = 2y$. At what point, ordinate increases at the same rate as abscissa increases ?

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2. The side of an equilateral triangle is increasing at the rate of 2 cm/s. At what rate is its area increasing when the side of the triangle is 20 cm?

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3. The length x of a rectangle is decreasing at the rate of 3 cm/minute and the width y is increasing at the rate of 2cm/minute. When

$x = 10$cm and $y = 6$cm, find the rates of change of (a) the perimeter and (b) the area of the rectangle.

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4. The volume of a sphere is increasing at the rate of 3 cubic centimeter per second. Find the rate of increase of its surface area, when the radius is 2 cm.

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5. For the curve $y = 5x - 2x^3$, if x increases at the rate of 2 units/sec, then how fast is the slope of the curve changing when $x = 3$?

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6. A water tank has the shape of an inverted right - circular cone with its axis vertical and vertex lower most. Its semi - vertical angle is $\tan^{-1}\left(\frac{1}{2}\right)$. Water is poured into it at a constant rate of 5 cubic meter per minute. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 10 m.

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7. Water is leaking from a conical funnel at the rate of $5c\frac{m^3}{\sec}$. If the radius of the base of the funnel is 5 cm and its altitude is 10 cm, find the rate at which the water level is dropping when it is 2.5 cm from the top.

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8. A stone is dropped into a quiet lake and waves move in circles at a speed of 4cm per second. At the instant, when the radius of the circular wave is 10 cm, how fast is the enclosed area increasing?

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9. A man of 2 metres height walks at a uniform speed of 6 km/hr away from a lamp post of 6 metres high. Find the rate at which the length of his shadow increases.

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10. A man is moving away from a tower 41.6 m high at the rate of 2 m/sec. Find the rate at which the angle of elevation of the top of tower is changing, when he is at a distance of 30m from the foot of the tower. Assume that the eye level of the man is 1.6m from the ground.

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11. The amount of pollution content added in air in a city due to $x - diesel$ vehicles is given by $P(x) = 0.005x^3 + 0.02x^2 + 30x$. Find the arginal increase in pollution content when 3 diesel vehicles are added and write which value is indicated in the above question.

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12. The money to be spend for the welfare of the employees of a firm is proportional to the rate of change of its total revenue (Marginal revenue). If the total revenue (in rupees) received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$, find the marginal revenue, when $x = 5$, and write which value does the question indicate.

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13. Using differentials, find the approximate value of $\sqrt{26}$

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14. Using differentials, find the approximate value of $\sqrt[3]{0.026}$, upto three places of decimals.

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15. Find the approximate change in the volume V of a cube of side x metres caused by increasing the side by 1%.

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16. If the radius of a sphere is measured as 9 m with an error of 0.03 m, then find the approximate error in calculating its surface area.

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17. If $y = x^4 + 10$ and x change from 2 to 1.99, find the approximate change in y.

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18. Find the approximate value of $\log_e(9 \cdot 01)$ given $\log_e 3 = 1 \cdot 0986$

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19. Using differentials find the approximate value of $\tan 46^0$, if it is being given that $1^0 = 0.01745$ radians.

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20. If in a triangle ABC, the side c and the angle C remain constant, while the remaining elements are changed slightly, using differentials show that $\dfrac{da}{csA} + \dfrac{db}{\cos B} = 0$

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21. The time t of a complete oscillation of a simple pendulum of length l is given by the equation $T = 2\pi\sqrt{\dfrac{1}{g}}$ where g is constant. What is the percentage error in T when l is increased by 1%?

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22. Find the maximum and minimum values, if any of the function given by :

(i) $f(x) = x^2, x \in R$

(ii) $f(x) = |x|, x \in R.$

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23. Find the maximum and minimum value, if any, of the following function without using derivatives:

(i) $f(x) = (2x - 1)^2 + 3$

(ii) $f(x) = 16x^2 - 16x + 28$

(iii) $f(x) = -|x + 1| + 3$

(iv) $f(x) = \sin 2x + 5$

(v) $f(x) = \sin(\sin x)$.

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24. Determine the absolute maximum and absolute minimum values of each of the following in the stated domains :

(i) $y = \frac{1}{2}x^2 + 5x + \frac{3}{2}, \ -6 \leq x \leq -2$

(ii) $f(x) = (x + 1)^{2/3}, 0 \leq x \leq 8.$

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25. Calculate the absolute maximum and absolute minimum value of the function $f(x) = \frac{x+1}{\sqrt{x^2+1}}, 0 \le x \le 2.$

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26. Find the absolute maximum and the absolute minimum value of the function given by :

$f(x) = \sin^2 x - \cos x, x \in [0, \pi].$

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27. Find the points of local maxima or local minima, if any, of the following function, using the first derivative test. Also, find the local maximum or local minimum values, as the case may be: $f(x) = (x-1)(x+2)^2$

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28. Find the points of local mixima and local minima, if any, of the following function :

$f(x) = \sin x + \frac{1}{2}\cos 2x : 0 \leq x \leq \frac{\pi}{2}.$

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29. Find all the points of local maxima and local minima of the function f given by $f(x) = 2x^3 - 6x^2 + 6x + 5.$

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Frequently Asked Questions

1. Without using the derivative show that the function $f(x) = 7x - 3$ is strictly increasing function on R .

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2. Show that the function $f(x) = 4x^3 - 18x^2 + 27x - 7$ is always increasing on .

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3. Find the intervals in which the function $\mathrm{f\ (x) = \ x^3 - \ 12x^2 + \ 36x + 17\ }$ is (a)

increasing, (b) decreasing.

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4. Find the intervals in which the function $f(x) = \frac{x^4}{4} - x^3 - 5x^2 + 24x + 12$ is (a) strictly increasing (b) strictly decreasing

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5. Separate the interval $\left[0, \frac{\pi}{2}\right]$ into sub intervals in which function

$f(x) = \sin^4(x) + \cos^4(x)$ is strictly increasing or decreasing.

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6. Find the intervals in which $f(x) = \sin 3x - \cos 3x, 0 < 0 < \pi,$ is strictly increasing or decreasing.

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7. Find the values of 'x' for which $f(x) = x^x, x > 0$ is strictly increasing or decreasing.

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8. If a, b, c are real numbers, then find the intervals in which :

$f(x) = \begin{vmatrix} x + a^2 & ab & ac \\ ab & x + b^2 & bc \\ ac & bc & x + c^2 \end{vmatrix}$ is strictly increasing or decreasing.

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9. Prove that

$$\frac{x}{(1-x)} < \log(1+x) < x \quad \text{for} \quad x > 0$$

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10. The point at which the tangent to the curve $y = \sqrt{4x-3} - 10$ has slope $\frac{2}{3}$ is

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11. Find the equation of all lines having slope 2 and being tangent to the curve $y + \frac{2}{x-3} = 0$.

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12. Find the equation of the tangent to the curve $y = \frac{x-7}{(x-2(x-3)}$ at the point where it cuts the x-axis.

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13. Find the equations of the tangent and the normal to the curve $16x^2 + 9y^2 = 145$ at the point (x_1, y_1), where $x_1 = 2$ and $y_1 > 0$

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14. Find the equations of the tangent and normal to the curve $x = a\sin^3\theta andy = a\cos^3\theta$ at $\theta = \frac{\pi}{4}$.

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15. Find the equations of tangent to the curve $x = 1 - \cos\theta, y = \theta - \sin\theta$ at $\theta = \frac{\pi}{4}$

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16. Find the equation of the tangent to the curve $x^2 + 3y = 3$, which is parallel to the line $y - 4x + 5 = 0$

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17. Determine the points on the curve $x^2 + y^2 = 13$, where the tangents are perpendicular to the line $3x - 2y = 0$.

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18. Show that the equation of normal at any point t on the curve $x = 3\cos t - \cos^3 t$ and $y = 3\sin t - \sin^3 t$ is $4(y\cos^3 t - x\sin^3 t) = 3\sin 4t$.

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19. Find the value of $n \in N$ such that the curve $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$ touches the straight line $\frac{x}{a} + \frac{y}{b} = 2$ at the point (a, b).

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20. At what point will be tangents to the curve $y = 2x^3 - 15x^2 + 36x - 21$ by parallel to x=axis? Also, find the equations of the tangents to the curve at these points.

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21. Show that the curves $xy = a^2 and x^2 + y^2 = 2a^2$ touch each other

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22. Find the angle of intersection of the following curves: $xy = 6 and x^2 y = 12$

$y^2 = 4x and x^2 = 4y$

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23. If the curve $ax^2 + by^2 = 1$ and $a'x^2 + b'y^2 = 1$ intersect orthogonally, then

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24. Find the values of x for which $f(x) = [x(x-2)]^2$ is an increasing unction. Also, find the points on the curve, where the tangent is parallel to x-axis.

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25. Find two positive number whose sum is 24 and their sum of square is minimum.

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26. Show that all the rectangles with a given perimeter, the square has the largest area.

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27. Show that the rectangle of maximum perimeter which can be inscribed in a circle of radius a is a square of side $\sqrt{2}a$.

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28. Show that the triangle of maximum area that can be inscribed in a given circle is an equilateral triangle.

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29. An open box with a square base is to be made out of a given iron sheet of area 27 sq. m.

Show that the maximum volume of the box is 13.5 cu.cm.

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30. 40. A given quantity of metal is to be cast into a half cylinder with a rectangular base and semicircular ends. Show that in order that the total surface area may be minimum, the ratio of the length of the cylinder to the diameter of its semi-circular ends is w `(

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31. A window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12 m, find the dimensions of the rectangle that will produce the largest area of the window.

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32. Show that the height of the cylinder of maximum volume that can be inscribed in a cone of height h is $\frac{1}{3}h$

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33. Let AP and BQ be two vertical poles at points A and B, respectively. If $AP = 16m, BQ = 22m and AB = 20m,$ then find the distance of a point R on AB from the point A such that $RP^2 + RQ^2$ is minimum.

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34. If length of three sides of a trapezium other than base are equal to 10cm, then find the area of the trapezium when it is maximum.

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35. A square piece of tin of side 24 cm is to be made into a box without top by cutting a square from each corner and foding up the flaps to form a box. What should be the side of the square to be cut off so that the volume of the box is maximum ? Also, find this maximum volume.

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36. A metal box with a square base and vertical sides is to contain 1024 cm3 of water, the material for the top and bottom costs Rs 5 per cm2 and the material for the sides costs Rs 2.50 per cm2. Find the least cost of the box.

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37. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is 8 m3. If building of tank costs Rs 70 per square metre for the base and Rs 45 per square metre for sides, what is the cost of least expensive tank?

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38. Show that the right circular cylinder, open at the top, and of given surface area and maximum volume is such that its height is equal to the radius of the base.

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39. Find the point on the curve $y^2 = 4x$ which is nearest to the point (2, 1).

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40. An Apache helicopter of enemy is flying along the curve given by $y = x^2 + 7$. A soldier, placed at (3, 7), wants to shoot down the helicopter when it is nearest to him. Find the nearest distance.

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41. An open tank with a square base and vertical sides is to be constructed from a metal sheet so as to hold a given quantity of water. Show that the cost of the material will be least when depth of the tank is half of its width.

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42. A cuboidal shaped godown with square base is to be constructed. Three times as much cost per square metre is incurred for constructing the roof as compared to the walls. Find the dimensions of the godown if it is to enclose a given volume and minimize the cost of constructing the roof and the walls.

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43. A manufacturer can sell x items at a price of Rs. $\left(5 - \frac{x}{100}\right)$ each. The cost price of x items is Rs. $\left(\frac{x}{5} + 500\right)$. Find the number of items he should sell to earn maximum profit.

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44. A person wants to plant some trees in his community park. The local nursery has to perform this task. It charges the cost of planting trees by the formula :

$$C(x) = x^3 - 45x^2 + 600x,$$

where 'x' is the number of trees and C(x) is cost of planting 'x' trees in rupees. The local authority has imposed a restriction that it can plant 10 to 20 trees in one community park for a fair - districution. For how many trees should the person place the order so that he has to spend the least amount? How much is the least amount? Use Calculate to answer these questions.

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Questions From NCERT Exemplar

1. For the curve $y = 5x - 2x^3$, if x increases at the rate of 2 units/sec, then how fast is the slope of the curve changing when $x = 3$?

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2. Prove that the function $f(x) = \tan x - 4x$ is strictly decreasing on $(-\pi/3, \pi/3)$.

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3. Show that the function $f(x) = 4x^3 - 18x^2 + 27x - 7$ has neither maxima nor minima.

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4. Find the condition for the curve $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $xy = c^2$ to interest orthogonally.

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5. Find the difference between the greatest and least values of the function $f(x) = \sin 2x - x$ on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

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6. Find the area of the greatest rectangle that can be inscribed in an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

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7. An isosceles triangle of vertical angle 2θ is inscribed in a circle of radius a . Show that the area of the triangle is maximum when $\theta = \frac{\pi}{6}$.

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EXERCISE 6 (a) (Short Answer Type Questions)

1. An edge of a variable cube is increasing at the rate of $3cm/s$. How fast is the volume of the cube increasing when the edge is $10cm$ long?

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2. The radius of a soap - bubble is increasing at the rate of $0.7cm/s$. Find the rate of increase of :

its volume when the radius is 5 cm.

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3. 6. The radius of a circle is increasing at the rate of 0.7 cm/s. What is the rate of increase of its circumference?

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4. The radius of a circle is increasing uniformly at the rate of 4 cm/sec. Find the rate at which the area of the circle is increasing when the radius is 8 cm.

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5. If the area of a circle increases at a uniform rate, then prove that perimeter varies inversely as the radius.

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6. The radius of a circle is increasing uniformly at the rate of 3 cm/s. Find the rate at which the area of the circle is increasing when the radius is 10 cm.

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7. The radius of an air bubble is increasing at the rate of $\frac{1}{2}cm/s$. At what rate is the volume of the bubble increasing when the radius is 1 cm?

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8. A balloon, which always remains spherical, has a variable diameter $\frac{3}{2}(2x+1)$.Find the rate of change of its volume with respect to x.

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9. A balloon which always remains spherical, is being inflated by pumping in 900 cubic centimetres of gas per second. Find the rate at which the radius of the balloon is increasing when the radius is 15 cm.

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10. The volume of a cube is increasing at a rate of 9 cubic centimetres per second. How fast is the surface area increasing when the length of an edge is 10 centimetres?

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11. The volume of a cube is increasing at the rate of $8cm^3/s$. How fast is the surface area increasing when the length of an edge is 12 cm?

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12. The volume of a cube is increasing at the rate of increasing at the instant when the length of an edge of the cube is 24 cm?

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13. A particle moves along the curve $y = \frac{4}{3}x^3 + 5$. Find the points on the curve at which the y - coordinate changes as fast as the x - coordinate.

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14. A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate

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15. A particle moves along the parabola $y^2 = 4x$. Find the co - ordinates of the point on the parabola where the rate of increment of abscissa is twice the rate of increment of the ordinate.

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EXERCISE 6 (a) (Long Answer Type Questions (I))

1. The radius of a cylinder increases at the rate of 1 cm/s and its height decreases at the rate of 1 cm/s. Find the rate of change of its volume when the radius is 5 cm and the height is 5 cm.

If the volume should not change even when the radius and height are changed, what is the relation between the radius and height?

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2. The total cost C(x) in Rupees, associated with the production of x units of an item is given by

$C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$. Find the marginal cost when 3 units are produced, where by marginal cost we mean the instantaneous rate of cha

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3. The total revenue in rupees received from the sale of 'x' units of a product is given by:

$R(x) = 13x^2 + 26x + 20.$

Find the marginal revenue when x = 7

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4. Total revenue from the sale of 'x' units of a product is given by :

$$R(x) = 40x - \frac{x^2}{2}.$$

Find the marginal revenue when x = 6 and interpret it.

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5. A man 160 cm tall, walks away from a source of light situated at the top of a pole 6m high at the rate of 1.1 m/sec. How fast is the length of his shadow increasing when he is 1 metre away from the pole.

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6. A man 2 metres high walks at a uniform speed of 5 km/hr away from a lamp-post 6 metres high. Find the rate at which the length of his shadow increases.

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7. The length x of a rectangle is decreasing at the rate of 5 cm/minute and the width y is increasing at the rate of 4 cm/minute. When x = 8cm and y = 6cm, find the rates of change of (a) the perimeter, and (b) the area of the rectangle

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8. The volume of a sphere is increasing at the rate of $8cm^3/s$. Find the rate at which its surface area is increasing when the radius of the sphere is 12 cm.

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9. The length 'x' of a rectangle is decreasing at the rate of 3 cm/m and the width 'y' is increasing at the rate of 2 cm/m. Find the rates of change of :

(a) the perimeter (b) the area of the rectangle when x = 8 cm and y = 6 cm.

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10. An edge of a variable cube is increasing at the rate of 3 cm per second. How fast is the volume of the cube increasing when the edge is 10 cm long?

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11. Water is drpping out from a conical funnel at the uniform rate of $2cm^3/s$ through a tiny hole at the vertex at the bottom. When the slant height of the water is 5 cm, find the rate of decrease of the slant higher of the water. Given that α is semi-vertical angle of the cone.

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12. An inverted conical vessel whose height is 10 cm and the radius of whose base is 5 cm is being filled with water at the uniform rate of $1.5cm^3 / \min$. Find the rate at which the level of water in the vessel is rising when the depth is 4 cm.

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13. A ladder 5m long is leaning against a wall. The foot of the ladder is pulled out along the ground away from from the wall at a rate of $2m/s$. How fast is the height of ladder on the decreasing at the instant when the foot of the ladder is $4m$ away from the wall?

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14. A 13-m long ladder is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2 m/s.

How fast is its height on the wall decreasing when the foot of the ladder is 5m aways from the wall ?

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15. The radius of a circular soap bubble is increasing at the rate of 0.2cm/s. Find the rate of change of its:

(I) Volume (II) Surface area

when the radius is 4 cm.

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EXERCISE 6 (a) (Long Answer Type Questions (II))

1. Water is running into a conical vessel, 15cm deep and 5cm in radius, at the rate of 0.1 cm^3 / sec . When the water is 6cm deep, find at what rate it. the water level rising? the water-surface area increasing? the wetted surface of the vessel increasing?

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EXERCISE 6 (b) (Short Answer Type Questions)

1. Show that the following functins are strictly increasing on R:

(a) $f(x) = 3x + 17$

(b) (i) $f(x) = e^{x}$

(ii) $f(x) = e^{2x}$.

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2. Without using the derivative, show that the function $f(x) = |x|$ is (a) strictly increasing in $(0, \infty)$ (b) strictly decreasing in $(-\infty, 0)$

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3. Show that the function f given by $f(x) = x^3 - 3x^2 + 4x, x \in R$is strictly increasing on R.

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4. Prove the following

(i) $f(x) = x^2$ is a decreasing function for $x < 0$, where $x \in R$

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5. Prove the following

$f(x) = x^2 - 8x, x \leq 4$ is a decreasing function

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6. Prove that the function

$f(x) = x^3 - 3x^2 + 3x - 100$ is increasing on R

.

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7. Prove that the function given by $f(x) = \cos x$ is

(a) strictly decreasing in $(0, \pi)$.

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8. Prove the following

(i) $f(x) = \sin x$ is :

(I) strictly increasing in $\left(0, \frac{\pi}{2}\right)$

(II) strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$

(III) neither increasing nor decreasing in $(0, \pi)$

(ii) $f(x) = 2\sin x + 1$ is an increasing function on $\left[0, \frac{\pi}{2}\right]$.

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9. Prove the following

$f(x) = \tan^{-1}(\sin x + \cos x)$ is strictly decreasing function on $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$.

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10. Prove that the logarithmic function is strictly increasing on $(0, \infty)$.

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11. Prove that the function f given by $f(x) = \log s \in x$

$f(x) = \log s \in x$ is strictly increasing on $\left(0, \frac{\pi}{2}\right)$ and strictly decreasing on $\left(\frac{\pi}{2}, \pi\right)$.

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12. Prove that the function f given by $f(x) = \log \cos x$ is strictly decreasing on $\left(0, \frac{\pi}{2}\right)$ and strictly increasing on $\left(\frac{\pi}{2}, \pi\right)$.

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13. Show that the function f(x) = x3– 3x^2 + 6x – 100 is increasing on .

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14. Find the intervals in which the following functions are increasing :

(i) $2x^3 - 3x$

(ii) $10 - 6x - 2x^2$.

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15. Find the interval in which $2x^3 + 9x^2 + 12x - 1$ is strictly increasing.

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16. Find the intervals in which the functions :

(i) $f(x) = x^3 + 2x^2 - 1$

(ii) $30 - 24x + 15x^2 - 2x^3$ are strictly decreasing.

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17. Prove that the function $f(x) = x^2 - x + 1$ is neither increasing nor decreasing on $(-1, 1)$.

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18. Find the values of 'a' for which the function :

$f(x) = x^2 - 2ax + 6$ is increasing when $x > 0$.

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19. Find the values of 'a' for which $f(x) = \sin x - ax + b$ is decreasing function on R.

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20. Find the values of x for which $y = [x(x-2)]^2$ is an increasing function

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21. Determine for which values of x, the following functions are increasing or decreasing :

$f(x) = -3x^2 + 12x + 8.$

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22. Determine for which values of x, the following functions are increasing or decreasing :

$f(x) = x^3 - 12x$

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23. Determine for which values of x, the following functions are increasing or decreasing :

$f(x) = 2x^3 - 24x + 107.$

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24. Determine for which values of x, the following functions are increasing or decreasing :

$$f(x) = x^4 - 2x^2$$

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25. Determine for which values of x, the following functions are increasing or decreasing :

$$f(x) = x^3 + \frac{1}{x^3}, x \neq 0$$

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26. For what values of 'x' are the following functions increasing or decreasing?

$$y = x + \frac{1}{x}, x \neq 0$$

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27. For what values of 'x' are the following functions increasing or decreasing?

$$y = 5x^{3/2} - 3x^{5/2}, x > 0$$

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EXERCISE 6 (b) (Long Answer Type Questions (I))

1. Find the intervals in which the function f given by $f(x) = 2x^2 - 3x$is(a) strictly increasing (b) strictly decreasing

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2. Determine the intervals in which the following functions are strictly increasing or strictly decreasing :

$$f(x) = x^2 + 2x - 5$$

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3. Find the intervals on which the function $f(x) = 10 - 6x - 2x^2$ is (a) strictly increasing (b) strictly decreasing.

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4. Determine the intervals in which the following functions are strictly increasing or strictly decreasing :

$f(x) = 6 - 9x - 2x^2$

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5. Find the intervals in which the function f given by $f(x) = x^2 - 4x + 6$is (a) strictly increasing (b) strictly decreasing

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6. Determine the intervals in which the following functions are strictly increasing or strictly decreasing :

$$f(x) = \frac{1}{4}x^4 + \frac{2}{3}x^3 - \frac{5}{2}x^2 - 6x + 7$$

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7. Determine the intervals in which the following functions are strictly increasing or strictly decreasing :

$$f(x) = -3\log(1+x) + 4\log(2+x) - \frac{4}{2+x}$$

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8. Determine the intervals in which the following functions are strictly increasing or strictly decreasing :

$f(x) = 20 - 9x + 6x^2 - x^3$

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9. Determine the intervals in which the following functions are strictly increasing or strictly decreasing :

$f(x) = 2x^3 - 15x^2 + 36x + 17.$

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10. Determine the intervals in which the following functions are strictly increasing or strictly decreasing :

$f(x) = 2x^3 - 9x^2 + 12x + 15$

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11. Determine the intervals in which the following functions are strictly increasing or

strictly decreasing :

$f(x) = 2x^3 - 3x^2 - 36x + 7.$

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12. Determine the intervals in which the following functions are strictly increasing or strictly decreasing :

$f(x) = x^3 - 6x^2 + 9x + 8$

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13. Determine the intervals in which the following functions are strictly increasing or strictly decreasing :

$f(x) = 4x^3 - 6x^2 - 72x + 30.$

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14. Determine the intervals in which the following functions are strictly increasing or strictly decreasing :

$f(x) = 2x^3 - 12x^2 + 18x + 5$

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15. Determine the intervals in which the following functions are strictly increasing or strictly decreasing :

$$f(x) = \frac{4x^2 + 1}{x}.$$

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16. Find the intervals in which the given functions are strictly increasing decreasing:

$$-2x^3 - 9x^2 - 12x + 1$$

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17. Determine the intervals in which the following functions are strictly increasing or strictly decreasing :

$f(x) = x^3 + 3x^2 - 4.$

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18. Determine the intervals in which the following functions are strictly increasing or strictly decreasing :

$f(x) = 2x^3 - 15x^2 + 36x + 6$

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19. Determine the intervals in which the following functions are strictly increasing or strictly decreasing :

$f(x) = (x-1)(x-2)^2$.

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20. Determine the intervals in which the following functions are strictly increasing or

strictly decreasing :

$$f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$$

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21. Determine the intervals in which the following functions are strictly increasing or strictly decreasing :

$f(x) = 3x^4 - 4x^3 - 12x^2 + 5.$

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22. Determine the intervals in which the following functions are strictly increasing or strictly decreasing :

$f(x) = (x+1)^3(x-3)^3.$

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23. Determine the intervals in which the following functions are strictly increasing or strictly decreasing :

$f(x) = x^8 + 6x^2.$

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24. On which of the following intervals is the function 'f' given by $f(x) = x^{100} + \sin x - 1$ strictly increasing?

A. $(-1, 1)$

B. $(0, 1)$

C. $\left(\frac{\pi}{2}, \pi\right)$

D. $\left(0, \frac{\pi}{2}\right)$.

Answer: b,c,d

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25. Find the intervals in which $f(x) = \sin x - \cos x$, where $0 < x < 2\pi$, is strictly increasing or decreasing.

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26. Find the intervals in which the function f given by $f(x) = \sin x + \cos x$, $0 \leq x \leq 2\pi$ is strictly increasing or strictly decreasing.

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27. Find the intervals in which the function 'f' given by :

$f(x) = \sin x - \cos x, 0 \le x \le 2\pi$

is strictly increasing or striclty decreasing.

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28. Find the intervals in which the function $f(x) = 2x^3 - 9x^2 + 12x + 29$ is :

(i) monotonic increasing (ii) monotonic decreasing.

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29. Find the intervals in which the function given by $f(x) = \sin 3x, x \in \left[0, \frac{\pi}{2}\right]$ is :

(a) increasing (b) decreasing.

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30. which of the following functinon are strictly decreasing on (0 , $\pi/2$) a) cosx b)cos2x c) cos3x d) tanx

A. $\cos x$

B. $\cos 2x$

C. $\cos 3x$

D. $\tan x$.

Answer: (i), (ii)

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EXERCISE 6 (b) (Long Answer Type Questions (II))

1. If $x > -1$, show that : $\frac{x}{\sqrt{1+x}} - \log(1+x) + 9$ is an increasing function of x.

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EXERCISE 6 (c) (Short Answer Type Questions)

1. Find the slope of the tangent to the curve :

$y = x^3 - 2x + 8$ at the point $(1, 7)$.

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2. Find the slope of the normal to the curve :

$y = x^3 - x + 1 \text{ at } x = 2$

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3. Find the slope of the normal to the curve :

$y = \tan^2 x + \sec x \text{ at } x = \frac{\pi}{4}$

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4. The slope of the normal to the curve

$x = a(\theta - \sin\theta), y = a(1 - \cos\theta)$ at $\theta = \frac{\pi}{2}$

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5. Find te slope of the normal to the curve $x = a\cos^2\theta$ and $y = a\sin^3\theta$ at $\theta = \frac{\pi}{4}$

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6. Equation of the tangent to the curve $2x^2 + 3y^2 - 5 = 0$ at (1, 1) is

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7. Find the equation of the tangent line to the curve :

$y = x^3 - 3x + 5$ at the point (2, 7)

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8. Find the equation of the tangent line to the curve $y = \cot^2 x - 2\cot x + 2$ at $x = \frac{\pi}{4}$

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9. Find the equation of the tangent of the tangent to the curve $y = (\sec^4 x - \tan^4 x)$ at $x = \frac{\pi}{3}$

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10. Find the equation of the tangent line to the following curves :

$y = x^2$ at $(0, 0)$

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11. Find the equation of the tangent line to the following curves :

$y = x^3$ at $(1, 1)$

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12. Find the equations of the tangent and the normal to the curve $y = 2x^2 - 3x - 1$ at $(1, -2)$ at the indicated points

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13. Find the equations of the tangent and the normal to the curve $x^{2/3} + y^{2/3} = 2$ at $(1, 1)$ at indicated points.

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14. Find the equations of the tangent and the normal to the curve $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at the point (1, 3)

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15. Find the equation of the tangent line to the following curves :

$y = x^4 - 6x^3 + 13x^2 - 10x + 5 \text{ at } (0, 5).$

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16. Find the equation of the tangent line to the following curves :

$x = \cos t, y = \sin t \text{ at } t = \frac{\pi}{4}.$

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17. The equation of tangent to the curve $x = a\cos^3\theta, y = a\sin^3\theta$ at $\theta = \frac{\pi}{4}$ is

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18. Find the equation of the normal line to the curve :

(i) $y = 2x^2 + 3\sin x$ at $x = 0$

(ii) $y(x-2)(x-3) - x + 7 = 0$ at the point, where it meets x - axis.

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EXERCISE 6 (c) (Long Answer Type Questions (I))

1. Find the equations of the tangent and normal lines to the following curves :

$$y = \sin^2 x \quad \text{at} \quad x = \frac{\pi}{2}.$$

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2. Find the equations of the tangent and normal lines to the following curves :

$$y = \frac{1 + \sin x}{\cos x} \quad \text{at} \quad x = \frac{\pi}{4}.$$

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3. Find the equations of the tangent and normal to the parabola :

$y^2 = 4ax \quad \text{at} \quad (at^2, 2at)$

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4. Find the equations of tangent and normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at (x_1, y_1)

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5. Find the equations of the tangent and the normal to the curve $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at $\left(\sqrt{2}a,\ b\right)$ at indicated points.

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6. Find the equation of the normal to the curve $ay^2 = x^3$ at the point $\left(am^2, am^3\right)$.

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7. Find the equation of tangent to the curve $2x^2 - y = 7$, which is parallel to the line $4x - y + 3 = 0$.

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8. Find the equation of the tangent to the curve $y = \sqrt{3x - 2}$, which is parallel to the line $4x - 2y + 5 = 0$.

Also, write the equation of normal to the curve at the point of contact.

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9. Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$ which is parallel to the line $2x - y + 9 = 0$

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10. Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$ which is perpendicular to the line $5y - 15x = 13$.

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11. Find the equation of the tangents to the curve :

$y = x^3 + 2x - 4$,

Which are perpendicular to line $x + 14y + 3 = 0$.

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12. Find the equation of the normals to the curve $y = x^3 + 2x + 6$which are parallel to the line $x + 14y + 4 = 0$.

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13. Find the equation(s) of normal(s) to the curve $3x^2 - y^2 = 8$ which is (are) parallel to the line $x + 3y = 4$.

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14. Find the equations of the normals to the curve : $2x^2 - y^2 = 14$, which are parallel to the line $x + 3y = 6$.

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15. If a normal of curve$x^{2/3} + y^{2/3} = a^{2/3}$ makes an angle ϕ from X-axis then show that its equation is $y \cos \phi - x \sin \phi = a \cos 2\phi$.

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16. (i) Find the equation of the normal to the curve : $x^2 = 4y$, which passes through the point (1, 2).

(ii) Find the equation of the normal to the curve : $y^2 = 4x$, which passes through the point (1, 2).

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17. Find the equation of the tangent to the curve $: y = 3x^2 - 2x + 5$, which is parallel to the line $4x - y = 10$.

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18. Find the points on the curve $x^3 - 2x^2 - 2x$ at which the tangent lines are parallel to the line $y = 2x - 3$.

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19. Find the equation of the tangent to the curve $y = \sqrt{5x - 3}$, which is parallel to the line $4x - 2y + 3 = 0$.

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20. Find the equations of tangent lines to the curve $y = 4x^3 - 3x + 5$, which are perpendicular to the line 9y +x+3=0

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21. Find the equations of the normal to the curve $y = x^3 + 2x + 6$ which are parallel to the line $x + 14y + 4 = 0$.

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22. Find the equation of the normal to the curve : $y = x^3 + 5x^2 - 10x + 10$, where the normal is parallel to the line $x - 2y + 10 = 0$.

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23. Find the equations of the normal to the curve $y = 4x^3 - 3x + 5$, which are perpendicular to the line : $9x - y + 5 = 0$.

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24. Find the equation of tangent to the curve given by$x = a\sin^3 t, y = b\cos^3 t$ ... (1)at a point where $t = \frac{\pi}{2}$.

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25. Find the equation of the tangent at $t = \frac{\pi}{4}$ to the curve : $x = \sin 3t, y = \cos 2t$.

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26. Find the point(s) on the curve :

(i) $y = 3x^2 - 12x + 6$

(ii) $x^2 + y^2 - 2x - 3 = 0$

at which the tangent is parallel to x - axis.

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27. Find the point(s) on the curve :

(i) $y = \frac{1}{4}x^2$, where the slope of the tangent is $\frac{16}{3}$

(ii) $y = x^2 + 1$, at which the slope of the tangent is equal to :

(I) x - coordinate (II) y - coordinate.

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28. Find the point on the curve $y = x^3 - 11x + 5$ at which the tangent is $y = x \quad 11$.

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29. For the curve $y = 4x^3 - 2x^5$, find all the points at which the tangent passes through the origin.

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30. Find the points on the following curve at which the tangents are parallel to x - axis : $y = x^3 - 3x^2 - 9x + 7$.

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31. At what point on the circle $x^2 + y^2 - 2x - 4y + 1 = 0,$ the tangent is parallel to $x - a\xi s\cdot$

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32. Find points on the curve $\frac{x^2}{4} + \frac{y^2}{25} = 1at$ which the tangents are (i) parallel to x-axis (ii) parallel to y-axis.

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33. Show that the tangents to the curve $y = 7x^3 + 11$ at the points $x = 2$ and $x = -2$ are parallel.

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34. Find the equations of all lines having slope 0 which are tangent to the curve $y = \frac{1}{x^2 - 2x + 3}$.

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35. Find the equations of all lines :

having slope -1 and that are tangents to the curve : $y = \dfrac{1}{x-1}, x \neq 1$

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36. Find the equations of all lines having slope 2 and that are tangent to the curve $y = \dfrac{1}{x-3}, \ \ x \neq 3$.

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37. Find the point of intersection of the tangent lines to the curve $y = 2x^2$ at the points $(1, 2)$ and $(-1, 2)$.

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38. Prove that the tangents to the curve $y = x^2 - 5x + 6$ at the points (2, 0) and (3, 0) are at right angles.

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39. Find the angle of intersection of the curves :

(i) $y^2 = 4x$ and $x^2 = 4y$

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40. Find the angle of intersection of the curves :

(ii)

$x^2 + y^2 - 4x - 1 = 0$ and $x^2 + y^2 - 2y - 9 = 0.$

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41. Show that the following set of curves intersect orthogonally: $y = x^3 and 6y = 7 - x^2$

$x^3 - 3xy^2 = - 2and3x^2y - y = 2.$

$x^2 + 4y^2 = 8andx^2 - 2y^2 = 4$

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42. If the curves :

$\alpha x^2 + \beta y^2 = 1$ and $\alpha' x^2 + \beta' y^2 = 1$

intersect orthogonally, prove that

$(\alpha - \alpha')\beta\beta' = (\beta - \beta')\alpha\alpha'.$

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EXERCISE 6 (c) (Long Answer Type Questions (I)) (HOTS)

1. Show that the curves $4x = y^2 and 4xy = k$ cut at right angles, if $k^2 = 512$.

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2. Show that the curves $2x = y^2 and 2xy = k$ cut at right angles, if $k^2 = 8$.

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3. Prove that the curves $y^2 = 4ax$ and $xy = c^2$ cut at right angles if $c^4 = 32a^4$.

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4. The normal to the curve $x = a(\cos\theta + \theta\sin\theta), y = a(\sin\theta - \theta\cos\theta)$ at any θ is such that

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5. Find the equation of the tangent to the curve $y = (x^3 - 1)(x - 2)$ at the points where the curve cuts the x-axis.

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6. Find a point on the graph of $y = x^3$, where the tangent is parallel to the chord joining (1, 1) and (3, 27).

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7. Find a point on the parabola $y = (x-2)^2$, where the tangent is parallel to the line joining (2, 0) and (4, 4).

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8. The area bounded by the curve $y^2(2a - x) = x^3$ and the line x = 2a is

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9. Find the equation of the normal at a point on the curve $x^2 = 4y$, which passes through the point (1,2). Also find the equation of the corresponding tangent.

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10. Find the equation of the tangents to the curve $3x^2 - y^2 = 8$, which passes through the point $\left(\frac{4}{3}, 0\right)$.

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11. Deterine the values of 'x' for which the function $f(x) = x^2 + 2x - 3$ is an increasing. Also, find the co - ordinates of the point on the curve $y = x^2 + 2x - 3$, where the normal is parallel to the line $x - 4y + 7 = 0$.

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12. Determine the intervals in which the function $f(x) = (x - 1)(x + 1)^2$ is increasing or decreasing. Find also the points at which the tangents to the curve are parallel to x - axis.

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EXERCISE 6 (d) (Long Answer Type Questions (I))

1. In the following find the approximate values, using differentials :

$\sqrt{50}$

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2. In the following find the approximate values, using differentials :

$\sqrt{360}$.

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3. In the following find the approximate values, using differentials :

$\sqrt{1.037}$.

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4. In the following find the approximate values, using differentials :

$\sqrt{0.0037}$.

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5. Use differentials to approximate $\sqrt{25.2}$

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6. In the following find the approximate values, using differentials :

$\sqrt{25.3}$

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7. In the following find the approximate values, using differentials :

$\sqrt{49.3}$

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8. Use differential to approximate $\sqrt{36.6}$

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9. In the following find the approximate values, using differentials :

$\sqrt{16.3}$

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10. In the following find the approximate values, using differentials :

$\sqrt{0.6}$.

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11. In the following find the approximate values, using differentials :

$\sqrt{0.17}$

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12. In the following find the approximate values, using differentials :

$\sqrt{0.26}$

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13. In the following find the approximate values, using differentials :

$\sqrt{0.82}$

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14. In the following find the approximate values, using differentials :

$\sqrt{0.26}$

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15. In the following find the approximate values, using differentials :

$\sqrt{0.50}$.

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16. In the following find the approximate values, using differentials :

$(17)^{1/4}$

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17. In the following find the approximate values, using differentials :

$(28)^{1/3}$

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18. In the following find the approximate values, using differentials :

$(255)^{1/4}$

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19. In the following find the approximate values, using differentials :

$(401)^{1/2}$

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20. In the following find the approximate values, using differentials :

$(26.57)^{1/3}$

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21. In the following find the approximate values, using differentials :

$(0.731)^{1/3}$

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22. The approximate value of $\sqrt[3]{0.009}$ is

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23. In the following find the approximate values, using differentials :

$\sqrt[3]{0.007}$

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24. In the following find the approximate values, using differentials :

$(15)^{1/4}$

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25. In the following find the approximate values, using differentials :

$(82)^{1/4}$

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26. In the following find the approximate values, using differentials :

$(255)^{1/4}$

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27. In the following find the approximate values, using differentials :

$(81.5)^{1/4}$

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28. In the following find the approximate values, using differentials :

$\left(\frac{17}{81}\right)^{1/4}$.

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29. In the following find the approximate values, using differentials :

$(32.15)^{1/5}$

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30. In the following find the approximate values, using differentials :

$(0.999)^{1/10}$

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31. Find approximation value of $(3.968)^{\frac{3}{2}}$ using differentials.

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32. In the following find the approximate values, using differentials :

$(33)^{-1/5}$

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33. Find the approximate value of :

$f(3.02), \quad \text{where} \quad f(x) = 3x^2 + 15x + 3$

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34. Find the approximate value of $f(5.001)$, where $f(x) = x^3 - 7x^2 + 15$.

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35. Find the approximate change in the volume V of a cube of side x meters caused by increasing the side by 2%.

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36. Find the approximate change in the surface area of a cube of side x metres caused by decreasing the side by 1%.

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37. If the radius of a sphere is measured as 9 cm with an error of 0.03 cm, then find the approximating error in calculating its volume.

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38. $\cos 61^\circ$, it being given that $\sin 60^\circ = 0.86603$ and $1^\circ = 0.01745$ radian.

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39. Find the approximate change in the value of $\frac{1}{x^2}$. when x changes from $x = 2$ to $x = 2.002$.

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40. Using differentiation, find the approximate value of $f(3.01)$, where $f(x) = 4x^2 + 5x + 2$.

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41. Use differentials, find the approximate value of the following :

$\sin\frac{22}{14}$

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42. Use differentials, find the approximate value of the following :

$\cos\frac{11\pi}{36}$.

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43. If $y = \sin x and x$ change from $\frac{\pi}{2} \rightarrow \frac{22}{14}$, what is the approximate change in y?

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44. A circular metal plate expands under heating so that its radius increases by 2%. Find the approximate increase in the area of the plate if the radius of the plate before heating is 10 cm.

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45. Find the percentage error in calculating the surface area of a cubical box if an error of 1% is made in measuring the lengths of edges of the cube.

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46. The radius of a spherical diamond is measured as 6 cm with an error of 0.04 cm. Obtain the approximate error in calculating its volume. If the cost of $1cm^3$ diamond is Rs 1600, what is the loss to the buyer of the diamond?

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EXERCISE 1 (e) (Short Answer Type Questions)

1. Find the maximum or minimum values, if any, of the following funcitons without using the derivatives :

$f(x) = -(x-1)^2 + 10$

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2. Find the maximum or minimum values, if any, of the following functions without using the derivatives :

$f(x) = (2x - 1)^2 + 3$

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3. Find the maximum or minimum values, if any, of the following funcitons without using the derivatives :

$f(x) = x + 1, . x \in [-1, 1]$

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4. Find the maximum or minimum values, if any, of the following funcitons without using the derivatives :

$g(x) = x^3 + 1.$

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5. Find the maximum or minimum values, if any, of the following funcitons without using the derivatives :

$f(x) = |x + 2| - 1$

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6. Find the maximum or minimum values, if any, of the following funcitons without using the derivatives :

$g(x) = -|x-1| + 3.$

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7. Find the maximum or minimum values, if any, of the following funcitons without using the derivatives :

$f(x) = \sin 2x + 5$

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8. Find the maximum or minimum values, if any, of the following funcitons without using the derivatives :

$f(x) = |\sin 4x + 3|.$

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9. Find the points of absolute maximum and minimum of each of the following :

$y = x(1 + 10x - x^2), 3 \le x \le 9$

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10. Find the points of absolute maximum and minimum of each of the following :

$y = \frac{1}{3}x^{3/2} - 4x, 0 \le x \le 64$

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11. Find the points of absolute maximum and minimum of each of the following :

$$y = \sqrt{5}\left(\sin x + \frac{1}{2}\cos 2x\right), 0 \leq x \leq \frac{\pi}{2}.$$

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12. Find the maximum and the minimum values, if any, of the function given by $f(x) = x, x \in (0, 1)$

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13. Find the absolute minimum value of $y = x^2 - 3x$ in $0 \leq x \leq 2$.

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14. Find the minimum value of $\sin^2 x$

A. -1

B. 0

C. 1

D. 1/2

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15. Find the maximum and minimum values of the function :

$f(x) = 2x^3 - 15x^2 + 36x + 11.$

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16. Find local minimum value of the function f given by $f(x) = 3 + |x|, x \in R.$

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EXERCISE 1 (e) (Long Answer Type Questions (I))

1. Find the absolute maximum and minimum values of each of the following in the given intervals :

$f(x) = x^{50} - x^{20}, [0, 1]$

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2. Find the absolute maximum and minimum values of each of the following in the given intervals :

$$f(x) = x^2 + \frac{16}{x}, x \in [1, 3]$$

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3. Find absolute maximum and minimum values of a function f given by $f(x) = 12x^{\frac{4}{3}} - 6x^{\frac{1}{3}}, x \in [-1, 1]$.

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4. Find the absolute maximum and minimum values of each of the following in the given intervals :

$$f(x) = x^3 - 3x, \ -3 \le x \le 3$$

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5. Find the absolute maximum and minimum values of each of the following in the given intervals :

$$f(x) = x^3 - \frac{5}{2}x^2 - 2x + 1, 0 \le x \le 3.$$

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6. Find the absolute maximum and minimum values of each of the following in the given intervals :

$f(x) = x^3$ in $[-2, 2]$.

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7. Find the absolute maximum and minimum values of each of the following in the given intervals :

$f(x) = (x-1)^2 + 3$ in $[-3, 1]$

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8. Find the absolute maximum and minimum values of each of the following in the given intervals :

$f(x) = 2x^3 - 15x^2 + 36x + 1 \text{ in } [1, 5]$

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9. Find the absolute maximum and minimum values of each of the following in the given intervals :

$f(x) = \sin x + \cos x \quad \text{in} \quad [0, \pi]$

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10. Find the absolute maximum and minimum values of each of the following in the given intervals :

$f(x) = \cos^2 x + \sin x \quad \text{in} \quad [0, \pi].$

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11. Find the maximum and minimum values of each of the following in the given intervals :

$y = \sec x + \log(\cos^2 x), \quad \text{in} \quad (0, 2\pi).$

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12. Find the absolute maximum and minimum values of each of the following in the given intervals :

$y = 2\cos 2x - \cos 4x, 0 \le x \le \pi.$

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13. Find the maximum value and the minimum value and the minimum value of $3x^4 - 8x^3 + 12x^2 - 48x + 25$ on the interval $[0, 3]$.

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14. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximum and local minimum values :

The constant function α.

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15. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximum and local minimum values :

$f(x) = x^2$

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16. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximum and local minimum values :

$f(x) = x^3 - 3x.$

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17. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximum and local minimum values :

$f(x) = \cos x, 0 < x < \pi$

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18. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximum and local minimum values :

$$f(x) = \sin x + \cos x, 0 < x < \frac{\pi}{2}$$

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19. Find the local maxima and local minima, if any, of the followig functions. Find also the local maximum and the local minimum values, as the case may be :

$$f(x) = \sin x - \cos x, 0 < x < 2\pi$$

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20. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximum and local minimum values :

$$g(x) = \frac{x}{5} + \frac{5}{x}, x > 0,$$

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21. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximum and local minimum values :

$$g(x) = \frac{1}{x^2 + 2}$$

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22. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximum and local minimum values :

$$f(x) = x\sqrt{1-x}, x > 0.$$

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23. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximum and local minimum values :

$f(x) = x^3 - 12x^2 + 36x - 4$

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24. Find the points of local maxima and local minima, if any, of the following functions. Find

also the local maximum and local minimum values :

$$g(x) = x^3 - 3x$$

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25. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximum and local minimum values :

$$f(x) = x^3 - 3x + 3.$$

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26. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximum and local minimum values :

$$f(x) = -\frac{3}{4}x^4 - 8x^3 - \frac{45}{2}x^2 + 105$$

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27. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximum and local minimum values :

$f(x) = x^3 - 6x^2 + 9x + 15. \, 0 \leq x \leq 6$

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28. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximum and local minimum values :

$f(x) = \; - x + 2\sin x, 0 \leq x \leq 2\pi$

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29. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximum and local minimum values :

$f(x) = \sin^4 x + \cos^4 x, 0 < x < \frac{\pi}{2}$.

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30. Prove that $\left(\frac{1}{x}\right)^x$ has maximum value is $(e)^{\frac{1}{e}}$

.

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31. The curve $y = ax^2 + bx$ has a turning point at (1, -2). Find the values of 'a and 'b' and also show that y is minimum at this point .

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32. If $y = \dfrac{ax - b}{(x-1)(x-4)}$ has a turning point $P(2, \ -1)$, find the value of $a and b$ and show that y is maximum at P.

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EXERCISE 1 (f) (Long Answer Type Questions (I))

1. Find two positive numbers whose sum is 16 and product is maximum.

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2. Amongest all pairs of positive numbers with product (i) 256 (ii) 64, find those whose sum is least.

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3. Find two numbers whose sum is 15 and the square of one multiplied by the cube of the other is maximum.

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4. Find two positive numbers whose product is 64 and the sum is minimum.

Watch Video Solution

5. Find two positive numbers x and y such that their sum is 35 and the product x^2y^5 is a maximum.

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6. Find two positive numbers whose sum is 16 and the sum of whose cubes is minimum.

Watch Video Solution

7. Find two positive numbers x and y such that $x + y = 60$ and xy^3 is maximum.

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8. How should we choose two numbers, each greater than or equal to -2, whose sum is $1/2$ so that the sum of the first and the cube of the second is minimum?

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9. Find the maximum slope of the curve $y = -x^3 + 3x^2 + 2x - 27$.

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10. Two sides of a triangle are given. The angle between them such that the area is maximum, is given by

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11. A wire of length 36m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the lengths of the two pieces, so that the combined area of the square and the circle is minimum?

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12. A wire of length 36cm is cut into the two pieces, one of the pieces is turned in the form of a square and other in form of an equilateral triangle. Find the length of each piece so that the sum of the areas of the two be minimum

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13. Prove that the perimeter of a right - angled triangle of given hypotenuse equal to 5 cm is maximum when the triangle is isosceles.

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14. Prove that the area of right-angled triangle of given hypotenuse is maximum when the triangle is isosceles.

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15. Prove that the least perimeter of an isosceles triangle in which a circle of radius r can be inscribed is $6\sqrt{3}r$.

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16. Show that of all the rectangles of given area, the square has the smallest perimeter.

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17. Show that the rectangle of maximum perimeter which can be inscribed in a circle of radius a is a square of side $\sqrt{2}a$.

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18. Show that the rectangle of maximum area that can be inscribed in a circle is a square.

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19. Show that of all the rectangles inscribed in a given fixed circle, the square has the maximum area.

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20. A rectangle is inscribed in a semi-circle of radius r with one of its sides on diameter of semi-circle. Find the dimensions of the rectangle so that its area is maximum. Find also the area.

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21. Of all rectangles, each of which has perimeter :

(i) 40 cm

(ii) 60 cm

Find the one having maximum area. Also, find that area.

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22. An open box with a square base is to be made out of a given quantity of card board of area c2 square units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units.

Watch Video Solution

23. Show that the semi - vertical angle of the right - circular cone of maximum volume and of given slant height is :

(i) $\tan^{-1}\sqrt{2}$

(ii) $\cos^{-1}\frac{1}{\sqrt{3}}$.

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24. Prove that the semi-vertical angle of the right circular cone of given volume and least curved surface is $\cot^{-1}(\sqrt{2})$.

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EXERCISE 1 (f) (Long Answer Type Questions (II))

1. Show that the volume of the greatest cylinder, which can be inscribed in a cone of height 'h' and semi - vertical angle 30° is $\frac{4}{81}\pi h^3$

Watch Video Solution

2. Show that the altitude of the right circulau cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$. also show that the maximum volume of cone is $\frac{8}{27}$ of the volume of the sphere.

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3. Find the volume of the larges cylinder that can be inscribed in a sphere of radius r

Watch Video Solution

4. Find the volume of the larges cylinder that can be inscribed in a sphere of radius r

Watch Video Solution

5. Show that the right-circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.

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6. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.

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7. Find the height of right circular cylinder of maximum volume that can be inscribed in a sphere of radius $10\sqrt{3}cm$.

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8. Show that the radius of right - circular cylinder of maximum volume, that can be inscribed in a sphere of radius 18 cm, is $6\sqrt{6}cm$.

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9. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

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10. Of all the closed cylinderical cans (right - circular), which enclose a given volume of :

(i) 100 cubic centimeters

(ii) 128π cubic centimeters,

find the dimensions of the can, which has the minimum surface area.

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11. Show that the surface area of a closed cuboid with square base and given volume is minimum, when it is a cube.

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12. A figure consists of a semi-circle with a rectangle on its diameter. Given the perimeter of the figure, find its dimensions in order that the area may be maximum.

Watch Video Solution

13. A window is the in the form of a reactangle, surmounted by a semi - circle. If the perimeter be 15 metres, find the dimesnions so that greatest possible amount of light may be admitted in order that its area may be maximum.

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14. Show that a cylinder of a given volume which is open at the top has minimum total surface area, when its height is equal to the radius of its base.

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15. The height of a closed cylinder of given volume and the minimum surface area is (a) equal to its diameter (b) half of its diameter (c) double of its diameter (d) None of these

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16. Rectangles are inscribed inside a semicircle of radius r. Find the rectangle with maximum area.

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17. A square-based tank of capacity 250 cu m has to bedug out. The cost of land is Rs 50 per sq m. The cost of digging increases with the depth and for the whole tank the cost is Rs $400 \times (depth)^2$. Find the dimensions of the tank for the least total cost.

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18. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is 8 m3. If building of tank costs Rs 70 per square metre for the base and Rs 45 per square metre for sides, what is the cost of least expensive tank?

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19. A rectangular sheet of tin 45 cm by 24 cm is to be made into a box without top, by cutting off square from each corner and folding up the flaps. What should be the side of the square to be cut off so that the volume of the box is maximum ?

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20. An open box is to be made of square sheet of tin with side 20 cm, by cutting off small squares from each corner and foding the flaps. Find the side of small square, which is to be cut off, so that volume of box is maximum.

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21. A canon is fired at an angle $\theta\left(0 \leq \theta \leq \frac{\pi}{2}\right)$ with the horizontal. If 'v' is the intial velocity of the canon ball, the height 'h' of the ball at time 't', ignoring wind resistance, is given by $h = (v \sin \theta)t - 4.9t^2$.

(a) Will be ball return to the ground?

(b) How far will the ball have travelled horizontally at the time it hits the ground, assuming there are no forces in the horizontal direction?

(c) Determine $'\theta'$ so that the horizontal range of the ball is maximum.

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22. Find the maximum profit that a company can make, if the profit function isgiven by $p(x) = 41 - 24x - 18x^2$

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23. Find the maximum profit that a company can make, if the profit function is given by :

$P(x) = 41 - 72x - 18x^2$

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24. Find the maximum profit that a company can make, if the profit function is given by :

$P(x) = 41 - 24x - 6x^2$.

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25. Find the point on the curve $y^2 = 4x$ which is nearest to the point $(2; \ -8)$

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26. Find the point on the curve $y^2 = 2x$ which is at a minimum distance from the point $(1, 4)$

Watch Video Solution

27. Find the point on the curve $y^2 = 2x$, which is nearest to the point $(1, -4)$.

Watch Video Solution

28. Find the point on the parabola $x^2 = 8y$, which is nearest to the point (2, 4).

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29. A helicopter is flying along the curve $y = x^2 + 2$. A soldier is placed at the point (3, 2). Find the nearest distance between the solider and the helicopter.

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30. A manufacturer can sell 'x' items at a price of Rs $(250 - x)$ each. The cost of producing 'x' items is Rs $(x^2 - 50x + 12)$. Determine the number of items to be sold so that he can make maximum profit.

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31. A factory can shell 'x' items per week at price of Rs $\left(20 - \frac{x}{1000}\right)$ each. If the cost price of one item is Rs $\left(5 + \frac{2000}{x}\right)$, find the number of items, the factory should produce every week for maximum profit. If price is reduced, how it will effect the sale? Give reasons.

(ii) Profit function of a company is given as :

$$P(x) = \frac{24x}{5} - \frac{x^2}{100} - 500.$$

where 'x' is the number of units produced.

What is the maximum profit of the company?

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32. Let 'p' be the price per unit of a certain product, when there is a sale of 'x' units. The total revenue function is :

$$P(x) = \frac{100x}{3x+1} - 4x.$$

(i) Find the marginal revenue function, rate of change of total revenue function with respect to x.

(ii) When x = 10, find the relative change of revenue R, i.e., Rate of change of R with respect to x and also the percentage rate of change of R at $x = 10$.

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33. If performance of the students 'y' depends on the number of hours 'x' given by the relation :

$$y = 4x - \frac{x^2}{2}.$$

find the number of hours, the students work to have the best performance.

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Objective Type Questions (A. Multiple Choice Questions)

1. The rate of change of the area of a circle with respect to its radius r at r = 6 cm is:

A. 10π

B. 12π

C. 8π

D. 11π.

Answer: b

Watch Video Solution

2. The total revenue in Rupees received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$. The marginal revenue, when $x = 15$is (A) 116 (B) 96 (C) 90 (D) 126

A. 116

B. 96

C. 90

D. 126

Answer: d

Watch Video Solution

3. The interval in which $y = x^2 e^{-x}$ is increasing is (A) $(-\infty, \infty)$ (B) (2, 0) (C) $(2, \infty)$ (D) (0, 2)

A. $(-\infty, \infty)$

B. $(-2, 0)$

C. $(2, \infty)$

D. $(0, 2)$

Answer: d

Watch Video Solution

4. Find the slopes of the tangent and the normal to the curve $y = 2x^2 + 3\sin x$ at $x = 0$

A. 2

B. -3

C. $\frac{1}{2}$

D. $-\frac{1}{3}$.

Answer: d

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5. The line $y = x + 1$ is a tangent to the curve $y^2 = 4x$ at the point:

A. $(1, 2)$

B. $(2, 1)$

C. $(1, -2)$

D. $(-1, 2)$

Answer: a

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6. If $f(x) = 3x^2 + 15x + 5,$ then the approximate value of $f(3.02)$ is :

A. 47.66

B. 57.66

C. 67.66

D. 77.66.

Answer: d

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7. The approximate change in the volume of a cube of side x metres caused by increasing the side by 3% is(A) 0.06 x^3m^3 (B) 0.6 x^3m^3 (C) 0.09 x^3m^3 (D) 0.9 x^3m^3

A. $0.06x^3m^3$

B. $0.6x^3m^3$

C. $0.09x^3m^3$

D. $0.9x^3m^3$.

Answer: c

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8. The point on the curve $x^2 = 2y$which is nearest to the point (0, 5) is(A) $\left(2\sqrt{2}, 4\right)$ (B) $\left(2\sqrt{2}, 0\right)$ (C) (0, 0) (D) (2, 2)

A. $\left(2\sqrt{2}, 4\right)$

B. $\left(2\sqrt{2}, 0\right)$

C. $(0, 0)$

D. $(2, 2)$

Answer: a

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9. For all real values of x, the minimum value of $\frac{1-x+x^2}{1+x+x^2}$ is(A) 0 (B) 1 (C) 3 (D) $\frac{1}{3}$

A. 0

B. 1

C. 3

D. $\frac{1}{3}$

Answer: d

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10. The maximum value of $[x(x-1)+1]^{1/3}.0 \leq x \leq 1$ is

A. $\left(\frac{1}{3}\right)^{\frac{1}{3}}$

B. $\frac{1}{2}$

C. 1

D. 0

Answer: c

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11. A cylindrical tank of radius 10 m is being filled with wheat at the rate of 314 cubic metre per hour. Then the depth of the wheat is increasing at the rate of(A) 1 m^3/h (B) 0.1 m^3/h (C) 1.1 m^3/h (D) 0.5 m^3/h

A. $1\ \mathrm{m}^3/\mathrm{minute}$

B. $0.1m^3/\mathrm{minute.}$

C. $1.1m^3/\mathrm{minute}$

D. $0.5m^3/\mathrm{minute.}$

Answer: a

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12. The slope of the tangent to the curve $x = t^2 + 3t - 8, y = 2t^2 - 2t - 5$ at the point (2, -1), is

A. $\frac{22}{7}$

B. $\frac{6}{7}$

C. $\frac{7}{6}$

D. $-\frac{6}{7}$.

Answer: b

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13. The line $y = mx + 1$is a tangent to the curve $y^2 = 4x$if the value of m is(A) 1 (B) 2 (C) 3 (D) $\frac{1}{2}$

A. 1

B. 2

C. 3

D. $\frac{1}{2}$.

Answer: a

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14. The normal at the point (1,1) on the curve $2y + x^2 = 3$is(A) $x + y = 0$ (B) $xy = 0$ (C) $x + y + 1 = 0$(D) $xy = 0$

A. $x + y = 0$

B. $x - y = 0$

C. $x + y + 1 = 0$

D. $x - y + 1 = 0$

Answer: b

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15. Find the equation of the normal to curve $x^2 = 4y$ which passes through the point (1, 2).

A. $x + y = 3$

B. $x - y = 3$

C. $x + y = 1$

D. $x - y = 1$.

Answer: a

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16. The points on the curve $9y^2 = x^3$, where the normal to the curve makes equal intercepts with the axes are(A) $\left(4, \pm \frac{8}{3}\right)$ (B) $\left(4, \frac{-8}{3}\right)$ (C) $\left(4, \pm \frac{3}{8}\right)$ (D) $\left(\pm 4, \frac{3}{8}\right)$

A. $\left(4, \pm \frac{8}{3}\right)$

B. $\left(4, -\frac{8}{3}\right)$

C. $\left(4, \pm \frac{3}{8}\right)$

D. $\left(\pm 4, \frac{3}{8}\right)$.

Answer: a

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17. The point on the curve $3y = 6x - 5x^3$ the normal at Which passes through the origin, is

A. 1

B. $\frac{1}{3}$

C. 2

D. $\frac{1}{2}$

Answer: a

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18. The two curves $x^3 - 3xy^2 + 2 = 0$ and $3x^2y - y^3 - 2 = 0$

A. touch each other

B. cut at right angle

C. cut at an angle $\frac{\pi}{3}$

D. cut at an angle $\frac{\pi}{4}$.

Answer: b

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19. If the parametric of a curve given by $x = e^t \cos t, y = et \sin t$, then the tangent to the curve at the point $t = \pi / 4$ makes with axis of x the angle

A. 0

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer: d

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20. The equation to the normal to the curve $y = \sin x$ at $(0, 0)$ is $x = 0$ (b) $y = 0$ (c) $x + y = 0$ (d) $x - y = 0$

A. $x = 0$

B. $y = 0$

C. $x + y = 0$

D. $x - y = 0$

Answer: c

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21. Write the coordinates of the point on the curve $y^2 = x$ where the tangent line makes an angle $\frac{\pi}{4}$ with x-axis.

A. $\left(\frac{1}{2}, \frac{1}{4}\right)$

B. $\left(\frac{1}{4}, \frac{1}{2}\right)$

C. $(4, 2)$

D. $(1, 1)$

Answer: b

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22. The slope of the normal to the curve $y = 2x^2 + 3\sin x$ at $x = 0$is(A) 3 (B) $\frac{1}{3}$ (C)-3 (D) $-\frac{1}{3}$

A. 3

B. $\frac{1}{3}$

C. -3

D. $-\frac{1}{3}$

Answer: d

Watch Video Solution

23. The line $y = x + 1$ is a tangent to the curve $y^2 = 4x$ at the point :

A. $(1, 2)$

B. $(2, 1)$

C. $(1, -2)$

D. $(-1, 2)$

Answer: d

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24. The rate of change of the area of a circle with respect to its radius r at r = 6 cm is:

A. $16\pi cm^2 / cm$

B. $12\pi cm^2 / cm$

C. $8\pi cm^2 / cm$

D. $11\pi cm^2 / cm.$

Answer: b

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25. The rate of change of the area of a circle with respect to its radius r when r = 3 cm is :

A. $6\pi cm^2 / cm$

B. $4\pi cm^2 / cm$

C. $5\pi cm^2 / cm$

D. None of these.

Answer: a

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26. The point on the curve $y = 2x^2$, where the slope of the tangent is 8, is :

A. $(0, 2)$

B. $(0, 8)$

C. $(2, 8)$

D. $(8, 2)$

Answer: c

Watch Video Solution

27. Find the equation of the normal to the curve

$y = 2x^2 + 3 \sin x at x = 0.$

A. 3

B. $\frac{1}{3}$

C. -3

D. $-\frac{1}{3}$

Answer: d

Watch Video Solution

28. The rate of change of the area of a circle with respect to its radius at r = 2 cm is :

A. 8π

B. 2π

C. 4π

D. 11π

Answer: c

Watch Video Solution

29. Find the slope of the tangent to the curve $y = x^3 - 3x + 2$ at the point whose x-coordinate is 3.

A. 20

B. 24

C. 30

D. 25

Answer: b

Watch Video Solution

30. The line $y = x + 1$ is tangent to the curve $y^2 = 4x$ at the point :

A. $(1, 2)$

B. $(2, 1)$

C. $(1, \ -2)$

D. $(-1, 2)$

Answer: a

Watch Video Solution

31. The interval, in which $y = 2x^2 e^{-2x}$ is increasing is :

A. $(-\infty, \infty)$

B. $(-1, 0)$

C. $(1, \infty)$

D. $(0, 1)$.

Answer: d

Watch Video Solution

32. The equation of tangent to the curve $x = a\cos^3\theta, y = a\sin^3\theta$ at $\theta = \frac{\pi}{4}$ is

A. 1

B. 2

C. -1

D. None of these.

Answer: c

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33. Find the slope of the normal to the curve $x = a\cos^3\theta, y = \sin^3\theta$at $\theta = \frac{\pi}{4}$.

A. 1

B. -1

C. 3

D. -2.

Answer: a

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34. The maximum and minimum values of function $f(x) = \sin 3x + 4$ are respectively:

A. 5 and 3

B. 6 and 4

C. 4 and 3

D. None of these.

Answer: A

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35. The function $f(x) = \cos x - \sin x$ has maximum or minimum value at $x = \ldots\ldots$

A. $\frac{\pi}{4}$

B. $\frac{3\pi}{4}$

C. $\frac{\pi}{2}$

D. $\frac{\pi}{3}$.

Answer: b

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36. Find an angle , which increases twice as fast as it sine.

A. $\frac{\pi}{3}$

B. $\frac{\pi}{2}$

C. π

D. $\frac{3\pi}{2}$.

Answer: a

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37. $f(x)$ is a strictly increasing function, if $f'(x)$ is :

A. positive

B. negative

C. 0

D. None of these.

Answer: a

Watch Video Solution

38. The approximate change in the voluem V of a cube of side x metres caused by increasing the side by 2% is :

A. $0.06x^3m^3$

B. $0.02x^3m^3$

C. $0.6x^3m^3$

D. $0.006x^3m^3$.

Answer: a

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39. The rate of change of the area of a circle with respect to its radius at r = 5, is :

A. 10π

B. 8π

C. 12π

D. 13π.

Answer: a

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40. The radius of a circle is increasing at the rate of 0.7 cm/sec. What is the rate of increase of its circumference?

A. $3.3\pi cm / s$

B. $1.4\pi cm / s$

C. $2.2\pi cm / s$

D. $4.4\pi cm / s.$

Answer: a

Watch Video Solution

41. Find the point on the curve $y = x^3 - 11x + 5$ at which the tangent is $y = x \qquad 11$.

A. $(-2, 0)$

B. $(3, 7)$

C. $(0, 2)$

D. $(2, -9)$

Answer: a

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42. If the rate of change of area of a circle is equal to the rate of change of its diameter, then its radius is equal to (a) unit (b) unit (c) units (d) units

A. $\frac{1}{\pi}$

B. $\frac{2}{\pi}$

C. $\frac{\pi}{2}$

D. π.

Answer: a

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43. The interval on which the function $f(x) = 2x^2 - 3x$ is increasing or decreasing in :

A. $\left[-\infty, \frac{3}{4}\right]$

B. $[3, \infty]$

C. $\left[\frac{3}{4}, 3\right]$

D. $\left[\frac{3}{4}, \infty\right]$

Answer: d

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44. The rate of change of volume of a sphere with respect to its radius when radius is 1 unit is :

A. 4π

B. 2π

C. π

D. $\frac{\pi}{2}$.

Answer: c

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45. Slope of the normal to the curve :

$y^2 = 4x$ at $(1, 2)$ is :

A. 1

B. $\frac{1}{2}$

C. 2

D. -1

Answer: b

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Objective Type Questions (B. Fill in the Blanks)

1. The radius of a sphere starts to increase at a rate of 0.1 cm/s . The rate of chane of a surface area of the sphere with time when radius is 10.

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2. Rate of change of the volume of a ball with respect to its radius is ....

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3. Without using the derivative, show that the function $f(x)=|x|$ is strictly increasing in $(0,\infty)$ strictly decreasing in $(-\infty,0)$.

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4. Logarithmic function is strictly ....in $(0,\pi)$.

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5. The value of 'a' for which $f(x)=\sin x - ax + 5$ is decreasing function on R if.

A. $a > 1$

B. $a < 13$

C. $a > \left(\frac{1}{2}\right)$

D. $a < \left(\frac{1}{2}\right)$

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6. Slope of the tangent to the curve $x = at^2, y = 2t$ at $t = 2$ is ....

Watch Video Solution

7. Slope of the normal to the curve $y = 2x^2 - 1$ at $(1, 1)$ is ....

Watch Video Solution

8. Find the equation of the tangent line to the curve $y = \cot^2 x - 2\cot x + 2$ at $x = \frac{\pi}{4}$

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9. If $x > 0, y > 0$ and $xy = 5$, then the minimum value of $x + y$ is ....

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10. Maximum value of $f(x) = -(x-1)^2 + 2$ is ....

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Objective Type Questions (C. True/False Questions)

1. For the curve $y = 5x - 2x^3$, if x increases at the rate of 2 units/sec, then how fast is the slope of the curve changing when $x = 3$?

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2. The function $f(x) = 4x^3 - 6x^2 - 72x + 30$ is strictly decreasing on interval.

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3. Slope of the tangent to the curve $y = 3x^4 - 4x$ at $x = 1$ is 6.

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4. Using differentials, the approximate valueof $\sqrt{25.5}$.

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5. The function $f(x) = x^2, x \in R$ has no maximum value.

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Objective Type Questions (D. Very Short Answer Types Questions)

1. Find the rate of change of the area of a circle with respect to its radius 'r' when r = 6 cm.

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2. The radius of spherical balloon is increasing at the rate of 5 cm per second. At what rate is the surface of the balloon increasing, when the radius is 10 cm?

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3. The radius of an air bubble in increasing at the rate of 0.5cm/s. Find the rate of change of its volume, when the radius is 1.5 cm.

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4. What are the values of 'a' for which the function $f(x) = a^x$ is :

(i) increasing

(ii) decreasing in R?

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5. What are the values of "a for which the function $f(x) = \log_a x$ is :

(i) increasing

(ii) decreasing in its domain?

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6. Find the value of 'k' such for $f(x) = k(x + \sin x) + k$ is increasing in R.

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7. Find the set of values of 'a' such that $f(x) = ax - \sin x$ is increasing on R.

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8. Examine whether the function given by $f(x) = x^3 - 3x^2 + 3x - 5$ is increasing in R.

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9. Write the interval in which the function $f(x) = \cos x$ is strictly decreasing.

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10. Find the point on the curve $y = x^2 - 2x + 5$, where the tangent is paralle to x - axis.

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11. Write the value of $\frac{dy}{dx}$, if the normal to the curve $y = f(x)$ at (x, y) is parallel to y - axis.

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12. Find the slope of the tangent to the curve $y = x^3 \quad x \quad at \quad x = 2$.

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13. Find the slope of tangent line to the curve :

$y = x^2 - 2x + 1.$

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14. Find the slope of the normal to the curve to

$y = x^3 - x + 1$ at $x = 2.$

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15. Find the slope of the normal to the curve

$x = 1 - a\sin\theta, y = b\cos^2\theta$at $\theta = \frac{\pi}{2}.$

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16. Find the equation of the tangent line to the curve $y = \sin x \quad \text{at} \quad x = \frac{\pi}{4}$.

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17. Find the equation of the tangent line to the curve $y = x \tan^2 x \quad \text{at} \quad x = \frac{\pi}{4}$.

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18. Find the equation of the normal line to the curve

$f(x) = 5x^3 - 2x^2 - 3x - 1$ at $x = -8$

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19. Find the angle between $y = f(x)$ and $y = 2e^{2x}$ at their point of intersection. Where $f(x) = x^2 f(1) - x f'(2) + f''(3)$ and $f(0) = 2$.

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20. Using differentials, find the approximate values of the following :

(i) $\sqrt{37}$

(ii) $\sqrt{401}$.

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21. Find the minimum values of $f(x) = x^2 + \dfrac{1}{x^2}, x > 0$

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22. Local maximum of $f(x) = x + \frac{1}{x}$, where $x < 0$, is

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23. Write the maximum value of $f(x) = \frac{\log x}{x}$, if it exists.

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24. Find the maximum and minimum values, if any, of the following functions without using the derivatives :

(i) $f(x) = -(x-2)^2 + 3$

(ii) $f(x) = 9x^2 + 12x + 2.$

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25. Show that the value of x^x is minimum when $x = \frac{1}{e}.$

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26. Find two positive numbers whose sum is 14 and product is maximum.

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NCERT - FILE (Question from NCERT Book) (Exercise 6.1)

1. Find the rate of change of the area of circle with respect to its radius r when :

(a) r = 3 cm (b) r = 4 cm.

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2. The volume of a cube is increasing at the rate of $8cm^3 / s$. How fast is the surface area increasing when the length of an edge is 12 cm?

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3. The radius of a circle is increasing uniformly at the rate of 3 cm/s. Find the rate at which the area of the circle is increasing when the radius is 10 cm.

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4. An edge of a variable cube is increasing at the rate of $3cm/s$. How fast is the volume of the cube increasing when the edge is $10cm$ long?

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5. A stone is dropped into a quiet lake and waves move in circles at the speed of $5cm/s$. At the instant when the radius of the circular wave is $8cm$, how fast is the enclosed area increasing?

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6. The radius of a circle is increasing at the rate of 0.7 cm/sec. What is the rate of increase of its circumference?

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7. The length x of a rectangle is decreasing at the rate of 5 cm/minute and the width y is increasing at the rate of 4 cm/minute. When x = 8cm and y = 6cm, find the rates of change of (a) the perimeter, and (b) the area of the rectangle

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8. A balloon, which always remains spherical on inflation, is being inflated by pumping in 900 cubic centimetres of gas per second. Find the rate at which the radius of the balloon increases when the radius is 15 cm.

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9. A balloon, which always remains spherical, has a variable radius. Find the rate at which its volume is increasing with the radius when the later is 10 cm

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10. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall ?

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11. A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate

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12. The radius of an air bubble is increasing at the rate of $\frac{1}{2}cm/s$. At what rate is the volume of the bubble increasing when the radius is 1 cm?

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13. A balloon, which always remains spherical, has a variable diameter $\frac{3}{2}(2x+1)$.Find the rate of change of its volume with respect to x.

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14. Sand is pouring from a pipe at the rate of 12 cm^3/s . The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the

base. How fast is the height of the sand cone increasing when t

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15. The total cost C (x) in Rupees associated with the production of x units of an item is given by $C(x) = 0.007x^3 - 0.003x^2 + 15x + 4000.$

Find the marginal cost when 17 units are produced

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16. The total revenue in Rupees received from the sale of x units of a product is given by $R(x) = 13x^2 + 26x + 15$. Find the marginal revenue when $x = 7$.

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17. The rate of change of the area of a circle with respect to its radius r at r = 6 cm is:

A. 10π

B. 12π

C. 8π

D. 11π.

Answer: B

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18. The total revenue in Rupees received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$. The marginal revenue, when $x = 15$is (A) 116 (B) 96 (C) 90 (D) 126

A. 116

B. 96

C. 90

D. 126

Answer: D

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NCERT - FILE (Question from NCERT Book) (Exercise 6.2)

1. Show that the function given by $f(x) = 3x + 17$is strictly increasing on R.

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2. Show that the function given by $f(x) = e^{2x}$ is strictly increasing on R.

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3. Show that the fucntion given by $f(x) = \sin x$ is :

(a) strictly increasing in $\left(0, \frac{\pi}{2}\right)$

(b) strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$

(c) neither increasing nor decreasing in $(0, \pi)$.

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4. Find the intervals in which the function f given by $f(x) = 2x^2 - 3x$is(a) strictly increasing (b) strictly decreasing

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5. Find the intervals in which the fucntion 'f' given by $f(x) = x^3 - 3x^2 + 5x + 7$ is strictly increasing

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6. Find the invervals in which the following functions are strictly increasing or decreasing :

$x^2 + 2x - 5$

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7. Find the invervals in which the following functions are strictly increasing or decreasing :

$10 - 6x - 2x^2$

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8. Find the invervals in which the following functions are strictly increasing or decreasing :

$-2x^3 - 9x^2 - 12x + 1$

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9. Find the invervals in which the following functions are strictly increasing or decreasing :

$6 - 9x - x^2$

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10. Find the invervals in which the following functions are strictly increasing or decreasing :

$(x+1)^3(x-3)^3$.

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11. Show that $y = \log(1+x) - \frac{2x}{2+x}, x \succ 1,$ is an increasing function of x throughout its domain.

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12. The function$y = 5 + 36x + 3x^2 - 2x^3$ is increasing in the interval.

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13. Prove that $y = \frac{4\sin\theta}{(2+\sin\theta)} - \theta$ is an increasing function of θ in $\left[0, \frac{\pi}{2}\right]$

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14. Prove that the exponential function is strictly increasing on R.

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15. Prove that the function f given by $f(x) = x^2 - x + 1$ is neither strictly increasing nor strictly decreasing on $(-1, 1)$.

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16. Which of the following functions are strictly increasing on $\left(0, \frac{\pi}{2}\right)$?

A. $\sin x$

B. $\sin 2x$

C. $\sin 3x$

D. $\tan x$.

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17. On which of the following intervals is the function 'f' given by $f(x) = x^{100} + \cos x - 1$ strictly decreasing?

A. $(0, 1)$

B. $\left(\frac{\pi}{2}, \pi\right)$

C. $\left(0, \frac{\pi}{2}\right)$

D. None of these.

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18. Find the least value of a such that the function f given by $f(x) = x^2 + ax + 1$is strictly increasing on $(1, 2)$.

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19. Let I be an interval disjointed from $[-1,\ 1]$. Prove that the function $f(x) = x + \frac{1}{x}$ is increasing on I .

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20. Prove that the function 'f' given by $f(x) = \log \sin x$ is strictly increasing on $\left(0, \frac{\pi}{2}\right)$

.

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21. Prove that the function f given by $f(x) = \log \cos x$ is strictly decreasing on $\left(0, \frac{\pi}{2}\right)$.

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22. Prove that the function given by $f(x) = 2x^3 - 6x^2 + 7x$ is strictly increasing in R.

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23. The interval in which $y = x^2 e^{-x}$ is decreasing is :

A. $(-\infty, \infty)$

B. $(-\infty, 0)\cup(0, \infty)$

C. $(-\infty, 0)\cup(2, \infty)$

D. $(0, 2)$.

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NCERT - FILE (Question from NCERT Book) (Exercise 6.3)

1. Find the slope of the tangent to the curve $y = 3x^2 - 5x + 2$ at $x = 3$.

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2. Find the slope of the tangent to the curve

$y = \frac{x-3}{x-5}, x \neq 5$ at $x = 10.$

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3. Find the slope of the tangent to curve

$y = x^3 - x + 1$ at the point whose x-coordinate is 2.

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4. Find the slope of the tangent to the curve $y = x^3 - x + 1$ at the point whose x - coordinate is 3.

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5. Find the slope of the normal to the curve $x = 1 - a\sin^3\theta, y = a\cos^2\theta$ at $\theta = \frac{\pi}{2}$.

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6. Find the slope of the normal to the curve $x = a\sin^2\theta, y = b\cos^3\theta \text{ at} \theta = \frac{\pi}{4}$.

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7. Find points at which the tangent to the curve $y = x^3 - 3x^2 - 9x + 7$ is parallel to the x-axis

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8. Find a point on the curve $y = (x-2)^2$ at which the the tangent is parallel to the chord joining the points (2, 0) and (4, 4).

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9. Find the point on the curve $y = x^3 + 5$ at which the tangent is parallel to line $y = 12x - 7$ is.

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10. Find the equation of all lines having slope -3 that are tangents to the curve

$$y = \frac{1}{x-2}, x \neq 2.$$

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11. Find the equation of all lines having slope 2 which are tangents to the curve

$$y = \frac{1}{x-3}, x \neq 3$$

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12. Find the equations of all lines having slope 0 which are tangent to the curve

$$y = \frac{1}{x^2 - 2x + 3}.$$

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13. Find points on the curve $\frac{x^2}{9} + \frac{y^2}{25} = 1$ at which the tangents are :

(i) parallel to x -axis

(ii) parallel to y - axis.

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14. Find the equations of the tangent and normal to the given curves at the indicated points :

(i) $y = x^4 - 6x + 13x^2 - 10x + 5$ at $(0, 5)$

(ii) $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at $(1, 3)$.

(iii) $y = x^3$ at $(1, 1)$

(iv) $y = x^2$ at $(1, 1)$

(v) $x = \cos t, y = \sin t$ at $y = \frac{\pi}{4}$.

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15. Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$ which is :

(a) parallel to the line $2x - y + 9 = 0$

(b) perpendicular to the line $5y - 15x = 13$.

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16. Show that the tangents to the curve $y = 2x^3 - 3$ at the points where $x = 2$ and $x = -2$ are parallel.

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17. Find the points on the curve $y = x^3 - 3x$ at which the tangents are parallel to the chord joining the points $(1, -2)$ and $(2, 2)$.

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18. For the curve $y = 4x^3 - 2x^5$, find all the points at which the tangent passes through the origin.

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19. Find the points on the curve $y = (x - 2)^2$ at which the tangents are parallel to the chord joining the points $(2, 0)$ and $(4, 4)$.

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20. Find the equation of the normal at the point (am^2, am^3) for the curve $ay^2 = x^3$.

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21. Find the equation of the normals to the curve $y = x^3 + 2x + 6$which are parallel to the line $x + 14y + 4 = 0$.

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22. Find the equations of the tangent and normal to the parabola $y^2 = 4ax$at the point $(at^2, 2at)$.

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23. Show that the curves $x = y^2$ and $xy = k$ cut at right angles; if $8k^2 = 1$

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24. Find the equation of the tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point (x_0, y_0).

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25. Find the equation of the tangent to the curve $y = \sqrt{3x - 2}$, which is parallel to the line $4x - 2y + 5 = 0.$

Also, write the equation of normal to the curve at the point of contact.

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26. The slope of the normal to the curve $y = 2x^2 + 3\sin x$ at $x = 0$ is :

A. 3

B. $\frac{1}{3}$

C. -3

D. $-\frac{1}{3}$.

Answer: D

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27. The line $y = x + 1$is a tangent to the curve $y^2 = 4x$at the point(A) $(1, 2)$ (B)$(2, 1)$ (C) $(1, 2)$ (D) $(1, 2)$

A. (1, 2)

B. $(2, 1)$

C. $(1, -2)$

D. $(-1, 2)$

Answer: A

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NCERT - FILE (Question from NCERT Book) (Exercise 6.4)

1. Using differentials, find the approximate value of each of the following up to 3 places of decimal.

$\sqrt{25.3}$

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2. Using differentials, find the approximate value of each of the following up to 3 places of decimal.

$\sqrt{49.5}$

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3. Using differentials, find the approximate value of each of the following up to 3 places of decimal.

$\sqrt{0.6}$

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4. Using differentials, find the approximate value of each of the following up to 3 places of decimal.

$(0.009)^{1/3}$

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5. Using differentials, find the approximate value of each of the following up to 3 places of decimal.

$(0.999)^{1/10}$

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6. Using differentials, find the approximate value of each of the following up to 3 places of decimal.

$(15)^{1/4}$

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7. Using differentials, find the approximate value of each of the following up to 3 places of decimal.

$(26)^{1/3}$

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8. Using differentials, find the approximate value of each of the following up to 3 places of decimal.

$$\left(\frac{17}{81}\right)^{1/4}$$

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9. Using differentials, find the approximate value of each of the following up to 3 places of decimal.

$$(82)^{1/4}$$

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10. Using differentials, find the approximate value of each of the following up to 3 places of decimal.

$(401)^{1/2}$

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11. Using differentials, find the approximate value of each of the following up to 3 places of decimal.

$(0.0037)^{1/2}$

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12. Using differentials, find the approximate value of each of the following up to 3 places of decimal.

$(26.57)^{1/3}$

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13. Using differentials, find the approximate value of each of the following up to 3 places of decimal.

$(81.5)^{1/4}$

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14. Using differentials, find the approximate value of each of the following up to 3 places of decimal.

$(3.968)^{3/2}$

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15. Using differentials, find the approximate value of each of the following up to 3 places of decimal.

$(32.15)^{1/5}$.

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16. Find the approximate value of $f(3.12)$, where $f(x) = 4x^2 + 5x + 2$.

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17. Find the approximate value of $f(3.02)$, where $f(x) = 3x^2 + 15x + 5.$

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18. Find the approximate change in the volume V of a cube of side x metres caused by increasing the side by 5%.

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19. Find the approximate change in the surface area of a cube of side 'x' metres caused by decreasing the side by 5%.

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20. If the radius of a sphere is measured as 5 m with an error of 0.03 m, then find the approximate error in calculating its volume.

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21. If the radius of a sphere is measured as 7 m with an error of 0.03m, then find the approximate error in calculating its volume.

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22. If $f(x) = 3x^2 + 5x + 2$, then the approximate value $f(5.02)$ is

A. 47.66

B. 57.66

C. 67.66

D. 102.76

Answer: D

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23. The apprximate change in the volume of a cube of side x metres caused by increasing the side by 5% is :

A. $0.06x^3m^3$

B. $0.6x^3m^3$

C. $0.15x^3m^3$

D. $0.9x^3m^3$.

Answer: C

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NCERT - FILE (Question from NCERT Book) (Exercise 6.5)

1. Find the maximum and minimum values, if any, of the following function given by :

$f(x) = (2x - 3)^2 + 5$

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2. Find the maximum and minimum values, if any, of the following function given by :

$f(x) = 9x^2 + 12x + 5$

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3. Find the maximum and minimum values, if any, of the following function given by :

$f(x) = -(x-1)^2 + 5$

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4. Find the maximum and minimum values, if any, of the following function given by :

$g(x) = x^3 + 5.$

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5. Find the maximum and minimum values, if any, of the following function given by :

$f(x) = |x + 2| - 1$

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6. Find the maximum and minimum values, if any, of the following function given by :

$g(x) = -|x+2|+3$

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7. Find the maximum and minimum values, if any, of the following function given by :

$h(x) = \sin(2x) + 5$

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8. Find the maximum and minimum values, if any, of the following function given by :

$f(x) = |\sin 4x + 3|$

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9. Find the maximum and minimum values, if any, of the following function given by :

$h(x) = x + 1, x \in (-1, 1)$

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10. Find the local maxima and local minima, if any, of the followig functions. Find also the local maximum and the local minimum values, as the case may be :

$f(x) = x^2$

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11. Find the local maxima and local minima, if any, of the followig functions. Find also the local maximum and the local minimum values, as the case may be :

$$g(x) = x^3 - 3x$$

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12. Find the local maxima and local minima, if any, of the followig functions. Find also the local maximum and the local minimum values, as the case may be :

$$h(x) = \sin x + \cos x, 0 < x < \frac{\pi}{2}$$

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13. Find the local maxima and local minima, if any, of the followig functions. Find also the local maximum and the local minimum values, as the case may be :

$f(x) = \sin x - \cos x, 0 < x < 2\pi$

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14. Find the local maxima and local minima, if any, of the followig functions. Find also the local maximum and the local minimum values, as the case may be :

$f(x) = x^3 - 6x^2 + 9x + 15$

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15. Find the local maxima and local minima, if any, of the followig functions. Find also the local maximum and the local minimum values, as the case may be :

$$g(x)=\frac{x}{2}+\frac{2}{x}, x>0$$

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16. Find the local maxima and local minima, if any, of the followig functions. Find also the local

maximum and the local minimum values, as the case may be :

$$g(x) = \frac{1}{x^2 + 2}$$

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17. Find the local maxima and local minima, if any, of the followig functions. Find also the local maximum and the local minimum values, as the case may be :

$$f(x) = x\sqrt{1 - x}, x > 0$$

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18. Prove that the following functions do not have maxima or minima :

$f(x) = e^x$

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19. Prove that the following functions do not have maxima or minima :

$g(x) = \log x$

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20. Prove that the following functions do not have maxima or minima :

$h(x) = x^3 + x^2 + x + 1.$

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21. Find the absolute maximum value and the absolute minimum value of the following functions is given intervals :

$f(x) = x^2, x \in [-2, 2]$

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22. Find the absolute maximum value and the absolute minimum value of the following functions is given intervals :

$f(x) = \sin x + \cos x, x \in [0, \pi]$

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23. Find the absolute maximum value and the absolute minimum value of the following functions is given intervals :

$f(x) = 4x - \frac{1}{2}x^2, x \in \left[-2, \frac{9}{2}\right]$

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24. Find the absolute maximum value and the absolute minimum value of the following functions is given intervals :

$f(x) = (x-1)^2 + 3, x \in [-3, 1]$.

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25. Find the maximum profit that a company can make, if the profit function is given by :

$p(x) = 41 - 72x - 18x^2$.

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26. Find both the maximum value and the minimum value of $3x^4 - 8x^3 + 12x^2 - 48x + 25$on the interval [0, 3].

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27. At what points in the interval $[0, 2\pi]$, does the function $s \in \quad 2x$ attain its maximum value?

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28. What is the maximum value of sinx+cosx

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29. Find the maximum value of $2x^3 - 24x + 107$ in the interval $[1, 3]$. Find the maximum value of the same function in $[-3, -1]$.

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30. It is given that at $x = 1$, the function $x^4 - 62x^2 + ax + 9$ attains its maximum value on the interval $[0, \ 2]$. Find the value of a .

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31. Find the maximum and minimum values of $x + s \in 2x$on $[0, 2\pi]$.

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32. Find the two numbers with maximum product and whose sum is 24.

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33. Find two positive numbers x and y such that $x + y = 60$and $xy^3$is maximum.

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34. Find two positive number m and n such that their sum is 35 and the product m^2n^5 is maximum.

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35. Find two positive numbers whose sum is 16 and the sum of whose cubes is minimum.

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36. A square piece of tin of side 18 cm is to be made into a box without top, by cutting a square from each corner and folding up the flaps to form the box. What should be the side of the square to be cut off so that the volume of the box is the maxi

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37. A rectangular sheet of tin 45 cm by 24 cm is to be made into a box without top, by cutting off square from each corner and folding up the flaps. What should be the side of the square to be cut off so that the volume of the box is maximum ?

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38. Show that of all the rectangles inscribed in a given fixed circle, the square has the maximum area.

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39. Show that the right circular cylinder of given surface and maximum volume is such that its height is equal to the diameter of the base.

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40. Of all the closed cylindrical cans (right circular), of a given volume of 100 cubic centimetres, find the dimensions of the can which has the minimum surface area?

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41. A wire of length 28 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the length of the two pieces so that the combined area of the square and the circle is minimum?

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42. Prove that the volume of the largest cone, that can be inscribed in a sphere of radius $R\cdot$ is $\frac{8}{27}$ of the volume of the sphere.

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43. Show that the right-circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.

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44. Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$.

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45. Show that semi-vertical angle of right circular cone of given surface area and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$.

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46. The point on the curve $x^2 = 2y$which is nearest to the point (0, 5) is(A) $\left(2\sqrt{2}, 4\right)$ (B) $\left(2\sqrt{2}, 0\right)$ (C) (0, 0) (D) (2, 2)

A. $\left(2\sqrt{2}, 4\right)$

B. $\left(2\sqrt{2}, 0\right)$

C. $(0, 0)$

D. $(2, 2)$

Answer: A

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47. For all real values of x, the maximum value of $\dfrac{1 - x + x^2}{1 + x + x^2}$ is :

A. 0

B. 1

C. 3

D. $\frac{1}{3}$

Answer: D

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48. The maximum value of $[x(x-1)+1]^{1/3}, 0 \leq x \leq 1$ is :

A. $\left(\frac{1}{3}\right)^{1/3}$

B. $\frac{1}{2}$

C. 1

D. 0

Answer: C

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Misellaneous Exercise on Chapter (6)

1. Using differentials, find the approximate value of each of the following :

$(17)^{1/4}$

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2. Using differentials, find the approximate value of $(33)^{1/5}$

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3. Show that the function given by $f(x) = \frac{\log x}{x}$ has maximum at $x = e$.

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4. The two equal sides of an isosceles triangle with fixed base b are decreasing at the rate of 3 cm per second. How fast is the area decreasing when the two equal sides are equal to the base ?

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5. Find the equation of the normal to curve $y^2 = 4x$ at the point $(1, 2)$.

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6. Show that the normal at any point θ to the curve

$x = a\cos\theta + a\theta\sin\theta,\ y = a\sin\theta - a\,\theta\cos\theta$ is at a constant distance from the origin.

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7. Find the intervals in which the function f given by $f(x) = \dfrac{4\sin x - 2x - xc\otimes}{2+\cos x}$ is (i) increasing (ii) decreasing.

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8. Find the intervals in which the function f given by $f(x) = x^3 + \frac{1}{x^3}, x \neq 0$is (i) increasing (ii) decreasing.

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9. Find the maximum are of the isosceles triangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$ with its vertex at one end of major axis.

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10. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is 8 m3. If building of tank costs Rs 70 per square metre for the base and Rs 45 per square metre for sides, what is the cost of least expensive tank?

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11. The sum of the perimeter of a circle and square is k, where k is some constant. Prove that the sum of their areas is least when the side of square is double the radius of the circle.

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12. A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.

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13. A point on the hypotenuse of a triangle is at distance a and b from the sides of the triangle.

Show that the maximum length of the hypotenuse is $\left(a^{\frac{2}{3}}+b^{\frac{2}{3}}\right)^{\frac{3}{2}}$.

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14. Find the points at which the function f given by $f(x)=(x-2)^{4}(x+1)^{3}$ has local maxima local minima point of inflexion

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15. Find the absolute maximum and minimum values of the function f given by $f(x) = \cos^2 x + \sin x$, $x \in [0, \pi]$.

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16. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$.

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17. Let f be a function defined on [a, b] such that $f'(x) > 0$, for all $x \in (a, b)$. Then prove that f is an increasing function on (a, b).

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18. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$.

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19. Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and semi vertical angle is one-third that of the cone and the greatest volume of cylinder is $\frac{4}{27}\pi h^3 \tan^2 \alpha$.

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20. A cylindrical tank of radius 10 m is being filled with wheat at the rate of 314 cubic metre per hour. Then the depth of the wheat is increasing at the rate of(A) 1 m^3/h (B) 0.1 m^3/h (C) 1.1 m^3/h (D) 0.5 m^3/h

A. $1m^3/h$

B. $0.1m^3/h$

C. $1.1m^3/h$

D. $0.5m^3/h$.

Answer: A

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21. The slope of the tangent to the curve $x = t^2 + 3t - 8, y = 2t^2 - 2t - 5$ at the point (2, -1), is

A. $\frac{22}{7}$

B. $\frac{6}{7}$

C. $\frac{7}{6}$

D. $\frac{-6}{7}$

Answer: B

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22. The line $y = mx + 1$is a tangent to the curve $y^2 = 4x$if the value of m is(A) 1 (B) 2 (C) 3 (D) $\frac{1}{2}$

A. 1

B. 2

C. 3

D. $\frac{1}{2}$

Answer: A

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23. The normal at the point (1,1) on the curve $2y + x^2 = 3$is(A) $x + y = 0$ (B) $xy = 0$ (C) $x + y + 1 = 0$(D) $xy = 0$

A. $x + y = 0$

B. $x - y = 0$

C. $x + y + 1 = 0$

D. $x - y = 0$

Answer: B

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24. The normal to the curve $x^2 = 4y$ passing $(1, 2)$ is :

A. $x+y=3$

B. $x-y=3$

C. $x+y=1$

D. $x-y=1.$

Answer: A

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25. The points on the curve $9y^2=x^3$, where the normal to the curve makes equal intercepts with the axes are(A) $\left(4, \pm\frac{8}{3}\right)$ (B) $\left(4, \frac{-8}{3}\right)$ (C) $\left(4, \pm\frac{3}{8}\right)$ (D) $\left(\pm 4, \frac{3}{8}\right)$

A. $\left(4, \pm\frac{8}{3}\right)$

B. $\left(4, \frac{-8}{3}\right)$

C. $\left(4, \pm\frac{3}{8}\right)$

D. $\left(\pm 4, \frac{3}{8}\right)$

Answer: A

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Exercise

1. A spherical ball of salt is dissolving in water in such a manner that the rate of decrease of volume at any instant is proportional to the surface. Prove that the radius is decreasing at a constant rate.

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2. A kite is moving horizontally at a height of $151.5m$. If the speed of the kite is $10\frac{m}{s}$, how fast is the string being let out, when the kite is 250 m away from the boy who is flying the kite?

The height of the boy is 1.5 m. (A) 8 m/s (B) 12 m/s (C) 16 m/s (D) 19 m/s

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3. The volume of a cube is increasing at a constant rate. Prove that the increase in surface area varies inversely as the length of the edge of the cube.

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4. If $x and y$ are the sides of two squares such that $y = x - x^2$. Find the change of the area of second square with respect to the area of the first square.

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5. Show that for $a \geq 1, \quad f(x) = \sqrt{3} \sin x - \cos x - 2ax + b$ is decreasing on R .

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6. Show that the function f given by $f(x) = \tan^{-1}(\sin x + \cos x), x > 0$is always an strictly increasing function in $\left(0, \frac{\pi}{4}\right)$.

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7. Determine for which values of x, the function $y = x^4 - \frac{4x^3}{3}$ is increasing and for which it is decreasing.

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8. Show that $f(x) = 2x + \cot^{-1} x + \log\left(\sqrt{1 + x^2} - x\right)$ is increasing in R

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9. Find the angle of intersection of the curves y $= 4 - x^2$ and $y = x^2$

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10. Prove that the curves $xy = 4andx^2 + y^2 = 8$ touch each other.

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11. The slope of the tangent to the curve $x = t^2 + 3t - 8, y = 2t^2 - 2t - 5$ at the point (2, -1), is

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12. Using differentials, find the approximate value of $\sqrt{0.082}$

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13. Using differentials, find the approximate value of $(1.999)^5$

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14. Find the approximate volume of metal in a hollow spherical shell whose internal and external radii are 3 cm and 3.0005 cm, respectively.

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15. The maximum slope of curve y $= -x^3 + 3x^2 + 9x - 27$ is

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Revision Exercise

1. A car starts from a point P at time t = 0 seconds and stops at point Q. The distance x, in metres, covered by it, in t seconds is given by $x = t^2\left(2 - \frac{t}{3}\right)$ Find the time taken by it to reach Q and also find distance between P and Q.

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2. A water tank has the shape of an inverted righ circular cone with its axis vertical and vertex lowermost . Its semi-vertical angle is $\tan^{-1}(0.5)$. Water is poured into it at a constant rate of 4 cubic meter per hour . Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 2 m.

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3. The two equal sides of an isosceles triangle with fixed base b are decreasing at the rate of 3 cm per second. How fast is the area decreasing when the two equal sides are equal to the base ?

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4. The bottom of a rectangular swimming tank is 25 m by 40m. Water is pumped into the tank at the rate of 500 cubic metres per minute. Find the rate at which the level of water in the tank is rising.

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5. A ladder 13m long leans against a wall. The foot of the ladder is pulled along the ground away from the wall, at the rate of 1.5m/sec. How fast is the angle θ between the ladder and the ground is changing when the foot of the ladder is 12m away from the wall.

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6. The radius of a cylinder is increasing at the rate 2cm/sec. and its altitude is decreasing at the rate of 3cm/sec. Find the rate of change of volume when radius is 3 cm and altitude 5 cm.

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7. A kit is 120m high and 130 m of string is out. If the kite is moving away horizontally at the rate of 52 m/sec, find the rate at which the string is being paid out.

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8. $f(x) = \tan^{-1}(\sin x + \cos x), x > 0$ is always and increasing function on the interval

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9. The equation of tangents to the curve $y = \cos(x + y), \ -2\pi \le x \le 2\pi$ that are parallel to the line $x + 2y = 0$, is

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10. Find the angle between the parabolas $y^2 = 4ax$ and $x^2 = 4by$ at their point of intersection other than the origin.

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11. Tangents are drawn from the origin to the curve $y = \sin x$. Prove that their points of contact lie on the curve $x^2y^2 = (x^2 - y^2)$

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12. Show that the line $\frac{d}{a} + \frac{y}{b} = 1$ touches the curve $y = be^{-\frac{x}{a}}$ at the point where it crosses the the y-axis.

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13. If the straight line $x\cos\alpha + y\sin\alpha = p$ touches the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then prove that $a^2\cos^2\alpha + b^2\sin^2\alpha = p^2$.

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14. if the straight line $x\cos\alpha + y\sin\alpha = p$ touches the curve $x^m y^n = a^{m+n}$ prove that $p^{m+n} m^m n^n = (m+n)^{m+n} a^{m+n} \sin^n\alpha\cos^m\alpha$

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15. Find the point on the curve $y = 3x^2 - 9x + 8$ at which the tangents are equally inclined with the axes.

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16. The equation of the tangent at (2,3) on the curve $y^2 = ax^3 + b$ is $y = 4x - 5$. Find the values of $a and b\cdot$

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17. A circular disc of radius 3 cm is being heated. Due to expansion, its radius increases at the rate of 0.05 cm/s. Find the rate at which its area is increasing when radius is 3.2 cm.

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18. The radius of a sphere shrinks from 10 to 9.8 cm. Find approximately the decrease in its volume.

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19. If the error committed in measuring the radius of a circle is 0.01%, find the corresponding error in calculating the area.

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20. If a triangle ABC, inscribed in a fixed circle, be slightly varied in such away as to have its vertices always on the circle, then show that $\frac{da}{cas A} + \frac{db}{\cos B} + \frac{dc}{\cos C} = 0.$

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21. The area S of a triangle is calculated by measuring the sides b and c, and $\angle A$. If there be an error δA in the measurement of $\angle A$, show that the relative error in area is given by

$$\frac{\delta S}{S} = \cot A. \delta A$$

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22. The pressure p and the volume v of a gas are connected by the relation $pv^{1.4} = const.$ Find the percentage error in p corresponding to a decrease of % in $v.$

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23. Show that the function given by $f(x) = \frac{\log x}{x}$ has maximum at $x = e$.

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24. The sum of the perimeter of a circle and square is k, where k is some constant. Prove that the sum of their areas is least when the side of square is double the radius of the circle.

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25. Find the points at which the function f given by $f(x) = (x-2)^4(x+1)^3$ has local maxima local minima point of inflexion

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26. Show that $s \in^p \theta \cos^q \theta$ attains a maximum, when $\theta = \tan^{-1}\sqrt{\frac{p}{q}}$.

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27. The fraction exceeds its p^{th} power by the greatest number possible, where $p \geq 2$ is

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28. If the sum of the lengths of the hypotenues and a side of a right angled triangle is given, show that the area of the triangle is maximum when the angle between them is $\pi / 3$.

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29. Divide 4 into two positive numbers such that the sum of the square of one and cube of the other is a minimum.

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30. A cylindrical can to be made to hold 1 litres of oil. Find the dimensions which will minimize the cost of the metal to make the can.

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31. Find the shortest distance of the point $(0, c)$ from the curve $y = x^2$, where $0 \leq c \leq 5$.

View Text Solution

32. A beam of length l is supported at one end. If W is the uniform load per unit length, the bending moment M at a distance x from the end is given by $M = \frac{1}{2}lx - \frac{1}{2}Wx^2$. Find the point on the beam at which the bending moment has the maximum value.

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33. Find the maximum area of an isosceles triangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with its vertex at one end of the major axis.

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34. Find the area of the greatest rectangle that can be inscribed in an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

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35. A window of perimeter P (including the base of the arch) is in the form of a rectangle surrounded by a semi-circle. The semi-circular portion is fitted with the colored glass while the rectangular part is fitted with the clear glass that transmits three times as much light per square meter as the colored glass does. What is the ratio for the sides of the rectangle so that the window transmits the maximum light?

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CHECK YOUR UNDERSTANDING

1. The radius of a soap bubble is increasing at the rate of $0.2cm / s$. Find the rate of increase of its surface area when radius = 5 cm.

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2. Is the function $f(x) = x^2, x \in R$ increasing?

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3. The function $f(x) = x^2 - 6x + 9$ is increasing for $x > 3$.

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4. Find the slope of the tangent to the curve $y = 3x^2 - 4x$ at the point, whose x - co - ordinate is 2.

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5. Find the equation of the tangent of the curve $y = 3x^2$ at (1, 1).

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6. The function $f(x) = x^2, x \in R$ has no minimum value. (True/False)

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7. What is the absolute minimum value of $y = x^2 - 3x$ in $[0, 2]$?

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8. What are the maximum and minimum values, if any, of $f(x) = x, x \in (0, 1)$?

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9. Has the function $f(x) = x^n$ minimum value at $x = \frac{1}{e}$?

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10. Find two positive numbers whose product is 49 and the sum is minimum.

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COMPLETITION FILE (Questions from JEE Main)

1. Given P(x) $= x^4 + ax^3 + bx^2 + cx + d$ such that x=0 is the only real root of P'(x) =0 . If P(-1) lt P(1),$then \in the \int erval$[-1,1]`

A. $P(-1)$ is the minimum and P(1) is the maximum of P

B. $P(-1)$ is not minimum but P(1) is the maximum of P

C. $P(-1)$ is the minimum but P(1) is not the maximum of P

D. Neither $P(-1)$ is the minimum nor P(1) is the maximum of P.

Answer: B

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2. The equation of the tangent to the curve $y = x + \frac{4}{x^2}$, that is parallel to the x-axis, is (1) $y = 1$ (2) $y = 2$ (3) $y = 3$ (4) $y = 0$

A. $y = 0$

B. $y = 1$

C. $y = 2$

D. $y = 3.$

Answer: D

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3. Let $f: R \to R$ be a positive increasing function with $\lim_{x \to \infty} \frac{f(3x)}{f(x)} = 1$.Then $\lim_{x \to \infty} \frac{f(2x)}{f(x)}$ is

A. 1

B. $\frac{2}{3}$

C. $\frac{3}{2}$

D. 3

Answer: A

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4. Let $f: R\overrightarrow{R}$ be defined by $f(x) = \{k - 2x, \quad \text{if} \quad x \leq -12x + 3, fx \succ 1\}$. If f has a local minimum at $x = 1$, then a possible value of k is (1) 0 (2) $-\frac{1}{2}$ (3) -1 (4) 1

A. 1

B. 0

C. $-\frac{1}{2}$

D. $-1.$

Answer: D

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5. The shortest distance between line y-x=1 and curve $x = y^2$ is

A. $\dfrac{\sqrt{3}}{4}$

B. $\dfrac{3\sqrt{2}}{8}$

C. $\dfrac{8}{3\sqrt{2}}$

D. $\dfrac{4}{\sqrt{3}}$

Answer: B

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6. A spherical balloon is filled with 4500p cubic meters of helium gas. If a leak in the balloon causes the gas to escape at the rate of 72π

cubic meters per minute, then the rate (in meters per minute) at which the radius of the balloon decreases 49 minutes after the leakage began is (1) $\frac{9}{7}$ (2) $\frac{7}{9}$ (3) $\frac{2}{9}$ (4) $\frac{9}{2}$

A. $\frac{9}{7}$

B. $\frac{7}{9}$

C. $\frac{2}{9}$

D. $\frac{9}{2}$.

Answer: C

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7. The real number k for which the equation, $2x^3 + 3x + k = 0$ has two distinct real roots in [0, 1] (1) lies between 2 and 3 (2) lies between -1 and 0 (3) does not exist (4) lies between 1 and 2

A. lies between 2 and 3

B. lies between -1 and 0

C. does not exist

D. lies between 1 and 2.

Answer: C

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8. If f and g are differentiable functions in [0, 1] satisfying $f(0) = 2 = g(1), g(0) = 0$ and $f(1) = 6$, then for some $c \in]0, 1[$ (1) $2f'(c) = g'(c)$ (2) $2f'(c) = 3g'(c)$ (3) $f'(c) = g'(c)$ (4) $f'(c) = 2g'(c)$

A. $2f'(c) = 3g'(c)$

B. $f'(c) = g'(c)$

C. $f'(c) = 2g'(c)$

D. $2f'(c) = g'(c)$

Answer: C

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9. If x = -1 and x = 2 are extreme points of f(x) = $\alpha \log|x| + \beta x^2 + x$, then

A. $\alpha = -6, \beta = \frac{-1}{2}$

B. $\alpha = 2, \beta = \frac{-1}{2}$

C. $\alpha = 2, \beta = \frac{1}{2}$

D. $\alpha = -6, \beta = \frac{1}{2}$.

Answer: B

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10. The normal to the curve $x^2 + 2xy - 3y^2 = 0$, at (1, 1):

A. does not meet the curve again

B. meets the curve again in the second quadrant

C. meets the curve again in the third quadrant

D. meets the curve again in the fourth quadrant.

Answer: D

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11. Let f(x) be a polynomial of degree four having extreme values at x=1 and x=2. If $\lim_{x \to 0}\left(1+\frac{f(x)}{x^2}\right)=3$, then f(2) is equal to

A. -8

B. -4

C. 0

D. 4

Answer: C

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12. Consider $f(x)=\tan^{-1}\left(\sqrt{\dfrac{1+\sin x}{1-\sin x}}\right), x\in\left(0,\dfrac{\pi}{2}\right)$. A normal to $y=f(x)$ at $x=\dfrac{\pi}{6}$ also passes through the point: (1) (0, 0) (2) $\left(0, \frac{2\pi}{3}\right)$ (3) $\left(\frac{\pi}{6}, 0\right)$ (4) $\left(\frac{\pi}{4}, 0\right)$

A. $\left(0, \frac{2\pi}{3}\right)$

B. $\left(\frac{\pi}{6}, 0\right)$

C. $\left(\frac{\pi}{4}, 0\right)$

D. $(0, 0)$

Answer: A

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13. The normal to the curve $y(x-2)(x-3)=x+6$ at the point where the curve intersects the $y-a\xi s,$ passes through the point : $\left(\frac{1}{2},-\frac{1}{3}\right)$ (2) $\left(\frac{1}{2},\frac{1}{3}\right)$ (3) $\left(-\frac{1}{2},-\frac{1}{2}\right)$ (4) $\left(\frac{\frac{1}{2,1}}{2}\right)$

A. $\left(\frac{1}{2},-\frac{1}{3}\right)$

B. $\left(\frac{1}{2},\frac{1}{3}\right)$

C. $\left(-\frac{1}{2},-\frac{1}{2}\right)$

D. $\left(\frac{1}{2},\frac{1}{2}\right)$

Answer: D

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14. Twenty metres of wire is available for fencing off a flower-bed in the form of a circular sector. Then the maximum area (in $sq\dot{m})$ of the flower-bed is: 25 (2) 30 (3) 12.5 (4) 10

A. 25

B. 30

C. 12.5

D. 10

Answer: B

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15. If the curves $y^2 = 6x, \quad 9x^2 + by^2 = 16$ intersect each other at right angles then the value of b is: (1) 6 (2) $\frac{7}{2}$ (3) 4 (4) $\frac{9}{2}$

A. 6

B. $\frac{7}{2}$

C. 4

D. $\frac{9}{2}$

Answer: D

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16. Let $f(x) = x^2 + \left(\frac{1}{x^2}\right)$ and $g(x) = x - \frac{1}{x}$ $\xi n R - \{-1, 0, 1\}$. If $h(x) = \left(\frac{f(x)}{g(x)}\right)$ then the local minimum value of $h(x)$ is: (1) 3 (2) -3 (3) $-2\sqrt{2}$ (4) $2\sqrt{2}$

A. 3

B. -3

C. $-2\sqrt{2}$

D. $2\sqrt{2}$.

Answer: D

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17. The maximum values of $3\cos\theta + 5\sin\left(\theta - \frac{\pi}{6}\right)$ for any real value of θ is:

A. $\sqrt{34}$

B. $\sqrt{19}$

C. $\frac{\sqrt{79}}{2}$

D. $\sqrt{31}$

Answer: B

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18. Let $f(x)$ be an non - zero polynomial of degree 4. Extremum points of $f(x)$ are $0, -1, 1$. If $f(k) = f(0)$ then,

A. k has one rational and two irrational roots

B. k has four rational roots

C. k has four irrational roots

D. k has three irrational roots.

Answer: A

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CHAPTER TEST (1)

1. The approximate change in the volume of a cube of side x metres caused by increasing the side by 3% is :

A. $0.06x^3m^3$

B. $0.6x^3m^3$

C. $0.09x^3m^3$

D. $0.9x^3x^3$

Answer: C

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2. Find the slope of the tangent to the curve $y = x^3 - 3x + 2$ at the point whose x-coordinate is 3.

A. 20

B. 24

C. 30

D. 25

Answer: B

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3. The radius of a circle is increasing at the rate of 0.7 cm/sec. What is the rate of increase of its circumference?

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4. Show that the function f given by $f(x) = x^3 - 3x^2 + 4x, x \in R$is strictly increasing on R.

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5. Find the slope of the tangent to the curve $y = x^3 - 3x + 2$ at the point whose x-coordinate is 3.

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6. A man 2 metres high walks at a uniform speed of 5 km/hr away from a lamp-post 6 metres high. Find the rate at which the length of his shadow increases.

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7. Find the intervals in which the function f given by $f(x) = \sin x + \cos x, \quad 0 \leq x \leq 2\pi$ is strictly increasing or strictly decreasing.

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8. Show that the curves $x = y^2$ and $xy = k$ cut at right angles; if $8k^2 = 1$

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9. Evaluate $\sqrt{402}$, using differentials.

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10. It is given that at $x = 1$, the function $x^4 - 62x^2 + ax + 9$ attains its maximum value on the interval $[0, 2]$. Find the value of a.

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11. Find the equation of the tangents to the curve $3x^2 - y^2 = 8$, which passes through the point $\left(\frac{4}{3}, 0\right)$.

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12. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $2\frac{R}{\sqrt{3}}$. Also find maximum volume.

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