MATHS

BOOKS - ARIHANT MATHS (HINGLISH)

APPLICATIONS OF DERIVATIVES

ILLUSTRATIVE EXAMPLES

1. A particle moves along the curve $x^2 = 2y$. At what point, ordinate increases at the same rate as abscissa increases ?

Watch Video Solution

2. The side of an equilateral triangle is increasing at the rate of 2 cm/s. At what rate is its area increasing when the side of the triangle is 20 cm?

Watch Video Solution

3. The length x of a rectangle is decreasing at the rate of 3 cm/minute and the width y is increasing at the rate of 2cm/minute. When

$x = 10$cm and $y = 6$cm, find the rates of change of (a) the perimeter and (b) the area of the rectangle.

Watch Video Solution

4. The volume of a sphere is increasing at the rate of 3 cubic centimeter per second. Find the rate of increase of its surface area, when the radius is 2 cm.

Watch Video Solution

5. For the curve $y = 5x - 2x^3$, if x increases at the rate of 2 units/sec, then how fast is the slope of the curve changing when $x = 3$?

Watch Video Solution

6. A water tank has the shape of an inverted right - circular cone with its axis vertical and vertex lower most. Its semi - vertical angle is $\tan^{-1}\left(\frac{1}{2}\right)$. Water is poured into it at a constant rate of 5 cubic meter per minute. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 10 m.

Watch Video Solution

7. Water is leaking from a conical funnel at the rate of $5c\frac{m^3}{\sec}$. If the radius of the base of the funnel is 5 cm and its altitude is 10 cm, find the rate at which the water level is dropping when it is 2.5 cm from the top.

Watch Video Solution

8. A stone is dropped into a quiet lake and waves move in circles at a speed of 4cm per second. At the instant, when the radius of the circular wave is 10 cm, how fast is the enclosed area increasing?

Watch Video Solution

9. A man of 2 metres height walks at a uniform speed of 6 km/hr away from a lamp post of 6 metres high. Find the rate at which the length of his shadow increases.

Watch Video Solution

10. A man is moving away from a tower 41.6 m high at the rate of 2 m/sec. Find the rate at which the angle of elevation of the top of tower is changing, when he is at a distance of 30m from the foot of the tower. Assume that the eye level of the man is 1.6m from the ground.

11. The amount of pollution content added in air in a city due to $x - diesel$ vehicles is given by $P(x) = 0.005x^3 + 0.02x^2 + 30x$. Find the arginal increase in pollution content when 3 diesel vehicles are added and write which value is indicated in the above question.

Watch Video Solution

12. The money to be spend for the welfare of the employees of a firm is proportional to the rate of change of its total revenue (Marginal revenue). If the total revenue (in rupees) received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$, find the marginal revenue, when $x = 5$, and write which value does the question indicate.

Watch Video Solution

13. Using differentials, find the approximate value of $\sqrt{26}$

Watch Video Solution

14. Using differentials, find the approximate value of $\sqrt[3]{0.026}$, upto three places of decimals.

Watch Video Solution

15. Find the approximate change in the volume V of a cube of side x metres caused by increasing the side by 1%.

Watch Video Solution

16. If the radius of a sphere is measured as 9 m with an error of 0.03 m, then find the approximate error in calculating its surface area.

Watch Video Solution

17. If $y = x^4 + 10$ and x change from 2 to 1.99, find the approximate change in y.

Watch Video Solution

18. Find the approximate value of $\log_e(9 \cdot 01)$ given $\log_e 3 = 1 \cdot 0986$

Watch Video Solution

19. Using differentials find the approximate value of $\tan 46^0$, if it is being given that $1^0 = 0.01745$ radians.

Watch Video Solution

20. If in a triangle ABC , the side c and the angle C remain constant, while the remaining elements are changed slightly, using differentials show that $\frac{da}{cs A} + \frac{db}{\cos B} = 0$

Watch Video Solution

21. The time t of a complete oscillation of a simple pendulum of length l is given by the equation $T = 2\pi\sqrt{\frac{1}{g}}$ where g is constant. What is the percentage error in T when l is increased by 1%?

22. Find the maximum and minimum values, if any of the function given by :

(i) $f(x) = x^2, x \in R$

(ii) $f(x) = |x|, x \in R.$

23. Find the maximum and minimum value, if any, of the following function without using derivatives:

(i) $f(x) = (2x - 1)^2 + 3$

(ii) $f(x) = 16x^2 - 16x + 28$

(iii) $f(x) = -|x + 1| + 3$

(iv) $f(x) = \sin 2x + 5$

(v) $f(x) = \sin(\sin x)$.

Watch Video Solution

24. Determine the absolute maximum and absolute minimum values of each of the following in the stated domains :

(i) $y = \frac{1}{2}x^2 + 5x + \frac{3}{2}, \; -6 \le x \le -2$

(ii) $f(x) = (x + 1)^{2/3}, 0 \le x \le 8.$

View Text Solution

25. Calculate the absolute maximum and absolute minimum value of the function $f(x) = \frac{x+1}{\sqrt{x^2+1}}, 0 \leq x \leq 2.$

Watch Video Solution

26. Find the absolute maximum and the absolute minimum value of the function given by :

$f(x) = \sin^2 x - \cos x, x \in [0, \pi].$

Watch Video Solution

Watch Video Solution

27. Find the points of local maxima or local minima, if any, of the following function, using the first derivative test. Also, find the local maximum or local minimum values, as the case may be: $f(x) = (x-1)(x+2)^2$

Watch Video Solution

28. Find the points of local mixima and local minima, if any, of the following function :

$f(x) = \sin x + \frac{1}{2}\cos 2x : 0 \leq x \leq \frac{\pi}{2}$.

View Text Solution

29. Find all the points of local maxima and local minima of the function f given by $f(x) = 2x^3 - 6x^2 + 6x + 5.$

Watch Video Solution

Frequently Asked Questions

1. Without using the derivative show that the function $f(x) = 7x - 3$ is strictly increasing function on R .

Watch Video Solution

2. Show that the function $f(x)=4x^3-18x^2+27x-7$ is always increasing on .

Watch Video Solution

3. Find the intervals in which the function $f\ (x)=\ x^3-\ 12x^2+\ 36x+17\ $ is (a) increasing, (b) decreasing.

Watch Video Solution

4. Find the intervals in which the function $f(x) = \frac{x^4}{4} - x^3 - 5x^2 + 24x + 12$ is (a) strictly increasing (b) strictly decreasing

Watch Video Solution

5. Separate the interval $\left[0, \frac{\pi}{2}\right]$ into sub intervals in which function

$f(x) = \sin^4(x) + \cos^4(x)$ is strictly increasing or decreasing.

Watch Video Solution

6. Find the intervals in which $f(x) = \sin 3x - \cos 3x, 0 < 0 < \pi,$ is strictly increasing or decreasing.

Watch Video Solution

7. Find the values of 'x' for which $f(x) = x^x, x > 0$ is strictly increasing or decreasing.

Watch Video Solution

8. If a, b, c are real numbers, then find the intervals in which :

$$f(x) = \begin{vmatrix} x+a^2 & ab & ac \\ ab & x+b^2 & bc \\ ac & bc & x+c^2 \end{vmatrix}$$ is strictly increasing or decreasing.

Watch Video Solution

9. Prove that

$$\frac{x}{(1-x)} < \log(1+x) < x \quad \text{for} \quad x > 0$$

Watch Video Solution

10. The point at which the tangent to the curve $y = \sqrt{4x-3} - 10$ has slope $\frac{2}{3}$ is

Watch Video Solution

11. Find the equation of all lines having slope 2 and being tangent to the curve $y + \frac{2}{x-3} = 0$.

Watch Video Solution

12. Find the equation of the tangent to the curve $y = \frac{x-7}{(x-2(x-3)}$ at the point where it cuts the x-axis.

Watch Video Solution

13. Find the equations of the tangent and the normal to the curve $16x^2 + 9y^2 = 145$ at the point (x_1, y_1), where $x_1 = 2$ and $y_1 > 0$

Watch Video Solution

14. Find the equations of the tangent and normal to the curve $x = a\sin^3\theta andy = a\cos^3\theta$ at $\theta = \frac{\pi}{4}$.

Watch Video Solution

15. Find the equations of tangent to the curve $x = 1 - \cos\theta, y = \theta - \sin\theta$ at $\theta = \frac{\pi}{4}$

Watch Video Solution

16. Find the equation of the tangent to the curve $x^2 + 3y = 3$, which is parallel to the line $y - 4x + 5 = 0$

Watch Video Solution

17. Determine the points on the curve $x^2 + y^2 = 13$, where the tangents are perpendicular to the line $3x - 2y = 0$.

Watch Video Solution

18. Show that the equation of normal at any point t on the curve $x = 3\cos t - \cos^3 t$ and $y = 3\sin t - \sin^3 t$ is $4\left(y\cos^3 t - x\sin^3 t\right) = 3\sin 4t$.

Watch Video Solution

19. Find the value of $n \in N$ such that the curve $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$ touches the straight line $\frac{x}{a} + \frac{y}{b} = 2$ at the point (a, b).

Watch Video Solution

20. At what point will be tangents to the curve $y = 2x^3 - 15x^2 + 36x - 21$ by parallel to x=axis? Also, find the equations of the tangents to the curve at these points.

Watch Video Solution

21. Show that the curves $xy = a^2 and x^2 + y^2 = 2a^2$ touch each other

Watch Video Solution

22. Find the angle of intersection of the following curves: $xy = 6 and x^2 y = 12$

$y^2 = 4x and x^2 = 4y$

Watch Video Solution

23. If the curve $ax^2 + by^2 = 1$ and $a'x^2 + b'y^2 = 1$ intersect orthogonally, then

Watch Video Solution

24. Find the values of x for which $f(x) = [x(x-2)]^2$ is an increasing unction. Also, find the points on the curve, where the tangent is parallel to x-axis.

Watch Video Solution

25. Find two positive number whose sum is 24 and their sum of square is minimum.

Watch Video Solution

26. Show that all the rectangles with a given perimeter, the square has the largest area.

Watch Video Solution

27. Show that the rectangle of maximum perimeter which can be inscribed in a circle of radius a is a square of side $\sqrt{2}a$.

Watch Video Solution

28. Show that the triangle of maximum area that can be inscribed in a given circle is an equilateral triangle.

Watch Video Solution

29. An open box with a square base is to be made out of a given iron sheet of area 27 sq. m.

Show that the maximum volume of the box is 13.5 cu.cm.

View Text Solution

30. 40. A given quantity of metal is to be cast into a half cylinder with a rectangular base and semicircular ends. Show that in order that the total surface area may be minimum, the ratio of the length of the cylinder to the diameter of its semi-circular ends is w `(

Watch Video Solution

31. A window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12 m, find the dimensions of the rectangle that will produce the largest area of the window.

Watch Video Solution

32. Show that the height of the cylinder of maximum volume that can be inscribed in a cone of height h is $\frac{1}{3}h$

Watch Video Solution

33. Let AP and BQ be two vertical poles at points A and B, respectively. If $AP = 16m, BQ = 22m and AB = 20m,$ then find the distance of a point R on AB from the point A such that $RP^2 + RQ^2$ is minimum.

Watch Video Solution

34. If length of three sides of a trapezium other than base are equal to 10cm, then find the area of the trapezium when it is maximum.

Watch Video Solution

35. A square piece of tin of side 24 cm is to be made into a box without top by cutting a square from each corner and foding up the flaps to form a box. What should be the side of the square to be cut off so that the volume of the box is maximum ? Also, find this maximum volume.

Watch Video Solution

36. A metal box with a square base and vertical sides is to contain 1024 cm3 of water, the material for the top and bottom costs Rs 5 per cm2 and the material for the sides costs Rs 2.50 per cm2. Find the least cost of the box.

Watch Video Solution

37. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is 8 m3. If building of tank costs Rs 70 per square metre for the base and Rs 45 per square metre for sides, what is the cost of least expensive tank?

Watch Video Solution

38. Show that the right circular cylinder, open at the top, and of given surface area and maximum volume is such that its height is equal to the radius of the base.

Watch Video Solution

39. Find the point on the curve $y^2 = 4x$ which is nearest to the point (2, 1).

Watch Video Solution

40. An Apache helicopter of enemy is flying along the curve given by $y = x^2 + 7$. A soldier, placed at (3, 7), wants to shoot down the helicopter when it is nearest to him. Find the nearest distance.

Watch Video Solution

41. An open tank with a square base and vertical sides is to be constructed from a metal sheet so as to hold a given quantity of water. Show that the cost of the material will be least when depth of the tank is half of its width.

Watch Video Solution

42. A cuboidal shaped godown with square base is to be constructed. Three times as much cost per square metre is incurred for constructing the roof as compared to the walls. Find the dimensions of the godown if it is to enclose a given volume and minimize the cost of constructing the roof and the walls.

Watch Video Solution

43. A manufacturer can sell x items at a price of Rs. $\left(5 - \frac{x}{100}\right)$ each. The cost price of x items is Rs. $\left(\frac{x}{5} + 500\right)$. Find the number of items he should sell to earn maximum profit.

Watch Video Solution

44. A person wants to plant some trees in his community park. The local nursery has to perform this task. It charges the cost of planting trees by the formula :

$C(x) = x^3 - 45x^2 + 600x,$

where 'x' is the number of trees and C(x) is cost of planting 'x' trees in rupees. The local authority has imposed a restriction that it can plant 10 to 20 trees in one community park for a fair - districution. For how many trees should the person place the order so that he has to spend the least amount? How much is the least amount? Use Calculate to answer these questions.

Watch Video Solution

Questions From NCERT Exemplar

1. For the curve $y = 5x - 2x^3$, if x increases at the rate of 2 units/sec, then how fast is the slope of the curve changing when $x = 3$?

Watch Video Solution

2. Prove that the function $f(x) = \tan x - 4x$ is strictly decreasing on $(-\pi/3, \pi/3)$.

Watch Video Solution

3. Show that the function $f(x) = 4x^3 - 18x^2 + 27x - 7$ has neither maxima nor minima.

Watch Video Solution

4. Find the condition for the curve $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $xy = c^2$ to interest orthogonally.

Watch Video Solution

5. Find the difference between the greatest and least values of the function $f(x) = \sin 2x - x$ on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Watch Video Solution

6. Find the area of the greatest rectangle that can be inscribed in an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Watch Video Solution

7. An isosceles triangle of vertical angle 2θ is inscribed in a circle of radius a . Show that the area of the triangle is maximum when $\theta=\frac{\pi}{6}$.

Watch Video Solution

EXERCISE 6 (a) (Short Answer Type Questions)

1. An edge of a variable cube is increasing at the rate of $3cm/s$. How fast is the volume of the cube increasing when the edge is $10cm$ long?

Watch Video Solution

2. The radius of a soap - bubble is increasing at the rate of $0.7cm/s$. Find the rate of increase of :

its volume when the radius is 5 cm.

Watch Video Solution

3. 6. The radius of a circle is increasing at the rate of 0.7 cm/s. What is the rate of increase of its circumference?

Watch Video Solution

4. The radius of a circle is increasing uniformly at the rate of 4 cm/sec. Find the rate at which the area of the circle is increasing when the radius is 8 cm.

Watch Video Solution

5. If the area of a circle increases at a uniform rate, then prove that perimeter varies inversely as the radius.

Watch Video Solution

6. The radius of a circle is increasing uniformly at the rate of 3 cm/s. Find the rate at which the area of the circle is increasing when the radius is 10 cm.

Watch Video Solution

7. The radius of an air bubble is increasing at the rate of $\frac{1}{2}cm/s$. At what rate is the volume of the bubble increasing when the radius is 1 cm?

Watch Video Solution

8. A balloon, which always remains spherical, has a variable diameter $\frac{3}{2}(2x+1)$.Find the rate of change of its volume with respect to x.

Watch Video Solution

9. A balloon which always remains spherical, is being inflated by pumping in 900 cubic centimetres of gas per second. Find the rate at which the radius of the balloon is increasing when the radius is 15 cm.

Watch Video Solution

10. The volume of a cube is increasing at a rate of 9 cubic centimetres per second. How fast is the surface area increasing when the length of an edge is 10 centimetres?

Watch Video Solution

11. The volume of a cube is increasing at the rate of $8cm^3 / s$. How fast is the surface area increasing when the length of an edge is 12 cm?

Watch Video Solution

12. The volume of a cube is increasing at the rate of increasing at the instant when the length of an edge of the cube is 24 cm?

View Text Solution

13. A particle moves along the curve $y = \frac{4}{3}x^3 + 5$. Find the points on the curve at which the y - coordinate changes as fast as the x - coordinate.

Watch Video Solution

14. A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate

Watch Video Solution

15. A particle moves along the parabola $y^2 = 4x$. Find the co - ordinates of the point on the parabola where the rate of increment of abscissa is twice the rate of increment of the ordinate.

Watch Video Solution

EXERCISE 6 (a) (Long Answer Type Questions (I))

1. The radius of a cylinder increases at the rate of 1 cm/s and its height decreases at the rate of 1 cm/s. Find the rate of change of its volume when the radius is 5 cm and the height is 5 cm.

If the volume should not change even when the radius and height are changed, what is the relation between the radius and height?

Watch Video Solution

2. The total cost C(x) in Rupees, associated with the production of x units of an item is given by

$C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$. Find the marginal cost when 3 units are produced, where by marginal cost we mean the instantaneous rate of cha

Watch Video Solution

3. The total revenue in rupees received from the sale of 'x' units of a product is given by:

$R(x) = 13x^2 + 26x + 20.$

Find the marginal revenue when x = 7

Watch Video Solution

4. Total revenue from the sale of 'x' units of a product is given by :

$$R(x) = 40x - \frac{x^2}{2}.$$

Find the marginal revenue when x = 6 and interpret it.

Watch Video Solution

5. A man 160 cm tall, walks away from a source of light situated at the top of a pole 6m high at the rate of 1.1 m/sec. How fast is the length of his shadow increasing when he is 1 metre away from the pole.

Watch Video Solution

6. A man 2 metres high walks at a uniform speed of 5 km/hr away from a lamp-post 6 metres high. Find the rate at which the length of his shadow increases.

Watch Video Solution

7. The length x of a rectangle is decreasing at the rate of 5 cm/minute and the width y is increasing at the rate of 4 cm/minute. When x = 8cm and y = 6cm, find the rates of change of (a) the perimeter, and (b) the area of the rectangle

Watch Video Solution

8. The volume of a sphere is increasing at the rate of $8cm^3/s$. Find the rate at which its surface area is increasing when the radius of the sphere is 12 cm.

Watch Video Solution

9. The length 'x' of a rectangle is decreasing at the rate of 3 cm/m and the width 'y' is increasing at the rate of 2 cm/m. Find the rates of change of :

(a) the perimeter (b) the area of the rectangle when x = 8 cm and y = 6 cm.

Watch Video Solution

10. An edge of a variable cube is increasing at the rate of 3 cm per second. How fast is the volume of the cube increasing when the edge is 10 cm long?

Watch Video Solution

11. Water is drpping out from a conical funnel at the uniform rate of $2cm^3 / s$ through a tiny hole at the vertex at the bottom. When the slant height of the water is 5 cm, find the rate of decrease of the slant higher of the water. Given that α is semi-vertical angle of the cone.

Watch Video Solution

12. An inverted conical vessel whose height is 10 cm and the radius of whose base is 5 cm is being filled with water at the uniform rate of $1.5cm^3 / \min$. Find the rate at which the level of water in the vessel is rising when the depth is 4 cm.

Watch Video Solution

13. A ladder 5m long is leaning against a wall. The foot of the ladder is pulled out along the ground away from from the wall at a rate of $2m/s$. How fast is the height of ladder on the decreasing at the instant when the foot of the ladder is $4m$ away from the wall?

Watch Video Solution

14. A 13-m long ladder is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2 m/s.

How fast is its height on the wall decreasing when the foot of the ladder is 5m aways from the wall ?

Watch Video Solution

15. The radius of a circular soap bubble is increasing at the rate of 0.2cm/s. Find the rate of change of its:

(I) Volume (II) Surface area

when the radius is 4 cm.

Watch Video Solution

EXERCISE 6 (a) (Long Answer Type Questions (II))

1. Water is running into a conical vessel, 15cm deep and 5cm in radius, at the rate of 0.1 cm^3/sec . When the water is 6cm deep, find at what rate it. the water level rising? the water-surface area increasing? the wetted surface of the vessel increasing?

Watch Video Solution

EXERCISE 6 (b) (Short Answer Type Questions)

1. Show that the following functins are strictly increasing on R:

(a) $f(x) = 3x + 17$

(b) (i) $f(x) = e^{x}$

(ii) $f(x) = e^{2x}$.

Watch Video Solution

2. Without using the derivative, show that the function $f(x) = |x|$ is (a) strictly increasing in $(0, \infty)$ (b) strictly decreasing in $(-\infty, 0)$

Watch Video Solution

3. Show that the function f given by $f(x) = x^3 - 3x^2 + 4x, x \in R$is strictly increasing on R.

Watch Video Solution

4. Prove the following

(i) $f(x) = x^2$ is a decreasing function for $x < 0$, where $x \in R$

Watch Video Solution

5. Prove the following

$f(x) = x^2 - 8x, x \leq 4$ is a decreasing function

Watch Video Solution

6. Prove that the function

$f(x) = x^3 - 3x^2 + 3x - 100$ is increasing on R

.

Watch Video Solution

7. Prove that the function given by $f(x) = \cos x$ is

(a) strictly decreasing in $(0, \pi)$.

Watch Video Solution

8. Prove the following

(i) $f(x) = \sin x$ is :

(I) strictly increasing in $\left(0, \frac{\pi}{2}\right)$

(II) strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$

(III) neither increasing nor decreasing in $(0, \pi)$

(ii) $f(x) = 2\sin x + 1$ is an increasing function on $\left[0, \frac{\pi}{2}\right]$.

Watch Video Solution

9. Prove the following

$f(x) = \tan^{-1}(\sin x + \cos x)$ is strictly decreasing function on $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$.

Watch Video Solution

10. Prove that the logarithmic function is strictly increasing on $(0, \infty)$.

Watch Video Solution

11. Prove that the function f given by

$f(x) = \log s \in x$

$f(x) = \log s \in x$ is strictly increasing on $\left(0, \frac{\pi}{2}\right)$ and strictly decreasing on $\left(\frac{\pi}{2}, \pi\right)$.

Watch Video Solution

12. Prove that the function f given by $f(x) = \log \cos x$ is strictly decreasing on $\left(0, \frac{\pi}{2}\right)$ and strictly increasing on $\left(\frac{\pi}{2}, \pi\right)$.

Watch Video Solution

13. Show that the function f(x) = x3– 3x^2 + 6x – 100 is increasing on .

Watch Video Solution

14. Find the intervals in which the following functions are increasing :

(i) $2x^3 - 3x$

(ii) $10 - 6x - 2x^2$.

Watch Video Solution

15. Find the interval in which $2x^3 + 9x^2 + 12x - 1$ is strictly increasing.

Watch Video Solution

16. Find the intervals in which the functions :

(i) $f(x) = x^3 + 2x^2 - 1$

(ii) $30 - 24x + 15x^2 - 2x^3$ are strictly decreasing.

Watch Video Solution

17. Prove that the function $f(x) = x^2 - x + 1$ is neither increasing nor decreasing on $(-1, 1)$.

Watch Video Solution

18. Find the values of 'a' for which the function :

$f(x) = x^2 - 2ax + 6$ is increasing when $x > 0$.

Watch Video Solution

19. Find the values of 'a' for which $f(x) = \sin x - ax + b$ is decreasing function on R.

Watch Video Solution

20. Find the values of x for which $y = [x(x-2)]^2$ is an increasing function

Watch Video Solution

21. Determine for which values of x, the following functions are increasing or decreasing :

$f(x) = -3x^2 + 12x + 8.$

Watch Video Solution

22. Determine for which values of x, the following functions are increasing or decreasing :

$f(x) = x^3 - 12x$

Watch Video Solution

23. Determine for which values of x, the following functions are increasing or decreasing :

$f(x) = 2x^3 - 24x + 107.$

Watch Video Solution

24. Determine for which values of x, the following functions are increasing or decreasing :

$$f(x) = x^4 - 2x^2$$

Watch Video Solution

25. Determine for which values of x, the following functions are increasing or decreasing :

$$f(x) = x^3 + \frac{1}{x^3}, x \neq 0$$

View Text Solution

26. For what values of 'x' are the following functions increasing or decreasing?

$$y = x + \frac{1}{x}, x \neq 0$$

Watch Video Solution

27. For what values of 'x' are the following functions increasing or decreasing?

$$y = 5x^{3/2} - 3x^{5/2}, x > 0$$

Watch Video Solution

EXERCISE 6 (b) (Long Answer Type Questions (I))

1. Find the intervals in which the function f given by $f(x) = 2x^2 - 3x$is(a) strictly increasing (b) strictly decreasing

Watch Video Solution

2. Determine the intervals in which the following functions are strictly increasing or strictly decreasing :

$f(x) = x^2 + 2x - 5$

Watch Video Solution

3. Find the intervals on which the function $f(x) = 10 - 6x - 2x^2$ is (a) strictly increasing (b) strictly decreasing.

Watch Video Solution

4. Determine the intervals in which the following functions are strictly increasing or strictly decreasing :

$f(x) = 6 - 9x - 2x^2$

Watch Video Solution

5. Find the intervals in which the function f given by $f(x) = x^2 - 4x + 6$is (a) strictly increasing (b) strictly decreasing

Watch Video Solution

6. Determine the intervals in which the following functions are strictly increasing or strictly decreasing :

$$f(x) = \frac{1}{4}x^4 + \frac{2}{3}x^3 - \frac{5}{2}x^2 - 6x + 7$$

Watch Video Solution

7. Determine the intervals in which the following functions are strictly increasing or strictly decreasing :

$$f(x) = -3\log(1+x) + 4\log(2+x) - \frac{4}{2+x}$$

Watch Video Solution

8. Determine the intervals in which the following functions are strictly increasing or strictly decreasing :

$f(x) = 20 - 9x + 6x^2 - x^3$

Watch Video Solution

9. Determine the intervals in which the following functions are strictly increasing or strictly decreasing :

$f(x) = 2x^3 - 15x^2 + 36x + 17.$

Watch Video Solution

10. Determine the intervals in which the following functions are strictly increasing or strictly decreasing :

$f(x) = 2x^3 - 9x^2 + 12x + 15$

Watch Video Solution

11. Determine the intervals in which the following functions are strictly increasing or

strictly decreasing :

$f(x) = 2x^3 - 3x^2 - 36x + 7.$

Watch Video Solution

12. Determine the intervals in which the following functions are strictly increasing or strictly decreasing :

$f(x) = x^3 - 6x^2 + 9x + 8$

Watch Video Solution

13. Determine the intervals in which the following functions are strictly increasing or strictly decreasing :

$f(x) = 4x^3 - 6x^2 - 72x + 30.$

Watch Video Solution

14. Determine the intervals in which the following functions are strictly increasing or strictly decreasing :

$f(x) = 2x^3 - 12x^2 + 18x + 5$

Watch Video Solution

15. Determine the intervals in which the following functions are strictly increasing or strictly decreasing :

$$f(x) = \frac{4x^2 + 1}{x}.$$

Watch Video Solution

16. Find the intervals in which the given functions are strictly increasing decreasing:

$$-2x^3 - 9x^2 - 12x + 1$$

Watch Video Solution

17. Determine the intervals in which the following functions are strictly increasing or strictly decreasing :

$f(x) = x^3 + 3x^2 - 4.$

Watch Video Solution

18. Determine the intervals in which the following functions are strictly increasing or strictly decreasing :

$f(x) = 2x^3 - 15x^2 + 36x + 6$

Watch Video Solution

19. Determine the intervals in which the following functions are strictly increasing or strictly decreasing :

$f(x) = (x - 1)(x - 2)^2$.

Watch Video Solution

20. Determine the intervals in which the following functions are strictly increasing or strictly decreasing :

$$f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$$

Watch Video Solution

21. Determine the intervals in which the following functions are strictly increasing or strictly decreasing :

$$f(x) = 3x^4 - 4x^3 - 12x^2 + 5.$$

Watch Video Solution

22. Determine the intervals in which the following functions are strictly increasing or strictly decreasing :

$f(x) = (x+1)^3(x-3)^3.$

Watch Video Solution

23. Determine the intervals in which the following functions are strictly increasing or strictly decreasing :

$f(x) = x^8 + 6x^2.$

Watch Video Solution

24. On which of the following intervals is the function 'f' given by $f(x) = x^{100} + \sin x - 1$ strictly increasing?

A. $(-1, 1)$

B. $(0, 1)$

C. $\left(\frac{\pi}{2}, \pi\right)$

D. $\left(0, \frac{\pi}{2}\right)$.

Answer: b,c,d

Watch Video Solution

25. Find the intervals in which $f(x) = \sin x - \cos x$, where $0 < x < 2\pi$, is strictly increasing or decreasing.

Watch Video Solution

26. Find the intervals in which the function f given by $f(x) = \sin x + \cos x$, $0 \leq x \leq 2\pi$ is strictly increasing or strictly decreasing.

Watch Video Solution

27. Find the intervals in which the function 'f' given by :

$f(x) = \sin x - \cos x, 0 \leq x \leq 2\pi$

is strictly increasing or striclty decreasing.

Watch Video Solution

28. Find the intervals in which the function $f(x) = 2x^3 - 9x^2 + 12x + 29$ is :

(i) monotonic increasing (ii) monotonic decreasing.

Watch Video Solution

29. Find the intervals in which the function given by $f(x) = \sin 3x, x \in \left[0, \frac{\pi}{2}\right]$ is :

(a) increasing (b) decreasing.

Watch Video Solution

30. which of the following functinon are strictly decreasing on (0 , π/2) a) cosx b)cos2x c) cos3x d) tanx

A. $\cos x$

B. $\cos 2x$

C. $\cos 3x$

D. $\tan x$.

Answer: (i), (ii)

Watch Video Solution

EXERCISE 6 (b) (Long Answer Type Questions (II))

1. If $x > -1$, show that : $\frac{x}{\sqrt{1+x}} - \log(1+x) + 9$ is an increasing function of x.

Watch Video Solution

EXERCISE 6 (c) (Short Answer Type Questions)

1. Find the slope of the tangent to the curve :

$y = x^3 - 2x + 8$ at the point $(1, 7)$.

Watch Video Solution

2. Find the slope of the normal to the curve :

$y = x^3 - x + 1$ at $x = 2$

Watch Video Solution

3. Find the slope of the normal to the curve :

$y = \tan^2 x + \sec x$ at $x = \frac{\pi}{4}$

Watch Video Solution

4. The slope of the normal to the curve

$x = a(\theta - \sin\theta), y = a(1 - \cos\theta)$ at $\theta = \frac{\pi}{2}$

Watch Video Solution

5. Find te slope of the normal to the curve $x = a\cos^2\theta$ and $y = a\sin^3\theta$ at $\theta = \frac{\pi}{4}$

Watch Video Solution

6. Equation of the tangent to the curve $2x^2 + 3y^2 - 5 = 0$ at (1, 1) is

Watch Video Solution

7. Find the equation of the tangent line to the curve :

$y = x^3 - 3x + 5$ at the point (2, 7)

Watch Video Solution

8. Find the equation of the tangent line to the curve $y = \cot^2 x - 2\cot x + 2$ at $x = \frac{\pi}{4}$

Watch Video Solution

9. Find the equation of the tangent of the tangent to the curve

$y = (\sec^4 x - \tan^4 x)$ at $x = \frac{\pi}{3}$

Watch Video Solution

10. Find the equation of the tangent line to the following curves :

$y = x^2$ at $(0, 0)$

Watch Video Solution

11. Find the equation of the tangent line to the following curves :

$y = x^3$ at $(1, 1)$

Watch Video Solution

12. Find the equations of the tangent and the normal to the curve $y = 2x^2 - 3x - 1$ at $(1, -2)$ at the indicated points

Watch Video Solution

13. Find the equations of the tangent and the normal to the curve $x^{2/3} + y^{2/3} = 2$ at $(1, 1)$ at indicated points.

Watch Video Solution

14. Find the equations of the tangent and the normal to the curve $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at the point (1, 3)

Watch Video Solution

15. Find the equation of the tangent line to the following curves :

$y = x^4 - 6x^3 + 13x^2 - 10x + 5 \text{ at } (0, 5).$

Watch Video Solution

16. Find the equation of the tangent line to the following curves :

$x = \cos t, y = \sin t \text{ at } t = \frac{\pi}{4}.$

Watch Video Solution

17. The equation of tangent to the curve $x = a\cos^3\theta, y = a\sin^3\theta$ at $\theta = \frac{\pi}{4}$ is

Watch Video Solution

18. Find the equation of the normal line to the curve :

(i) $y = 2x^2 + 3\sin x$ at $x = 0$

(ii) $y(x-2)(x-3) - x + 7 = 0$ at the point, where it meets x - axis.

Watch Video Solution

EXERCISE 6 (c) (Long Answer Type Questions (I))

1. Find the equations of the tangent and normal lines to the following curves :

$$y = \sin^2 x \quad \text{at} \quad x = \frac{\pi}{2}.$$

Watch Video Solution

2. Find the equations of the tangent and normal lines to the following curves :

$$y = \frac{1 + \sin x}{\cos x} \quad \text{at} \quad x = \frac{\pi}{4}.$$

Watch Video Solution

3. Find the equations of the tangent and normal to the parabola :

$y^2 = 4ax \quad \text{at} \quad (at^2, 2at)$

Watch Video Solution

4. Find the equations of tangent and normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at (x_1, y_1)

Watch Video Solution

5. Find the equations of the tangent and the normal to the curve $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at $(\sqrt{2}a,\ b)$ at indicated points.

Watch Video Solution

6. Find the equation of the normal to the curve $ay^2 = x^3$ at the point (am^2, am^3).

Watch Video Solution

7. Find the equation of tangent to the curve $2x^2 - y = 7$, which is parallel to the line $4x - y + 3 = 0$.

Watch Video Solution

8. Find the equation of the tangent to the curve $y = \sqrt{3x - 2}$, which is parallel to the line $4x - 2y + 5 = 0$.

Also, write the equation of normal to the curve at the point of contact.

Watch Video Solution

9. Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$ which is parallel to the line $2x - y + 9 = 0$

Watch Video Solution

10. Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$ which is perpendicular to the line $5y - 15x = 13$.

Watch Video Solution

11. Find the equation of the tangents to the curve :

$y = x^3 + 2x - 4,$

Which are perpendicular to line

$x + 14y + 3 = 0.$

Watch Video Solution

12. Find the equation of the normals to the curve $y = x^3 + 2x + 6$which are parallel to the line $x + 14y + 4 = 0.$

Watch Video Solution

13. Find the equation(s) of normal(s) to the curve $3x^2 - y^2 = 8$ which is (are) parallel to the line $x + 3y = 4.$

Watch Video Solution

14. Find the equations of the normals to the curve : $2x^2 - y^2 = 14$, which are parallel to the line $x + 3y = 6.$

Watch Video Solution

15. If a normal of curve $x^{2/3} + y^{2/3} = a^{2/3}$ makes an angle ϕ from X-axis then show that its equation is $y\cos\phi - x\sin\phi = a\cos 2\phi$.

Watch Video Solution

16. (i) Find the equation of the normal to the curve : $x^2 = 4y$, which passes through the point (1, 2).

(ii) Find the equation of the normal to the curve : $y^2 = 4x$, which passes through the point (1, 2).

Watch Video Solution

17. Find the equation of the tangent to the curve : $y = 3x^2 - 2x + 5$, which is parallel to the line $4x - y = 10$.

Watch Video Solution

18. Find the points on the curve $x^3 - 2x^2 - 2x$ at which the tangent lines are parallel to the line $y = 2x - 3$.

Watch Video Solution

19. Find the equation of the tangent to the curve $y = \sqrt{5x - 3}$, which is parallel to the line $4x - 2y + 3 = 0$.

Watch Video Solution

20. Find the equations of tangent lines to the curve $y = 4x^3 - 3x + 5$, which are perpendicular to the line 9y +x+3=0

Watch Video Solution

21. Find the equations of the normal to the curve $y = x^3 + 2x + 6$ which are parallel to the line $x + 14y + 4 = 0$.

Watch Video Solution

22. Find the equation of the normal to the curve : $y = x^3 + 5x^2 - 10x + 10$, where the normal is parallel to the line $x - 2y + 10 = 0$.

Watch Video Solution

23. Find the equations of the normal to the curve $y = 4x^3 - 3x + 5$, which are perpendicular to the line : $9x - y + 5 = 0$.

Watch Video Solution

24. Find the equation of tangent to the curve given by $x = a\sin^3 t$, $y = b\cos^3 t$ ... (1)at a point where $t = \frac{\pi}{2}$.

Watch Video Solution

25. Find the equation of the tangent at $t = \frac{\pi}{4}$ to the curve : $x = \sin 3t, y = \cos 2t.$

Watch Video Solution

26. Find the point(s) on the curve :

(i) $y = 3x^2 - 12x + 6$

(ii) $x^2 + y^2 - 2x - 3 = 0$

at which the tangent is parallel to x - axis.

Watch Video Solution

27. Find the point(s) on the curve :

(i) $y = \frac{1}{4}x^2$, where the slope of the tangent is $\frac{16}{3}$

(ii) $y = x^2 + 1$, at which the slope of the tangent is equal to :

(I) x - coordinate (II) y - coordinate.

Watch Video Solution

28. Find the point on the curve $y = x^3 - 11x + 5$ at which the tangent is $y = x \quad 11$.

Watch Video Solution

29. For the curve $y = 4x^3 - 2x^5$, find all the points at which the tangent passes through the origin.

Watch Video Solution

30. Find the points on the following curve at which the tangents are parallel to x - axis : $y = x^3 - 3x^2 - 9x + 7.$

Watch Video Solution

31. At what point on the circle $x^2 + y^2 - 2x - 4y + 1 = 0$, the tangent is parallel to $x - a\xi s$.

Watch Video Solution

32. Find points on the curve $\frac{x^2}{4} + \frac{y^2}{25} = 1at$ which the tangents are (i) parallel to x-axis (ii) parallel to y-axis.

Watch Video Solution

33. Show that the tangents to the curve $y = 7x^3 + 11$ at the points $x = 2$ and $x = -2$ are parallel.

Watch Video Solution

34. Find the equations of all lines having slope 0 which are tangent to the curve $y = \frac{1}{x^2 - 2x + 3}$.

Watch Video Solution

35. Find the equations of all lines :

having slope -1 and that are tangents to the curve : $y = \frac{1}{x-1}, x \neq 1$

Watch Video Solution

36. Find the equations of all lines having slope 2 and that are tangent to the curve $y = \frac{1}{x-3}, \quad x \neq 3$.

Watch Video Solution

37. Find the point of intersection of the tangent lines to the curve $y = 2x^2$ at the points $(1, 2)$ and $(-1, 2)$.

Watch Video Solution

38. Prove that the tangents to the curve $y = x^2 - 5x + 6$ at the points (2, 0) and (3, 0) are at right angles.

Watch Video Solution

39. Find the angle of intersection of the curves :

(i) $y^2 = 4x$ and $x^2 = 4y$

Watch Video Solution

40. Find the angle of intersection of the curves :

(ii)

$x^2 + y^2 - 4x - 1 = 0$ and $x^2 + y^2 - 2y - 9 = 0.$

Watch Video Solution

41. Show that the following set of curves intersect orthogonally: $y = x^3 and 6y = 7 - x^2$

$x^3 - 3xy^2 = -2 and 3x^2y - y = 2.$

$x^2 + 4y^2 = 8 and x^2 - 2y^2 = 4$

Watch Video Solution

42. If the curves :

$\alpha x^2 + \beta y^2 = 1$ and $\alpha' x^2 + \beta' y^2 = 1$

intersect orthogonally, prove that

$(\alpha - \alpha')\beta\beta' = (\beta - \beta')\alpha\alpha'$.

Watch Video Solution

EXERCISE 6 (c) (Long Answer Type Questions (I)) (HOTS)

1. Show that the curves $4x = y^2 and 4xy = k$ cut at right angles, if $k^2 = 512$.

Watch Video Solution

2. Show that the curves $2x = y^2 and 2xy = k$ cut at right angles, if $k^2 = 8$.

Watch Video Solution

3. Prove that the curves $y^2 = 4ax$ and $xy = c^2$ cut at right angles if $c^4 = 32a^4$.

Watch Video Solution

4. The normal to the curve $x = a(\cos\theta + \theta\sin\theta), y = a(\sin\theta - \theta\cos\theta)$ at any θ is such that

Watch Video Solution

5. Find the equation of the tangent to the curve $y = (x^3 - 1)(x - 2)$ at the points where the curve cuts the x-axis.

Watch Video Solution

6. Find a point on the graph of $y = x^3$, where the tangent is parallel to the chord joining (1, 1) and (3, 27).

Watch Video Solution

7. Find a point on the parabola $y = (x - 2)^2$, where the tangent is parallel to the line joining (2, 0) and (4, 4).

Watch Video Solution

8. The area bounded by the curve $y^2(2a - x) = x^3$ and the line x = 2a is

Watch Video Solution

9. Find the equation of the normal at a point on the curve $x^2 = 4y$, which passes through the point (1,2). Also find the equation of the corresponding tangent.

Watch Video Solution

10. Find the equation of the tangents to the curve $3x^2 - y^2 = 8$, which passes through the point $\left(\frac{4}{3}, 0\right)$.

Watch Video Solution

11. Deterine the values of 'x' for which the function $f(x) = x^2 + 2x - 3$ is an increasing. Also, find the co - ordinates of the point on the curve $y = x^2 + 2x - 3$, where the normal is parallel to the line $x - 4y + 7 = 0$.

Watch Video Solution

12. Determine the intervals in which the function $f(x) = (x - 1)(x + 1)^2$ is increasing or decreasing. Find also the points at which the tangents to the curve are parallel to x - axis.

Watch Video Solution

EXERCISE 6 (d) (Long Answer Type Questions (I))

1. In the following find the approximate values, using differentials :

$\sqrt{50}$

Watch Video Solution

2. In the following find the approximate values, using differentials :

$\sqrt{360}$.

Watch Video Solution

3. In the following find the approximate values, using differentials :

$\sqrt{1.037}$.

Watch Video Solution

4. In the following find the approximate values, using differentials :

$\sqrt{0.0037}$.

Watch Video Solution

5. Use differentials to approximate $\sqrt{25.\ 2}$

Watch Video Solution

6. In the following find the approximate values, using differentials :

$\sqrt{25.3}$

Watch Video Solution

7. In the following find the approximate values, using differentials :

$\sqrt{49.3}$

Watch Video Solution

8. Use differential to approximate $\sqrt{36.6}$

Watch Video Solution

9. In the following find the approximate values, using differentials :

$\sqrt{16.3}$

Watch Video Solution

10. In the following find the approximate values, using differentials :

$\sqrt{0.6}$.

Watch Video Solution

11. In the following find the approximate values, using differentials :

$\sqrt{0.17}$

Watch Video Solution

12. In the following find the approximate values, using differentials :

$\sqrt{0.26}$

Watch Video Solution

13. In the following find the approximate values, using differentials :

$\sqrt{0.82}$

Watch Video Solution

14. In the following find the approximate values, using differentials :

$\sqrt{0.26}$

Watch Video Solution

15. In the following find the approximate values, using differentials :

$\sqrt{0.50}$.

Watch Video Solution

16. In the following find the approximate values, using differentials :

$(17)^{1/4}$

Watch Video Solution

17. In the following find the approximate values, using differentials :

$(28)^{1/3}$

Watch Video Solution

18. In the following find the approximate values, using differentials :

$(255)^{1/4}$

Watch Video Solution

19. In the following find the approximate values, using differentials :

$(401)^{1/2}$

Watch Video Solution

20. In the following find the approximate values, using differentials :

$(26.57)^{1/3}$

Watch Video Solution

21. In the following find the approximate values, using differentials :

$(0.731)^{1/3}$

Watch Video Solution

22. The approximate value of $\sqrt[3]{0.009}$ is

Watch Video Solution

23. In the following find the approximate values, using differentials :

$\sqrt[3]{0.007}$

Watch Video Solution

24. In the following find the approximate values, using differentials :

$(15)^{1/4}$

Watch Video Solution

25. In the following find the approximate values, using differentials :

$(82)^{1/4}$

Watch Video Solution

26. In the following find the approximate values, using differentials :

$(255)^{1/4}$

Watch Video Solution

27. In the following find the approximate values, using differentials :

$(81.5)^{1/4}$

Watch Video Solution

28. In the following find the approximate values, using differentials :

$$\left(\frac{17}{81}\right)^{1/4}.$$

Watch Video Solution

29. In the following find the approximate values, using differentials :

$(32.15)^{1/5}$

Watch Video Solution

30. In the following find the approximate values, using differentials :

$(0.999)^{1/10}$

Watch Video Solution

31. Find approximation value of $(3.968)^{\frac{3}{2}}$ using differentials.

Watch Video Solution

32. In the following find the approximate values, using differentials :

$(33)^{-1/5}$

Watch Video Solution

33. Find the approximate value of :

$f(3.02), \quad \text{where} \quad f(x) = 3x^2 + 15x + 3$

Watch Video Solution

34. Find the approximate value of $f(5.001)$, where $f(x) = x^3 - 7x^2 + 15$.

Watch Video Solution

35. Find the approximate change in the volume V of a cube of side x meters caused by increasing the side by 2%.

Watch Video Solution

36. Find the approximate change in the surface area of a cube of side x metres caused by decreasing the side by 1%.

Watch Video Solution

37. If the radius of a sphere is measured as 9 cm with an error of 0.03 cm, then find the approximating error in calculating its volume.

Watch Video Solution

38. $\cos 61^\circ$, it being given that $\sin 60^\circ = 0.86603$ and $1^\circ = 0.01745$ radian.

Watch Video Solution

39. Find the approximate change in the value of $\frac{1}{x^2}$. when x changes from $x = 2$ to $x = 2.002$.

Watch Video Solution

40. Using differentiation, find the approximate value of $f(3.01)$, where $f(x) = 4x^2 + 5x + 2$.

Watch Video Solution

41. Use differentials, find the approximate value of the following :

$$\sin \frac{22}{14}$$

Watch Video Solution

42. Use differentials, find the approximate value of the following :

$$\cos \frac{11\pi}{36}.$$

Watch Video Solution

43. If $y = \sin x and x$ change from $\frac{\pi}{2} \rightarrow \frac{22}{14}$, what is the approximate change in y?

Watch Video Solution

44. A circular metal plate expands under heating so that its radius increases by 2%. Find the approximate increase in the area of the plate if the radius of the plate before heating is 10 cm.

Watch Video Solution

45. Find the percentage error in calculating the surface area of a cubical box if an error of 1% is made in measuring the lengths of edges of the cube.

Watch Video Solution

46. The radius of a spherical diamond is measured as 6 cm with an error of 0.04 cm. Obtain the approximate error in calculating its volume. If the cost of $1 cm^3$ diamond is Rs 1600, what is the loss to the buyer of the diamond?

Watch Video Solution

EXERCISE 1 (e) (Short Answer Type Questions)

1. Find the maximum or minimum values, if any, of the following funcitons without using the derivatives :

$f(x) = -(x-1)^2 + 10$

Watch Video Solution

2. Find the maximum or minimum values, if any, of the following functions without using the derivatives :

$f(x) = (2x - 1)^2 + 3$

Watch Video Solution

3. Find the maximum or minimum values, if any, of the following funcitons without using the derivatives :

$f(x) = x + 1, . x \in [-1, 1]$

Watch Video Solution

4. Find the maximum or minimum values, if any, of the following funcitons without using the derivatives :

$g(x) = x^3 + 1.$

Watch Video Solution

5. Find the maximum or minimum values, if any, of the following funcitons without using the derivatives :

$f(x) = |x + 2| - 1$

Watch Video Solution

6. Find the maximum or minimum values, if any, of the following funcitons without using the derivatives :

$g(x) = -|x-1| + 3.$

Watch Video Solution

7. Find the maximum or minimum values, if any, of the following funcitons without using the

derivatives :

$f(x) = \sin 2x + 5$

Watch Video Solution

8. Find the maximum or minimum values, if any, of the following funcitons without using the derivatives :

$f(x) = |\sin 4x + 3|.$

Watch Video Solution

9. Find the points of absolute maximum and minimum of each of the following :

$y = x(1 + 10x - x^2), 3 \le x \le 9$

Watch Video Solution

10. Find the points of absolute maximum and minimum of each of the following :

$y = \frac{1}{3}x^{3/2} - 4x, 0 \le x \le 64$

Watch Video Solution

11. Find the points of absolute maximum and minimum of each of the following :

$$y = \sqrt{5}\left(\sin x + \frac{1}{2}\cos 2x\right), 0 \leq x \leq \frac{\pi}{2}.$$

Watch Video Solution

12. Find the maximum and the minimum values, if any, of the function given by $f(x) = x, x \in (0, 1)$

Watch Video Solution

13. Find the absolute minimum value of $y = x^2 - 3x$ in $0 \leq x \leq 2.$

Watch Video Solution

14. Find the minimum value of $\sin^2 x$

A. -1

B. 0

C. 1

D. 1/2

Watch Video Solution

15. Find the maximum and minimum values of the function :

$f(x) = 2x^3 - 15x^2 + 36x + 11.$

Watch Video Solution

16. Find local minimum value of the function f given by $f(x) = 3 + |x|, x \in R.$

Watch Video Solution

EXERCISE 1 (e) (Long Answer Type Questions (I))

1. Find the absolute maximum and minimum values of each of the following in the given intervals :

$f(x) = x^{50} - x^{20}, [0, 1]$

Watch Video Solution

2. Find the absolute maximum and minimum values of each of the following in the given intervals :

$$f(x) = x^2 + \frac{16}{x}, x \in [1, 3]$$

Watch Video Solution

3. Find absolute maximum and minimum values of a function f given by $f(x) = 12x^{\frac{4}{3}} - 6x^{\frac{1}{3}}, x \in [-1, 1]$.

Watch Video Solution

4. Find the absolute maximum and minimum values of each of the following in the given intervals :

$f(x) = x^3 - 3x, \ -3 \leq x \leq 3$

Watch Video Solution

5. Find the absolute maximum and minimum values of each of the following in the given intervals :

$f(x) = x^3 - \frac{5}{2}x^2 - 2x + 1, 0 \leq x \leq 3.$

Watch Video Solution

6. Find the absolute maximum and minimum values of each of the following in the given intervals :

$f(x) = x^3$ in $[-2, 2]$.

Watch Video Solution

7. Find the absolute maximum and minimum values of each of the following in the given intervals :

$f(x) = (x-1)^2 + 3$ in $[-3, 1]$

Watch Video Solution

8. Find the absolute maximum and minimum values of each of the following in the given intervals :

$f(x) = 2x^3 - 15x^2 + 36x + 1 \text{ in } [1, 5]$

Watch Video Solution

9. Find the absolute maximum and minimum values of each of the following in the given

intervals :

$f(x) = \sin x + \cos x \quad \text{in} \quad [0, \pi]$

Watch Video Solution

10. Find the absolute maximum and minimum values of each of the following in the given intervals :

$f(x) = \cos^2 x + \sin x \quad \text{in} \quad [0, \pi].$

Watch Video Solution

11. Find the maximum and minimum values of each of the following in the given intervals :

$y = \sec x + \log(\cos^2 x), \quad \text{in} \quad (0, 2\pi).$

Watch Video Solution

12. Find the absolute maximum and minimum values of each of the following in the given intervals :

$y = 2\cos 2x - \cos 4x, 0 \leq x \leq \pi.$

Watch Video Solution

13. Find the maximum value and the minimum value and the minimum value of $3x^4 - 8x^3 + 12x^2 - 48x + 25$ on the interval $[0, 3]$.

Watch Video Solution

14. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximum and local minimum values :

The constant function α.

Watch Video Solution

15. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximum and local minimum values :

$f(x) = x^2$

Watch Video Solution

16. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximum and local minimum values :

$f(x) = x^3 - 3x.$

Watch Video Solution

17. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximum and local minimum values :

$f(x) = \cos x, 0 < x < \pi$

Watch Video Solution

18. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximum and local minimum values :

$$f(x) = \sin x + \cos x, 0 < x < \frac{\pi}{2}$$

Watch Video Solution

19. Find the local maxima and local minima, if any, of the followig functions. Find also the local maximum and the local minimum values, as the case may be :

$$f(x) = \sin x - \cos x, 0 < x < 2\pi$$

Watch Video Solution

20. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximum and local minimum values :

$$g(x) = \frac{x}{5} + \frac{5}{x}, x > 0,$$

Watch Video Solution

21. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximum and local minimum values :

$$g(x) = \frac{1}{x^2 + 2}$$

Watch Video Solution

22. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximum and local minimum values :

$$f(x) = x\sqrt{1 - x}, x > 0.$$

Watch Video Solution

23. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximum and local minimum values :

$f(x) = x^3 - 12x^2 + 36x - 4$

Watch Video Solution

24. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximum and local minimum values :

$g(x) = x^3 - 3x$

Watch Video Solution

25. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximum and local minimum values :

$f(x) = x^3 - 3x + 3.$

Watch Video Solution

26. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximum and local minimum values :

$$f(x) = -\frac{3}{4}x^4 - 8x^3 - \frac{45}{2}x^2 + 105$$

Watch Video Solution

27. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximum and local minimum values :

$f(x) = x^3 - 6x^2 + 9x + 15. \, 0 \leq x \leq 6$

Watch Video Solution

28. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximum and local minimum values :

$f(x) = \; - x + 2\sin x, 0 \leq x \leq 2\pi$

Watch Video Solution

29. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximum and local minimum values :

$$f(x) = \sin^4 x + \cos^4 x, 0 < x < \frac{\pi}{2}.$$

Watch Video Solution

30. Prove that $\left(\frac{1}{x}\right)^x$ has maximum value is $(e)^{\frac{1}{e}}$

.

Watch Video Solution

31. The curve $y = ax^2 + bx$ has a turning point at (1, -2). Find the values of 'a and 'b' and also show that y is minimum at this point .

Watch Video Solution

32. If $y = \frac{ax - b}{(x-1)(x-4)}$ has a turning point $P(2, -1)$, find the value of $a and b$ and show that y is maximum at P.

Watch Video Solution

EXERCISE 1 (f) (Long Answer Type Questions (I))

1. Find two positive numbers whose sum is 16 and product is maximum.

Watch Video Solution

2. Amongest all pairs of positive numbers with product (i) 256 (ii) 64, find those whose sum is least.

Watch Video Solution

3. Find two numbers whose sum is 15 and the square of one multiplied by the cube of the other is maximum.

Watch Video Solution

4. Find two positive numbers whose product is 64 and the sum is minimum.

Watch Video Solution

5. Find two positive numbers x and y such that their sum is 35 and the product x^2y^5 is a maximum.

Watch Video Solution

6. Find two positive numbers whose sum is 16 and the sum of whose cubes is minimum.

Watch Video Solution

7. Find two positive numbers x and y such that $x + y = 60$and $xy^3$is maximum.

Watch Video Solution

8. How should we choose two numbers, each greater than or equal to -2, whose sum is $1/2$ so that the sum of the first and the cube of the second is minimum?

Watch Video Solution

9. Find the maximum slope of the curve $y = -x^3 + 3x^2 + 2x - 27$.

Watch Video Solution

10. Two sides of a triangle are given. The angle between them such that the area is maximum, is given by

Watch Video Solution

11. A wire of length 36m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the lengths of the two pieces, so that the combined area of the square and the circle is minimum?

Watch Video Solution

12. A wire of length 36cm is cut into the two pieces, one of the pieces is turned in the form of a square and other in form of an equilateral triangle. Find the length of each piece so that the sum of the areas of the two be minimum

Watch Video Solution

13. Prove that the perimeter of a right - angled triangle of given hypotenuse equal to 5 cm is maximum when the triangle is isosceles.

Watch Video Solution

14. Prove that the area of right-angled triangle of given hypotenuse is maximum when the triangle is isosceles.

Watch Video Solution

15. Prove that the least perimeter of an isosceles triangle in which a circle of radius r can be inscribed is $6\sqrt{3}r$.

Watch Video Solution

16. Show that of all the rectangles of given area, the square has the smallest perimeter.

Watch Video Solution

17. Show that the rectangle of maximum perimeter which can be inscribed in a circle of radius a is a square of side $\sqrt{2}a$.

Watch Video Solution

18. Show that the rectangle of maximum area that can be inscribed in a circle is a square.

Watch Video Solution

19. Show that of all the rectangles inscribed in a given fixed circle, the square has the maximum area.

Watch Video Solution

20. A rectangle is inscribed in a semi-circle of radius r with one of its sides on diameter of semi-circle. Find the dimensions of the rectangle so that its area is maximum. Find also the area.

Watch Video Solution

21. Of all rectangles, each of which has perimeter :

(i) 40 cm

(ii) 60 cm

Find the one having maximum area. Also, find that area.

Watch Video Solution

22. An open box with a square base is to be made out of a given quantity of card board of area c2 square units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units.

Watch Video Solution

23. Show that the semi - vertical angle of the right - circular cone of maximum volume and of given slant height is :

(i) $\tan^{-1}\sqrt{2}$

(ii) $\cos^{-1}\frac{1}{\sqrt{3}}$.

Watch Video Solution

24. Prove that the semi-vertical angle of the right circular cone of given volume and least curved surface is $\cot^{-1}(\sqrt{2})$.

Watch Video Solution

EXERCISE 1 (f) (Long Answer Type Questions (II))

1. Show that the volume of the greatest cylinder, which can be inscribed in a cone of height 'h' and semi - vertical angle 30° is $\frac{4}{81}\pi h^3$

Watch Video Solution

2. Show that the altitude of the right circulau cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$. also show that the maximum volume of cone is $\frac{8}{27}$ of the volume of the sphere.

Watch Video Solution

3. Find the volume of the larges cylinder that can be inscribed in a sphere of radius r

Watch Video Solution

4. Find the volume of the larges cylinder that can be inscribed in a sphere of radius r

Watch Video Solution

5. Show that the right-circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.

Watch Video Solution

6. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.

Watch Video Solution

7. Find the height of right circular cylinder of maximum volume that can be inscribed in a sphere of radius $10\sqrt{3}cm$.

Watch Video Solution

8. Show that the radius of right - circular cylinder of maximum volume, that can be inscribed in a sphere of radius 18 cm, is $6\sqrt{6}cm$.

Watch Video Solution

9. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

Watch Video Solution

10. Of all the closed cylinderical cans (right - circular), which enclose a given volume of :

(i) 100 cubic centimeters

(ii) 128π cubic centimeters,

find the dimensions of the can, which has the minimum surface area.

Watch Video Solution

11. Show that the surface area of a closed cuboid with square base and given volume is minimum, when it is a cube.

Watch Video Solution

12. A figure consists of a semi-circle with a rectangle on its diameter. Given the perimeter of the figure, find its dimensions in order that the area may be maximum.

Watch Video Solution

13. A window is the in the form of a reactangle, surmounted by a semi - circle. If the perimeter be 15 metres, find the dimesnions so that greatest possible amount of light may be admitted in order that its area may be maximum.

Watch Video Solution

14. Show that a cylinder of a given volume which is open at the top has minimum total surface area, when its height is equal to the radius of its base.

Watch Video Solution

15. The height of a closed cylinder of given volume and the minimum surface area is (a) equal to its diameter (b) half of its diameter (c) double of its diameter (d) None of these

Watch Video Solution

16. Rectangles are inscribed inside a semicircle of radius r. Find the rectangle with maximum area.

Watch Video Solution

17. A square-based tank of capacity 250 cu m has to bedug out. The cost of land is Rs 50 per sq m. The cost of digging increases with the depth and for the whole tank the cost is Rs $400 \times (depth)^2$. Find the dimensions of the tank for the least total cost.

Watch Video Solution

18. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is 8 m3. If building of tank costs Rs 70 per square metre for the base and Rs 45 per square metre for sides, what is the cost of least expensive tank?

Watch Video Solution

19. A rectangular sheet of tin 45 cm by 24 cm is to be made into a box without top, by cutting off square from each corner and folding up the flaps. What should be the side of the square to be cut off so that the volume of the box is maximum ?

Watch Video Solution

20. An open box is to be made of square sheet of tin with side 20 cm, by cutting off small squares from each corner and foding the flaps. Find the side of small square, which is to be cut off, so that volume of box is maximum.

Watch Video Solution

21. A canon is fired at an angle $\theta\left(0 \leq \theta \leq \frac{\pi}{2}\right)$ with the horizontal. If 'v' is the intial velocity of the canon ball, the height 'h' of the ball at time 't', ignoring wind resistance, is given by $h = (v \sin \theta)t - 4.9t^2$.

(a) Will be ball return to the ground?

(b) How far will the ball have travelled horizontally at the time it hits the ground, assuming there are no forces in the horizontal direction?

(c) Determine $'\theta'$ so that the horizontal range of the ball is maximum.

Watch Video Solution

22. Find the maximum profit that a company can make, if the profit function isgiven by $p(x) = 41 - 24x - 18x^2$

Watch Video Solution

23. Find the maximum profit that a company can make, if the profit function is given by :

$P(x) = 41 - 72x - 18x^2$

Watch Video Solution

24. Find the maximum profit that a company can make, if the profit function is given by :

$P(x) = 41 - 24x - 6x^2$.

Watch Video Solution

25. Find the point on the curve $y^2 = 4x$ which is nearest to the point $(2; \ -8)$

Watch Video Solution

26. Find the point on the curve $y^2 = 2x$ which is at a minimum distance from the point $(1, 4)$

Watch Video Solution

27. Find the point on the curve $y^2 = 2x$, which is nearest to the point $(1, -4)$.

Watch Video Solution

28. Find the point on the parabola $x^2 = 8y$, which is nearest to the point (2, 4).

Watch Video Solution

29. A helicopter is flying along the curve $y = x^2 + 2$. A soldier is placed at the point (3, 2). Find the nearest distance between the solider and the helicopter.

Watch Video Solution

30. A manufacturer can sell 'x' items at a price of Rs $(250 - x)$ each. The cost of producing 'x' items is Rs $(x^2 - 50x + 12)$. Determine the

number of items to be sold so that he can make maximum profit.

Watch Video Solution

31. A factory can shell 'x' items per week at price of Rs $\left(20 - \frac{x}{1000}\right)$ each. If the cost price of one item is Rs $\left(5 + \frac{2000}{x}\right)$, find the number of items, the factory should produce every week for maximum profit. If price is reduced, how it will effect the sale? Give reasons.

(ii) Profit function of a company is given as :

$$P(x) = \frac{24x}{5} - \frac{x^2}{100} - 500.$$

where 'x' is the number of units produced.

What is the maximum profit of the company?

View Text Solution

32. Let 'p' be the price per unit of a certain product, when there is a sale of 'x' units. The total revenue function is :

$$P(x) = \frac{100x}{3x+1} - 4x.$$

(i) Find the marginal revenue function, rate of change of total revenue function with respect to x.

(ii) When x = 10, find the relative change of revenue R, i.e., Rate of change of R with respect to x and also the percentage rate of change of R at $x = 10$.

Watch Video Solution

33. If performance of the students 'y' depends on the number of hours 'x' given by the relation :

$$y = 4x - \frac{x^2}{2}.$$

find the number of hours, the students work to have the best performance.

Watch Video Solution

Objective Type Questions (A. Multiple Choice Questions)

1. The rate of change of the area of a circle with respect to its radius r at r = 6 cm is:

A. 10π

B. 12π

C. 8π

D. 11π.

Answer: b

Watch Video Solution

2. The total revenue in Rupees received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$. The marginal revenue, when $x = 15$is (A) 116 (B) 96 (C) 90 (D) 126

A. 116

B. 96

C. 90

D. 126

Answer: d

Watch Video Solution

3. The interval in which $y = x^2 e^{-x}$ is increasing is (A) $(-\infty, \infty)$ (B) (2, 0) (C) $(2, \infty)$ (D) (0, 2)

A. $(-\infty, \infty)$

B. $(-2, 0)$

C. $(2, \infty)$

D. $(0, 2)$

Answer: d

Watch Video Solution

4. Find the slopes of the tangent and the normal to the curve $y = 2x^2 + 3\sin x$ at $x = 0$

A. 2

B. -3

C. $\frac{1}{2}$

D. $-\frac{1}{3}$.

Answer: d

Watch Video Solution

5. The line $y = x + 1$ is a tangent to the curve $y^2 = 4x$ at the point:

A. $(1, 2)$

B. $(2, 1)$

C. $(1, -2)$

D. $(-1, 2)$

Answer: a

Watch Video Solution

6. If $f(x) = 3x^2 + 15x + 5,$ then the approximate value of $f(3.02)$ is :

A. 47.66

B. 57.66

C. 67.66

D. 77.66.

Answer: d

Watch Video Solution

7. The approximate change in the volume of a cube of side x metres caused by increasing the side by 3% is(A) 0.06 x^3m^3 (B) 0.6 x^3m^3 (C) 0.09 x^3m^3 (D) 0.9 x^3m^3

A. $0.06x^3m^3$

B. $0.6x^3m^3$

C. $0.09x^3m^3$

D. $0.9x^3m^3$.

Answer: c

Watch Video Solution

8. The point on the curve $x^2 = 2y$which is nearest to the point (0, 5) is(A) $\left(2\sqrt{2}, 4\right)$ (B) $\left(2\sqrt{2}, 0\right)$ (C) (0, 0) (D) (2, 2)

A. $\left(2\sqrt{2}, 4\right)$

B. $\left(2\sqrt{2}, 0\right)$

C. $(0, 0)$

D. $(2, 2)$

Answer: a

Watch Video Solution

9. For all real values of x, the minimum value of $\frac{1-x+x^2}{1+x+x^2}$ is(A) 0 (B) 1 (C) 3 (D) $\frac{1}{3}$

A. 0

B. 1

C. 3

D. $\frac{1}{3}$

Answer: d

Watch Video Solution

10. The maximum value of $[x(x-1)+1]^{1/3}.0 \le x \le 1$ is

A. $\left(\frac{1}{3}\right)^{\frac{1}{3}}$

B. $\frac{1}{2}$

C. 1

D. 0

Answer: c

Watch Video Solution

11. A cylindrical tank of radius 10 m is being filled with wheat at the rate of 314 cubic metre per hour. Then the depth of the wheat is increasing at the rate of(A) 1 m^3/h (B) 0.1 m^3/h (C) 1.1 m^3/h (D) 0.5 m^3/h

A. $1\ \mathrm{m}^3/\mathrm{minute}$

B. $0.1m^3/\mathrm{minute.}$

C. $1.1m^3/\mathrm{minute}$

D. $0.5m^3/\mathrm{minute.}$

Answer: a

Watch Video Solution

12. The slope of the tangent to the curve $x = t^2 + 3t - 8, y = 2t^2 - 2t - 5$ at the point (2, -1), is

A. $\frac{22}{7}$

B. $\frac{6}{7}$

C. $\frac{7}{6}$

D. $-\frac{6}{7}$.

Answer: b

Watch Video Solution

13. The line $y = mx + 1$is a tangent to the curve $y^2 = 4x$if the value of m is(A) 1 (B) 2 (C) 3 (D) $\frac{1}{2}$

A. 1

B. 2

C. 3

D. $\frac{1}{2}$.

Answer: a

Watch Video Solution

14. The normal at the point (1,1) on the curve $2y + x^2 = 3$is(A) $x + y = 0$ (B) $xy = 0$ (C) $x + y + 1 = 0$(D) $xy = 0$

A. $x + y = 0$

B. $x - y = 0$

C. $x + y + 1 = 0$

D. $x - y + 1 = 0$

Answer: b

Watch Video Solution

15. Find the equation of the normal to curve $x^2 = 4y$which passes through the point (1, 2).

A. $x + y = 3$

B. $x - y = 3$

C. $x + y = 1$

D. $x - y = 1.$

Answer: a

Watch Video Solution

16. The points on the curve $9y^2 = x^3$, where the normal to the curve makes equal intercepts with the axes are(A) $\left(4, \pm \frac{8}{3}\right)$ (B) $\left(4, \frac{-8}{3}\right)$ (C) $\left(4, \pm \frac{3}{8}\right)$ (D) $\left(\pm 4, \frac{3}{8}\right)$

A. $\left(4, \pm \frac{8}{3}\right)$

B. $\left(4, -\frac{8}{3}\right)$

C. $\left(4, \pm \frac{3}{8}\right)$

D. $\left(\pm 4, \frac{3}{8}\right)$.

Answer: a

Watch Video Solution

17. The point on the curve $3y = 6x - 5x^3$ the normal at Which passes through the origin, is

A. 1

B. $\frac{1}{3}$

C. 2

D. $\frac{1}{2}$

Answer: a

Watch Video Solution

18. The two curves $x^3 - 3xy^2 + 2 = 0$ and $3x^2y - y^3 - 2 = 0$

A. touch each other

B. cut at right angle

C. cut at an angle $\frac{\pi}{3}$

D. cut at an angle $\frac{\pi}{4}$.

Answer: b

Watch Video Solution

19. If the parametric of a curve given by $x = e^t \cos t, y = et \sin t,$ then the tangent to the curve at the point $t = \pi / 4$ makes with axis of x the angle

A. 0

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer: d

Watch Video Solution

20. The equation to the normal to the curve $y = \sin x$ at $(0,\ 0)$ is $x = 0$ (b) $y = 0$ (c) $x + y = 0$ (d) $x - y = 0$

A. $x = 0$

B. $y = 0$

C. $x + y = 0$

D. $x - y = 0$

Answer: c

Watch Video Solution

21. Write the coordinates of the point on the curve $y^2 = x$ where the tangent line makes an angle $\frac{\pi}{4}$ with x-axis.

A. $\left(\frac{1}{2}, \frac{1}{4}\right)$

B. $\left(\frac{1}{4}, \frac{1}{2}\right)$

C. $(4, 2)$

D. $(1, 1)$

Answer: b

Watch Video Solution

22. The slope of the normal to the curve $y = 2x^2 + 3\sin x$ at $x = 0$is(A) 3 (B) $\frac{1}{3}$ (C)-3 (D) $-\frac{1}{3}$

A. 3

B. $\frac{1}{3}$

C. -3

D. $-\frac{1}{3}$

Answer: d

Watch Video Solution

23. The line $y = x + 1$ is a tangent to the curve $y^2 = 4x$ at the point :

A. $(1, 2)$

B. $(2, 1)$

C. $(1, -2)$

D. $(-1, 2)$

Answer: d

Watch Video Solution

24. The rate of change of the area of a circle with respect to its radius r at r = 6 cm is:

A. $16\pi cm^2 / cm$

B. $12\pi cm^2 / cm$

C. $8\pi cm^2 / cm$

D. $11\pi cm^2 / cm.$

Answer: b

Watch Video Solution

25. The rate of change of the area of a circle with respect to its radius r when r = 3 cm is :

A. $6\pi cm^2 / cm$

B. $4\pi cm^2 / cm$

C. $5\pi cm^2 / cm$

D. None of these.

Answer: a

Watch Video Solution

26. The point on the curve $y = 2x^2$, where the slope of the tangent is 8, is :

A. $(0, 2)$

B. $(0, 8)$

C. $(2, 8)$

D. $(8, 2)$

Answer: c

Watch Video Solution

27. Find the equation of the normal to the curve

$y = 2x^2 + 3 \sin x at x = 0.$

A. 3

B. $\frac{1}{3}$

C. -3

D. $-\frac{1}{3}$

Answer: d

Watch Video Solution

28. The rate of change of the area of a circle with respect to its radius at r = 2 cm is :

A. 8π

B. 2π

C. 4π

D. 11π

Answer: c

Watch Video Solution

29. Find the slope of the tangent to the curve $y = x^3 - 3x + 2$ at the point whose x-coordinate is 3.

A. 20

B. 24

C. 30

D. 25

Answer: b

Watch Video Solution

30. The line $y = x + 1$ is tangent to the curve $y^2 = 4x$ at the point :

A. $(1, 2)$

B. $(2, 1)$

C. $(1, -2)$

D. $(-1, 2)$

Answer: a

Watch Video Solution

31. The interval, in which $y = 2x^2 e^{-2x}$ is increasing is :

A. $(-\infty, \infty)$

B. $(-1, 0)$

C. $(1, \infty)$

D. $(0, 1)$.

Answer: d

Watch Video Solution

32. The equation of tangent to the curve $x = a\cos^3\theta, y = a\sin^3\theta$ at $\theta = \frac{\pi}{4}$ is

A. 1

B. 2

C. -1

D. None of these.

Answer: c

Watch Video Solution

33. Find the slope of the normal to the curve $x = a\cos^3\theta, y = \sin^3\theta \text{at } \theta = \frac{\pi}{4}$.

A. 1

B. -1

C. 3

D. $-2.$

Answer: a

Watch Video Solution

34. The maximum and minimum values of function $f(x) = \sin 3x + 4$ are respectively:

A. 5 and 3

B. 6 and 4

C. 4 and 3

D. None of these.

Answer: A

Watch Video Solution

35. The function $f(x) = \cos x - \sin x$ has maximum or minimum value at $x = \ldots\ldots$

A. $\frac{\pi}{4}$

B. $\frac{3\pi}{4}$

C. $\frac{\pi}{2}$

D. $\frac{\pi}{3}$.

Answer: b

Watch Video Solution

36. Find an angle , which increases twice as fast as it sine.

A. $\frac{\pi}{3}$

B. $\frac{\pi}{2}$

C. π

D. $\frac{3\pi}{2}$.

Answer: a

Watch Video Solution

37. $f(x)$ is a strictly increasing function, if $f'(x)$ is :

A. positive

B. negative

C. 0

D. None of these.

Answer: a

Watch Video Solution

38. The approximate change in the voluem V of a cube of side x metres caused by increasing the side by 2% is :

A. $0.06x^3m^3$

B. $0.02x^3m^3$

C. $0.6x^3m^3$

D. $0.006x^3m^3$.

Answer: a

Watch Video Solution

39. The rate of change of the area of a circle with respect to its radius at r = 5, is :

A. 10π

B. 8π

C. 12π

D. 13π.

Answer: a

Watch Video Solution

40. The radius of a circle is increasing at the rate of 0.7 cm/sec. What is the rate of increase of its circumference?

A. $3.3\pi cm / s$

B. $1.4\pi cm / s$

C. $2.2\pi cm / s$

D. $4.4\pi cm / s$.

Answer: a

Watch Video Solution

41. Find the point on the curve $y = x^3 - 11x + 5$ at which the tangent is $y = x \quad 11$.

A. $(-2, 0)$

B. $(3, 7)$

C. $(0, 2)$

D. $(2, -9)$

Answer: a

Watch Video Solution

42. If the rate of change of area of a circle is equal to the rate of change of its diameter, then its radius is equal to (a) unit (b) unit (c) units (d) units

A. $\frac{1}{\pi}$

B. $\frac{2}{\pi}$

C. $\frac{\pi}{2}$

D. π.

Answer: a

Watch Video Solution

43. The interval on which the function $f(x) = 2x^2 - 3x$ is increasing or decreasing in :

A. $\left[-\infty, \frac{3}{4}\right]$

B. $[3, \infty]$

C. $\left[\frac{3}{4}, 3\right]$

D. $\left[\frac{3}{4}, \infty\right]$

Answer: d

Watch Video Solution

44. The rate of change of volume of a sphere with respect to its radius when radius is 1 unit is :

A. 4π

B. 2π

C. π

D. $\frac{\pi}{2}$.

Answer: c

Watch Video Solution

45. Slope of the normal to the curve :

$y^2 = 4x$ at $(1, 2)$ is :

A. 1

B. $\frac{1}{2}$

C. 2

D. -1

Answer: b

Watch Video Solution

Objective Type Questions (B. Fill in the Blanks)

1. The radius of a sphere starts to increase at a rate of 0.1 cm/s . The rate of chane of a surface area of the sphere with time when radius is 10.

Watch Video Solution

2. Rate of change of the volume of a ball with respect to its radius is ....

Watch Video Solution

3. Without using the derivative, show that the function $f(x) = |x|$ is strictly increasing in $(0, \infty)$ strictly decreasing in $(-\infty, 0)$.

Watch Video Solution

4. Logarithmic function is strictly ....in $(0, \pi)$.

Watch Video Solution

5. The value of 'a' for which $f(x) = \sin x - ax + 5$ is decreasing function on R if.

A. $a > 1$

B. $a < 13$

C. $a > \left(\frac{1}{2}\right)$

D. $a < \left(\frac{1}{2}\right)$

Watch Video Solution

6. Slope of the tangent to the curve $x = at^2, y = 2t$ at $t = 2$ is ....

Watch Video Solution

7. Slope of the normal to the curve $y = 2x^2 - 1$ at $(1, 1)$ is ....

Watch Video Solution

8. Find the equation of the tangent line to the curve $y = \cot^2 x - 2\cot x + 2$ at $x = \frac{\pi}{4}$

Watch Video Solution

9. If $x > 0, y > 0$ and $xy = 5$, then the minimum value of $x + y$ is ....

Watch Video Solution

10. Maximum value of $f(x) = -(x-1)^2 + 2$ is ....

Watch Video Solution

Objective Type Questions (C. True/False Questions)

1. For the curve $y = 5x - 2x^3$, if x increases at the rate of 2 units/sec, then how fast is the slope of the curve changing when $x = 3$?

Watch Video Solution

2. The function $f(x) = 4x^3 - 6x^2 - 72x + 30$ is strictly decreasing on interval.

Watch Video Solution

3. Slope of the tangent to the curve $y = 3x^4 - 4x$ at $x = 1$ is 6.

Watch Video Solution

4. Using differentials, the approximate valueof $\sqrt{25.5}$.

Watch Video Solution

5. The function $f(x) = x^2, x \in R$ has no maximum value.

Watch Video Solution

Objective Type Questions (D. Very Short Answer Types Questions)

1. Find the rate of change of the area of a circle with respect to its radius 'r' when r = 6 cm.

Watch Video Solution

2. The radius of spherical balloon is increasing at the rate of 5 cm per second. At what rate is the surface of the balloon increasing, when the radius is 10 cm?

Watch Video Solution

3. The radius of an air bubble in increasing at the rate of 0.5cm/s. Find the rate of change of its volume, when the radius is 1.5 cm.

Watch Video Solution

4. What are the values of 'a' for which the function $f(x) = a^x$ is :

(i) increasing

(ii) decreasing in R?

Watch Video Solution

5. What are the values of "a for which the function $f(x) = \log_a x$ is :

(i) increasing

(ii) decreasing in its domain?

Watch Video Solution

6. Find the value of 'k' such for $f(x) = k(x + \sin x) + k$ is increasing in R.

Watch Video Solution

7. Find the set of values of 'a' such that $f(x) = ax - \sin x$ is increasing on R.

Watch Video Solution

8. Examine whether the function given by $f(x) = x^3 - 3x^2 + 3x - 5$ is increasing in R.

Watch Video Solution

9. Write the interval in which the function $f(x) = \cos x$ is strictly decreasing.

Watch Video Solution

10. Find the point on the curve $y = x^2 - 2x + 5$, where the tangent is paralle to x - axis.

Watch Video Solution

11. Write the value of $\frac{dy}{dx}$, if the normal to the curve $y = f(x)$ at (x, y) is parallel to y - axis.

Watch Video Solution

12. Find the slope of the tangent to the curve $y = x^3 \quad x \quad at \quad x = 2$.

Watch Video Solution

13. Find the slope of tangent line to the curve :

$y = x^2 - 2x + 1.$

Watch Video Solution

14. Find the slope of the normal to the curve to

$y = x^3 - x + 1$ at $x = 2$.

Watch Video Solution

15. Find the slope of the normal to the curve

$x = 1 - a\sin\theta, y = b\cos^2\theta$ at $\theta = \frac{\pi}{2}$.

Watch Video Solution

16. Find the equation of the tangent line to the curve $y = \sin x \quad \text{at} \quad x = \frac{\pi}{4}$.

Watch Video Solution

17. Find the equation of the tangent line to the curve $y = x \tan^2 x \quad \text{at} \quad x = \frac{\pi}{4}$.

Watch Video Solution

18. Find the equation of the normal line to the curve

$f(x) = 5x^3 - 2x^2 - 3x - 1$ at $x = -8$

Watch Video Solution

19. Find the angle between $y = f(x)$ and $y = 2e^{2x}$ at their point of intersection. Where $f(x) = x^2 f(1) - x f'(2) + f''(3)$ and $f(0) = 2.$

Watch Video Solution

20. Using differentials, find the approximate values of the following :

(i) $\sqrt{37}$

(ii) $\sqrt{401}$.

Watch Video Solution

21. Find the minimum values of $f(x) = x^2 + \frac{1}{x^2}, x > 0$

Watch Video Solution

22. Local maximum of $f(x) = x + \frac{1}{x}$, where $x < 0$, is

Watch Video Solution

23. Write the maximum value of $f(x) = \frac{\log x}{x}$, if it exists.

Watch Video Solution

24. Find the maximum and minimum values, if any, of the following functions without using the derivatives :

(i) $f(x) = -(x-2)^2 + 3$

(ii) $f(x) = 9x^2 + 12x + 2.$

Watch Video Solution

25. Show that the value of x^x is minimum when $x = \frac{1}{e}$.

Watch Video Solution

26. Find two positive numbers whose sum is 14 and product is maximum.

Watch Video Solution

NCERT - FILE (Question from NCERT Book) (Exercise 6.1)

1. Find the rate of change of the area of circle with respect to its radius r when :

(a) r = 3 cm (b) r = 4 cm.

Watch Video Solution

2. The volume of a cube is increasing at the rate of $8cm^3/s$. How fast is the surface area increasing when the length of an edge is 12 cm?

Watch Video Solution

3. The radius of a circle is increasing uniformly at the rate of 3 cm/s. Find the rate at which the area of the circle is increasing when the radius is 10 cm.

Watch Video Solution

4. An edge of a variable cube is increasing at the rate of $3cm/s$. How fast is the volume of the cube increasing when the edge is $10cm$ long?

Watch Video Solution

5. A stone is dropped into a quiet lake and waves move in circles at the speed of $5cm/s$. At the instant when the radius of the circular wave is $8cm$, how fast is the enclosed area increasing?

Watch Video Solution

6. The radius of a circle is increasing at the rate of 0.7 cm/sec. What is the rate of increase of its circumference?

Watch Video Solution

7. The length x of a rectangle is decreasing at the rate of 5 cm/minute and the width y is increasing at the rate of 4 cm/minute. When x = 8cm and y = 6cm, find the rates of change of (a) the perimeter, and (b) the area of the rectangle

Watch Video Solution

8. A balloon, which always remains spherical on inflation, is being inflated by pumping in 900 cubic centimetres of gas per second. Find the rate at which the radius of the balloon increases when the radius is 15 cm.

Watch Video Solution

9. A balloon, which always remains spherical, has a variable radius. Find the rate at which its volume is increasing with the radius when the later is 10 cm

Watch Video Solution

10. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall ?

Watch Video Solution

11. A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate

Watch Video Solution

12. The radius of an air bubble is increasing at the rate of $\frac{1}{2} cm / s$. At what rate is the volume of the bubble increasing when the radius is 1 cm?

Watch Video Solution

13. A balloon, which always remains spherical, has a variable diameter $\frac{3}{2}(2x+1)$.Find the rate of change of its volume with respect to x.

Watch Video Solution

14. Sand is pouring from a pipe at the rate of 12 cm^3/s . The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the

base. How fast is the height of the sand cone increasing when t

Watch Video Solution

15. The total cost C (x) in Rupees associated with the production of x units of an item is given by $C(x) = 0.007x^3 - 0.003x^2 + 15x + 4000.$

Find the marginal cost when 17 units are produced

Watch Video Solution

16. The total revenue in Rupees received from the sale of x units of a product is given by $R(x) = 13x^2 + 26x + 15$. Find the marginal revenue when $x = 7$.

Watch Video Solution

17. The rate of change of the area of a circle with respect to its radius r at r = 6 cm is:

A. 10π

B. 12π

C. 8π

D. 11π.

Answer: B

Watch Video Solution

18. The total revenue in Rupees received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$. The marginal revenue, when $x = 15$is (A) 116 (B) 96 (C) 90 (D) 126

A. 116

B. 96

C. 90

D. 126

Answer: D

Watch Video Solution

NCERT - FILE (Question from NCERT Book) (Exercise 6.2)

1. Show that the function given by $f(x) = 3x + 17$is strictly increasing on R.

Watch Video Solution

Watch Video Solution

2. Show that the function given by $f(x) = e^{2x}$ is strictly increasing on R.

Watch Video Solution

3. Show that the fucntion given by $f(x) = \sin x$ is :

(a) strictly increasing in $\left(0, \frac{\pi}{2}\right)$

(b) strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$

(c) neither increasing nor decreasing in $(0, \pi)$.

Watch Video Solution

Watch Video Solution

4. Find the intervals in which the function f given by $f(x) = 2x^2 - 3x$is(a) strictly increasing (b) strictly decreasing

Watch Video Solution

5. Find the intervals in which the fucntion 'f' given by $f(x) = x^3 - 3x^2 + 5x + 7$ is strictly increasing

Watch Video Solution

6. Find the invervals in which the following functions are strictly increasing or decreasing :

$x^2 + 2x - 5$

Watch Video Solution

7. Find the invervals in which the following functions are strictly increasing or decreasing :

$10 - 6x - 2x^2$

Watch Video Solution

8. Find the invervals in which the following functions are strictly increasing or decreasing :

$-2x^3 - 9x^2 - 12x + 1$

Watch Video Solution

9. Find the invervals in which the following functions are strictly increasing or decreasing :

$6 - 9x - x^2$

Watch Video Solution

10. Find the invervals in which the following functions are strictly increasing or decreasing :

$(x+1)^3(x-3)^3$.

Watch Video Solution

11. Show that $y = \log(1+x) - \frac{2x}{2+x}, x \succ 1$, is an increasing function of x throughout its domain.

Watch Video Solution

12. The function $y = 5 + 36x + 3x^2 - 2x^3$ is increasing in the interval.

Watch Video Solution

13. Prove that $y = \frac{4\sin\theta}{(2+\sin\theta)} - \theta$ is an increasing function of θ in $\left[0, \frac{\pi}{2}\right]$

Watch Video Solution

14. Prove that the exponential function is strictly increasing on R.

Watch Video Solution

15. Prove that the function f given by $f(x) = x^2 - x + 1$ is neither strictly increasing nor strictly decreasing on $(-1, 1)$.

Watch Video Solution

16. Which of the following functions are strictly increasing on $\left(0, \frac{\pi}{2}\right)$?

A. $\sin x$

B. $\sin 2x$

C. $\sin 3x$

D. $\tan x$.

Watch Video Solution

17. On which of the following intervals is the function 'f' given by $f(x) = x^{100} + \cos x - 1$ strictly decreasing?

A. $(0, 1)$

B. $\left(\frac{\pi}{2}, \pi\right)$

C. $\left(0, \frac{\pi}{2}\right)$

D. None of these.

Watch Video Solution

18. Find the least value of a such that the function f given by $f(x) = x^2 + ax + 1$is strictly increasing on $(1, 2)$.

Watch Video Solution

19. Let I be an interval disjointed from $[-1, 1]$. Prove that the function $f(x) = x + \frac{1}{x}$ is increasing on I.

Watch Video Solution

20. Prove that the function 'f' given by $f(x) = \log \sin x$ is strictly increasing on $\left(0, \frac{\pi}{2}\right)$

.

Watch Video Solution

21. Prove that the function f given by $f(x) = \log \cos x$ is strictly decreasing on $\left(0, \frac{\pi}{2}\right)$.

Watch Video Solution

22. Prove that the function given by $f(x) = 2x^3 - 6x^2 + 7x$ is strictly increasing in R.

Watch Video Solution

23. The interval in which $y = x^2 e^{-x}$ is decreasing is :

A. $(-\infty, \infty)$

B. $(-\infty, 0)$U$(0, \infty)$

C. $(-\infty, 0)$U$(2, \infty)$

D. $(0, 2)$.

Watch Video Solution

NCERT - FILE (Question from NCERT Book) (Exercise 6.3)

1. Find the slope of the tangent to the curve $y = 3x^2 - 5x + 2$ at $x = 3$.

Watch Video Solution

2. Find the slope of the tangent to the curve

$y = \frac{x-3}{x-5}, x \neq 5 \text{ at } x = 10.$

Watch Video Solution

3. Find the slope of the tangent to curve

$y = x^3 - x + 1$ at the point whose x-coordinate is 2.

Watch Video Solution

4. Find the slope of the tangent to the curve $y = x^3 - x + 1$ at the point whose x - coordinate is 3.

Watch Video Solution

5. Find the slope of the normal to the curve $x = 1 - a\sin^3\theta, y = a\cos^2\theta$ at $\theta = \frac{\pi}{2}$.

Watch Video Solution

6. Find the slope of the normal to the curve $x = a\sin^2\theta, y = b\cos^3\theta$ at $\theta = \frac{\pi}{4}$.

Watch Video Solution

7. Find points at which the tangent to the curve $y = x^3 - 3x^2 - 9x + 7$ is parallel to the x-axis

Watch Video Solution

8. Find a point on the curve $y = (x-2)^2$ at which the the tangent is parallel to the chord joining the points (2, 0) and (4, 4).

Watch Video Solution

9. Find the point on the curve $y = x^3 + 5$ at which the tangent is parallel to line $y = 12x - 7$ is.

Watch Video Solution

10. Find the equation of all lines having slope -3 that are tangents to the curve

$$y = \frac{1}{x-2}, x \neq 2.$$

Watch Video Solution

11. Find the equation of all lines having slope 2 which are tangents to the curve

$$y = \frac{1}{x-3}, x \neq 3$$

Watch Video Solution

12. Find the equations of all lines having slope 0 which are tangent to the curve

$$y = \frac{1}{x^2 - 2x + 3}.$$

Watch Video Solution

13. Find points on the curve $\frac{x^2}{9} + \frac{y^2}{25} = 1$ at which the tangents are :

(i) parallel to x -axis

(ii) parallel to y - axis.

Watch Video Solution

14. Find the equations of the tangent and normal to the given curves at the indicated points :

(i) $y = x^4 - 6x + 13x^2 - 10x + 5 \text{ at } (0, 5)$

(ii) $y = x^4 - 6x^3 + 13x^2 - 10x + 5 \text{ at } (1, 3)$.

(iii) $y = x^3 \text{ at } (1, 1)$

(iv) $y = x^2 \text{ at } (1, 1)$

(v) $x = \cos t, y = \sin t \text{ at } y = \frac{\pi}{4}$.

15. Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$ which is :

(a) parallel to the line $2x - y + 9 = 0$

(b) perpendicular to the line $5y - 15x = 13$.

Watch Video Solution

16. Show that the tangents to the curve $y = 2x^3 - 3$ at the points where $x = 2$ and $x = -2$ are parallel.

Watch Video Solution

17. Find the points on the curve $y = x^3 - 3x$ at which the tangents are parallel to the chord joining the points $(1, -2)$ and $(2, 2)$.

Watch Video Solution

18. For the curve $y = 4x^3 - 2x^5$, find all the points at which the tangent passes through the origin.

Watch Video Solution

19. Find the points on the curve $y = (x - 2)^2$ at which the tangents are parallel to the chord joining the points $(2, 0)$ and $(4, 4)$.

Watch Video Solution

20. Find the equation of the normal at the point (am^2, am^3) for the curve $ay^2 = x^3$.

Watch Video Solution

21. Find the equation of the normals to the curve $y = x^3 + 2x + 6$which are parallel to the line $x + 14y + 4 = 0$.

Watch Video Solution

22. Find the equations of the tangent and normal to the parabola $y^2 = 4ax$at the point $(at^2, 2at)$.

Watch Video Solution

23. Show that the curves $x = y^2$ and $xy = k$ cut at right angles; if $8k^2 = 1$

Watch Video Solution

24. Find the equation of the tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point (x_0, y_0).

Watch Video Solution

25. Find the equation of the tangent to the curve $y = \sqrt{3x - 2}$, which is parallel to the line $4x - 2y + 5 = 0$.

Also, write the equation of normal to the curve at the point of contact.

Watch Video Solution

26. The slope of the normal to the curve $y = 2x^2 + 3\sin x$ at $x = 0$ is :

A. 3

B. $\frac{1}{3}$

C. -3

D. $-\frac{1}{3}$.

Answer: D

Watch Video Solution

27. The line $y = x + 1$is a tangent to the curve $y^2 = 4x$at the point(A) $(1, 2)$ (B)$(2, 1)$ (C) $(1, 2)$ (D) $(1, 2)$

A. (1, 2)

B. $(2, 1)$

C. $(1, -2)$

D. $(-1, 2)$

Answer: A

Watch Video Solution

NCERT - FILE (Question from NCERT Book) (Exercise 6.4)

1. Using differentials, find the approximate value of each of the following up to 3 places of decimal.

$\sqrt{25.3}$

Watch Video Solution

2. Using differentials, find the approximate value of each of the following up to 3 places of decimal.

$\sqrt{49.5}$

Watch Video Solution

3. Using differentials, find the approximate value of each of the following up to 3 places of decimal.

$\sqrt{0.6}$

Watch Video Solution

4. Using differentials, find the approximate value of each of the following up to 3 places of decimal.

$(0.009)^{1/3}$

Watch Video Solution

5. Using differentials, find the approximate value of each of the following up to 3 places of decimal.

$(0.999)^{1/10}$

Watch Video Solution

6. Using differentials, find the approximate value of each of the following up to 3 places of

decimal.

$(15)^{1/4}$

Watch Video Solution

7. Using differentials, find the approximate value of each of the following up to 3 places of decimal.

$(26)^{1/3}$

Watch Video Solution

8. Using differentials, find the approximate value of each of the following up to 3 places of decimal.

$$\left(\frac{17}{81}\right)^{1/4}$$

Watch Video Solution

9. Using differentials, find the approximate value of each of the following up to 3 places of decimal.

$$(82)^{1/4}$$

Watch Video Solution

Watch Video Solution

10. Using differentials, find the approximate value of each of the following up to 3 places of decimal.

$(401)^{1/2}$

Watch Video Solution

11. Using differentials, find the approximate value of each of the following up to 3 places of decimal.

$(0.0037)^{1/2}$

Watch Video Solution

12. Using differentials, find the approximate value of each of the following up to 3 places of decimal.

$(26.57)^{1/3}$

Watch Video Solution

13. Using differentials, find the approximate value of each of the following up to 3 places of decimal.

$(81.5)^{1/4}$

Watch Video Solution

14. Using differentials, find the approximate value of each of the following up to 3 places of decimal.

$(3.968)^{3/2}$

Watch Video Solution

15. Using differentials, find the approximate value of each of the following up to 3 places of decimal.

$(32.15)^{1/5}$.

Watch Video Solution

16. Find the approximate value of $f(3.12)$, where $f(x) = 4x^2 + 5x + 2$.

Watch Video Solution

17. Find the approximate value of $f(3.02)$, where

$f(x) = 3x^2 + 15x + 5.$

Watch Video Solution

18. Find the approximate change in the volume V of a cube of side x metres caused by increasing the side by 5%.

Watch Video Solution

19. Find the approximate change in the surface area of a cube of side 'x' metres caused by decreasing the side by 5%.

Watch Video Solution

20. If the radius of a sphere is measured as 5 m with an error of 0.03 m, then find the approximate error in calculating its volume.

Watch Video Solution

21. If the radius of a sphere is measured as 7 m with an error of 0.03m, then find the approximate error in calculating its volume.

Watch Video Solution

22. If $f(x) = 3x^2 + 5x + 2$, then the approximate value $f(5.02)$ is

A. 47.66

B. 57.66

C. 67.66

D. 102.76

Answer: D

Watch Video Solution

23. The apprximate change in the volume of a cube of side x metres caused by increasing the side by 5% is :

A. $0.06x^3m^3$

B. $0.6x^3m^3$

C. $0.15x^3m^3$

D. $0.9x^3m^3$.

Answer: C

Watch Video Solution

NCERT - FILE (Question from NCERT Book) (Exercise 6.5)

1. Find the maximum and minimum values, if any, of the following function given by :

$f(x) = (2x-3)^2 + 5$

Watch Video Solution

2. Find the maximum and minimum values, if any, of the following function given by :

$f(x) = 9x^2 + 12x + 5$

Watch Video Solution

3. Find the maximum and minimum values, if any, of the following function given by :

$f(x) = -(x-1)^2 + 5$

Watch Video Solution

4. Find the maximum and minimum values, if any, of the following function given by :

$g(x) = x^3 + 5.$

Watch Video Solution

5. Find the maximum and minimum values, if any, of the following function given by :

$f(x) = |x + 2| - 1$

Watch Video Solution

6. Find the maximum and minimum values, if any, of the following function given by :

$g(x) = -|x+2| + 3$

Watch Video Solution

7. Find the maximum and minimum values, if any, of the following function given by :

$h(x) = \sin(2x) + 5$

Watch Video Solution

8. Find the maximum and minimum values, if any, of the following function given by :

$f(x) = |\sin 4x + 3|$

Watch Video Solution

9. Find the maximum and minimum values, if any, of the following function given by :

$h(x) = x + 1, x \in (-1, 1)$

Watch Video Solution

10. Find the local maxima and local minima, if any, of the followig functions. Find also the local maximum and the local minimum values, as the case may be :

$f(x) = x^2$

Watch Video Solution

11. Find the local maxima and local minima, if any, of the followig functions. Find also the local maximum and the local minimum values, as the case may be :

$$g(x) = x^3 - 3x$$

Watch Video Solution

12. Find the local maxima and local minima, if any, of the followig functions. Find also the local maximum and the local minimum values, as the case may be :

$$h(x) = \sin x + \cos x, 0 < x < \frac{\pi}{2}$$

Watch Video Solution

13. Find the local maxima and local minima, if any, of the followig functions. Find also the local maximum and the local minimum values, as the case may be :

$f(x) = \sin x - \cos x, 0 < x < 2\pi$

Watch Video Solution

14. Find the local maxima and local minima, if any, of the followig functions. Find also the local maximum and the local minimum values, as the case may be :

$f(x) = x^3 - 6x^2 + 9x + 15$

Watch Video Solution

15. Find the local maxima and local minima, if any, of the followig functions. Find also the local maximum and the local minimum values, as the case may be :

$g(x) = \frac{x}{2} + \frac{2}{x}, x > 0$

Watch Video Solution

16. Find the local maxima and local minima, if any, of the followig functions. Find also the local maximum and the local minimum values, as the case may be :

$$g(x) = \frac{1}{x^2 + 2}$$

Watch Video Solution

17. Find the local maxima and local minima, if any, of the followig functions. Find also the local maximum and the local minimum values, as the case may be :

$$f(x) = x\sqrt{1 - x}, x > 0$$

Watch Video Solution

18. Prove that the following functions do not have maxima or minima :

$f(x) = e^x$

Watch Video Solution

19. Prove that the following functions do not have maxima or minima :

$g(x) = \log x$

Watch Video Solution

20. Prove that the following functions do not have maxima or minima :

$h(x) = x^3 + x^2 + x + 1.$

Watch Video Solution

21. Find the absolute maximum value and the absolute minimum value of the following functions is given intervals :

$f(x) = x^2, x \in [-2, 2]$

Watch Video Solution

22. Find the absolute maximum value and the absolute minimum value of the following functions is given intervals :

$f(x) = \sin x + \cos x, x \in [0, \pi]$

Watch Video Solution

23. Find the absolute maximum value and the absolute minimum value of the following functions is given intervals :

$f(x) = 4x - \frac{1}{2}x^2, x \in \left[-2, \frac{9}{2}\right]$

Watch Video Solution

24. Find the absolute maximum value and the absolute minimum value of the following functions is given intervals :

$f(x) = (x - 1)^2 + 3, x \in [-3, 1]$.

Watch Video Solution

25. Find the maximum profit that a company can make, if the profit function is given by :

$p(x) = 41 - 72x - 18x^2$.

Watch Video Solution

Watch Video Solution

26. Find both the maximum value and the minimum value of $3x^4 - 8x^3 + 12x^2 - 48x + 25$on the interval [0, 3].

Watch Video Solution

27. At what points in the interval $[0, 2\pi]$, does the function $s \in \quad 2x$ attain its maximum value?

Watch Video Solution

Watch Video Solution

28. What is the maximum value of sinx+cosx

Watch Video Solution

29. Find the maximum value of $2x^3 - 24x + 107$ in the interval $[1, 3]$. Find the maximum value of the same function in $[-3, -1]$.

Watch Video Solution

30. It is given that at $x = 1$, the function $x^4 - 62x^2 + ax + 9$ attains its maximum value on the interval $[0, \ 2]$. Find the value of a .

Watch Video Solution

31. Find the maximum and minimum values of $x + s \in 2x$on $[0, 2\pi]$.

Watch Video Solution

32. Find the two numbers with maximum product and whose sum is 24.

Watch Video Solution

33. Find two positive numbers x and y such that $x+y=60$and $xy^3$is maximum.

Watch Video Solution

34. Find two positive number m and n such that their sum is 35 and the product m^2n^5 is maximum.

Watch Video Solution

35. Find two positive numbers whose sum is 16 and the sum of whose cubes is minimum.

Watch Video Solution

36. A square piece of tin of side 18 cm is to be made into a box without top, by cutting a square from each corner and folding up the flaps to form the box. What should be the side of the square to be cut off so that the volume of the box is the maxi

Watch Video Solution

37. A rectangular sheet of tin 45 cm by 24 cm is to be made into a box without top, by cutting off square from each corner and folding up the flaps. What should be the side of the square to be cut off so that the volume of the box is maximum ?

Watch Video Solution

38. Show that of all the rectangles inscribed in a given fixed circle, the square has the maximum area.

Watch Video Solution

39. Show that the right circular cylinder of given surface and maximum volume is such that its height is equal to the diameter of the base.

Watch Video Solution

40. Of all the closed cylindrical cans (right circular), of a given volume of 100 cubic centimetres, find the dimensions of the can which has the minimum surface area?

Watch Video Solution

41. A wire of length 28 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the length of the two pieces so that the combined area of the square and the circle is minimum?

Watch Video Solution

42. Prove that the volume of the largest cone, that can be inscribed in a sphere of radius $R\cdot$ is $\frac{8}{27}$ of the volume of the sphere.

Watch Video Solution

43. Show that the right-circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.

Watch Video Solution

44. Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$.

Watch Video Solution

45. Show that semi-vertical angle of right circular cone of given surface area and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$.

Watch Video Solution

46. The point on the curve $x^2 = 2y$which is nearest to the point (0, 5) is(A) $\left(2\sqrt{2}, 4\right)$ (B) $\left(2\sqrt{2}, 0\right)$ (C) (0, 0) (D) (2, 2)

A. $\left(2\sqrt{2}, 4\right)$

B. $\left(2\sqrt{2}, 0\right)$

C. $(0, 0)$

D. $(2, 2)$

Answer: A

Watch Video Solution

47. For all real values of x, the maximum value of $\frac{1-x+x^2}{1+x+x^2}$ is :

A. 0

B. 1

C. 3

D. $\frac{1}{3}$

Answer: D

Watch Video Solution

48. The maximum value of $[x(x-1)+1]^{1/3}, 0 \leq x \leq 1$ is :

A. $\left(\frac{1}{3}\right)^{1/3}$

B. $\frac{1}{2}$

C. 1

D. 0

Answer: C

Watch Video Solution

Misellaneous Exercise on Chapter (6)

1. Using differentials, find the approximate value of each of the following :

$(17)^{1/4}$

Watch Video Solution

2. Using differentials, find the approximate value of $(33)^{1/5}$

Watch Video Solution

3. Show that the function given by $f(x) = \frac{\log x}{x}$ has maximum at $x = e$.

Watch Video Solution

4. The two equal sides of an isosceles triangle with fixed base b are decreasing at the rate of 3 cm per second. How fast is the area decreasing when the two equal sides are equal to the base ?

Watch Video Solution

5. Find the equation of the normal to curve $y^2 = 4x$ at the point $(1, 2)$.

Watch Video Solution

6. Show that the normal at any point θ to the curve

$x = a\cos\theta + a\theta\sin\theta,\ y = a\sin\theta - a\,\theta\cos\theta$ is at a constant distance from the origin.

Watch Video Solution

7. Find the intervals in which the function f given by $f(x) = \frac{4\sin x - 2x - xc \otimes}{2 + \cos x}$ is (i) increasing (ii) decreasing.

Watch Video Solution

8. Find the intervals in which the function f given by $f(x) = x^3 + \frac{1}{x^3}, x \neq 0$is (i) increasing (ii) decreasing.

Watch Video Solution

9. Find the maximum are of the isosceles triangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$ with its vertex at one end of major axis.

Watch Video Solution

10. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is 8 m3. If building of tank costs Rs 70 per square metre for the base and Rs 45 per square metre for sides, what is the cost of least expensive tank?

Watch Video Solution

11. The sum of the perimeter of a circle and square is k, where k is some constant. Prove that the sum of their areas is least when the side of square is double the radius of the circle.

Watch Video Solution

12. A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.

Watch Video Solution

13. A point on the hypotenuse of a triangle is at distance a and b from the sides of the triangle.

Show that the maximum length of the hypotenuse is $\left(a^{\frac{2}{3}}+b^{\frac{2}{3}}\right)^{\frac{3}{2}}$.

Watch Video Solution

14. Find the points at which the function f given by $f(x)=(x-2)^4(x+1)^3$ has local maxima local minima point of inflexion

Watch Video Solution

15. Find the absolute maximum and minimum values of the function f given by $f(x) = \cos^2 x + \sin x, x \in [0, \pi]$.

Watch Video Solution

16. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$.

Watch Video Solution

17. Let f be a function defined on [a, b] such that $f'(x) > 0$, for all $x \in (a, b)$. Then prove that f is an increasing function on (a, b).

Watch Video Solution

18. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$.

Watch Video Solution

19. Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and semi vertical angle is one-third that of the cone and the greatest volume of cylinder is $\frac{4}{27}\pi h^3 \tan^2 \alpha$.

Watch Video Solution

20. A cylindrical tank of radius 10 m is being filled with wheat at the rate of 314 cubic metre per hour. Then the depth of the wheat is increasing at the rate of(A) 1 m^3 / h (B) 0.1 m^3 / h (C) 1.1 m^3 / h (D) 0.5 m^3 / h

A. $1m^3/h$

B. $0.1m^3/h$

C. $1.1m^3/h$

D. $0.5m^3/h.$

Answer: A

Watch Video Solution

21. The slope of the tangent to the curve $x = t^2 + 3t - 8, y = 2t^2 - 2t - 5$ at the point (2, -1), is

A. $\frac{22}{7}$

B. $\frac{6}{7}$

C. $\frac{7}{6}$

D. $\frac{-6}{7}$

Answer: B

Watch Video Solution

22. The line $y = mx + 1$is a tangent to the curve $y^2 = 4x$if the value of m is(A) 1 (B) 2 (C) 3 (D) $\frac{1}{2}$

A. 1

B. 2

C. 3

D. $\frac{1}{2}$

Answer: A

Watch Video Solution

23. The normal at the point (1,1) on the curve $2y+x^2=3$is(A) $x+y=0$ (B) $xy=0$ (C) $x+y+1=0$(D) $xy=0$

A. $x + y = 0$

B. $x - y = 0$

C. $x + y + 1 = 0$

D. $x - y = 0$

Answer: B

Watch Video Solution

24. The normal to the curve $x^2 = 4y$ passing $(1, 2)$ is :

A. $x + y = 3$

B. $x - y = 3$

C. $x + y = 1$

D. $x - y = 1$.

Answer: A

Watch Video Solution

25. The points on the curve $9y^2 = x^3$, where the normal to the curve makes equal intercepts with the axes are(A) $\left(4, \pm\frac{8}{3}\right)$ (B) $\left(4, \frac{-8}{3}\right)$ (C) $\left(4, \pm\frac{3}{8}\right)$ (D) $\left(\pm 4, \frac{3}{8}\right)$

A. $\left(4, \pm\frac{8}{3}\right)$

B. $\left(4, \frac{-8}{3}\right)$

C. $\left(4, \pm\frac{3}{8}\right)$

D. $\left(\pm 4, \frac{3}{8}\right)$

Answer: A

Watch Video Solution

Exercise

1. A spherical ball of salt is dissolving in water in such a manner that the rate of decrease of volume at any instant is proportional to the surface. Prove that the radius is decreasing at a constant rate.

Watch Video Solution

2. A kite is moving horizontally at a height of $151.5m$. If the speed of the kite is $10\frac{m}{s}$, how fast is the string being let out, when the kite is 250 m away from the boy who is flying the kite?

The height of the boy is 1.5 m. (A) 8 m/s (B) 12 m/s (C) 16 m/s (D) 19 m/s

Watch Video Solution

3. The volume of a cube is increasing at a constant rate. Prove that the increase in surface area varies inversely as the length of the edge of the cube.

Watch Video Solution

4. If $x and y$ are the sides of two squares such that $y = x - x^2$. Find the change of the area of second square with respect to the area of the first square.

Watch Video Solution

5. Show that for $a \geq 1, \ \ f(x) = \sqrt{3}\sin x - \cos x - 2ax + b$ is decreasing on R .

Watch Video Solution

6. Show that the function f given by $f(x) = \tan^{-1}(\sin x + \cos x), x > 0$is always an strictly increasing function in $\left(0, \frac{\pi}{4}\right)$.

Watch Video Solution

7. Determine for which values of x, the function $y = x^4 - \frac{4x^3}{3}$ is increasing and for which it is decreasing.

Watch Video Solution

8. Show that $f(x) = 2x + \cot^{-1} x + \log\left(\sqrt{1 + x^2} - x\right)$ is increasing in R

Watch Video Solution

9. Find the angle of intersection of the curves y $= 4 - x^2$ and $y = x^2$

Watch Video Solution

10. Prove that the curves $xy = 4 and x^2 + y^2 = 8$ touch each other.

Watch Video Solution

11. The slope of the tangent to the curve $x = t^2 + 3t - 8, y = 2t^2 - 2t - 5$ at the point (2, -1), is

Watch Video Solution

12. Using differentials, find the approximate value of $\sqrt{0.082}$

Watch Video Solution

13. Using differentials, find the approximate value of $(1.999)^5$

Watch Video Solution

14. Find the approximate volume of metal in a hollow spherical shell whose internal and external radii are 3 cm and 3.0005 cm, respectively.

Watch Video Solution

15. The maximum slope of curve y $= -x^3 + 3x^2 + 9x - 27$ is

Watch Video Solution

Revision Exercise

1. A car starts from a point P at time t = 0 seconds and stops at point Q. The distance x, in metres, covered by it, in t seconds is given by $x = t^2\left(2 - \frac{t}{3}\right)$ Find the time taken by it to reach Q and also find distance between P and Q.

Watch Video Solution

2. A water tank has the shape of an inverted righ circular cone with its axis vertical and vertex lowermost . Its semi-vertical angle is $\tan^{-1}(0.5)$. Water is poured into it at a constant rate of 4 cubic meter per hour . Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 2 m.

Watch Video Solution

3. The two equal sides of an isosceles triangle with fixed base b are decreasing at the rate of 3 cm per second. How fast is the area decreasing when the two equal sides are equal to the base ?

Watch Video Solution

4. The bottom of a rectangular swimming tank is 25 m by 40m. Water is pumped into the tank at the rate of 500 cubic metres per minute. Find the rate at which the level of water in the tank is rising.

Watch Video Solution

5. A ladder 13m long leans against a wall. The foot of the ladder is pulled along the ground away from the wall, at the rate of 1.5m/sec. How fast is the angle θ between the ladder and the ground is changing when the foot of the ladder is 12m away from the wall.

Watch Video Solution

6. The radius of a cylinder is increasing at the rate 2cm/sec. and its altitude is decreasing at the rate of 3cm/sec. Find the rate of change of volume when radius is 3 cm and altitude 5 cm.

Watch Video Solution

7. A kit is 120m high and 130 m of string is out. If the kite is moving away horizontally at the rate of 52 m/sec, find the rate at which the string is being paid out.

Watch Video Solution

8. $f(x) = \tan^{-1}(\sin x + \cos x), x > 0$ is always and increasing function on the interval

Watch Video Solution

9. The equation of tangents to the curve $y = \cos(x + y), \; -2\pi \le x \le 2\pi$ that are parallel to the line $x + 2y = 0$, is

Watch Video Solution

10. Find the angle between the parabolas $y^2 = 4ax$ and $x^2 = 4by$ at their point of intersection other than the origin.

Watch Video Solution

11. Tangents are drawn from the origin to the curve $y = \sin x$. Prove that their points of contact lie on the curve $x^2 y^2 = (x^2 - y^2)$

Watch Video Solution

12. Show that the line $\frac{d}{a} + \frac{y}{b} = 1$ touches the curve $y = be^{-\frac{x}{a}}$ at the point where it crosses the y-axis.

Watch Video Solution

13. If the straight line $x\cos\alpha + y\sin\alpha = p$ touches the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then prove that $a^2\cos^2\alpha + b^2\sin^2\alpha = p^2$.

Watch Video Solution

14. if the straight line $x\cos\alpha + y\sin\alpha = p$ touches the curve $x^m y^n = a^{m+n}$ prove that $p^{m+n} m^m n^n = (m+n)^{m+n} a^{m+n} \sin^n\alpha\cos^m\alpha$

Watch Video Solution

15. Find the point on the curve $y = 3x^2 - 9x + 8$ at which the tangents are equally inclined with the axes.

Watch Video Solution

16. The equation of the tangent at (2,3) on the curve $y^2 = ax^3 + b$ is $y = 4x - 5$. Find the values of $a and b\cdot$

Watch Video Solution

17. A circular disc of radius 3 cm is being heated. Due to expansion, its radius increases at the rate of 0.05 cm/s. Find the rate at which its area is increasing when radius is 3.2 cm.

Watch Video Solution

18. The radius of a sphere shrinks from 10 to 9.8 cm. Find approximately the decrease in its volume.

Watch Video Solution

19. If the error committed in measuring the radius of a circle is 0.01%, find the corresponding error in calculating the area.

Watch Video Solution

20. If a triangle ABC, inscribed in a fixed circle, be slightly varied in such away as to have its vertices always on the circle, then show that $\frac{da}{cas A}+\frac{db}{\cos B}+\frac{dc}{\cos C}=0.$

Watch Video Solution

21. The area S of a triangle is calculated by measuring the sides b and c, and $\angle A$. If there be an error δA in the measurement of $\angle A$, show that the relative error in area is given by

$$\frac{\delta S}{S} = \cot A.\ \delta A$$

Watch Video Solution

22. The pressure p and the volume v of a gas are connected by the relation $pv^{1.4} = const.$ Find the percentage error in p corresponding to a decrease of % in $v.$

Watch Video Solution

23. Show that the function given by $f(x) = \frac{\log x}{x}$ has maximum at $x = e$.

Watch Video Solution

24. The sum of the perimeter of a circle and square is k, where k is some constant. Prove that the sum of their areas is least when the side of square is double the radius of the circle.

Watch Video Solution

25. Find the points at which the function f given by $f(x) = (x-2)^4(x+1)^3$ has local maxima local minima point of inflexion

Watch Video Solution

26. Show that $s \in^p \theta \cos^q \theta$ attains a maximum, when $\theta = \tan^{-1}\sqrt{\frac{p}{q}}$.

Watch Video Solution

27. The fraction exceeds its p^{th} power by the greatest number possible, where $p \geq 2$ is

Watch Video Solution

28. If the sum of the lengths of the hypotenues and a side of a right angled triangle is given, show that the area of the triangle is maximum when the angle between them is $\pi / 3$.

Watch Video Solution

29. Divide 4 into two positive numbers such that the sum of the square of one and cube of the other is a minimum.

Watch Video Solution

30. A cylindrical can to be made to hold 1 litres of oil. Find the dimensions which will minimize the cost of the metal to make the can.

Watch Video Solution

31. Find the shortest distance of the point $(0, c)$ from the curve $y = x^2$, where $0 \leq c \leq 5$.

View Text Solution

32. A beam of length l is supported at one end. If W is the uniform load per unit length, the bending moment M at a distance x from the end is given by $M = \frac{1}{2}lx - \frac{1}{2}Wx^2$. Find the point on the beam at which the bending moment has the maximum value.

Watch Video Solution

33. Find the maximum area of an isosceles triangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with its vertex at one end of the major axis.

Watch Video Solution

34. Find the area of the greatest rectangle that can be inscribed in an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Watch Video Solution

35. A window of perimeter P (including the base of the arch) is in the form of a rectangle surrounded by a semi-circle. The semi-circular portion is fitted with the colored glass while the rectangular part is fitted with the clear glass that transmits three times as much light per square meter as the colored glass does. What is the ratio for the sides of the rectangle so that the window transmits the maximum light?

Watch Video Solution

CHECK YOUR UNDERSTANDING

1. The radius of a soap bubble is increasing at the rate of $0.2cm/s$. Find the rate of increase of its surface area when radius = 5 cm.

Watch Video Solution

2. Is the function $f(x) = x^2, x \in R$ increasing?

Watch Video Solution

3. The function $f(x) = x^2 - 6x + 9$ is increasing for $x > 3$.

Watch Video Solution

4. Find the slope of the tangent to the curve $y = 3x^2 - 4x$ at the point, whose x - co - ordinate is 2.

Watch Video Solution

5. Find the equation of the tangent of the curve $y = 3x^2$ at (1, 1).

Watch Video Solution

6. The function $f(x) = x^2, x \in R$ has no minimum value. (True/False)

Watch Video Solution

7. What is the absolute minimum value of $y = x^2 - 3x$ in $[0, 2]$?

Watch Video Solution

8. What are the maximum and minimum values, if any, of $f(x) = x, x \in (0, 1)$?

Watch Video Solution

9. Has the function $f(x) = x^n$ minimum value at $x = \frac{1}{e}$?

View Text Solution

10. Find two positive numbers whose product is 49 and the sum is minimum.

Watch Video Solution

COMPLETITION FILE (Questions from JEE Main)

1. Given P(x) $= x^4 + ax^3 + bx^2 + cx + d$ such that x=0 is the only real root of P'(x) =0 . If P(-1) lt P(1),$then \in the \int erval$[-1,1]`

A. $P(-1)$ is the minimum and P(1) is the maximum of P

B. $P(-1)$ is not minimum but P(1) is the maximum of P

C. $P(-1)$ is the minimum but P(1) is not the maximum of P

D. Neither $P(-1)$ is the minimum nor P(1) is the maximum of P.

Answer: B

Watch Video Solution

2. The equation of the tangent to the curve $y = x + \frac{4}{x^2}$, that is parallel to the x-axis, is (1) $y = 1$ (2) $y = 2$ (3) $y = 3$ (4) $y = 0$

A. $y = 0$

B. $y = 1$

C. $y = 2$

D. $y = 3.$

Answer: D

Watch Video Solution

3. Let $f : R \to R$ be a positive increasing function with $\lim_{x \to \infty} \frac{f(3x)}{f(x)} = 1$.Then $\lim_{x \to \infty} \frac{f(2x)}{f(x)}$ is

A. 1

B. $\frac{2}{3}$

C. $\frac{3}{2}$

D. 3

Answer: A

Watch Video Solution

4. Let $f:\overrightarrow{RR}$ be defined by $f(x) = \{k - 2x, \quad \text{if} \quad x \leq -12x + 3, fx \succ 1\}$. If f has a local minimum at $x = 1$, then a possible value of k is (1) 0 (2) $-\frac{1}{2}$ (3) -1 (4) 1

A. 1

B. 0

C. $-\frac{1}{2}$

D. $-1.$

Answer: D

Watch Video Solution

5. The shortest distance between line y-x=1 and curve $x = y^2$ is

A. $\dfrac{\sqrt{3}}{4}$

B. $\dfrac{3\sqrt{2}}{8}$

C. $\dfrac{8}{3\sqrt{2}}$

D. $\dfrac{4}{\sqrt{3}}$

Answer: B

Watch Video Solution

6. A spherical balloon is filled with 4500p cubic meters of helium gas. If a leak in the balloon causes the gas to escape at the rate of 72π

cubic meters per minute, then the rate (in meters per minute) at which the radius of the balloon decreases 49 minutes after the leakage began is (1) $\frac{9}{7}$ (2) $\frac{7}{9}$ (3) $\frac{2}{9}$ (4) $\frac{9}{2}$

A. $\frac{9}{7}$

B. $\frac{7}{9}$

C. $\frac{2}{9}$

D. $\frac{9}{2}$.

Answer: C

Watch Video Solution

7. The real number k for which the equation, $2x^3 + 3x + k = 0$ has two distinct real roots in [0, 1] (1) lies between 2 and 3 (2) lies between -1 and 0 (3) does not exist (4) lies between 1 and 2

A. lies between 2 and 3

B. lies between -1 and 0

C. does not exist

D. lies between 1 and 2.

Answer: C

Watch Video Solution

8. If f and g are differentiable functions in [0, 1] satisfying $f(0) = 2 = g(1), g(0) = 0$ and $f(1) = 6$, then for some $c \in]0, 1[$ (1) $2f'(c) = g'(c)$ (2) $2f'(c) = 3g'(c)$ (3) $f'(c) = g'(c)$ (4) $f'(c) = 2g'(c)$

A. $2f'(c) = 3g'(c)$

B. $f'(c) = g'(c)$

C. $f'(c) = 2g'(c)$

D. $2f'(c) = g'(c)$

Answer: C

Watch Video Solution

9. If x = -1 and x = 2 are extreme points of f(x) = $\alpha \log|x| + \beta x^2 + x$, then

A. $\alpha = -6, \beta = \frac{-1}{2}$

B. $\alpha = 2, \beta = \frac{-1}{2}$

C. $\alpha = 2, \beta = \frac{1}{2}$

D. $\alpha = -6, \beta = \frac{1}{2}$.

Answer: B

Watch Video Solution

10. The normal to the curve $x^2 + 2xy - 3y^2 = 0$, at (1, 1):

A. does not meet the curve again

B. meets the curve again in the second quadrant

C. meets the curve again in the third quadrant

D. meets the curve again in the fourth quadrant.

Answer: D

Watch Video Solution

11. Let f(x) be a polynomial of degree four having extreme values at x=1 and x=2. If $\lim_{x \to 0} \left(1 + \frac{f(x)}{x^2}\right) = 3$, then f(2) is equal to

A. -8

B. -4

C. 0

D. 4

Answer: C

Watch Video Solution

12. Consider $f(x) = \tan^{-1}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right), x \in \left(0, \frac{\pi}{2}\right)$. A normal to $y = f(x)$ at $x = \frac{\pi}{6}$ also passes through the point: (1) (0, 0) (2) $\left(0, \frac{2\pi}{3}\right)$ (3) $\left(\frac{\pi}{6}, 0\right)$ (4) $\left(\frac{\pi}{4}, 0\right)$

A. $\left(0, \frac{2\pi}{3}\right)$

B. $\left(\frac{\pi}{6}, 0\right)$

C. $\left(\frac{\pi}{4}, 0\right)$

D. $(0, 0)$

Answer: A

Watch Video Solution

13. The normal to the curve $y(x-2)(x-3)=x+6$ at the point where the curve intersects the $y-a\xi s,$ passes through the point : $\left(\frac{1}{2},-\frac{1}{3}\right)$ (2) $\left(\frac{1}{2},\frac{1}{3}\right)$ (3) $\left(-\frac{1}{2},-\frac{1}{2}\right)$ (4) $\left(\frac{\frac{1}{2,1}}{2}\right)$

A. $\left(\frac{1}{2},-\frac{1}{3}\right)$

B. $\left(\frac{1}{2},\frac{1}{3}\right)$

C. $\left(-\frac{1}{2},-\frac{1}{2}\right)$

D. $\left(\frac{1}{2},\frac{1}{2}\right)$

Answer: D

Watch Video Solution

14. Twenty metres of wire is available for fencing off a flower-bed in the form of a circular sector. Then the maximum area (in $sq\dot{m}$) of the flower-bed is: 25 (2) 30 (3) 12.5 (4) 10

A. 25

B. 30

C. 12.5

D. 10

Answer: B

Watch Video Solution

15. If the curves $y^2 = 6x, \quad 9x^2 + by^2 = 16$ intersect each other at right angles then the value of b is: (1) 6 (2) $\frac{7}{2}$ (3) 4 (4) $\frac{9}{2}$

A. 6

B. $\frac{7}{2}$

C. 4

D. $\frac{9}{2}$

Answer: D

Watch Video Solution

16. Let $f(x) = x^2 + \left(\frac{1}{x^2}\right)$ and $g(x) = x - \frac{1}{x}$ $\xi n R - \{-1, 0, 1\}$. If $h(x) = \left(\frac{f(x)}{g(x)}\right)$ then the local minimum value of $h(x)$ is: (1) 3 (2) -3 (3) $-2\sqrt{2}$ (4) $2\sqrt{2}$

A. 3

B. -3

C. $-2\sqrt{2}$

D. $2\sqrt{2}$.

Answer: D

Watch Video Solution

17. The maximum values of $3\cos\theta + 5\sin\left(\theta - \frac{\pi}{6}\right)$ for any real value of θ is:

A. $\sqrt{34}$

B. $\sqrt{19}$

C. $\frac{\sqrt{79}}{2}$

D. $\sqrt{31}$

Answer: B

Watch Video Solution

18. Let $f(x)$ be an non - zero polynomial of degree 4. Extremum points of $f(x)$ are $0, \ -1, 1$. If $f(k) = f(0)$ then,

A. k has one rational and two irrational roots

B. k has four rational roots

C. k has four irrational roots

D. k has three irrational roots.

Answer: A

Watch Video Solution

CHAPTER TEST (1)

1. The approximate change in the volume of a cube of side x metres caused by increasing the side by 3% is :

A. $0.06x^3m^3$

B. $0.6x^3m^3$

C. $0.09x^3m^3$

D. $0.9x^3x^3$

Answer: C

Watch Video Solution

2. Find the slope of the tangent to the curve $y = x^3 - 3x + 2$ at the point whose x-coordinate is 3.

A. 20

B. 24

C. 30

D. 25

Answer: B

Watch Video Solution

3. The radius of a circle is increasing at the rate of 0.7 cm/sec. What is the rate of increase of its circumference?

Watch Video Solution

4. Show that the function f given by $f(x) = x^3 - 3x^2 + 4x, x \in R$is strictly increasing on R.

Watch Video Solution

5. Find the slope of the tangent to the curve $y = x^3 - 3x + 2$ at the point whose x-coordinate is 3.

Watch Video Solution

6. A man 2 metres high walks at a uniform speed of 5 km/hr away from a lamp-post 6 metres high. Find the rate at which the length of his shadow increases.

Watch Video Solution

7. Find the intervals in which the function f given by $f(x) = \sin x + \cos x$, $0 \leq x \leq 2\pi$ is strictly increasing or strictly decreasing.

Watch Video Solution

8. Show that the curves $x = y^2$ and $xy = k$ cut at right angles; if $8k^2 = 1$

Watch Video Solution

9. Evaluate $\sqrt{402}$, using differentials.

Watch Video Solution

10. It is given that at $x = 1$, the function $x^4 - 62x^2 + ax + 9$ attains its maximum value on the interval $[0, \ 2]$. Find the value of a .

Watch Video Solution

11. Find the equation of the tangents to the curve $3x^2 - y^2 = 8$, which passes through the point $\left(\frac{4}{3}, 0\right)$.

Watch Video Solution

12. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $2\frac{R}{\sqrt{3}}$. Also find maximum volume.

Watch Video Solution