MATHS

BOOKS - ARIHANT MATHS (HINGLISH)

CONTINUITY AND DIFFERENTIABILITY

EXAMPLE

1. Determine the value of the constant k, so that the function f(x) = $\{\frac{kx}{|x|}$, if $x < 0$,3 if $x \geq 0$ is continuous at $x = 0$

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2. Prove that every rational function is continuous.

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3. Discuss the continuity of the function f given by $f(x) = |x| at x = 0$.

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4. Prove that the greatest integer function $[x]$ is continuous at all points except at integer points.

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5. Find all points of discontinuity of the function 'f' defined by :

$$f(x) = \begin{cases} x + 2, x \leq 1 \\ x - 2, 1 < x < 2 \\ 0, x \geq 2. \end{cases}$$

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6. Find the values of k so that the function f is continuous at the indicated point in$f(x) = \{kx + 1, \quad \text{if} \quad x \le 53x - 5, \quad \text{if} \quad x > 5$at $x = 5$

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7. Find the value of a for which the function f defined by

$$f9x\Big) = \left\{a\frac{\sin\pi}{2}(x+1), x \le 0\frac{\tan x - \sin x}{x^3}, x > 0\text{i sc o n t i n u o u } sat x = 0\right.$$

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8. Find the values of a and b such that the function defined by

`f(x)={5, ifxlt=2a x+b , if2

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9. Find the relationship between a and b so that the function f defined by $f(x) = ax + 1, \quad \text{if} x \le 3 \quad f(x) = bx + 3, \quad \text{if} \quad x > 3$ is continuous at x =3.

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10. Show that the function f(x) defined by :

$$f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, \ x > 0 \\ 2, \ x = 0 \\ \frac{4\left(1 - \sqrt{1 - x}\right)}{x}, \ x < 0 \end{cases}$$

is continuous at x = 0.

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11. Given that $f(x) = \begin{cases} \dfrac{\sqrt{1+kx}-\sqrt{1-kx}}{x} & \text{if } -1 \leq x < 0 \\ \dfrac{2x+1}{x-1} & \text{if } 0 \leq x \leq 1 \end{cases}$ is continous at $x = 0$. Find k

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12. If $f(x) = \begin{cases} \dfrac{\sin(a+1)x + 2\sin x}{x}, & x < 0 \\ 2, & x = 0 \\ \dfrac{\sqrt{1+bx}-1}{x}, & x > 0 \end{cases}$

is continuous at x = 0, then find the values of 'a' and 'b'.

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13. Show that the function f defined by $f(x) = |1 - x + |x||$

, where x is any real number, is a continuous function.

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14. Is f(x) = |x - 1| + |x - 2| differentiable at x = 2 ?

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15. Find 'a' and 'b', if the function given by:

$$f(x) = \begin{cases} ax^2 + b, \text{ if } x < 1 \\ 2x + 1, \text{ if } x \geq 1 \end{cases}$$

is differentiable at x = 1.

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16. If f is derivable at x =a,then $\lim_{x \to a}\left(\frac{xf(a) - af(x)}{x - a}\right)$

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17. If $f(x) = \begin{cases} x^2\sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$, then

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18. If $f(x) = x + 1$, find $\frac{d}{dx}(\text{fof})(x)$.

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19. Find $\frac{dy}{dx}$ when $\sin^2 x + \cos^2 y = 1$.

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20. Use Chain rule of find $\frac{dy}{dx}$, if $y = \left(\frac{2x - 1}{2x + 1}\right)^2$.

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21. Differentiate $s \in \left(\cos\left(x^2\right)\right)$ with respect to x.

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22. If $y = \sqrt{\frac{1 - x}{1 + x}}, provethat\left(1 - x^2\right)\frac{dy}{dx} + y = 0$

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23. $If y = \left\{x + \sqrt{x^2 + a^2}\right\}^n, provethat \frac{dy}{dx} = \frac{ny}{\sqrt{x^2 + a^2}}$ +a

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24. Find $\frac{dy}{dx}$, if $y + \sin y = \cos x$.

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25. If $\left(x^2 + y^2\right)^2 = xy$, find $\frac{dy}{dx}$

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26. If $x\sin(a + y) + \sin a.\cos(a + y) = 0$, then prove that

$$\frac{dy}{dx} = \frac{\sin^2(a + y)}{\sin a}$$

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27. If $x^2 + y^2 = t$ and $x^4 + y^4 = t^2 + \frac{1}{t^2}$, then prove that $\frac{dy}{dx} = \frac{1}{x^3 y}$.

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28. Differentiate $\sin^{-1}\left(\sqrt{\cos x}\right)$w.r.t. x, using chain rule.

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29. If $y = \sqrt{\cot^{-1}\sqrt{x}}$, find $\frac{dy}{dx}$.

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30. If $y = \sin\left(2\sin^{-1}x\right)$, show that:

$$\frac{dy}{dx} = 2\sqrt{\frac{1-y^2}{1-x^2}}.$$

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31. If $y = \sec^{-1}\left(\frac{1}{2x^2-1}\right)$ then $\frac{dy}{dx} = ?$

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32. If $y = \sin^{-1}\left[x(1-x) - \sqrt{x}\sqrt{1-x^2}\right]$, find $\frac{dy}{dx}$,

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33. Differentiate $\tan^{-1}\left(\frac{1+\cos x}{\sin x}\right)$ with respect to x.

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34. If $y=\sin^{-1}\left(6x\sqrt{1-9x^2}\right),\ -\frac{1}{3\sqrt{2}}<x<\frac{1}{3\sqrt{2}}$, then find $\frac{dy}{dx}$.

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35. Find $\frac{dy}{dx}$ if $y=\sin^{-1}\left[\frac{6x-4\sqrt{1-4x^2}}{5}\right]$

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36. सिद्ध करें कि (Prove that) :

$$\frac{d}{dx}\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right] = \sqrt{a^2 - x^2}$$

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37. If $\log\left(x^2 + y^2\right) = 2\tan^{-1}\left(\frac{y}{x}\right), showthat \frac{dy}{dx} = \frac{x + y}{x - y}$.

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38. If $\sqrt{1 - x^2} + \sqrt{1 - y^2} = a(x - y)$, then prove that $\frac{dy}{dx} = \sqrt{\frac{1 - y^2}{1 - x^2}}$

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39. Is it true that $x = e^{\log x}$ for all real

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40. Differentiate the following w.r.t. x: 3^{x+2}.

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41. If $xy = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{y(x-1)}{x(y+1)}$.

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42. Differentiate $\sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$ w.r.t. x.

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43. Differentiate the following with respect to x: $\sin^{-1}\left(\frac{2^{x+1}.3^x}{1+(36)^x}\right)$

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44. If $y = \log\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$, show that :

$\frac{dy}{dx} - \sec x = 0.$

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45. If $y = \log_{10}x + \log_x 10 + \log_x x + \log_{10}10$, find $\frac{dy}{dx}$.

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46. If $e^{x} + e^{y} = e^{x+y}$, prove that :

$$\frac{dy}{dx} = -\frac{e^{x}\left(e^{y} - 1\right)}{e^{y}\left(e^{x} - 1\right)}.$$

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47. Find the derivative of log(sin(logx))

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48. Find $\frac{dy}{dx}$, when $x = \frac{1 - t^{2}}{1 + t^{2}}$ and $y = \frac{2t}{1 + t^{2}}$

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49. Find $\frac{dy}{dx}$ when :

$x = a(1 + \cos\theta), y = a(\theta + \sin\theta)$.

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50. Find $\frac{dy}{dx}$, if $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$.

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51. Find $\frac{dy}{dx}$, $when\ x = e^{\theta}\left(\theta + \frac{1}{\theta}\right) and y = e^{-\theta}\left(\theta - \frac{1}{\theta}\right)$

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52. If $x = a(2\theta - \sin 2\theta)$ and $y = a(1 - \cos 2\theta)$ find $\frac{dy}{dx}$ when $\theta = \frac{\pi}{3}$

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53. Find the value of $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$, if :

$x = ae^{\theta}(\sin\theta - \cos\theta)$ and $y = ae^{\theta}(\sin\theta + \cos\theta)$.

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54. If $x = a\sin 2t(1 + \cos 2t)$ and $y = b\cos 2t(1 - \cos 2t)$, find the values of $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$ and $t = \frac{\pi}{3}$.

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55. Differentiate $\sin^2\left(x^2\right)$ w.r.t. x^2.

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56. Differentiate $\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$ w.r.t. $\tan^{-1}x$.

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57. Differentiate $\sin^{-1}\left(2x\sqrt{1-x^2}\right)$ w.r.t. $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$.

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58. If $x \in \left(\frac{1}{\sqrt{2}}, 1\right)$. Differentiate $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right) w.r.t. \cos^{-1}\left(2x\sqrt{1-x^2}\right)$.

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59. If $y = x^x$, find $\frac{dy}{dx}$.

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60. Differentiate x^{x^x} w.r.t. x.

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61. Find f'(x), if $f(x) = (\sin x)^{\sin x}$ for all $0 < x < \pi$.

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62. Differentiate $x^{\tan x} + (\sin x)^{\cos x}$ w.r.t. x.

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63. If $x^y y^x = 1$, prove that $\frac{dy}{dx} = -\frac{y(y + x\log y)}{x(y\log x + x)}$

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64. If $x^y = e^{x-y}$ then prove that $\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$

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65. If $x^y = e^{x-y}$, show that $\frac{dy}{dx} = \frac{\log x}{\{\log(xe)\}^2}$

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66. If $x^y + y^x = a^b$, then find $\frac{dy}{dx}$.

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67. Find $\frac{dy}{dx}$, if $y^x + x^y + x^x = a^b$.

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68. If $f(x) = \left(\frac{3+x}{1+x}\right)^{2+3x}$, find f'(0)

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69. If $y = \sqrt{2^x + \sqrt{2^x + \sqrt{2^x + \text{......to } \infty}}}$, then prove that :

$(2y - 1)\frac{dy}{dx} = 2^x \log 2.$

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70. $\sqrt{\log x}$

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71. If $y = e^x + e^x + e\left(\left(x + \dot{\rightarrow} \quad \infty\right)\right)$, show that $\frac{dy}{dx} = \frac{y}{1 - y}$.

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72. If $y = \sqrt{x}^{\sqrt{x}^{\sqrt{x} \dots \infty}}$ then prove that `dy/dx=(y^2)/(x(2-y log x)).

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73. Find the second derivative of $\sin^{-1}x$.

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74. If $y = 500e^{7x} + 600e^{-7x}$, show that $\frac{d^2y}{dx^2} = 49y$.

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75. If $x = a(\cos t + t\sin t)$ and $y = a(\sin t - t\cos t)$, find $\frac{d^2y}{dx^2}$.

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76. If $y = \sin(\sin x)$, prove that $\frac{d^2y}{dx^2} + \tan x\frac{dy}{dx} + y\cos^2 x = 0$.

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77. If $x = \tan\left(\frac{1}{a}\log y\right)$, show that $\left(1 + x^2\right)\frac{d^2y}{dx^2} + (2x - a)\frac{dy}{dx} = 0$.

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78. If $x = a\cos\theta + b\sin\theta, y = a\sin\theta - b\cos\theta,$ show :

$$y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0.$$

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79. If $x = \sin t$ and $y = \sin pt,$ prove that

$$\left(1 - x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0.$$

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80. If $y = a\sin(\log x) + b\cos(\log x)$, prove that :

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0.$$

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81. If $x\cos(a+y)=\cos y$, then prove that $\frac{dy}{dx}=\frac{\cos^2(a+y)}{\sin a}$.

Hence, show that $\sin a\frac{d^2y}{dx^2}+\sin 2(a+y)\frac{dy}{dx}=0$.

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82. If $y=\left[\log\left(x+\sqrt{1+x^2}\right)\right]^2$, show that :

$\left(1+x^2\right)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-2=0$.

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83. If $y=\log\left(1+2t^2+t^4\right),\ x=\tan^{-1}t,\ \text{ find }\frac{d^2y}{dx^2}$.

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84. If $y = x^x$, prove that $\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$

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85. If $y = \operatorname{cosec}^{-1}x,\ x > 1$, then show that $x\left(x^2 - 1\right)\frac{d^2y}{dx^2} + \left(2x^2 - 1\right)\frac{dy}{dx} = 0$.

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86. If $y = e^{2\tan^{-1}x}$, the show that : $\left(1 + x^2\right)^2\frac{d^2y}{dx^2} + 2x\left(1 + x^2\right)\frac{dy}{dx} = 4y$.

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87. At what points on the curve $y = (\cos x - 1)$ in $[0, 2\pi]$, is the tangent parallel to the x-axis?

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88. It is given that for the function f given by $f(x) = x^3 + bx^2 + ax$, $x \in [1, 3]$. Rolles theorem holds with $c = 2 + \frac{1}{\sqrt{3}}$. Find the values of a and b.

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89. Discuss the applicability of Lagrange's Mean Value Theorem to:

$f(x) = x(x - 1)(x - 2)$ in $\left[0, \frac{1}{2}\right]$.

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90. Check whether Lagrange's Mean Value Theorem is applicable on :

$f(x) = \sin x + \cos x$ in interval $\left[0, \frac{\pi}{2}\right]$.

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91. Find a point on the parabola $y = (x - 3)^2$, where the tangent is parallel to the chord joining (3, 0) and (4, 1).

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92. Using Lagranges mean value theorem, find a point on the curve $y = \sqrt{x - 2}$ defined on the interval [2,3], where the tangent is parallel to the chord joining the end points of the curve.

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93. $f(x) = \begin{cases} \dfrac{x^3 + x^2 - 16x + 20}{(x - 2)^2}, x \neq 2k, k, x = 2 \end{cases}$. Find the value of k, so that the following function is continuous at x=2.

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94. Let f(x) = x|x|, for all $x \in R$. Discuss the derivability of f(x) at x = 0.

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95. If $y = (\tan x + \sec x)$, prove that $\dfrac{d^2y}{dx^2} = \dfrac{\cos x}{(1 - \sin x)^2}$

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96. If f(x)= |cosxl, then $f'\left(\frac{3\pi}{4}\right)$ equal to -

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97. If $f(x) = \frac{\sqrt{2}\cos x - 1}{\cot x - 1}, x \neq \frac{\pi}{4}$. Find the value of $f\left(\frac{\pi}{4}\right)$ so that $f(x)$ becomes continuous at $x = \frac{\pi}{4}$.

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98. Given that , $f(x) = \begin{cases} \frac{1 - \cos 4x}{x^2} & \text{if } x < 0 \\ a & \text{if } x = 0 \\ \frac{\sqrt{x}}{\sqrt{16 + \sqrt{x}} - 4} & \text{if } x > 0 \end{cases}$

If f (x) is continuous at x= 0 , find the value of a

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99. Examine the differentiability of the function 'f' defined by :

$$f(x) = \begin{cases} 2x + 3, \text{if } -3 \le x < -2 \\ x + 1, \text{if } -2 \le x < 0 \\ x + 2, \text{if } 0 \le x \le 1 \end{cases}.$$

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100. Differentiate $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ with respect to $\cos^{-1}\left(2x\sqrt{1-x^2}\right)$, when $x \ne 0$.

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EXERCISE 5(a) (SHORT ANSWER TYPE QUESTIONS)

1. If $f(x) = \begin{cases} kx^2 & \text{if} x \leq 2 \\ 3 & \text{if} x > 2 \end{cases}$ is continuous at x = 2, then the value of k is

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2. Check the continuity of the function f given by $f(x) = 2x + 3atx = 1.$

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3. Check the continuity of the following functions :

$f(x) = x^2 \text{ at } x = 0.$

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4. Examine the continuity of the function $f(x) = 2x^2$ - 1at $x = 3.$

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5. Examine the function $f(x) = 2x^2 - 5$ for its continuity at the point$x = 3$.

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6. is $f(x) = |x|$ a continuous function?

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7. Examine the following functions for continuity :

$f(x) = x - 5$

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8. Examine the following functions for continuity :

$$f(x) = x^3 + x^2 - 1$$

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9. Examine the following functions for continuity :

$$f(x) = \frac{1}{x - 5}, x \neq 5$$

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10. Examine the following functions for continuity :

$$f(x) = \frac{x^2 - 25}{x + 5}, x \neq -5$$

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11. Examine the following functions for continuity:

(d) $f(x) = |x - 5|$.

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12. Prove that the following functions are continuous at all points of their domains :

$f(x) = \cos x$

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13. Prove that the following functions are continuous at all points of their domains :

$f(x) = e^x + e^{-x}$

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14. Prove that the following functions are continuous at all points of their domains :

$f(x) = \tan x$

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15. Prove that the following functions are continuous at all points of their domains :

$f(x) = \cos\left(x^2\right).$

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16. Discuss the continuity of the following functions a) $f(x) = \sin x + \cos x$

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17. Discuss the continuity of $f(x) = \sin x - \cos x$

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18. Discuss the continuity of $f(x) = \sin x \cos x$

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19. Discuss the continuity of the following functions :

$f(x) = \frac{\sin x}{\cos x}$.

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20. Prove that $f(x) = |\sin x|$ is continuous at all points of its domain.

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21. Examine if $\sin|x|$ is a continuous function.

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22. Is the function defined by $f(x) = x^2 - \sin x + 5$continuous at $x = \pi$?

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23. Show that $f(x) = x - |x|, x \in R$ is continuous at x = 0.

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24. Show that the function defined by $g(x) = x - [x]$ is discontinuous at all integral points which [x] denotes the greatest integer function.

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25. Find all points of discontinuity of f, where f is defined by

$$f(x) = \begin{cases} x + 1 & \text{if} \quad x \geq 1 \\ x^2 + 1 & \text{if} \quad x < 1 \end{cases}$$

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26. Find all points of discontinuity of f, where f is defined by

$$f(x) = \begin{cases} x^{10} - 1 & \text{if} \quad x \leq 1 \\ x^2 & \text{if} \quad x > 1 \end{cases}$$

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27. Find all points of discontinuity of f, where f is defined by

$$f(x) = \begin{cases} 2x + 3 & \text{if} \quad x \leq 2 \\ 2x - 3 & \text{if} \quad x > 2 \end{cases}$$

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28. Find all points of discontinuity of f, where f is defined by

$$f(x) = \left\{ \frac{|x|}{x}, \quad \text{if} \quad x \neq 00, \quad \text{if} \quad x = 0 \right.$$

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29. Find all points of discontinuity of f, where f is defined by

$$f(x) = \left\{ \frac{x}{|x|}, \quad \text{if} \quad x < 01, \quad \text{if} \quad x \geq 0 \right.$$

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30. Is the function defined by

$f(x) = \{x + 5, \quad \text{if} \quad x \leq 1x - 5, \quad \text{if} \quad x > 1$ a continuous function?

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31. Is the function f defined by

$$f(x) = \begin{cases} x & \text{if} \quad x \leq 1 \\ 5 & \text{if} \quad x > 1 \end{cases}$$

continuous at x = 0? At x = 2 ?

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32. Show that the function f given by

$f(x) = \{x^3 + 3 \quad \text{if} \quad x \neq 01 \quad \text{if} \quad x = 0$ is not continuous at $x = 0$.

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33. Discuss the continuity of the function f defined by

$$f(x) = \frac{1}{x}, x \neq 0.$$

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34. Discuss the continuity of the function :

$$f(x) = \begin{cases} x, \text{if } x \geq 0 \\ x^2, \text{if } x < 0 \end{cases}$$

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35. Discuss the continuity of the function defined by

$$f(x) = \{x + 2, \quad \text{if} \quad x < 0 - x + 2, \quad \text{if} \quad x > 0$$

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36. Examine the continuity of the function :

$$f(x) = \begin{cases} x + 1 \ , \ x \leq 2 \\ 2x - 1 \ , \ x > 2 \end{cases}$$ at x = 2.

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37. $f(x) = \begin{cases} \dfrac{x^2 - 25}{x - 5}, & \text{when} \quad x \neq 5 \\ 10, & \text{when} \quad x = 5 \end{cases}$ is continuous at x =5

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38. Discuss the continuity of the function :

$$f(x) = \begin{cases} \dfrac{|x - 2|}{x - 2}, \ x \neq 2 \\ 1 \ , x = 2 \end{cases}$$ at x = 2

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39. Discuss the continuity of the function :

$$f(x) = \begin{cases} \frac{|x-2|}{2-x}, & x \neq 2 \\ -1, & x = 2 \end{cases} \text{ at } x = 2$$

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40. Discuss the continuity of the function :

$$f(x) = \begin{cases} \frac{|x-a|}{x-a}, & \text{when } x \neq a \\ 1, & \text{when } x = a \end{cases} \text{ at } x = a.$$

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41. Discuss the continuity of the function f, where f is defined by

$$f(x)\begin{cases}3, & \text{if } 0 \le x \le 1\\ 4, & \text{if } 1 < x < 3\\ 5, & \text{if } 3 \le x \le 10\end{cases}$$

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42. Discuss the continuity of the function f, where f is defined by`f(x)={-2, ifxlt=-1 2x , if-11`

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43. Discuss the continuity of the function :

$$f(x) = \begin{cases} x, \ 0 \le x < \frac{1}{2} \\ \frac{1}{2}, \ x = \frac{1}{2} \\ 1 - x, \ \frac{1}{2} < x \le 1 \end{cases} \quad \text{at } x = \frac{1}{2}$$

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44. Discuss the continuity of the function :

$$f(x) = \begin{cases} \frac{1 - \cos x}{x^2}, \ x \ne 0 \\ 1, \ x = 0 \end{cases} \quad \text{at x = 0.}$$

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45. Show that the function $f(x)$defined as $f(x) = x\cos\left(\frac{1}{x}\right), x \neq 0; 0, x = 0$is continuous at $x = 0$ but not differentiable at x=0..

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46. Show that the following functions are continuous at x = 0 :

$$f(x) = \begin{cases} x\cos\frac{1}{x} \text{ when } x \neq 0 \\ 0 \text{ when } x = 0 \end{cases}$$

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47. Test the continuity of the function f (x) :

$$f(x) = \begin{cases} x^2\sin.\frac{1}{x} & \text{when } x \neq 0 \\ 0 & \text{when } x = 0 \end{cases}$$

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48. $f(x) = \{(\cos x,, \text{when}, x \geq 0,), (- \cos x,, \text{when}, x < 0)$ is discontinuous at x =0

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49. Examine the continuity of the function f(x) at x = 0.

$$f(x) = \begin{cases} \dfrac{\sin x}{x} & \text{when } x \neq 0 \\ 2 & \text{when } x = 0 \end{cases}$$

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50. Examine the continuity of the funcation

$$f(x) = \begin{cases} \frac{|\sin x|}{x}, & x \neq 0 \\ 1, & x = 0 \text{ at } x = 0 \end{cases}$$

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51. Examine the continuity of the function f(x) at x = 0.

$$f(x) = \begin{cases} \frac{\tan 2x}{3x} \text{when } x \neq 0 \\ \frac{3}{2} \text{ when } x = 0 \end{cases}$$

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52. Discuss the continuity of the cosine, cosecant, secant and cotangent functions.

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EXERCISE 5(a) (LONG ANSWER TYPE QUESTIONS (I))

1. Test for continuity of the following function at $x = a$: f(x)={(x-, a)

$\sin\left(\frac{1}{x - a}\right.$, where $x \neq a$ (x- a) 0 ,when $x = a$

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2. Examine the function for continuity at x = 0 :

$$f(x) = \begin{cases} \frac{\sin x}{x} & \text{when } x < 0 \\ x + 1 & \text{when } x \geq 0 \end{cases}.$$

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3. Discuss the continuity of f(x) at x = 0 if :

$$f(x) = \begin{cases} \dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sin x}, \text{ if } x \neq 0 \\ 1 \text{ , if } x = 0 \end{cases}.$$

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4. $f(x) = \begin{cases} kx^2, & \text{if } x \leq 2 \\ 3, & \text{if } x > 2 \end{cases}$ at x = 2

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5. The function is defined by $f(x) = \begin{cases} kx+1 & \text{if} & x \leq \pi \\ \cos x & \text{if} & x > \pi \end{cases}$ at $x = \pi$.

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6. If the function $f(x) = \begin{cases} kx + 5, \text{ when } x \leq 2 \\ x - 1, \text{ when } x > 2 \end{cases}$ is continuous at x = 2

then k = ?

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7. $f(x) = \begin{cases} 3k - 2x, \text{ when } x < 1 \\ 2k + 1, \text{ when } x \geq 1 \end{cases}$ at x = 1

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8. $f(x) = \begin{cases} 3x - 8 & \text{if } x \leq 5 \\ 2k & \text{if } x > 5 \end{cases}$ at $x = 5$

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9. $f(x) = \begin{cases} \frac{x-1}{x+1}, & x \neq 1 \\ \lambda - 1, x = 1 \end{cases}$ at x = 1.

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10. $f(x) = \begin{cases} \frac{1-\cos Ax}{x\sin x} & \text{if} \quad x \neq 0 \\ \frac{1}{2} & \text{if} \quad x = 0 \end{cases}$ at $x = 0$

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11. If the function $f(x) = \begin{cases} \frac{1-\cos(ax)}{x^2} & \text{when} \quad x \neq 0 \\ 1 & \text{when} \quad x = 0 \end{cases}$ be continuous at x=0, then find the value of a.

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12. $f(x) = \begin{cases} \dfrac{\sin 2x}{5x}, \text{when } x \neq 0 \\ m, \text{ when } x = 0 \end{cases}$ at x = 0

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13. Let $f(x) = \begin{cases} \dfrac{k\cos x}{\pi - 2x}, & x \neq \dfrac{\pi}{2} \\ 3, & x = \dfrac{\pi}{2}. \end{cases}$

If $\lim_{x \to \frac{\pi}{2}} f(x) = f\left(\dfrac{\pi}{2}\right)$, find the value of k.

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14. $f(x) = \begin{cases} \dfrac{k\cos x}{(\pi - 2x), \text{ if } x \neq \frac{\pi}{2}} \\ 3, \text{ if } x = \dfrac{\pi}{2} \end{cases}$

$at x = \dfrac{\pi}{2}$.

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15. $f(x) = \begin{cases} \dfrac{x^2 - 9}{x - 3}, \text{when } x \neq 3 \\ k, \text{ when } x = 3 \end{cases}$ at x = 3

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16. $f(x) = \begin{cases} \dfrac{(x+3)^2 - 36}{x - 3}, \ x \neq 3 \\ k\ , \ x = 3 \end{cases}$ at x = 3.

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17. For what value of 'k' is the function defined by :

$$f(x) = \begin{cases} k\left(x^2 + 2\right), \text{if } x \le 0 \\ 3x + 1 \ , \text{if } x > 0 \end{cases}$$

continuous at x = 0? Also, write whether the function is continuous at x = 1.

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18. If the function defined by :

$$f(x) = \begin{cases} 2x - 1, \ x < 2 \\ a, \ x = 2 \\ x + 1, \ x > 2 \end{cases}$$

is continuous at x = 2, find the value of 'a'.

Also discuss the continuity of f(x) at x = 3.

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19. Given that , $f(x) = \begin{cases} \dfrac{1-\cos 4x}{x^2} & \text{if } x < 0 \\ a & \text{if } x = 0 \\ \dfrac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} & \text{if } x > 0 \end{cases}$

If f (x) is continuous at x= 0 , find the value of a

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20. Find the values of a and b such that the function defined by

$f(x) = \begin{cases} 5 & \text{if} & x \leq 2 \\ ax+b & \text{if} & 2 < x < 10 \\ 21 & \text{if} & x \geq 10 \end{cases}$ is continuous function.

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21. Determine the constants 'a' and 'b' so that the function 'f' defined below is continuous everywhere :

$$f(x) = \begin{cases} x + 2, & x \le 2 \\ ax + b, & 2 < x < 5 \\ 3x - 2, & x \ge 5 \end{cases}.$$

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22. Determine the constants 'a' and 'b' so that the function 'f' defined below is continuous everywhere :

$$f(x) = \begin{cases} 5a, & x \le 0 \\ a\sin x + \cos x, & 0 < x < \frac{\pi}{2} \\ b - \frac{\pi}{2}, & x \ge \frac{\pi}{2} \end{cases}.$$

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23. If the function $f(x) = \begin{cases} 3ax + b & \text{for } x > 1 \\ 11 & \text{when } x = 1 \\ 5ax - 2b & \text{for } x < 1 \end{cases}$

is continuous at x = 1 , find the values of a and b .

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24. Find the values of 'a' and 'b' so that the following function is continuous at x = 3 and x = 5 :

$$f(x) = \begin{cases} 1, \text{ if } x \leq 3 \\ ax + b, \text{ if } 3 < x < 5 \\ 7, \text{ if } 5 \leq x \end{cases}.$$

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25. Find 'a' and 'b' if the function :

$$f(x) = \begin{cases} \dfrac{\sin x}{x}, & -2 \le x < 0 \\ a.2^x, & 0 \le x \le 1 \\ b + x, & 1 < x \le 2 \end{cases}$$

is a continuous function on [-2, 2].

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26. Determine the values of 'a' and 'b' such that the following function is continuous at x = 0 :

$$f(x) = \begin{cases} \dfrac{x + \sin x}{\sin(a+1)x}, \text{ if } -\pi < x < 0 \\ 2, \text{ if } x = 0 \\ 2\dfrac{e^{\sin bx} - 1}{bx}, \text{ if } x > 0 \end{cases}.$$

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27. Let $f(x) = \begin{cases} \dfrac{1-\sin^3 x}{3\cos^2 x} & \text{if } x < \dfrac{\pi}{2} \\ a & \text{if } x = \dfrac{\pi}{2} \\ \dfrac{b(1-\sin x)}{(\pi-2x)^2} & \text{if } x > \dfrac{\pi}{2} \end{cases}$,

if f (x) is continuous at $x = \dfrac{\pi}{2}$, find a and b .

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28. If the following function f (x) is continuous at x=0 , find the values of a , b and c, where

$$\begin{cases} \dfrac{\sin(a+1)x + \sin x}{x} & \text{if } x < 0 \\ c & \text{if } x = 0 \\ \dfrac{\sqrt{x+bx^2}-\sqrt{x}}{bx^{\frac{3}{2}}} & \text{if } x > 0 \end{cases}$$

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29. If f(x) $\begin{array}{l} = x^2 + ax + b, \text{ if } 0 \le x < 2 \\ = 3x + 2, \text{ if } 2 \le x \le 4 \\ = 2ax + 5b, \text{ if } 4 < x \le 8 \end{array}$ is continuous on $[0, 8]$ then $a - b = \ldots\ldots$

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30. Find all the points of discontinuity of the function 'f' defined by :

$$f(x) = \begin{cases} x + 2, \text{ if } x < 1 \\ 0, \text{ if } x = 1 \\ x - 2, \text{ if } x > 1 \end{cases}$$

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31. Find all points of discontinuity of f, where f is defined by`f(x)= {|x|+3, ifxlt=-3-2x , if-3

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32. Find all points of discontinuity of f, where f is defined by $f(x) = \{2x + 3, \quad \text{if} \quad x \leq 22x - 3, \quad \text{if} \quad x > 2$

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33. Show that the function defined by $f(x) \quad = \quad s \in \quad \left(x^2\right)$ is a continuous function.

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34. Show that the function defined by $f(x) = \cos\left(x^2\right)$is a continuous function.

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35. Show that the function defined by $f(x) = |\cos x|$is a continuous function.

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36. Show that the function 'f' given by :

$f(x) = |x| + |x - 1|, x \in R$

is continuous both at x = 0 and x = 1

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37. Discuss the continuity of the function of given by

$f(x) = \; + x - 1| + |x - 2 \; | \; atx = 1andx = 2$

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38. Find the points of discontinuity, if any, of the following

function: $f(x) = \begin{cases} \dfrac{x^4 - 16}{x - 2}, & \text{if} \quad x \neq 216, \quad \text{if} \quad x = 2 \end{cases}$

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39. Discuss continuity of $|x|$.

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EXERCISE 5(b) (LONG ANSWER TYPE QUESTIONS (I))

1. Examine the derivability of the following functions at the specified points :

$|x|$ at $x = 0$

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2. Examine the derivability of the following functions at the specified points :

$[x]$ at $x = 1$

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3. Examine the derivability of the following functions at the specified points :

$|x|^2$ at $x = 0$

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4. Examine the derivability of the following functions at the specified points :

x^3 at $x = 2$

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5. If $f(x)$ is differentiable at $x = a$, find $\lim_{x \to a} \frac{x^2 f(a) - a^2 f(x)}{x - a}$.

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6. If F(x) = f(ax) and f(ax) is differentiable, then prove that F'(x) = af'(ax) , $a \neq 0$.

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7. Show that $f(x) = \begin{cases} x \sin\frac{1}{x}, & \text{when } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases}$ is continuous but not differentiable at x = 0

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8. Show that the function defined by $f(x) = (3 - 2x), x < 2$ and $f(x) = 3x - 7, x \geq 2$ is not derivable at $x = 2$

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9. Discuss continuity $f(x) = \begin{cases} \dfrac{e^{1/x}}{1+e^{1/x}} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ *at*x = 0`

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10. Consider the following in respect of the function

$$f(x) = \begin{cases} 2 + x, x \geq 0 \\ 2 - x, x < 0 \end{cases}$$

1. $\lim x \to 1 f(x)$ does not exist.

2. f(x) is differentiable at x=0

3. f(x) is continuous at x=0

Which of the above statements is /aer correct?

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11. Show that the function 'f' defined as follows, is continuous at x = 2, but not differentiable there at :

$$f(x) = \begin{cases} 3x - 2, \ 0 < x \leq 1 \\ 2x^2 - x, \ 1 < x \leq 2 \\ 5x - 4, \ x > 2 \end{cases}.$$

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12. The function 'f' defined as :

$$f(x) = \begin{cases} x^2 + 3x + a, \text{ if } x \leq 1 \\ bx + c, \text{ if } x > 1 \end{cases}$$

is derivable for every x. Find the value of 'a' and 'b'.

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13. For what choice of $aandb$ is the function

$f(x) = \{x^2, x \leq cax + b, x > cisd \Leftrightarrow erentiableatx = \cdot$

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14. Let $f: R \rightarrow R$ (R is the set of real numbers) be defined as follows :

$$f(x) = \begin{cases} 2 - x, \text{ for } 1 \le x \le 2 \\ x - \frac{1}{2}x^2, \text{ for } x > 2 \end{cases}.$$

Examine the continuity and differentiability of f(x) at x = 2.

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15. Show that the function $f(x) = |x - 3|, \ x \in \ | R,$ is continuous but not differentiable at $x = 3$.

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16. Show that $f(x) = |x - 5|$ is continuous but not differentiable at $x = 5$.

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17. Write an example of a function which is everywhere continuous but fails to be differentiable exactly at five points.

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EXERCISE 5(c) (SHORT ANSWER TYPE QUESTIONS)

1. If $f(x) = x + 7$, and $g(x) = x - 7, x \in R$ them find $\frac{d}{dx}(fog)(x)$.

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2. $f(x) = \cos\sqrt{x}$. find $\frac{dy}{dx}$

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3. $f(x) = \left(3x^2 + 2\right)^3(5x - 1)^2$ find $\dfrac{dy}{dx}$

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4. $f(x) = \left(2x^2 + 3\right)^{\frac{5}{3}}(x + 5)^{\frac{-1}{3}}$

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5. $f(x) = \dfrac{3}{2 - x}, x \neq 2$ find $\dfrac{dy}{dx}$

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6. $h(x) = (x + 1)(x + 2)(x + 3)$. find $\dfrac{dy}{dx}$

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7. Use Chain Rule to find $\frac{dy}{dx}$, if $y = \left(\frac{3x - 1}{3x + 1}\right)^2$.

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8. $y = \left(4x^3 - 5x^2 + 1\right)^4$, Find $\frac{dy}{dx}$.

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9. Find $\frac{dy}{dx}$ if :

$y = (x - 1)^{-3}, x \neq 1$

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10. Find $\frac{dy}{dx}$ if :

$$y = 1 - 2\left(\frac{5x}{3x+2}\right)^2 + \left(\frac{5x}{3x+2}\right)^3$$

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11. Find $\frac{dy}{dx}$ if :

$$ax + by^2 = \cos y$$

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12. Differentiate the following w.r.t.x :

$$\sin\left(x^2\right)$$

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13. Differentiate the following w.r.t.x :

$\sin\left(x^2 + 5\right)$

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14. Differentiate the following w.r.t.x :

$\tan(2x + 3)$

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15. Differentiate the following w.r.t.x : $\sin\left(x^2\right)$

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16. Differentiate the following w.r.t.x :

$\sin^4(ax+b)^2$

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17. Differentiate the following w.r.t.x :

$\sin(\cot x)$

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18. Differentiate the following w.r.t.x :

$\operatorname{cos}ec\left(\cot\sqrt{x}\right)$

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19. Differentiate the following w.r.t.x :

$\sin^2\left(x^5\right)$

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20. Differentiate the following w.r.t.x :

$\cos^2\left(x^3\right)$

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21. Differentiate the following w.r.t.x :

$\cos x^3 . \sin^2\left(x^5\right)$

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22. Differentiate the following w.r.t.x :

$2\sqrt{\cot\left(x^2\right)}$

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23. Differentiate the following w.r.t x. (i) $\cos^{-1}(\sin x)$ (ii) $\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$ (iii) $\sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$

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24. Differentiate the following w.r.t.x :

$\sqrt{15x^2 - x + 1}$

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25. Differentiate the following w.r.t.x :

$$\frac{\sin(ax+b)}{\cos(cx+d)}$$

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26. Differentiate w.r.t. x the function

$\cos(a \cos x + b s \in x)$, for some constant a and b.

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27. Find $\frac{dy}{dx}$ if :

$y = 9u^2, u = 1 - \frac{3}{2}x^2$

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28. Find $\frac{dy}{dx}$ if :

$$y = \frac{3 - v}{2 + v}, v = \frac{4x}{1 - x^2}$$

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29. Find $\frac{dy}{dx}$ if :

$$y = at^2, t = \frac{x}{2a}$$

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30. Find $\frac{dy}{dx}$ at x=1,y=pi/4 if sin^2 y+cos xy0`

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31. If $x^{16}y^9 = \left(x^2 + y\right)^{17}$, prove that $\frac{dy}{dx} = \frac{2y}{x}$.

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32. Differentiate the following w.r.t. x:

$|2x - 1|$

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33. Differentiate the following w.r.t. x:

$\left|2x^2 - 3\right|$

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34. If $y + \sin y = \cos x$, then find the values of 'y' for which $\frac{dy}{dx}$ is valid.

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EXERCISE 5(d) (SHORT ANSWER TYPE QUESTIONS)

1. Find the derivative of y w.r.t. x in each of the following :

$xy + y^2 = \tan x + y$

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2. Find the derivative of y w.r.t. x in each of the following :

$x^2 + y^2 - 3xy + 1$

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3. Find the derivative of y w.r.t. x in each of the following :

$\sin^2 y + \cos xy = \pi.$

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4. Find the derivative of y w.r.t. x in each of the following :

$\sqrt{x} + \sqrt{y} = 20$

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5. Find the derivative of y w.r.t. x in each of the following :

$y(y+1) = x(x+1)(x+2)$

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6. Find the derivative of y w.r.t. x in each of the following :

$x^2 = \dfrac{x+3y}{x-3y}$

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7. Find the derivative of y w.r.t. x in each of the following :

$$\frac{y}{x+y} = 3 + x^3.$$

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8. Find the derivatives of f(x) w.r.t. x in the following :

$$f(x) = \sqrt[3]{2x^4 + x^2 - x}$$

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9. Find the derivatives of f(x) w.r.t. x in the following :

$$f(x) = \sqrt[3]{ax + b}$$

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10. Find the derivatives of f(x) w.r.t. x in the following :

$$f(x) = \left(x^2 + x + 5\right)^{1/3}\left(x^3 + 1\right)^{2/3}$$

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11. Find the derivatives of f(x) w.r.t. x in the following :

$$g(x) = \sqrt[3]{5x - 9}\sqrt[3]{3x - 4}.$$

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12. Obtain $\frac{dy}{dx}$ when :

$$x^2 + y^2 + 2axy = 0$$

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13. Find $\frac{dy}{dx}$ if $x^3 + y^3 - 3axy = 0$.

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14. If $x^2 + y^2 + 2gx + 2fy + c = 0$ then $\frac{dy}{dx} =$

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15. Obtain $\frac{dy}{dx}$ when :

$x^4 + y^4 + 4xy - 100 = 0$

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16. If $ax^2 + 2hxy + by^2 = 0$ then $\frac{dy}{dx} =$

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17. If $\sqrt{x}+\sqrt{y}=5$, find $\frac{dy}{dx}$ at (4, 9).

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18. If $x\sqrt{1+y}+y\sqrt{1+x}=0$, prove that $\frac{dy}{dx}=-\frac{1}{(x+1)^2}$.

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19. If $y=\sqrt{x}+\frac{1}{\sqrt{x}}$, then the value of $\left(2x\frac{dy}{dx}+y\right)$ is-

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20. Find $\frac{dy}{dx}$ for each of the following

$$y = \left(x^2 + 3x + 5\right)\left(x^2 - 2\right)^2$$

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21. Find $\frac{dy}{dx}$ for each of the following

$$y = \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)\left(1 + x + x^2\right)$$

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22. Find $\frac{dy}{dx}$ for each of the following

$$y = \left(\frac{x - \sqrt{x}}{1 - 2x}\right)^2$$

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23. Find $\frac{dy}{dx}$ for each of the following

$$y = \left(\frac{1}{1+x}\right)\left(x^{-2} + \frac{2}{x} - 1\right) + \sqrt[3]{x} - \frac{1}{\sqrt[3]{x}}$$

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24. Find $\frac{dy}{dx}$ for each of the following

$$y = \sqrt[3]{x^2\left(x^2+3\right)}.$$

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25. If $\cos y = x\cos(a+y)$, with $\cos a \neq \pm 1$, prove that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$.

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26. If $\sin y = x\sin(a + y)$, prove that, $\frac{dy}{dx} = \frac{\sin a}{1 - 2x\cos a + x^2}$.

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EXERCISE 5(e) (SHORT ANSWER TYPE QUESTIONS)

1. Differentiate the following w.r.t. x :

$$\cos^{-1}\left(\frac{x}{x + 1}\right)$$

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2. $\sin^{-1}\left(x\sqrt{x}\right), 0 \le x \le 1$

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3. Differentiate the following w.r.t. x :

$\left(\tan^{-1}x\right)^2$

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4. Differentiate the following w.r.t. x :

$x\sec^{-1}x.$

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5. Differentiate the following w.r.t. x :

$\cos^{-1}(\sin x)$

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6. Differentiate the following w.r.t. x:

$(i)\sin^{-1}2x \quad (ii)\tan^{-1}\sqrt{x} \quad (iii)\cos^{-1}(\cot x)$

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7. Differentiate the following w.r.t. x :

$\left(\cot^{-1}x\right)^2$

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8. Differentiate the following w.r.t. x :

$\sin\left(2\sin^{-1}x\right)$

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9. Find the derivatives w.r.t. x :

$$\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$$

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10. $\tan^{-1}\frac{\cos x}{1+\sin x}=$

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11. $\cot^{-1}\left(\frac{1+\cos x}{\sin x}\right)$

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12. Differentiate the following w.r.t. x :

$\sin^{-1}\left(1-2x^2\right)$

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13. $\sin^{-1}\left(3x-4x^3\right)$

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14. Differentiate the following w.r.t. x :

$\cos^{-1}\left(4x^3-3x\right), \ -1 < x < 1$

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15. Differentiate the following w.r.t. x :

$$\sin^{-1}\left(\frac{2x}{1+x^2}\right), \ -1 < x < 1$$

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16. $\operatorname{cosec}^{-1}\left(\frac{1+x^2}{2x}\right)$

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17. Differentiate the following w.r.t. x :

$$\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0 < x < 1$$

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18. Differentiate the following w.r.t. x :

$$\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0 < x < 1$$

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19. Differentiate the following w.r.t. x :

$$\cos^{-1}\left(\frac{2x}{1+x^2}\right), \; -1 < x < 1$$

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20. Differentiate the following w.r.t. x :

$$\tan^{-1}\left(\frac{2x}{1-x^2}\right), 0 < x < 1$$

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21. $\tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$

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22. निम्न फलन का x के सापेक्ष अवकल गुणांक ज्ञात कीजिए -

$\tan^{-1}\left[\frac{x^{1/3} + a^{1/3}}{1 - x^{1/3}a^{1/3}}\right]$

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23. $\tan^{-1}\frac{x}{\sqrt{a^2 - x^2}}$

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EXERCISE 5(e) (LONG ANSWER TYPE QUESTIONS (I))

1. Differentiate $\tan^{-1}\left\{\frac{\sqrt{1+x^2}-1}{x}\right\}$ w.r.t. x.

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2. $\tan^{-1}\left(\frac{\sqrt{1+x^2}+1}{x}\right)$

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3. Differentiate the following w.r.t. x :

$\tan^{-1}\left(\frac{\sqrt{1+a^2x^2}-1}{ax}\right)$

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4. Differentiate the following w.r.t. x :

$$\cot^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right), x \neq 0$$

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5. Differentiate the following w.r.t. x :

$$\cot^{-1}\left(\frac{1+x}{1-x}\right)$$

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6. Differentiate the following w.r.t. x :

$$\cot^{-1}\left(\sqrt{1+x^2}-x\right).$$

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7. Differentiate the following w.r.t. x :

$\tan^{-1}(\sec x + \tan x)$.

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8. Differentiate the following w.r.t. x :

$\tan^{-1}\sqrt{\frac{1 - \cos x}{1 + \cos x}}$

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9. Differentiate w.r.t. x:

$(i)\tan^{-1}\left\{\sqrt{\frac{1 + \cos x}{1 - \cos x}}\right\}$ $(ii)\tan^{-1}\left\{\sqrt{\frac{1 + \sin x}{1 - \sin x}}\right\}$

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10. Differentiate the following w.r.t. x :

$$\tan^{-1}\sqrt{\frac{1+\sin x}{1-\sin x}}$$

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11. Differentiate the following w.r.t. x :

$$\sin^{-1}\left(\sqrt{\frac{1+x^2}{2}}\right).$$

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12. If $y=\tan^{-1}\left(\frac{2y}{1-x^2}\right)+\sec^{-1}\left(\frac{1+x^2}{1-x^2}\right)$, $x>0$, prove that $\frac{dy}{dx}=\frac{4}{1+x^2}$.

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13. Find $\frac{dy}{dx}$ of $y = \cot^{-1}\left[\frac{\sqrt{1 + \sin x} + \sqrt{1 - \sin x}}{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}}\right]$

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14. Find $\frac{dy}{dx}$ of $y = \cot^{-1}\left[\frac{\sqrt{1 + \sin x} + \sqrt{1 - \sin x}}{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}}\right]$

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15. If $y = \tan^{-1}\frac{\left(\sqrt{1 + \sin x} + \sqrt{1 - \sin x}\right)}{\left(\sqrt{1 + \sin x} - \sqrt{1 - \sin x}\right)}$, find $\frac{dy}{dx}$.

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16. $\frac{d}{dx}\left[\cos^{-1}\sqrt{\frac{(1+x)}{2}}\right]$

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17. If $y = \tan^{-1}\left\{\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right\}$, `-1

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18. If $y = \tan^{-1}\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}$, then the value of $\frac{dy}{dx}$ is -

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19. If $y = \sin\left[2tan^{-1}\left\{\sqrt{\frac{1-x}{1+x}}\right\}\right]$, find $\frac{dy}{dx}$

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20. If $y = \tan^{-1}\left(\frac{5ax}{a^2 - 6x^2}\right)$, $then\frac{dy}{dx} =$

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21. Find $\frac{dy}{dx}$ if $y = \frac{\tan^{-1}(4x)}{1 + 5x^2} + \frac{\tan^{-1}(2 + 3x)}{3 - 2x}$

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22. If $y = \cos^{-1}\left(\frac{3x + 4\sqrt{1 - x^2}}{5}\right)$, find $\frac{dy}{dx}$

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EXERCISE 5(e) (LONG ANSWER TYPE QUESTIONS (II))

1. If $f(x) = \tan^{-1}x, g(x) = \tan^{-1}\left(\frac{1+x}{1-x}\right)$ for $|x| < 1$, show that $f'(x) = g'(x)$ and $g(x) - f(x) = \frac{\pi}{4}$.

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2. If $y = \frac{2}{\sqrt{a^2 - b^2}}\tan^{-1}\left[\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right]$, prove that $\frac{dy}{dx} = \frac{1}{a + b\cos x}, a > b > 0.$

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3. If $\sqrt{1-x^6}+\sqrt{1-x^6}=a\left(x^3-y^3\right)$, then prove that $\frac{dy}{dx}=\frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}}$

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4. $\frac{d}{dx}\left[\tan^{-1}(\cot x)+\cot^{-1}(\tan x)\right]=$

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5. If $y=\sin^{-1}(\cos x)+\cos^{-1}(\sin x)$, then prove that :

$\frac{dy}{dx}+2=0.$

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EXERCISE 5(f) (SHORT ANSWER TYPE QUESTIONS)

1. Differentiate the following w.r.t. x :

e^{-x}

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2. Differentiate the following w.r.t. x :

$e^{\sin^{-1}x}$

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3. Differentiate the following w.r.t. x :

$e^{\cot^{-1}x^2}$

Watch Video Solution

4. Differentiate the following w.r.t. x :

$e^{m \log x}$

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5. Differentiate the following w.r.t. x :

$\sqrt{e^{\sqrt{x}}}, x > 0$

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6. Differentiate the following w.r.t. x :

$\log(\sin x)$

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7. Differentiate the following w.r.t. x :

$\log\left(\cos e^{x}\right)$

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8. Differentiate the following w.r.t. x:$\sin\left(\tan^{-1}e^{-x}\right)$

Watch Video Solution

9. Differentiate the following w.r.t. x :

$\frac{\cos x}{\tan x}, x > 0$

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10. Differentiate the following w.r.t. x :

$\sqrt{\tan x a^x}$.

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11. Differentiate the following w.r.t. x :

$e^x \sin x$

Watch Video Solution

12. Differentiate the following w.r.t. x :

$\sqrt{x} \log x^2$

Watch Video Solution

13. Differentiate the following w.r.t. x :

$x^{-1/3}e^{x}$

Watch Video Solution

14. Differentiate the following w.r.t. x

$x \cdot \sin x \cdot e^{x}$

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15. Differentiate the following w.r.t. x :

$e^{\sin^{-1}(x+1)}$

Watch Video Solution

16. Differentiate the following w.r.t. x :

$\tan\{\log(\sin x)\}$

Watch Video Solution

17. Differentiate the following w.r.t. x :

$e^{\sin\sqrt{x}}$

Watch Video Solution

18. Differentiate the following w.r.t. x :

$e^{\cos^{-1}(x+1)}$

Watch Video Solution

19. Differentiate the following w.r.t. x :

$e^{\cos^{-1} x^2}$

Watch Video Solution

20. Differentiate the following w.r.t. x :

$\sqrt{1 - x^2} . e^{5x}$

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21. Differentiate the following w.r.t. x :

$e^{\sqrt{1 - x^2}} \cdot \tan x$

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22. Differentiate the following w.r.t. x :

$$e^{\sin^2}\left(2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right)$$

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23. Differentiate the following w.r.t. x :

$$\frac{\log x}{x}$$

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24. Differentiate the following w.r.t. x :

$$\frac{e^x}{x}$$

Watch Video Solution

25. Differentiate the following w.r.t. x :

$$\frac{\log x}{e^x}$$

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26. Differentiate the following w.r.t. x :

$\log(\cos 5x)$

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27. Differentiate the following w.r.t. x :

$$\frac{1}{\log \cos x}$$

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EXERCISE 5(f) (LONG ANSWER TYPE QUESTIONS (I))

1. Differentiate the following w.r.t. x :

$$\left(x^2 + 7x + 2\right)\left(e^x - \sin x\right)$$

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2. Differentiate the following w.r.t. x :

$$e^{-3x}\sin^2 3x$$

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3. Differentiate the following w.r.t. x :

$$e^{-x^2}\sin(\log x)$$

Watch Video Solution

4. Differentiate the following w.r.t. x :

$$\tan^{-1}\left(\frac{e^{2x}+1}{e^{2x}-1}\right)$$

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5. Differentiate the following w.r.t. x :

$$\log\left((x+3)+\sqrt{x^2+6x+3}\right)$$

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6. Differentiate the following w.r.t.x. $\log\left(x+\sqrt{a^2+x^2}\right)$

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7. Differentiate the following w.r.t. x :

$$x\sqrt{x^2 + 1} + \log\left(x + \sqrt{x^2 + 1}\right)$$

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8. Differentiate the following w.r.t. x :

$$\sqrt{x^2 + 1} - \log\left(\frac{1}{x} + \sqrt{1 + \frac{1}{x^2}}\right)$$

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9. Differentiate the following w.r.t. x :

$$\frac{e^x(x - 1)}{\left(x^2 + 1\right)}$$

Watch Video Solution

10. Differentiate the following w.r.t. x :

$$\frac{e^{ax}}{\sin(bx + c)}$$

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11. Differentiate the following w.r.t. x :

$$\frac{1}{3}e^{x} - 5e$$

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12. Differentiate the following w.r.t. x :

$$e^{x} + 2\cos x$$

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13. Differentiate the following w.r.t. x :

$x^2 e^x \sin x$

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14. Differentiate the following w.r.t. x :

$e^{\sec^2 x} + 3\cos^{-1} x.$

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15. Differentiate the following w.r.t. x :

$\log\left(\sin\sqrt{1+x^2}\right)$

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16. Differentiate the following w.r.t. x :

$\sin(\log x), x > 0$

Watch Video Solution

17. Differentiate the following w.r.t. x :

$\log(\cos 5x)$

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18. Differentiate the following w.r.t. x :

$\cot\left(\log x + e^{\sqrt{x}}\right)$

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19. Differentiate the following w.r.t. x :

$$2l_n\left(\frac{x - 1}{x + 1}\right)$$

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20. Differentiate the following w.r.t. x :

$$x^2 l_n\left(\sqrt{\frac{x^2 + 9}{x^2 + 4}}\right)$$

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21. Differentiate the following w.r.t. x :

$$\ln(\sec x + \tan x)$$

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22. Differentiate the following w.r.t. x :

$$l_n\left(\sqrt{\frac{1 - \cos x}{1 + \cos x}}\right)$$

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23. Differentiate the following w.r.t. x :

$$\log\left(\frac{1 + x}{1 - x}\right)$$

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24. Differentiate the following w.r.t. x :

$$\log\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$$

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25. Differentiate the following w.r.t. x :

$$\log\left(\frac{x+\sqrt{x^2-a^2}}{x-\sqrt{x^2-a^2}}\right)$$

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26. Differentiate the following w.r.t. x :

$$\log\sin^{-1}\left(2x\sqrt{1-x^2}\right)$$

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27. Differentiate the following w.r.t. x :

$$\sqrt{\log\left(\sin\left(\frac{x^2}{3}-1\right)\right)}$$

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28. Find $\frac{dy}{dx}$ when :

$\sin y + \log y = x^2 + 18x + 3$

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29. Find $\frac{dy}{dx}$ when :

$xy + xe^{-y} + ye^x = x^2.$

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30. if $e^{x+y} = xy$, show that $\frac{dy}{dx} = \frac{y(1-x)}{x(y-1)}$

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31. if $y = \frac{\sin^{-1}x}{\sqrt{1-x^2}}$, prove that $\left(1-x^2\right)\frac{dy}{dx} = xy + 1$

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32. If $x = \tan\left(\frac{1}{a}\log y\right)$, show that $\left(1 + x^2\right)\frac{dy}{dx} = ay.$

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33. Differentiate $\tan^{-1}\left(\frac{2^{x+1}}{1 - 4^x}\right)$ with respect to x.

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EXERCISE 5(f) (LONG ANSWER TYPE QUESTIONS (II))

1. Prove that :

$$\frac{d}{dx}\left[\log\left(\frac{x^2 + x + 1}{x^2 - x + 1}\right) + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x\sqrt{3}}{1 - x^2}\right)\right] = \frac{4}{1 + x^2 + x^4}$$

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2. Prove that :

$$\frac{d}{dx}\left[\frac{x\sin^{-1}x}{\sqrt{1-x^2}}+\log\sqrt{1-x^2}\right]=\frac{\sin^{-1}x}{\left(1-x^2\right)^{3/2}}.$$

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3. If $y=\log_{10}\sin x$, prove that :

$$\frac{dy}{dx}=\left(\log_{10}e\right)\cot x.$$

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4. If $y=\sin[\sin(\log 3x)]$, find $\frac{dy}{dx}$.

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5. Find, from first principle, the derivative of the following w.r.t. x :

e^{2x}

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6. Find, from first principle, the derivative of the following w.r.t. x :

$e^{\sqrt{x}}$

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7. Find, from first principle, the derivative of the following w.r.t. x :

xe^{x}

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8. Find, from first principle, the derivative of the following w.r.t. x :

$e^{\sin x}$

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9. Find, from first principle, the derivative of the following w.r.t. x :

$e^{\sqrt{\tan x}}$

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10. Find, from first principle, the derivative of the following w.r.t. x :

$\log(\sin x)$

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11. Find, from first principle, the derivative of the following w.r.t. x :

$\log(\cos x)$

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12. Find, from first principle, the derivative of the following w.r.t. x :

$\log x^2$

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13. Find, from first principle, the derivative of the following w.r.t. x :

$\cos(\log x)$, where $x > 0$.

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EXERCISE 5(g) (SHORT ANSWER TYPE QUESTIONS)

1. If x and y are connected parametrically by the equations given, without eliminating the parameter, Find $\frac{dy}{dx}.x = 2at^2, y = at^4$

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2. Find $\frac{dy}{dx}$, if x and y are connected parametrically by the equations, given below without eliminating the parameter:

$x = \frac{2t}{1 + t^2}, y = \frac{1 - t^2}{1 + t^2}$

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3. Find $\frac{dy}{dx}$, if x and y are connected parametrically by the equations, given below without eliminating the parameter:

$x = \frac{1 - t^2}{1 + t^2}, y = \frac{2t}{1 + t^2}$

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4. Find $\frac{dy}{dx}$, if x and y are connected parametrically by the equations, given below without eliminating the parameter:

$$x = \frac{a\left(1 + t^2\right)}{1 - t^2}, y = \frac{2t}{1 - t^2}$$

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5. Find $\frac{dy}{dx}$, if x and y are connected parametrically by the equations, given below without eliminating the parameter:

$$x = a\sqrt{\frac{t^2 - 1}{t^2 + 1}}, y = at\sqrt{\frac{t^2 - 1}{t^2 + 1}}.$$

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6. If x and y are connected parametrically by the equations given, without eliminating the parameter, Find $\frac{dy}{dx}$.$x = \sin t, y = \cos 2t$

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7. Find $\frac{dy}{dx}$, if x and y are connected parametrically by the equations, given below without eliminating the parameter:

$x = \log t, y = \sin t$

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8. Find $\frac{dy}{dx}$, if x and y are connected parametrically by the equations, given below without eliminating the parameter:

$x = e^t \cos t, y = e^t \sin t$ at $t = \frac{\pi}{2}$.

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9. Find $\frac{dy}{dx}$, if $x = a\cos\theta$, $y = a\sin\theta$

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10. Find $\frac{dy}{dx}$, if x and y are connected parametrically by the equations, given below without eliminating the parameter:

$x = a\cos\theta, y = b\sin\theta$

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11. If x and y are connected parametrically by the equations given, without eliminating the parameter, Find $\frac{dy}{dx}$.$x = a\cos\theta, y = b\cos\theta$

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12. Find $\frac{dy}{dx}$, if x and y are connected parametrically by the equations, given below without eliminating the parameter:

$x = a\cos^2\theta, y = b\sin^2\theta$

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13. Find $\frac{dy}{dx}$, if x and y are connected parametrically by the equations, given below without eliminating the parameter:

$x = 2\cos^2\theta, y = 2\sin^2\theta$

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14. Find $\frac{dy}{dx}$, if x and y are connected parametrically by the equations, given below without eliminating the parameter:

$x = 2\cos^3\theta, y = 2\sin^3\theta$

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15. Find $\frac{dy}{dx}$, if x and y are connected parametrically by the equations, given below without eliminating the parameter:

$x = 3\cos^3\theta, y = 3\sin^3\theta$

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16. Find $\frac{dy}{dx}$, if x and y are connected parametrically by the equations, given below without eliminating the parameter:

$x = c\tan\theta, y = c\cot\theta$

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17. Find $\frac{dy}{dx}$, if x and y are connected parametrically by the equations, given below without eliminating the parameter:

$x = a\tan^2\theta, y = b\sec^2\theta$

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18. If x and y are connected parametrically by the equations given, without eliminating the parameter, Find $\frac{dy}{dx}.x = a\sec\theta, y = b\tan\theta$

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19. If x and y are connected parametrically by the equations given, without eliminating the parameter, Find $\frac{dy}{dx}$.

$x = \frac{\sin^3 t}{\sqrt{\cos 2t}}, y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$

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20. If $y = a(\theta + \sin\theta), x = a(1 - \cos\theta)$, then $\frac{dy}{dx} =$

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21. Find $\frac{dy}{dx}$ if $x = a(\theta - \sin\theta)$ and $y = a(1 - \cos\theta)$.

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22. If x and y are connected parametrically by the equations given, without eliminating the parameter, Find $\frac{dy}{dx}$.

$x = a(\theta - \sin\theta), y = a(1 + \cos\theta)$

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23. Find $\frac{dy}{dx}$, if x and y are connected parametrically by the equations, given below without eliminating the parameter:

$y = a(\theta + \sin\theta), x = a(1 + \cos\theta)$

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24. Find $\frac{dy}{dx}$, if x and y are connected parametrically by the equations, given below without eliminating the parameter:

$x = a(\theta - \sin\theta), y = b(1 + \cos\theta)$

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25. If x and y are connected parametrically by the equations given, without eliminating the parameter, Find $\frac{dy}{dx}$.

$x = \cos\theta - \cos 2\theta, y = \sin\theta - \sin 2\theta$

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26. Find $\frac{dy}{dx}$, if x and y are connected parametrically by the equations, given below without eliminating the parameter:

$x = a(\cos\theta - \cos 2\theta), y = a(\sin\theta - \sin 2\theta)$

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27. If x and y are connected parametrically by the equations given, without eliminating the parameter, Find $\frac{dy}{dx}$.

$x = a(\cos\theta + \theta\sin\theta), y = a(\sin\theta - \theta\cos\theta)$

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28. Find $\frac{dy}{dx}$, if x and y are connected parametrically by the equations, given below without eliminating the parameter:

$x = \cos 2\theta + 2\cos\theta, y = \sin 2\theta - 2\sin\theta$.

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EXERCISE 5(g) (LONG ANSWER TYPE QUESTIONS (I))

1. Find $\frac{dy}{dx}$, if x and y are connected parametrically by the equations, given below without eliminating the parameter:

$$x = a\sin\theta, y = a\left(\cos\theta + \log\tan\frac{\theta}{2}\right)$$

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2. Find $\frac{dy}{dx}$, if x and y are connected parametrically by the equations, given below without eliminating the parameter:

$$y = 12(1 - \cos t), x = 10(t - \sin t), \ -\frac{\pi}{2} < t < \frac{\pi}{2}.$$

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3. If $x = 2\cos\theta - \cos 2\theta, y = 2\sin\theta - \sin 2\theta$, find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{2}$.

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4. If $x = a\left(\cos\theta + \log\tan\frac{\theta}{2}\right), y = a\sin\theta$, find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{3}$.

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5. Find $\frac{dy}{dx}$, where $x = t^3 + \frac{1}{t}$ and $y = \left(t + t^2\right)^3$.

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6. For a positive constant a find $\frac{dy}{dx}$, where $y = a^{t + \frac{1}{t}}$ and $x = \left(t + \frac{1}{t}\right)^a$.

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7. If $x = a\sin 2t(1 + \cos 2t)$ and $y = b\cos 2t(1 - \cos 2t)$, show that at $\frac{\pi}{4}, \frac{dy}{dx} = \frac{b}{a}$.

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8. If $x = \frac{1 + \log t}{t^2}, y = \frac{3 + 2\log t}{t}, t > 0$, prove that :

$\frac{dy}{dx} = t$

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9. If $x = \frac{1 + \log t}{t^2}, y = \frac{3 + 2\log t}{t}, t > 0$, prove that :

$y\frac{dy}{dx} - 2x\left(\frac{dy}{dx}\right)^2 = 1.$

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10. Find $\frac{dy}{dx}$, when :

$x = e^t(\sin t + \cos t)$ and $y = e^t(\sin t - \cos t)$.

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EXERCISE 5(h) (SHORT ANSWER TYPE QUESTIONS)

1. Differentiate the following w.r.t. as indicated :

x^2 w.r.t. x^3.

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2. Differentiate the following w.r.t. as indicated :

$$\frac{ax+b}{cx+d} \text{ w.r.t. } \frac{a'x+b'}{c'x+d'}$$

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3. Differentiate the following w.r.t. as indicated :

e^{2x} w.r.t. e^x

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4. Differentiate the following w.r.t. as indicated :

$$\frac{x^2}{1+x^2} \text{ w.r.t. } x^2$$

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5. Differentiate the following w.r.t. as indicated :

$$\left(x^2-1\right)^{4/5} \text{ w.r.t. } |x|$$

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6. Differentiate the following w.r.t. as indicated :

$$\cos^{-1}\theta \text{ w.r.t. } \log(1+\theta)$$

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7. Differentiate the following w.r.t. as indicated :

$\sin^{-1}\theta$ w.r.t. $\log(1+\theta)$

Watch Video Solution

8. Differentiate the following w.r.t. as indicated :

$\tan x$ w.r.t. $\cos x$

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9. Differentiate the following w.r.t. as indicated :

$\sec x$ w.r.t. $\operatorname{cosec} x$

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10. Differentiate the following w.r.t. as indicated :

$\sin^2 x$ w.r.t. $e^{\cos x}$

Watch Video Solution

11. Differentiate the following w.r.t. as indicated :

$\tan^2 x$ w.r.t. $\sec^2\left(x^2\right)$

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EXERCISE 5(h) (LONG ANSWER TYPE QUESTIONS (I))

1. Differentiate w.r.t. as indicated :

$$\cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right) \text{ w.r.t. } \tan^{-1}x$$

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Watch Video Solution

2. Differentiate w.r.t. as indicated :

$$\sin^{-1}\left(\frac{2x}{1+x^2}\right) \text{ w.r.t. } \tan^{-1}x$$

Watch Video Solution

3. Differentiate w.r.t. as indicated :

$$\sin^{-1}\left(\frac{2x}{1+x^2}\right) \text{ w.r.t. } \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

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4. Differentiate w.r.t. as indicated :

$$\tan^{-1}\frac{3x-x^3}{1-3x^2} \text{ w.r.t. } \tan^{-1}\frac{2x}{1-x^2}$$

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5. Differentiate $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ with respect to $\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$.

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6. Differentiate w.r.t. as indicated :

$$\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) \text{ w.r.t. } \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

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7. Differentiate w.r.t. as indicated :

$$\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) \text{ w.r.t. } \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

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8. Differentiate w.r.t. as indicated :

$$\tan^{-1}\left(\frac{\sqrt{1+a^2x^2}-1}{ax}\right) \text{ w.r.t. } \tan^{-1}ax$$

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9. Differentiate w.r.t. as indicated :

$$\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right) \text{ w.r.t. } \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

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10. Differentiate w.r.t. as indicated :

$$\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right) \text{ w.r.t. } \sin^{-1}\left(2x\sqrt{1-x^2}\right).$$

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11. Differentiate w.r.t. as indicated :

$$\tan^{-1}\left(\frac{x}{1+\sqrt{1-x^2}}\right) \text{ w.r.t. } \sin\left(2\cot^{-1}\sqrt{\frac{1+x}{1-x}}\right)$$

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12. Write the derivative of e^x wrt. $\sqrt{x}$

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13. Differentiate w.r.t. as indicated :

$\log_{10}x$ w.r.t. x^2

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14. Differentiate w.r.t. as indicated :

$\sin x^2$ w.r.t. x^3

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15. Differentiate w.r.t. as indicated :

$\sqrt{1+x^2}$ w.r.t. $\tan^{-1}x$

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16. Prove that derivative of $\tan^{-1}\left(\frac{x}{1+\sqrt{1-x^2}}\right)$ w.r.t. $\sin^{-1}x$ is independent of x.

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17. Prove that the derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ w.r.t. $\tan^{-1}x$ is independent of x.

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18. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)w.r.t.\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

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19. Differentiate $\tan^{-1}\left\{\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right\}$ with respect to $\cos^{-1}x^2$

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20. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}+1}{x}\right)$ w.r.t. $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at x = 0.

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EXERCISE 5(i) (SHORT ANSWER TYPE QUESTIONS)

1. Differentiate the following w.r.t. x :

$\left(\sqrt{x}\right)^{\sqrt{x}}$

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2. Differentiate the following w.r.t. x :

$\left(\frac{1}{2}\right)^{x}$

Watch Video Solution

3. Differentiate the following w.r.t. x :

$$\left(x^{x}\right)^{x}$$

Watch Video Solution

4. Differentiate the following w.r.t. x :

$$x^{x^{2}}$$

Watch Video Solution

5. Differentiate the following w.r.t. x :

$$(5x)^{3\cos 2x}$$

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6. Differentiate the following w.r.t. x :

$x^{\sin x}, x > 0$

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7. Differentiate the following w.r.t. x :

$(\sin x)^{x}$

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8. Differentiate the following w.r.t. x :

$x^{\sin^{-1} x}$

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9. Differentiate the following w.r.t. x :

$x^x \sin^{-1}\sqrt{x}$

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10. Differentiate the following w.r.t. x :

$(\sin x)^{\log x}, \sin x > 0$

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11. Differentiate the following w.r.t. x :

$(\sin x)^{\tan x}$

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12. Differentiate the following w.r.t. x :

$(\sin x)^{\sin x}$

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13. Differentiate the following w.r.t. x :

$\left(\sec^2 x\right)^{1/x}$

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14. Differentiate the following w.r.t. x :

$(x\cos x)^x$

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15. Differentiate the following w.r.t. x :

$(x)^{\log x}$

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16. Differentiate the following w.r.t. x :

$(\log x)^{\log x}, x > 1$

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17. Differentiate the following w.r.t. x :

$x^{\sin 2x + \cos 2x}$

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18. Differentiate the following w.r.t. x :

$x^{\sin x + \cos x}$

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19. Differentiate the following w.r.t. x :

$(\log x)^{x}$

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20. Differentiate the following w.r.t. x :

$\left(\sin^{-1}x\right)^{x}$

Watch Video Solution

21. Differentiate the following w.r.t. x :

$$\left(\tan^{-1}x\right)^{x}$$

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22. Differentiate the following w.r.t. x :

$$x^{\cos^{-1}x}$$

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23. Differentiate the following w.r.t. x :

$$(\sin x)^{\cos^{-1}x}$$

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24. Differentiate the following w.r.t. x :

$(\sin x - \cos x)^{\sin x - \cos x}, \frac{\pi}{4} < x < \frac{3\pi}{4}$

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25. Differentiate the following w.r.t. x :

$\cos\left(x^x\right)$

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26. Differentiate the following w.r.t. x :

$(1 + x)^{\log x}$

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27. Differentiate the following w.r.t. x :

$(\log x)^{\cos x}$

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EXERCISE 5(i) (LONG ANSWER TYPE QUESTIONS (I))

1. Differentiate the following w.r.t. x :

$x^{\sin x} + (\sin x)^{x}$

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2. Differentiate the following w.r.t. x :

$(x)^{\log x} + (\log x)^{x}$

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3. Differentiate the following w.r.t. x :

$(x)^{\tan x} + (\tan x)^{x}$

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4. Differentiate the following w.r.t. x :

$x^{\cot x} + (\cos x)^{\sin x}$

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5. Differentiate the following w.r.t. x :

$x^{\cos x} + (\cos x)^{x}$

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6. Differentiate the following w.r.t. x :

$x^{\cos x} + (\cos x)^{\sin x}$

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7. Differentiate the following w.r.t. x :

$(\sin x)^{\cos x} + (\cos x)^{\sin x}$

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8. $y = (\sin x)^{\tan x} + (\cos x)^{\sec x}$

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9. Differentiate the following w.r.t. x :

$(\log x)^{x} + (x)^{\cos x}$

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10. Differentiate the following w.r.t. x :

$x^{\sin x} + (\log x)^{x}$

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11. Differentiate the following w.r.t. x :

$(x)^{\sin x} + \sin\left(x^{x}\right)$

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12. Differentiate the following w.r.t. x :

$(x)^{x} + (\sin x)^{x}$

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13. Differentiate the following w.r.t. x :

$(x)^{\sin x} + (\cos x)^{x}$

Watch Video Solution

14. Differentiate the following w.r.t. x :

$x^{\sin x} + (\sin x)^{\cos x}$

Watch Video Solution

15. Differentiate the following w.r.t. x :

$(\sin x)^{\sec x} + (\tan x)^{\cos x}$

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16. Differentiate the following w.r.t. x :

$(\tan x)^{\cot x} + x^{\tan x}, 0 < x < \frac{\pi}{4}$

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17. Differentiate the following w.r.t. x :

$x^{\sin x} + (\sin x)^{\cos x}$

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18. Differentiate the following w.r.t. x :

$(\sin x)^{x}+\sin^{-1}\sqrt{x}$

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19. Differentiate the following w.r.t. x:$(\log x)^{x}+x^{\log x}$

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20. Differentiate the following w.r.t. x :

$(\log x)^{\cos x}+\frac{x^{2}+1}{x^{2}-1}$

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21. Differentiate the following w.r.t. x:$(x\cos x)^{x}+(x\sin x)^{\frac{1}{x}}$

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22. Differentiate the following w.r.t. x :

$(\cos x)^{x} + (\sin x)^{1/x}$

Watch Video Solution

23. $y = e^{\sin x} + (\tan x)^{x}$

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24. Differentiate the following w.r.t. x :

$x^{x} - 2^{\sin x}$

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25. Differentiate the functions given w.r.t. x: $\left(x+\frac{1}{x}\right)^{x}+x^{\left(1+\frac{1}{x}\right)}$

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26. Differentiate the following w.r.t. x :

$x^{x^2-3}+(x-3)^{x^2}$, for $x>3$

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27. Differentiate the following w.r.t. x :

$x^{x}+(\sin x)^{x}$

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28. Differentiate the following w.r.t. x :

$$\frac{(ax+b)(cx+d)}{(ax-b)(cx-d)}, x \neq \frac{b}{a}, \frac{d}{c}$$

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29. Differentiate the following w.r.t. x :

$$\sqrt{\frac{(x-3)\left(x^2+4\right)}{3x^2+4x+5}}$$

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30. Differentiate the following w.r.t. x :

$x^2 e^x \sin x$

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31. Differentiate the following w.r.t. x :

$e^{x}\cos^{3}x\sin^{2}x$

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32. Differentiate the following w.r.t. x :

$(x+3)^{2}(x+4)^{3}(x+5)^{4}$

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33. Differentiate the following w.r.t. x :

$\sqrt{(x-1)(x-2)(x-3)(x-4)}$

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34. If $xy = e^{x-y}$, find $\frac{dy}{dx}$.

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35. If $(\sin x)^y = (\sin y)^x$, find $\frac{dy}{dx}$.

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36. Find $\frac{dy}{dx}$ if $(\sin x)^{\cos y} = (\cos y)^{\sin x}$.

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37. Differentiate $\log\left(x^x + \operatorname{cosec}^2 x\right)$ w.r.t. x.

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38. If $x^p y^q = (x + y)^{p+q}$, show that $\frac{dy}{dx} = \frac{y}{x}$.

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39. If $y = x^y$, show that $\frac{dy}{dx} = \frac{y^2}{x(1 - y\log x)}$

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40. If $y^x = e^{y-x}$, then prove that $\frac{dy}{dx} = \frac{(1 + \log y)^2}{\log y}$

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41. If $x^x + y^x = 1$, prove that $\frac{dy}{dx} = - \left\{ \frac{x^x(1 + \log x) + y^x \log y}{xy^{(x-1)}} \right\}$

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42. If $x^y + y^x = 1$, find $\dfrac{dy}{dx}$

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43. If $x^y + y^x = a^b$, then find $\dfrac{dy}{dx}$.

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44. If $x^y + y^x = 4$, find $\dfrac{dy}{dx}$

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45. If $x^y + y^x = \log a$, find $\dfrac{dy}{dx}$.

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46. Show that if $x^y + y^x = m^n$, then :

$$\frac{dy}{dx} = -\frac{y^x \log y + yx^{y-1}}{x^y \log x + xy^{x-1}}.$$

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47. Find the derivative of the function given by :

$f(x) = (1 + x)\left(1 + x^2\right)\left(1 + x^4\right).... \left(1 + x^{2n}\right)$ and hence, find f'(0).

A. 0

B. -1

C. 1

D. 2

Answer: c

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48. Differentiate $\left(x^2 - 5x + 8\right)\left(x^3 + 7x + 9\right)$ in three ways mentioned below:(i) by using product rule(ii) by expanding the product to obtain a single polynomial.(iii) by logarithmic differentiation.Do they all give the same answer?

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49. If u, v and w are functions of x, then show that $\frac{d}{dx}\left(u \cdot v \cdot w\right) = \frac{du}{dx} v \cdot w + u \cdot \frac{dv}{dx} \cdot w + u \cdot v \frac{dw}{dx}$ in two ways - first by repeated application of product rule, second by logarithmic differentiation.

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EXERCISE 5(j) (LONG ANSWER TYPE QUESTIONS (I))

1. If $y = \sqrt{x + \sqrt{x + \sqrt{x + \ldots \rightarrow \infty}}}$, prove that $\frac{dy}{dx} = \frac{1}{2y - 1}$

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2. If

$y = \sqrt{3^x + \sqrt{3^x + \sqrt{3^x + \ldots\text{to } \infty}}}$, prove that $: (2y - 1)\frac{dy}{dx} = 3^x \log 3.$

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3. If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \quad \rightarrow \infty}}}$, $provethat \frac{dy}{dx} = \frac{\cos x}{2y - 1}$

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4. If $y = (\sin x)^{(\sin x)^{(\sin x) \ldots \infty}}$, prove that $\frac{dy}{dx} = \frac{y^2 \cot x}{(1 - y \log \sin x)}$.

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5. If

$y = (\cos x)^{\cos x} \wedge (\cos x) \wedge ((((\infty)))), provethat \frac{dy}{dx} = - \frac{y^2 \tan x}{(1 - y\log\cos x)}.$

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6. If $y = (\tan x)^{(\tan x)^{(\tan x)^{\cdots \infty}}}$, then prove that $\frac{dy}{dx} = 2$ at $x = \frac{\pi}{4}$.

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7. If $y = x^{x^{x^{\dots \infty}}}$ then prove that $x\frac{dy}{dx} = \frac{y^2}{1 - y\log x}$

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8. If $y = a^{x^{a^{x \dots \infty}}}$, prove that $\frac{dy}{dx} = \frac{y^2(\log y)}{x[1 - y(\log x)(\log y)]}$.

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9. If $y = x + \cfrac{1}{x + \cfrac{1}{x + \cfrac{1}{x + \dots \infty}}}$, prove that $\frac{dy}{dx} = \frac{y}{(2y - x)}$.

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10. If $y = \cfrac{\sin x}{1 + \cfrac{\cos x}{1 + \cfrac{\sin x}{1 + \cfrac{\cos x}{1 + \ddot{\to} \infty}}}}$, prove that $\frac{dy}{dx} = \frac{(1 + y)\cos x + y\sin x}{1 + 2y + \cos x - \sin x}$

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EXERCISE 5(k) (SHORT ANSWER TYPE QUESTIONS)

1. Find $(a)\frac{dy}{dx}$ and $(b)\frac{d^2y}{dx^2}$ when y is given by :

1 + 2x.

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2. Find $(a)\frac{dy}{dx}$ and $(b)\frac{d^2y}{dx^2}$ when y is given by :

$ax^3 + bx^2 + cx + d$

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3. Find $(a)\frac{dy}{dx}$ and $(b)\frac{d^2y}{dx^2}$ when y is given by :

$\frac{1}{2x+3}, x \neq -\frac{3}{2}$

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4. Find $(a)\frac{dy}{dx}$ and $(b)\frac{d^2y}{dx^2}$ when y is given by :

$\log x$ - x

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5. Find $(a)\frac{dy}{dx}$ and $(b)\frac{d^2y}{dx^2}$ when y is given by :

$e^x + x^4$.

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6. Find the second derivatives of the following functions :

x^{20}

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7. Find the second derivatives of the following functions :

$x^2 + 3x + 2$

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8. Find the second derivatives of the following functions :

$x\cos x$

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9. Find the second derivatives of the following functions :

$x^3 + \tan x$

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10. Find the second derivatives of the following functions :

$\log x$

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11. Find the second derivatives of the following functions :

$x^3 \log x$

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12. Find the second derivatives of the following functions :

$\log(\log x)$

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13. Find the second derivatives of the following functions :

$\sin(\log x)$

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14. Find the second derivatives of the following functions :

$e^{x}\sin 5x$

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15. Find the second derivatives of the following functions :

$e^{6x}\cos 3x$

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16. Find the second derivatives of the following functions :

$e^{-x}\cos x$

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17. Find the second derivatives of the following functions :

$\tan x + \sec x$

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18. Find the second derivatives of the following functions :

$$\frac{\log x}{x}$$

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19. Find the second derivatives of the following functions :

x^x

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EXERCISE 5(k) (LONG ANSWER TYPE QUESTIONS (I))

1. If $y = \cos^{-1}x$, then show that :

$$\left(1 - x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$$

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2. If $y = a\sin^{-1}x$, then show that :

$$\left(1 - x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$$

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3. If $y = ae^{2x} + be^{-x}$, show that, $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$.

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4. If $y = 3\,e^{2x} + 2\,e^{3x}$, prove that $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$.

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5. If $y = Ae^{mx} + Be^{nx}$, show that $\frac{d^2y}{dx^2} - \frac{(m+n)dy}{dx} + mny = 0.$

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6. If $y = Pe^{ax} + Qe^{bx}$, show that :

$$\frac{d^2y}{dx^2} - (a+b)\frac{dy}{dx} + aby = 0.$$

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7. If $y = 2\sin x + 3\cos x$, prove that : $y + \frac{d^2y}{dx^2} = 0$

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8. If $y = 5\sin x$ - $3\sin x$, prove that : $\frac{d^2y}{dx^2} + y = 0.$

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9. If $e^y(x + 1) = 1, showthat \frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^{2 \cdot}$

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10. If $y = A\sin x + B\cos x$, prove that $\frac{d^2y}{dx^2} + y = 0.$

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11. Find $\frac{d^2y}{dx^2}$ in the following

$x = at^2, y = 2at$

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12. Find $\frac{d^2y}{dx^2}$ in the following

$x = \frac{2at^2}{1+t}, y = \frac{3at}{1+t}$

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13. Find $\frac{d^2y}{dx^2}$ in the following

$x = a\cos\theta, y = b\sin\theta$

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14. Find $\frac{d^2y}{dx^2}$ in the following

$x = a\cos^3\theta, y = a\sin^3\theta$

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15. Find $\frac{d^2y}{dx^2}$ in the following

$x = a\cos^3\theta, y = b\sin^3\theta$

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16. Find $\frac{d^2y}{dx^2}$ in the following

If $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$, find $\frac{d^2y}{dx^2}$

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17. Find $\frac{d^2y}{dx^2}$ in the following

If $x = a\cos^3\theta$ and $y = a\sin^3\theta$, then find the value of $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{6}$.

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18. Find $\frac{d^2y}{dx^2}$ in the following

$x = a(\cos t + t\sin t), y = a(\sin t - t\cos t)$

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19. Find $\frac{d^2y}{dx^2}$ in the following

$x = a(\theta + \sin\theta), y = a(1 + \cos\theta)$

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20. Find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{2}$ when :

$x = a(\theta + \sin\theta), y = a(1 - \cos\theta)$

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21. Find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{2}$ when :

$x = a(\theta - \sin\theta), y = a(1 + \cos\theta)$

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22. Find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{2}$ when :

$x = a(1 - \cos\theta), y = a(\theta + \sin\theta)$

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23. Find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{2}$ when :

$x = a(\theta - \sin\theta), y = a(1 - \cos\theta)$

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24. Find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$ when :

$x = a(\cos\theta + \log\tan\theta/2), y = a\sin\theta$

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25. If $x = \cos t + \frac{\log \tan t}{2}$, $y = \sin t$, then find the value of $\frac{d^2y}{dt^2}$ and $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$.

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26. Find $\frac{d^2y}{dx^2}$ when :

$x = 2\cos\theta - \cos 2\theta$ and $y = 2\sin\theta - \sin 2\theta$

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27. If $x = a(\cos 2\theta + 2\theta \sin 2\theta)$ and $y = a(\sin 2\theta - 2\theta \cos 2\theta)$, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{8}$.

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28. If $x = a\sin t$ and $y = a\left(\cos t + \frac{\log\tan t}{2}\right)$, find $\frac{d^2y}{dx^2}$

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29. If $x + y = \tan^{-1}y$ and $\frac{d^2y}{dx^2} = f(y)\frac{dy}{dx}$, then $f(y) =$

A. $-\frac{2}{y^3}$

B. $\frac{2}{y^3}$

C. $\frac{1}{y}$

D. $-\frac{1}{y}$

Answer: B

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30. If $y = \left(\sin^{-1}x\right)^2$ then prove that $\left(1 - x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0.$

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31. If $y = \left(\cos^{-1}x\right)^2$, then prove that :

$\left(1 - x^2\right)y_2 - xy_1 - 2 = 0.$

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32. If $y = \left(\tan^{-1}x\right)^2$, show that $\left(x^2 + 1\right)^2 y_2 + 2x\left(x^2 + 1\right)y_1 = 2$

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33. If $y = \left(\cot^{-1}x\right)^2$, then show that

$\left(x^2 + 1\right)^2\frac{d^2y}{dx^2} + 2x\left(x^2 + 1\right)\frac{dy}{dx} = 2.$

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34. If $y = \frac{s \in^{-1} x}{\sqrt{1 - x^2}}$, show that $\left(1 - x^2\right)\frac{d^2 y}{dx^2} - 3x\frac{dy}{dx} - y = 0$

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35. If $y = \log\left[x + \sqrt{x^2 + 1}\right]$, prove that $\left(x^2 + 1\right)\frac{d^2 y}{dx^2} + x\frac{dy}{dx} = 0$

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36. If $y = \log\left\{x + \sqrt{x^2 + a^2}\right\}$, prove that: $\left(x^2 + a^2\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$.

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37. If $y = \left\{x + \sqrt{x^2 + 1}\right\}^m$, then show that $\left(x^2 + 1\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - m^2y = 0.$

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38. If $y = \cos\left(m\cos^{-1}x\right)$, then prove that :

$\left(1 - x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2y = 0.$

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39. If $y = \sin\left(m\tan^{-1}x\right)$, prove that :

$\left(1 + x^2\right)^2y_2 + 2x\left(1 + x^2\right)y_1 + m^2y = 0.$

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40. If $y = e^{m\sin^{-1}x}, \ -1 \le x \le 1$, show that :

$\left(1 - x^2\right)y_2 - xy_1 - m^2y = 0.$

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41. If $y = e^{a\cos^{-1}x}, \ -1 \le x \le 1$, show that : $\dfrac{dy}{dx} = \dfrac{-ae^{a\cos^{-1}x}}{\sqrt{1 - x^2}}$

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42. If $y = e^{a}\cos^{(-1)x}, \ -1 \le x \le 1$, show that $\left(1 - x^2\right)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - a^2y = 0.$

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43. If $y = e^{m\tan^{-1}x}$, prove that :

$$\left(1 + x^2\right)\frac{d^2y}{dx^2} + (2x - m)\frac{dy}{dx} = 0.$$

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44. If $y = e^{ax}\cos bx$, then prove that :

$$\frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + \left(a^2 + b^2\right)y = 0.$$

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45. If $y = \sin^{-1}x$, show that $\left(1 - x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0.$

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46. If $y = \tan x$, then show that : $\frac{d^2y}{dx^2} = 2y\frac{dy}{dx}$

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47. If $y = \tan^{-1}x$, show that $\left(1 + x^2\right)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = 0$.

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48. If $x = f(t), y = g(t)$ possess second order derivatives for all t in (a, b) and $f'(t)$ is invertible $[f'(t) \neq 0]$, then prove that :

$$\frac{d^2y}{dx^2} = \frac{f'(t)g''(t) - g'(t)f''(t)}{[f'(t)]^3}.$$

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49. If $y^m x^n = (x + y)^{m+n}$, then find $\frac{dy}{dx}$

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50. If $x^m y^n = (x+y)^{m+n}$, prove that :

$$\frac{d^2y}{dx^2} = 0.$$

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51. If $p^2 = a^2\cos^2\theta + b^2\sin^2\theta$, then show that :

$$p + \frac{d^2p}{d\theta^2} = \frac{a^2b^2}{p^3}.$$

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EXERCISE 5(I) (SHORT ANSWER TYPE QUESTIONS)

1. Verify the truth of Rolle's Theorem for the following functions

$f(x) = x^2 + 2, a = -2, \text{ and } b = 2$

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2. Verify the truth of Rolle's Theorem for the following functions

$$f(x) = \frac{x^3}{3} - \frac{5}{3}x^2 + 2x, x \in [0, 3]$$

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3. Verify the truth of Rolle's Theorem for the following functions

$$f(x) = \frac{x(x - 2)}{x - 1} \quad \text{on} \quad [0, 2]$$

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4. Verify the truth of Rolle's Theorem for the following functions

$f(x) = x^2 + 2x - 8$ defined in the interval [-4, 2].

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5. Verify the truth of Rolle's Theorem for the following functions

$f(x) = x^{3/5}$ in $[-1, 1]$

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6. Verify the truth of Rolle's Theorem for the following functions

$f(x) = x^{1/3}$ in $[-1, 1]$

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7. Verify the truth of Rolle's Theorem for the following functions

$f(x) = |x|$ in $[-1, 1]$

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8. Verify the truth of Rolle's Theorem for the following functions

$f(x) = |x - 1|$ in $[1, 2]$

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9. Verify the truth of Rolle's Theorem for the following functions

$f(x) = \sqrt{x - 2}$ in $[1, 2]$

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10. Verify the truth of Rolle's Theorem for the following functions

$f(x) = [x]$ in $[-1, 1]$

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11. Verify the conditions of Rolle's Theorem in the following problems. In each case, find a point in the interval, where the derivative vanishes :

$x^2 - 1$ on $[-1, 1]$

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12. Verify the conditions of Rolle's Theorem in the following problems. In each case, find a point in the interval, where the derivative vanishes :

$\left(x^2 - 1\right)(x - 2)$ on $[-1, 2]$

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13. Verify the conditions of Rolle's Theorem in the following problems. In each case, find a point in the interval, where the derivative vanishes :

$\sin x - \sin 2x$ on $[0, \pi]$

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14. Verify the conditions of Rolle's Theorem in the following problems. In each case, find a point in the interval, where the derivative vanishes :

$\log\left(x^2 + 2\right) - \log 3$ on $[-1, 1]$

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15. Verify the conditions of Rolle's Theorem in the following problems. In each case, find a point in the interval, where the derivative vanishes :

$e^{1 - x^2}$ on $[-1, 1]$

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EXERCISE 5(I) (LONG ANSWER TYPE QUESTIONS (I))

1. Verify the truth of Rolle's Theorem for the following functions :

$f(x) = x^2 - 5x + 4$ on $[1, 4]$

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2. Verify the truth of Rolle's Theorem for the following functions :

$f(x) = x^2 - 4x + 3$ on $[1, 3]$

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3. Verify the truth of Rolle's Theorem for the following functions :

$f(x) = x^3 - 2x^2 - 3x$ in the interval [-1, 3].

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4. Verify the truth of Rolle's Theorem for the following functions :

$f(x) = x^2 + 2x - 8$ in the interval [-4, 2]

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5. Verify the truth of Rolle's Theorem for the following functions :

$f(x) = x^2 - x - 12$ in the interval [-3, 4]

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6. Verify the truth of Rolle's Theorem for the following functions :

$f(x) = (x - 2)(x - 3)(x - 4)$ in the interval $2 \leq x \leq 4$.

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7. Verify the truth of Rolle's Theorem for the following functions :

$f(x) = x^3 - 4x$ in the interval $-2 \leq x \leq 2$

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8. Verify the truth of Rolle's Theorem for the following functions :

$f(x) = (x - 1)(x - 2)^2$ in the interval [1, 2]

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9. Verify the truth of Rolle's Theorem for the following functions :

$f(x) = (x - 2)(x - 4)^2$ in the interval [2, 4].

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10. Verify Rolle's Theorem in the interval [a, b] for the function :

$f(x) = (x - a)^2(x - b)^2$. Find the value of 'c'.

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11. Examine the applicability of Rolle's Theorem for the function :

$f(x) = 2 + (x - 1)^{2/3}$ in the interval $0 \leq x \leq 2$.

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12. Verify Rolle's Theorem for the functions :

$f(x) = \sin^2 x$, defined in the interval $[0, \pi]$

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13. Verify Rolle's Theorem for the functions :

$f(x) = \cos x$, defined in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

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14. Verify Rolle's Theorem for the functions :

$f(x) = \tan x$, defined in the interval $[0, \pi]$.

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15. Verify Rolle's Theorem for the functions :

$f(x) = \sin x + \cos x$ in the interval $[0, 2\pi]$

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16. Verify Rolle's Theorem for the functions :

$f(x) = \sin x + \cos x + 5$ in the interval $[0, 2\pi]$

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17. Verify Rolle's Theorem for the functions :

$f(x) = \sin x \cos x \text{ in } \left[0, \frac{\pi}{2}\right]$

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18. Verify Rolle's Theorem for the functions :

$f(x) = \sin^3 x + \cos^3 x$ in the interval $\left[0, \frac{\pi}{2}\right]$.

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19. Verify Rolle's Theorem for the function :

$$f(x) = \begin{cases} -4x + 5, & 0 \le x \le 1 \\ 2x - 3, & 1 < x \le 2 \end{cases}$$

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20. At what points on the following curve, is the tangent parallel to x-axis ?

$y = x^2$ on $[-2, 2]$

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21. At what points on the following curve, is the tangent parallel to x-axis ?

$y = \cos x - 1$ on $[0, 2\pi]$

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22. For the function $f(x) = x^3 - 6x^2 + ax + b$, it is given that f(1) = f(3) = 0. Find the values of 'a' and 'b', and hence, verify Rolle's Theorem on [1, 3].

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23. Let $f(x) = (x - 1)(x - 2)(x - 3)$ on the interval [1, 3]. Prove that there is more than one c in (1, 3) such that f'(c) = 0.

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EXERCISE 5(m) (SHORT ANSWER TYPE QUESTIONS)

1. Discuss the applicability of Lagrange's Mean Value Theorem to the following :

$f(x) = x^2 - 1$ on $[1, 2]$.

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2. Discuss the applicability of Lagrange's Mean Value Theorem to the following :

$f(x) = x^2$ on $[2, 4]$

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3. Discuss the applicability of Lagrange's Mean Value Theorem to the following :

$f(x) = x^2 - 2x + 3$ in $[0, 4]$

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4. Verify Lagranges mean value theorem for the following function: $f(x) = \ x^2 + \ 2x + 3, \ f o r\ [4, 6]$

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5. Discuss the applicability of Lagrange's Mean Value Theorem to the following :

$f(x) = 2x^2 - 10x + 29 \text{ in } [2, 7]$

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6. Discuss the applicability of Lagrange's Mean Value Theorem to the following :

$f(x) = x^2 - 4x - 3 \text{ in } [1, 4]$

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7. Discuss the applicability of Lagrange's Mean Value Theorem to the following :

$f(x) = 2x - x^2$ in $[0, 1]$

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8. Discuss the applicability of Lagrange's Mean Value Theorem to the following :

$f(x) = x(2 - x)$ in $[0, 1]$

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9. $f(x) = x^3 - 2x^2 - x + 3$ in $[0, 1]$

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10. Discuss the applicability of Lagrange's Mean Value Theorem to the following :

$f(x) = x^3 - 5x^2 - 3x$ in $[1, 3]$

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11. Discuss the applicability of Lagrange's Mean Value Theorem to the following :

$f(x) = (x - 1)(x - 2)(x - 3)$ in $[0, 4]$

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EXERCISE 5(m) (LONG ANSWER TYPE QUESTIONS (I))

1. Verify the conditions of Mean Value Theorem in the following. In each case, find a point in the interval as stated by the Mean Value

Theorem :

$f(x) = x$ on $[a, b]$

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2. Verify the conditions of Mean Value Theorem in the following. In each case, find a point in the interval as stated by the Mean Value Theorem :

$f(x) = x + \frac{1}{x}$ on $[1, 3]$

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3. Verify the conditions of Mean Value Theorem in the following. In each case, find a point in the interval as stated by the Mean Value Theorem :

$f(x) = ax^2 + bx + c$ on $[0, 1]$

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4. Verify the conditions of Mean Value Theorem in the following. In each case, find a point in the interval as stated by the Mean Value Theorem :

$f(x) = ax^2 + ex + e$ on $[0, 1]$

Watch Video Solution

5. Verify the conditions of Mean Value Theorem in the following. In each case, find a point in the interval as stated by the Mean Value Theorem :

$f(x) = \sin x - \sin 2x$ on $[0, 2\pi]$

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6. Verify the conditions of Mean Value Theorem in the following. In each case, find a point in the interval as stated by the Mean Value Theorem :

$f(x) = \sin x - \sin 2x$ on $[0, \pi]$

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7. Verify Lagrange's Mean Value Theorem for the functions :

$f(x) = x^{1/3}$ in the interval [-1, 1]

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8. Verify Lagrange's Mean Value Theorem for the functions :

$f(x) = (x - 1)^{2/3}$ in the interval [1, 2].

Watch Video Solution

9. Verify Lagrange's Mean Value Theorem for the functions :

$f(x) = \frac{1}{x}$ in the interval [-1, 2]

Watch Video Solution

10. $f(x) = \frac{1}{4x - 1}$ in $[1, 4]$

Watch Video Solution

11. Verify Lagrange's Mean Value Theorem for the functions :

f(x) = |x| in the interval [-1, 1].

Watch Video Solution

12. Verify Lagrange's Mean Value Theorem for the functions :

$f(x) = \sqrt{x^2 - 4}$ in the interval [2, 4].

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13. Verify Lagrange's Mean Value Theorem for the functions :

$f(x) = \sqrt{25 - x^2}$ in the interval [-3, 4].

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14. Verify Lagrange's Mean Value Theorem for the functions :

$f(x) = \log_e x$ in the interval [1, 2]

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15. Verify Lagrange's Mean Value Theorem for the functions :

$f(x) = x \text{ on } [a, b]$

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16. Find 'c' of Lagrange's Mean Value Theorem for the functions :

$f(x) = 2x^2 - 1$ in the interval [1, 2]

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17. Find 'c' of Lagrange's Mean Value Theorem for the functions :

$f(x) = \log x$ in the interval [1, e]

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18. Find 'c' of Lagrange's Mean Value Theorem for the functions :

$f(x) = e^x$ in the interval [0, 1]

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19. Verify Mean Value Theorem, if $f(x) = x^3 - 5x^2 - 3x$in the interval [a, b], where $a = 1 and b = 3$. Find all $c \in (1, 3)$for which $f'(c) = 0$.

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20. If mean value theorem holds for the function $f(x) = (x - 1)(x - 2)(x - 3), x \in [0, 4]$, then c=

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21. Verify Lagrange's Mean Value Theorem for the function :

$f(x) = \begin{cases} 2 + x^3 \text{ if } x \leq 1 \\ 3x \text{ if } x > 1 \end{cases}$ on [-1, 2].

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22. Find a point on the parabola $y = (x - 2)^2$, where the tangent is parallel to the chord joining (2, 0) and (4, 4).

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23. Find a point on the graph of $y = x^3$, where the tangent is parallel to the chord joining (1, 1) and (3, 27).

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24. Find a point on the curve $y = x^3 - 3x$ where the tangent is parallel to the chord joining $(1, -2) and (2, 2)^{\cdot}$

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25. Find the co-ordinates of the point at which the tangent to the curve given by $f(x) = x^2 - 6x + 1$ is parallel to the chord joining the points (1, -4) and (3, -8).

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26. Use Lagrange's Mean value Theorem to determine a point P on the curve $y = \sqrt{x - 2}$, where the tangent is parallel to the chord joining (2, 0) and (3, 1).

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OBJECTIVE TYPE QUESTIONS (MULTIPLE CHOICE QUESTIONS)

1. The function $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, \text{ if } x \neq 0 \\ k, \text{ if } x = 0 \end{cases}$ is continuous at x = 0, then the value of 'k' is :

A. 3

B. 2

C. 1

D. 1.5

Answer: B

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2. The function $f(x) = [x]$, where $[x]$ denotes the greatest integer function,is continuous at

A. 4

B. -2

C. 1

D. 1.5

Answer: D

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3. If the function f(x) defined by

$$f(x) = \begin{cases} x\sin\frac{1}{x}, & \text{for } x \neq 0 \\ k, & \text{for } x = 0 \end{cases}$$

is continuous at x = 0, then k is equal to

A. 8

B. 8

C. -1

D. None of these

Answer: D

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4. Differential coefficient of $\sec\left(\tan^{-1}x\right)$ w.r.t x is

A. $\dfrac{x}{\sqrt{1+x^2}}$

B. $\dfrac{x}{1+x^2}$

C. $x\sqrt{1+x^2}$

D. $\dfrac{1}{\sqrt{1+x^2}}$

Answer: A

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5. If $y = \log\left(\frac{1 - x^2}{1 + x^2}\right)$, then $\frac{dy}{dx}$ is equal to

A. $\frac{4x^3}{1 - x^4}$

B. $\frac{-4x}{1 - x^4}$

C. $\frac{1}{4 - x^4}$

D. $\frac{-4x^3}{1 - x^4}$

Answer: B

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6. If $y = \sqrt{\sin x + y}$, then $\frac{dy}{dx}$ is equal to

A. $\frac{\cos x}{2y - 1}$

B. $\frac{\cos x}{1 - 2y}$

C. $\frac{\sin x}{1 - 2y}$

D. $\frac{\sin x}{2y - 1}$

Answer: A

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7. If $u = \sin^{-1}\left(\frac{2x}{1 + x^2}\right)$ and $v = \tan^{-1}\left(\frac{2x}{1 - x^2}\right)$, where `-1

A. $\frac{1}{2}$

B. x

C. $\dfrac{1 - x^2}{1 + x^2}$

D. 1

Answer: D

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8. If $x = t^2 y = t^3, then \dfrac{d^2y}{dx^2} = \dfrac{3}{2}$ (b) $\dfrac{3}{(4t)}$ (c) $\dfrac{3}{2(t)}$ (d) $\dfrac{3t}{2}$

A. $\dfrac{3}{2}$

B. $\dfrac{3}{4t}$

C. $\dfrac{3}{2t}$

D. $\dfrac{3t}{2}$

Answer: B

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9. The value of c in Rolle's theorem for the function $f(x) = x^3 - 3x$ in the interval $\left[0, \sqrt{3}\right]$ is

A. 1

B. -1

C. $\frac{3}{2}$

D. $\frac{1}{3}$

Answer: A

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10. The value of c in Largrange's mean value theorem for the function $f(x) = x(x - 2)$ when $x \in [1, 2]$ is

A. $\frac{3}{2}$

B. $\frac{2}{3}$

C. $\frac{1}{2}$

D. $\frac{3}{4}$

Answer: A

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11. If $f(x) = \begin{cases} \frac{x^2 - 9}{x - 3}, \text{ if } x \neq 3 \\ m, \text{ if } x = 3 \end{cases}$ is continuous at x = 3, then the value of 'm' is :

A. 3

B. 6

C. 2

D. 1

Answer: B

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12. If $y = \log(\tan x)$, then $\frac{dy}{dx}$ is :

A. $\frac{1}{\tan x}$

B. $\frac{\sec^2 x}{\tan x}$

C. $\sec^2 x$

D. 0

Answer: B

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13. Derivative of $\cos\sqrt{x}$ is :

A. $-\frac{\sin\sqrt{x}}{2\sqrt{x}}$

B. $\frac{\sin\sqrt{x}}{2\sqrt{x}}$

C. $-\frac{\sin\sqrt{x}}{\sqrt{x}}$

D. $-\frac{\sin\sqrt{x}}{2}$

Answer: A

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14. If $x - y = \pi$, then $\frac{dy}{dx}$ is :

A. 1

B. 0

C. π

D. -1

Answer: A

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15. $\frac{d}{dx}\left(\tan^{-1}x\right)$ is :

A. $\frac{1}{1+x^2}$

B. $\frac{1}{1-x^2}$

C. $\frac{-1}{1+x^2}$

D. $\frac{-1}{1-x^2}$

Answer: A

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16. If $y = a^x x^a$, then $\frac{dy}{dx}$ is equal to :

A. $a^{x}x^{a-1}(a - x\log a)$

B. $a^{x}x^{a-1}(a + x\log a)$

C. $a^{x}x^{a}(a + x\log a)$

D. $a^{x}x^{a-1}(x + a\log a)$

Answer: B

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17. $f(x) = \begin{cases} \frac{\sin x}{x}, & x \neq 0 \\ k - 1, & x = 0 \end{cases}$ is continuous at x = 0, then 'k' is :

A. 2

B. 0

C. -1

D. 1

Answer: A

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18. The derivative of a^x is :

A. a^x

B. $\frac{a^x}{\log a}$

C. $a^x \log a$

D. None of these

Answer: C

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19. Derivative of $\sin x^3$ w.r.t. x is :

A. $\cos x^3$

B. $3x^2 \cos x^3$

C. $3x^2 \cos x$

D. None of these

Answer: B

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20. At $x = 2$, $f(x) = [x]$ is

A. continuous but not differentiable

B. differentiable but not continuous

C. continuous as well as differentiable

D. None of these

Answer: B

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21. If $y = e^{\sin\sqrt{x}}$, then $\dfrac{dy}{dx}$ is :

A. $e^{\sin\sqrt{x}}\cos\sqrt{x}$

B. $\dfrac{e^{\sin\sqrt{x}}\cos\sqrt{x}}{2\sqrt{x}}$

C. $\dfrac{e^{\sin\sqrt{x}}}{2\sqrt{x}}$

D. None of these

Answer: B

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22. $\frac{d\left(a^{x}\right)}{dx} =$

A. a^{x}

B. $\log\left(a^{x}\right)$

C. $a^{x}\log a$

D. xa^{x-1}

Answer: C

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23. Which one of the following is true ?

For the real function :

$$f(x) = \begin{cases} x + 2 & \text{if } x \le 1 \\ x - 2 & \text{if } x > 1 \end{cases},$$

A. f is continuous at all real numbers $x > 1$ and $x < 1$

B. f is continuous at all real numbers $x \geq 1$

C. f is continuous at all real numbers $x \leq 1$

D. f is continuous at x = 1.

Answer: A

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24. If $f(x) = \begin{cases} mx + 1 & x \leq 5 \\ 3x - 5 & x > 5 \end{cases}$ is continuous, then the value of m is :

A. $\frac{9}{5}$

B. $\frac{5}{9}$

C. $\frac{5}{3}$

D. $\frac{3}{5}$

Answer: A

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25. If $y = \sin x$, then at $x = \frac{\pi}{2}, y_2$ is equal to :

A. -1

B. 1

C. 0

D. $\frac{1}{2}$

Answer: A

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26. The derivative of $\tan 45^\circ$ is :

A. 1

B. $\cot 45^\circ$

C. $-\cot 45^\circ$

D. 0

Answer: D

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27. If the function $f(x) = \begin{cases} kx + 1, \text{ if } x \leq \pi \\ \cos x, \text{ if } x > \pi \end{cases}$ is continuous at $x = \pi$, then the value of 'k' is :

A. -1

B. $-\frac{2}{\pi}$

C. -2

D. None of these

Answer: B

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28. If $y = \tan^{-1}\frac{x}{2} - \cot^{-1}\frac{x}{2}$, then $\frac{dy}{dx}$ is :

A. $\frac{4}{4 + x^2}$

B. $\frac{2}{4 + x^2}$

C. $\frac{1}{4 + x^2}$

D. $\frac{2}{1 + x^2}$

Answer: A

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29. let $y = \sqrt{x + \sqrt{x + \sqrt{x \ldots \infty}}}\Big)$ then dy/dx equals

A. $\dfrac{1}{2y - 1}$

B. $\dfrac{1}{y^2 - 1}$

C. $\dfrac{2y}{y^2 - 1}$

D. None of these

Answer: A

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30. The derivative of $e^{\sin^{-1}x}$ is :

A. $\dfrac{e^{\sin^{-1}x}}{\sqrt{1 - x^2}}$

B. $\dfrac{e^{\sin^{-1}x}}{\sqrt{1 + x^2}}$

C. $e^{\sin^{-1}x}\left(\sqrt{1 + x^2}\right)$

D. None of these

Answer: A

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OBJECTIVE TYPE QUESTIONS (FILL IN THE BLANKS)

1. If $f(x) = \begin{cases} kx^2 & \text{if} x \le 2 \\ 3 & \text{if} x > 2 \end{cases}$ is continuous at x = 2, then the value of k is

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2. If $f(x) = x + 7$, and $g(x) = x - 7, x \in R$ them find $\frac{d}{dx}(fog)(x)$.

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3. If $2x + 3y = \sin x$, then $\dfrac{dy}{dx} =$ ___________ .

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4. $\dfrac{dy}{dx}\left(\operatorname{cosec}^{-1}x\right) =$ _________.

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5. $\dfrac{d}{dx}\left(\sqrt{e^{\sqrt{x}}}\right) =$ __________.

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6. If $x = at^2, y = 2at,$ then $\dfrac{dy}{dx} =$

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7. The derivative of x^x w.r.t. x is __________.

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8. If $x^2 + 3x + 2$, then $\frac{d^2y}{dx^2} =$ ___________ .

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9. Find the value of c in Rolle's theorem for the function f(x) = x^3–3x in [–sqrt3, 0].

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10. Value of 'c' in LMV Theorem for $f(x) = x^2$ on $[2, 4]$ is ________.

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OBJECTIVE TYPE QUESTIONS (TRUE/FALSE QUESTION)

1. Is the function defined by $f(x) = x^2 - \sin x + 5$continuous at $x = \pi$?

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2. If $x = 2at^2, y = at^4$, then $\frac{dy}{dx} = t^2$.

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3. Differential of $\sin^2\left(x^2\right)$ w.r.t. x^2 is $\sin 2x^2$.

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OBJECTIVE TYPE QUESTIONS (VERY SHORT ANSWER TYPE QUESTIONS)

1. Find the points at which the function f(x) = [x] is not continuous in (-1, 4), where [x] is the largest integer function.

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2. Find all points of discontinuity of f, where f is defined by

$$f(x) = \{x^3 - 3, \quad \text{if} \quad x \le 2x^2 + 1, \quad \text{if} \quad x < 2$$

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3. Dicuss the continuity of the function:

$$f(x) = \begin{cases} 1 + x^2, \ 0 \le x \le 1 \\ 2 - x, \ x > 1 \end{cases}$$ at x = 1.

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4. Differentiate $\cos\left\{\sin(x)^2\right\}$ w.r.t. x.

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5. Find $\frac{dy}{dx}$ if $\sin^2 x + \cos^2 y = 1$.

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6. If $f(x) = x + 1$, then write the value of $\frac{d}{dx}(fof)(x)$.

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7. If $f(x) = x + 7$, and $g(x) = x - 7, x \in R$ them find $\frac{d}{dx}(fog)(x)$.

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8. If $y = \operatorname{cos}ec\left(\cot\sqrt{x}\right)$, then find $\frac{dy}{dx}$.

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9. Differentiate $\sin\left(x^2 + 5\right)$ w.r.t. x.

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10. Differentiate $\sqrt{\sin\left(e^x\right)}$ w.r.t. x.

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11. If $y = \log\left(\cos e^x\right)$, then find $\frac{dy}{dx}$.

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12. Differentiate $\left(x^2 + 3x + 4\right)^n, n \in I$ w.r.t. x.

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13. Find $\dfrac{dy}{dx}$ when $2x + 3y = \sin x$.

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14. Find the derivative of the following w.r.t. x :

$x - y = \pi$

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15. Find the derivative of the following w.r.t. x :

$x^3 + x^2y + xy^2 + y^3 = 81$

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16. Find the derivative of the following w.r.t. x :

$2x + 3y = \sin y$

Watch Video Solution

17. Find the derivative of the following w.r.t. x :

$ax + by^2 = \cos y$

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18. Find the derivative of the following w.r.t. x :

$\frac{1}{x} - \frac{1}{y} - 10 = 0$

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19. Find the derivative of the following w.r.t. x :

$$y = \frac{4}{3}x^{3/4}$$

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20. Find the derivative of the following w.r.t. x :

$$\tan^{-1}\sqrt{x}$$

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21. Find the derivative of the following w.r.t. x :

$$\tan\left(\sin^{-1}x\right)$$

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22. Find the derivative of the following w.r.t. x :

$x\tan^{-1}x$

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23. Find the derivative of the following w.r.t. x :

$\cos^{-1}\left(e^{x}\right)$

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24. Find the derivative of the following w.r.t. x :

$\log(\log x), x > 1$

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25. Find the derivative of the following w.r.t. x :

$e^{\cos x}$

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26. Find $\frac{dy}{dx}$, when $x = at^2$ and $y = 2\ at$

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27. Find $\frac{dy}{dx}$ when

$x = 4t, y = \frac{4}{t}.$

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28. Find $\frac{d^2y}{dx^2}$ when

$y = e^x + \sin x$

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29. Find $\frac{d^2y}{dx^2}$ when

$y = \tan^{-1}x$

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30. Find $\frac{d^2y}{dx^2}$ when

$y = \frac{\log x}{x}$.

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31. If $\frac{dy}{dx} = \frac{y}{x}$, prove that $\frac{d^2y}{dx^2} = 0.$

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32. If $2^x = 3^y$, then find $\frac{dy}{dx}$.

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33. Find the second derivative of $\sin^{-1}x.$

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34. Is Rolle's Theorem applicable to the function:

f(x) = |x| in the interval [-1, 1]?

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35. Is LMV Theorem applicable to the function:

$f(x) = \sin x \sin 2x$ in the interval $[0, \pi]$?

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EXERCISE 5.1

1. Prove that the function $f(x) = 5x - 3$is continuous at $x = 0$, at $x = -3$and at $x = 5$.

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2. Examine the continuity of the function $f(x) = 2x^2 - 1$at $x = 3$.

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3. Examine the following functions for continuity.(a) $f(x) = x - 5$ (b) $f(x) = \frac{1}{x - 5}$ (c) $f(x) = \frac{x^2 - 25}{x + 5}$(d) $f(x) = |x - 5|$

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4. Examine the following functions for continuity :

$f(x) = \frac{x^2 - 25}{x + 5}, x \neq -5$

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5. Examine the following functions for continuity:

(d) $f(x) = |x - 5|$.

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6. Prove that the function $f(x) = x^n$ is continuous at $x = n$, where n is a positive integer.

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7. Is the function f defined by

$$f(x) = \begin{cases} x & \text{if } x \leq 1 \\ 5 & \text{if } x > 1 \end{cases}$$

continuous at x = 0? At x = 2 ?

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8. Find all points of discontinuity of f, where f is defined by

$f(x) = \{2x + 3, \quad \text{if} \quad x \leq 22x - 3, \quad \text{if} \quad x > 2$

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9. Find all points of discontinuity of f, where f is defined by`f(x)=

{|x|+3, ifxlt=-3-2x , if-3

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10. Find all points of discontinuity of f, where f is defined by

$$f(x) = \begin{cases} \frac{x}{|x|}, & \text{if } x < 0 \\ 1, & \text{if } x \geq 0 \end{cases}$$

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11. Find all points of discontinuity of f, where f is defined by

$$f(x) = \begin{cases} \frac{x}{|x|}, & \text{if } x < 0 \\ 1, & \text{if } x \geq 0 \end{cases}$$

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12. Find all points of discontinuity of f, where f is defined by

$f(x) = \{x + 1, \quad \text{if} \quad x \geq 1x^2 + 1, \quad \text{if} \quad x < 1$

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13. Find all points of discontinuity of f, where f is defined by

$f(x) = \{x^3 - 3, \quad \text{if} \quad x \leq 2x^2 + 1, \quad \text{if} \quad x < 2$

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14. Find all points of discontinuity of f, where f is defined by

$f(x) = \{x^{10} - 1, \quad \text{if} \quad x \leq 1x^2, \quad \text{if} \quad x > 1$

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15. Is the function defined by $f(x) = \{x + 5, \quad \text{if} \quad x \leq 1x - 5, \quad \text{if} \quad x > 1$ a continuous function?

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16. Discuss the continuity of the function f, where f is defined by

$$f(x)\begin{cases} 3, & \text{if} \quad 0 \leq x \leq 1 \\ 4, & \text{if} \quad 1 < x < 3 \\ 5, & \text{if} \quad 3 \leq x \leq 10 \end{cases}$$

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17. Discuss the continuity of the function f, where f is defined by :

$$f(x) = \begin{cases} 2x, \text{ if } x < 0 \\ 0, \text{ if } 0 \leq x \leq 1 \\ 4x, \text{ if } x > 1 \end{cases}$$

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18. Discuss the continuity of the function f, where f is defined by`f(x)={-2, ifxlt=-1 2x , if-11`

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19. Find the relationship between a and b so that the function f defined by: $f(x) = \{ax + 1bx + 3 \quad \text{if } x \le 3\text{if } x > 3$

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20. For what value of $\lambda$is the function defined by $f(x) = \left\{\lambda\left(x^2 - 2x\right), \quad \text{if} \quad x \le 04x + 1, \quad \text{if} \quad x > 0\right.$continuous at $x = 0$? What about continuity at $x = 1$?

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21. Show that the function defined by $g(x) = x - [x]$ is discontinuous at all integral points which [x] denotes the greatest integer function.

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22. Is the function defined by $f(x) = x^2 - \sin x + 5$continuous at $x = \pi$?

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23. Discuss the continuity of the following functions a) $f(x) = \sin x + \cos x$

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24. Discuss the continuity of $f(x) = \sin x - \cos x$

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25. Discuss the continuity of the following functions :

$f(x) = \sin x \cdot \cos x$

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26. Discuss the continuity of the cosine, cosecant, secant and cotangent functions.

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27. Find all points of discontinuity of f, where

$f(x) = \begin{cases} \frac{\sin x}{x}, & \text{if } x < 0 x + 1, & \text{if } x \geq 0 \end{cases}$

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28. Determine if f defined by :

$$f(x) = \begin{cases} x^2 \sin\frac{1}{x}, \text{ if } x \neq 0 \\ 0, \text{ if } x = 0 \end{cases}$$ is a continuous function ?

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29. Examine the continuity of f, where f is defined by :

$$f(x) = \begin{cases} \sin x + \cos x, \text{ if } x \neq 0 \\ 1, \text{ if } x = 0 \end{cases}.$$

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30. Find the value of k so that the function f defined by

$f(x) = \begin{cases} \frac{k\cos x}{\pi - 2x}, 3 & , \end{cases}$ if $x \neq \frac{\pi}{2}$ if $x = \frac{\pi}{2}$ is continuous at $x = \frac{\pi}{2}$

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31. $f(x) = \begin{cases} kx^2, & \text{if } x \leq 2 \\ 3, & \text{if } x > 2 \end{cases}$ at x = 2

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32. Find the values of k so that the function f is continuous at the indicated point in $f(x) = \{kx + 1, \quad \text{if} \quad x \leq \pi\cos x, \quad \text{if} \quad x > \pi$ at $x \quad = \pi$

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33. Find the values of k so that the function f is continuous at the indicated point in$f(x) = \{kx + 1, \quad \text{if} \quad x \leq 53x - 5, \quad \text{if} \quad x > 5$at $x = 5$

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34. Find the value of 'a' and 'b' such that the function defined by :

$f(x) = \begin{cases} 5, \text{ if } x \leq 2 \\ ax + b, \text{ if } 2 < x < 5 \\ 20, \text{ if } x \geq 5 \end{cases}$ is a continuous function.

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35. If $f(x) = \left(\dfrac{3x + \tan^2 x}{x}\right)$ is continuous at $x = 0$, then $f(0)$ is equal to.

A. 3

B. 2

C. 4

D. 0

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36. Show that the function defined by $f(x) = |\cos x|$is a continuous function.

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37. Examine that $\sin|x|$ is a continuous function.

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38. Find all the points of discontinuity of f defined by

$f(x) = |x| - |x+1|$.

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EXERCISE 5.2

1. Differentiate the functions with respect to x in :

$\sin(x^2+5)$

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2. Differentiate the functions with respect to x in :

$\cos(\sin x)$

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3. Differentiate the functions with respect to x in :

$\sin(ax+b)$

Watch Video Solution

4. Differentiate the functions with respect to x in :

$\sec\left(\tan\sqrt{x}\right)$

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5. Differentiate the functions with respect to x in :

$$\frac{\sin(ax+b)}{\cos(cx+d)}$$

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6. Differentiate the functions with respect to x in :

$$\cos x^3 \cdot \sin^2\left(x^5\right)$$

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7. Differentiate the functions with respect to x in :

$$2\sqrt{\tan\left(x^2\right)}$$

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8. Differentiate the functions with respect to x in :

$$\cos\left(\sqrt{x}\right)$$

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9. Prove that the function f given by $f(x) = |x \quad 1|, \quad x \in R$ is not differentiable at $x = 1$

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10. Prove that the greatest integer function defined by $f(x) = [x], 0 < x < 3$is not differentiable at $x = 1 and x = 2$.

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EXERCISE 5.3

1. Find $\frac{dy}{dx}$ in the following :

$2y + 3x = \sin y$

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2. Find $\frac{dy}{dx}$ in the following: $2x \quad + \quad 3y \quad = \quad s \in \quad y$

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3. Find $\frac{dy}{dx}$ in the following :

$ax + by^2 = \cos y$

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4. Find $\frac{dy}{dx}$ in the following:$xy + y^2 = \tan x + y$

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5. Find $\frac{dy}{dx}$ in the following :

$x^2 + y^2 = 5xy$

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6. $x^3 + x^2y + xy^2 + Y^3 = 81$

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7. Find $\frac{dy}{dx}$ in the following :

$\cos xy = x^2y$

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8. Find $\frac{dy}{dx}$ in the following :

$\sin^3 x + \cos^3 y = 1$

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9. Find $\frac{dy}{dx}$ in the following:$y = \sin^{-1}\left(\frac{2x}{1 + x^2}\right)$

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10. Find $\frac{dy}{dx}$ in the following :

$y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right), -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$

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11. Find $\frac{dy}{dx}$ in the following:`y=cos^(-1)((1-x^2)/(1+x^2)),0

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12. Find $\frac{dy}{dx}$ in the following: `y=sin^(-1)((1-x^2)/(1+x^2)),0

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13. Find $\frac{dy}{dx}$ in the following:`y=cos^(-1)((2x)/(1+x^2)),-1

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14. Find $\frac{dy}{dx}$ in the following :

$y = \sin^{-1}\left(2x\sqrt{1 - x^2}\right), \ -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}.$

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15. Find $\frac{dy}{dx}$ in the following: `y=sec^(-1)(1/(2x^2-1)),0

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EXERCISE 5.4

1. Differentiate the following w.r.t. x :

$$\frac{e^x}{\sin x}$$

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2. Differentiate the following w.r.t. x :

$$e^{\sin^{-1}x}$$

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3. Differentiate the following w.r.t. x :

$$e^{x^3}$$

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4. Differentiate the following w.r.t. x :

$$\cos\left(\tan^{-1}e^{-x}\right)$$

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5. Differentiate the following w.r.t. x :

$$\log\left(\sin e^{x}\right)$$

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6. Differentiate the following w.r.t. x :

$e^{x}+e^{x^{2}}+\ldots+e^{x^{7}}$

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7. Differentiate the following w.r.t. x :

$\sqrt{e^{\sqrt{x}}}, x>0$

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8. Differentiate the following w.r.t. x :

$\log(\log x), x>1$

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9. Differentiate the following w.r.t. x :

$\frac{\cos x}{\log x}, x > 0$

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10. Differentiate the following w.r.t. x :

$\cos\left(\log x + e^x\right), x > 0$

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EXERCISE 5.5

1. Differentiate the functions given in w.r.t. x :

$\cos 2x + \sin 3x.$

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2. Differentiate the functions given w.r.t. x:

$$\sqrt{\left((x-1)\frac{x-2}{(x-3)(x-4)(x-5)}\right.}$$

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3. Differentiate the functions given in w.r.t. x :

$(\log x)^{\cos x}$

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4. Differentiate the functions given in w.r.t. x :

$x^{x}-2^{\sin x}$

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5. Differentiate the functions given in w.r.t. x :

$(x+7)^2 \cdot (x+4)^3.$

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6. Differentiate the functions given w.r.t. x: $\left(x+\frac{1}{x}\right)^x + x^{\left(1+\frac{1}{x}\right)}$

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7. Differentiate the functions given in w.r.t. x :

$(\log x)^x + x^{\log x}$

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8. Differentiate the functions given in w.r.t. x :

$(\sin x)^x + \sin^{-1}\sqrt{x}$

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9. Differentiate the functions given in w.r.t. x :

$x^{\sin x} + (\sin x)^{\cos x}$

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10. Differentiate the functions given in w.r.t. x :

$x^{x\cos x} + \frac{x^2 + 1}{x^2 - 1}$

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11. Differentiate the functions given in w.r.t. x :

$(x\cos x)^x + (x\sin x)^{1/x}$.

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12. Find $\frac{dy}{dx}$ of the function given in :

$x^y + y^x = b$

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13. Find $\frac{dy}{dx}$ of the function given in :

$y^x = x^y$

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14. Find $\frac{dy}{dx}$, when:

$(\cos x)^y = (\cos y)^x$

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15. Find $\frac{dy}{dx}$ of the functions given $xy = e^{(x-y)}$

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16. Find the derivative of the function given by $f(x) = (1+x)\left(1+x^2\right)\left(1+x^4\right)\left(1+x^8\right)$ and hence find $f'(1)$.

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17. Differentiate $\left(x^2-5x+8\right)\left(x^3+7x+9\right)$ in three ways mentioned below:(i) by using product rule(ii) by expanding the product to obtain a single polynomial.(iii) by logarithmic differentiation.Do they all give the same answer?

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18. If u, v and w are functions of x, then show that $\frac{d}{dx}\left(u.v.w\right) = \frac{du}{dx}v.w + u.\frac{dv}{dx}.w + u.v\frac{dw}{dx}$ in two ways - first by repeated application of product rule, second by logarithmic differentiation.

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EXERCISE 5.6

1. If x and y are connected parametrically by the equations given, without eliminating the parameter, Find $\frac{dy}{dx}.x = 2at^2, y = at^4$

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2. If x and y are connected parametrically by the equations given in, without eliminating the parameter, find $\frac{dy}{dx}$.

$x = a\cos\theta, y = b\sin\theta.$

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3. If x and y are connected parametrically by the equations given in, without eliminating the parameter, find $\frac{dy}{dx}$.

$x = a(\cos t + t\sin t), y = a(\sin t - t\cos t)$

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4. If x and y are connected parametrically by the equations given in, without eliminating the parameter, find $\frac{dy}{dx}$.

$x = t^2 - 2t, y = t^4 - 4t$

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5. If x and y are connected parametrically by the equations given in, without eliminating the parameter, find $\frac{dy}{dx}$.

$x = \cos\phi - \cos 2\phi, y = \sin\phi - \sin 2\phi$

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6. If x and y are connected parametrically by the equations given, without eliminating the parameter, Find $\frac{dy}{dx}$.

$x = a(\theta - \sin\theta), y = a(1 + \cos\theta)$

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7. If x and y are connected parametrically by the equations given, without eliminating the parameter, Find $\frac{dy}{dx}$.

$x = \frac{\sin^3 t}{\sqrt{\cos 2t}}, y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$

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8. If x and y are connected parametrically by the equations given in, without eliminating the parameter, find $\frac{dy}{dx}$.

$x = a\left(\cos\theta + \log\tan\frac{\theta}{2}\right), y = a\sin\theta.$

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9. If x and y are connected parametrically by the equations given in, without eliminating the parameter, find $\frac{dy}{dx}$.

$x = a\sec t, y = b\tan t.$

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10. If x and y are connected parametrically by the equations given, without eliminating the parameter, Find $\frac{dy}{dx}$.

$x = a(\cos\theta + \theta\sin\theta), y = a(\sin\theta - \theta\cos\theta)$

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11. If $x = \sqrt{a^{\sin} \wedge ((-1)t)}, y = \sqrt{a^{\cos} \wedge ((-1)t)}$, show that $\frac{dy}{dx} = -\frac{y}{x}$

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EXERCISE 5.7

1. Find the first and second order derivatives of the functions given in :

$x^2 + 3x + 2.$

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2. Find the second order derivatives of the functions given. x^{20}

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3. Find the second order derivatives of the functions given in :

$x.\cos x.$

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4. Find the second order derivatives of the functions given in :

$x \log x.$

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5. Find the second order derivatives of the functions given in :

$x^3 \log x$.

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6. Find the second order derivatives of the functions given in :

$e^{2x} \sin 5x$.

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7. Find the second order derivatives of the functions given in :

$e^{5x} \cos 3x$.

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8. Find the second order derivatives of the functions given in :

$\tan^{-1} x$.

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9. Find the second order derivatives of the functions given in :

$\log(\log x)$

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10. Find the second order derivatives of the functions given in :

$\sin(\log x)$.

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11. If $y = 5\cos x - 3\sin x$, prove that $\frac{d^2y}{dx^2} + y = 0$.

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12. If $y = \cos^{-1}x$, find $\frac{d^2y}{dx^2}$ in terms of y alone.

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13. If $y = 3\cos(\log x) + 4\sin(\log x)$, show that $x^2y_2 + xy_1 + y = 0$

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14. If $y = Ae^{mx} + Be^{nx}$, show that $\frac{d^2y}{dx^2} - (m + n)\frac{dy}{dx} + mny = 0$

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15. If $y = 500e^{7x} + 600e^{-7x}$, show that $\frac{d^2y}{dx^2} = 49y$.

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16. If $e^y(x+1) = 1$, show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$.

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17. If $y = \left(\tan^{-1}x\right)^2$, show that $\left(x^2+1\right)^2 y_2 + 2x\left(x^2+1\right)y_1 = 2$

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EXERCISE 5.8

1. Verify Rolles theorem for the function $f(x) = x^2 + 2x - 8$, $x \in [-4, 2]$.

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2. Examine if Rolles theorem is applicable to any of the following functions. Can you say something about the converse of Rolles theorem from these example?(i) $f(x) = [x]$for $x \in [5, 9]$(ii) $f(x) = [x]$ for $x \in [-2, 2]$(iii) `f(x)=x^

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3. If $f: [-5, 5]\vec{R}$ is differentiable function and if$f'(x)$ does not vanish anywhere, then prove that $f(-5) \neq f(5)$

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4. Verify Mean Value Theorem, if $f(x) = x^2 - 4x - 3$in the interval [a, b], where $a = 1 and b = 4$.

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5. Verify Mean Value theorem, if $f(x) = x^3 - 5x^2 - 3x$ in the interval [a, b], where a = 1 and b = 3. Find all $c \in (1, 3)$ for which f'(c) = 0.

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6. Differentiate $\frac{x^m}{x^n}$

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MISCELLANEOUS EXERCISE

1. Differentiate w.r.t. x the function in $\left(3x^2 + 9x + 7\right)^9$.

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2. Differentiate w.r.t. x the function in

$\sin^3 x + \cos^6 x$.

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3. Differentiate w.r.t. x the function$(5x)^{3\cos 2x}$.

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4. $\sin^{-1}\left(x\sqrt{x}\right), 0 \le x \le 1$

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5. Differentiate w.r.t. x the function `(cos^(-1)x/2)/(sqrt(2x+7)),-2

Watch Video Solution

6. Differentiate w.r.t. x the function in

$$\cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right], 0 < x < \frac{\pi}{2}$$

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7. Differentiate w.r.t. x the function in

$(\log x)^{\log x}, x > 1.$

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8. Differentiate w.r.t. x the function $\cos(a\cos x + b s \in x)$, for some constant a and b.

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9. Differentiate w.r.t. x the function in

$(\sin x - \cos x)^{(\sin x - \cos x)}, \frac{\pi}{4} < x < \frac{3\pi}{4}$.

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10. Differentiate w.r.t. x the function $x^x + x^a + a^x + a^a$, for some fixed $a > 0 and x > 0$.

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11. Differentiate w.r.t. x the function$x^x \wedge (2 - 3) + (x - 3)^x \wedge 2$for $x > 3.$

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12. Find $\frac{dy}{dx}$, if `y=12(1-cost), x=10(t-sint),-pi/2

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13. Find $\frac{dy}{dx}$, if $y = \sin^{-1}x + \sin^{-1}\sqrt{1 - x^2}, -1 \leq x \leq 1$

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14. If $x\sqrt{1 + y} + y\sqrt{1 + x} = 0$, prove that $\frac{dy}{dx} = -\frac{1}{(x + 1)^2}$.

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15. If $(x - a)^2 + (y - b)^2 = c^2$, for some $c > 0$, prove that

$$\frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$$ is a constant independent of a and b.

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16. If $\cos y = x\cos(a + y)$, with $\cos a \neq \pm 1$, prove that

$$\frac{dy}{dx} = \frac{\cos^2(a + y)}{\sin a}\cdot$$

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17. If $x = a(\cos t + t\sin t)$ and $y = a(\sin t - t\cos t)$, find $\frac{d^2y}{dx^2}$.

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18. If $f(x) = |x|^3$, show that $f(x)$ exists for all real x and find it.

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19. Using mathematical induction prove that $\frac{d}{dx}\left(x^n\right) = nx^{n-1}$ for all positive integers n.

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20. Using the fact that $s \in (A + B) = s \in A\cos B + \cos As \in B$and the differentiation, obtain the sum formula for cosines.

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21. Does there exist a function which is continuous everywhere but not differentiable at exactly two points? Justify your answer.

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22. If $y = |f(x)g(x)h(x)lmnabc|$, prove that $\frac{dy}{dx} = |f'(x)g'(x)h'(x)lmnabc|$

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23. If $y = e^{a}\cos^{(-1)x}, \ -1 \leq x < 1, showthat$ $\left(1 - x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$

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EXERCISE

1. Examine the continuity of the function :

$$f(x) = \begin{cases} \frac{|x-4|}{(x-4)}, \text{ if } x \neq 4 \\ 0 \text{ , if } x = 4 \end{cases}$$ at x = 4.

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2. For what value of k, $f(x) = \begin{cases} \frac{2^{x+2}-16}{4^x-16}, & x \neq 2 \\ k, & x = 2 \end{cases}$ is continuous at x = 2?

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3. Find the values of a and b sucht that the function f defined by

$$fx = \begin{cases} \frac{x-4}{|x-4|} + a & \text{if } x < 4 \\ a + b & \text{if } x = 4 \\ \frac{x-4}{|x-4|} + b & \text{if } x > 4 \end{cases}$$

is a continous function at $x = 4$.

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4. Given $f(x) = \frac{1}{x - 1}$. Find the points of discontinuity of the composite function $f(f(x))$.

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5. Find f'(x) when

$f(x) = 2^{\cos^2 x}$

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6. Find f'(x) when

$$f(x) = \sin^{-1}\left(\frac{1}{\sqrt{x+1}}\right).$$

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7. If $\sin x = \frac{2t}{1+t^2}, \tan y = \frac{2t}{1-t^2}$, find $\frac{dy}{dx}$

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8. If $y = \sec^{-1}\left(\frac{1}{4x^3 - 3x}\right)$, find $\frac{dy}{dx}$.

Watch Video Solution

9. Find $\frac{dy}{dx}$ when $\tan^{-1}\left(x^2 + y^2\right) = 0.$

Watch Video Solution

10. Examine the differentiability of f, where f is defined by

$f(x) = \begin{cases} x[x] & \text{if } \; 0 \le x < 2 \\ (x - 1)x & \text{if } \; 2 \le x < 3 \end{cases}$ at $x = 2$

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11. Show that $f(x) = |x - 5|$ is continuous but not differentiable at $x = 5.$

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12. A function $f: R \rightarrow R$ satisfies the equation $f(x+y) = f(x).f(y)$ for all $xy \in R, f(x) \neq 0$. Suppose that the function is differentiable at $x = 0$ and $f'(0) = 2$, then prove that $f' = 2f(x)$.

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13. If $x = e^{x/y}$, then prove that $\frac{dy}{dx} = \frac{x - y}{x\log x}$.

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14. $f(x) = \frac{1}{4x - 1}$ in $[1, 4]$

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15. Discuss the Lagrange's mean value theorem for the function

$f(x) = \sin x - \sin 2x$, in $[0, \pi]$.

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16. $f(x) = \sqrt{25 - x^2}$ in $[1, 5]$

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REVISION EXERCISE

1. Show that $f(x) = \begin{cases} |2x - 3|[x], & x \geq 1 \\ \sin\left(\frac{\pi x}{2}\right), & x < 1 \end{cases}$ is continuous but not differentiable at x = 1.

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2. Does there exist a function which is continuous everywhere but not differentiable at exactly two points? Justify your answer.

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3. Is $|\sin x|$ differentiable ?What about $\cos|x|$?

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4. Find, from first principle, the derivative of $\frac{\sin^{-1} x}{x}$ w.r.t. x.

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5. Differentiable the following w.r.t. x :

$$\sqrt{3x+2}+\frac{1}{\sqrt{2x^2+4}}$$

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6. Differentiable the following w.r.t. x :

$$e^{\sec^2 x}+3\cos^{-1}x.$$

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7. Differentiate the following w.r.t. x :

$$f(x)=\log\left[(2-x)^{1/2}\left(x^2-1\right)^{-1/4}\right]$$

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8. Differentiate the following function w.r.t x :

$$\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\left(\sqrt{1+x}+\sqrt{1-x}\right)}\right)$$

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9. Differentiate the following w.r.t. x:

$$\left(3x^2-9x+5\right)^9$$

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10. Differentiate the following w.r.t. x:

$$\sin^3 x+\cos^6 x$$

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11. Differentiate the following w.r.t. x:

$$e^{\log\left(x+\sqrt{x^2+a^2}\right)}$$

Watch Video Solution

12. Differentiate the following w.r.t. x:

$$e^{2\log x+3x}$$

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13. Prove that $\left(\cot^{-1}x + \cot^{-1}\frac{1}{x}\right)$ is a constant.

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14. If $y = f\left(\frac{2x - 1}{x^2 + 1}\right)$ and $f'(x) = \sin x^2$, find $\frac{dy}{dx}$.

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15. Find the derivative of the following w.e.t. x :

$$\frac{3}{\sqrt[3]{x}} - \frac{5}{\cos x} + \log_3 x + \frac{6}{\sin x} - \frac{2\tan x}{\sec x} + 7$$

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16. Find the derivative of the following w.e.t. x :

$$\log\left(\frac{1}{\sqrt{x}}\right) + 5x^a - 3a^x + \sqrt[3]{x^2} + 6\sqrt[4]{x^{-3}}.$$

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17. If $y = \tan^{-1}\left(\frac{e^{2x}+1}{e^{2x}-1}\right)$, prove that :

$\frac{dy}{dx} = -\frac{2e^{2x}}{1+e^{4x}}$.

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18. If the derivative of $\tan^{-1}(a+bx)$ takes the value 1 at $x = 0$, prove that $1 + a^2 = b^{\cdot}$

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19. Using the fact that $s \in (A+B) = s \in A \cos B + \cos A s \in B$and the differentiation, obtain the sum formula for cosines.

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20. If $\sqrt{y+x}+\sqrt{y-x}=c$ show that $\frac{dy}{dx}=\frac{y}{x}-\sqrt{\left(\frac{y^2}{x^2}\right)-1}$

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21. if $\sin x=y\sin(x+b)$ show that $\frac{dy}{dx}=\frac{\sin b}{\sin^2(x+b)}$

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22. If $x\sin(a+y)+\sin a\cos(a+y)=0, provethat\ \frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin a}$

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23. If $y=x\sin(a+y), provethat\frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin(a+y)-y\cos(a+y)}$.

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24. Differentiate $\log[\log(\log x)]$ w.r.t. x.

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25. if $y = e^{(x)^{e^x}} + x^{e^{e^x}} + e^{x^{x^e}}$, then dy/dx

$$= e^{(x)^{e^x}} x^{e^x}\left[e^x \log x + \frac{e^x}{x}\right] + x^{e^{e^x}} e^{e^x}\left[\frac{1}{x} + e^x \log x\right] + e^{x^{x^e}} x^{x^e} x^{e-1}[1 + e\log x]$$

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26. $y = \sqrt{x + \sqrt{x + \sqrt{x + \ldots . + \infty}}}$. Find $\frac{dy}{dx}$

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27. Find $\frac{dy}{dx}$ when :

$$y = x^{\sin x - \cos x} + \frac{x^2 - 1}{x^2 + 1}$$

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28. If $y = x^{\cot x} + \frac{2x^2 - 3}{x^2 + x + 2}$, find $\frac{dy}{dx}$.

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29. If $y = x^{\left(x^x\right)}$, prove that :

$$\frac{dy}{dx} = x^{x + x^x}\left[\frac{1}{x} + (1 + \log x)\log x\right].$$

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30. If $y = (\tan x)^{(\tan x)^{(\tan x)^{\cdots \infty}}}$, then prove that $\frac{dy}{dx} = 2$ at $x = \frac{\pi}{4}$.

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31. Differentiate $\tan x \tan 2x \tan 3x \tan 4x$ in two ways :

(i) by taking logarithums and

(ii) by repeatedly applying product rule.

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32. If $x = \sec\theta - \cos\theta$ and $y = \sec^n\theta - \cos^n\theta$ then show that

$$\left(x^2 + 4\right)\left(\frac{dy}{dx}\right)^2 = n^2\left(y^2 + 4\right)$$

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33. For what choices of a, b, c, if any, does the function

$$f(x) = \begin{cases} ax^2 + bx + x \ , \ 0 \le x \le 1 \\ bx - c \ , \ 1 \le x \le 2 \\ x \ , \ x > 2 \end{cases}$$

become differentiable at x = 1 and x = 2?

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34. If $y = Ae^{-kt}\cos(pt + c)$, then prove that $\frac{d^2y}{dt^2} + 2k\frac{dy}{dx} + n^2y = 0$, where $n^2 = p^2 + k^2$

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35. Using mathematical induction prove that :

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}f \text{ or } \backslash\ all\backslash\ n \in N$

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36. If $(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$, then prove that :

(i) $C_1 + 2C_2 + ... + nC_n = n \cdot 2^{n-1}$

(ii) $C_1 - 2C_2 + ... + (-1)^{n-1}nC_n = 0$

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37. If $y = \dfrac{ax^2}{(x-a)(x-b)(x-c)} + \dfrac{bx}{(x-b)(x-c)} + \dfrac{c}{x-c} + 1$ then $\dfrac{y'}{y} =$

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CHECK YOUR UNDERSTANDING

1. Are the following functions continuous at each point of their domains?

e^x

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2. Are the following functions continuous at each point of their domains?

$\sin x$

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3. Are the following functions continuous at each point of their domains?

$\cos x$

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4. Is greatest integer function [x] is continuous everywhere ?

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5. Find $\frac{dy}{dx}$ when $2x + 3y = \cos x$.

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6. Find the derivative of $\cos^{-1} x$ assuming that it exists.

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7. Differentiate $e^{\sin^{-1} x}$, w.r.t. x.

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8. If $y = \log\left(\cos e^x\right)$, then $\dfrac{dy}{dx}$ = ______.

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9. Find $\dfrac{d^2y}{dx^2}$ when $y = \log x + x$.

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10. Find $\dfrac{d^2y}{dx^2}$ when $y = e^x + \cos x$

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11. Verify Rolle's theorem for $f(x) = x, x \in [1, 2]$.

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12. Verify L.M.V. theorem for $f(x) = x(x - 2)$ on $[1, 3]$

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COMPETITION FILE

1. Let y be an implicit function of x defined by $x^{2x} - 2x^{x}\cot y - 1 = 0$.

Then $y'(1)$ equals: 1 b. $\log 2$ c. $-\log 2$ d. -1

A. -1

B. 1

C. $\log 2$

D. $-\log 2$

Answer: A

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2. Let f : (-1,1) $\rightarrow$ R be a differentiable function with f(0) =-1 and f'(0)=1

Let g(x)=$[f(f(2x)+1)]^2$. Then g'(0)=

A. 4

B. -4

C. log 2

D. $-\log 2$

Answer: B

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3. $\frac{d^2x}{dy^2}$ equals

A. $\left(\frac{d^2y}{dx^2}\right)^{-1}$

B. $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$

C. $\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-2}$

D. $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

Answer: D

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4. If $f(x)$ is differentiable at $x = a$, find $\lim_{x \to a} \frac{x^2 f(a) - a^2 f(x)}{x - a}$.

A. $a^2 f(a)$

B. $af(a) - a^2 f'(a)$

C. $2af(a) - a^2 f'(a)$

D. $2af(a) + a^2f'(a)$

Answer: C

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5. If $f: \mathbb{R} \rightarrow \mathbb{R}$ is a function defined by $f(x) = [x]\cos\left(\frac{2x - 1}{2}\right)\pi$, where [x] denotes the greatest integer function, then f is-

A. continuous for every real x

B. discontinuous only at x =0

C. discontinuous only at non-zero integral values of x

D. continuous only at x=0

Answer: A

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6. If $y = \sec\left(\tan^{-1}x\right)$, then $\frac{dy}{dx}$ at $x = 1$ is equal to

A. $\frac{1}{2}$

B. 1

C. $\sqrt{2}$

D. $\frac{1}{\sqrt{2}}$

Answer: D

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7. If g is the inverse of a function f and $f'(x) = \frac{1}{1 + x^5}$, then g'(x) is equal to

A. $5x^4$

B. $\frac{1}{1 + \{g(x)\}^5}$

C. $1 + \{g(x)\}^5$

D. $1 + x^5$

Answer: B

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8. if the function $g(x) = \begin{cases} k\sqrt{x+1}, 0 \le x \le 3 \\ mx + 2, 3 < x \le 5 \end{cases}$ Is differentiable , then the value of k+m is :

A. 2

B. $\frac{16}{5}$

C. $\frac{10}{3}$

D. 4

Answer: A

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9. For $x \in R, f(x) = |\log 2 - \sin x|$ and $g(x) = f(f(x))$, then

A. g is not differentiable at x =0

B. g'(0) = cos log 2

C. g'(0)=-cos (log 2)

D. g is differentiable at x =0 and g'(0) =-sin (log 2)

Answer: B

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10. If for $x\left(0, \frac{1}{4}\right)$, the derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x}g(x)$, then $g(x)$ equals: $\frac{3x}{1-9x^3}$ (2) $\frac{3}{1+9x^3}$ (3) $\frac{9}{1+9x^3}$ (4) $\frac{3x\sqrt{x}}{1-9x^3}$

A. $\frac{3x}{1-9x^3}$

B. $\frac{3}{1+9x^3}$

C. $\frac{9}{1+9x^3}$

D. $\frac{3x\sqrt{x}}{1-9x^3}$

Answer: C

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11. Let $S = \{t \in R : f(x) = |x-\pi|\left(e^{|x|}-1\right)\sin|x|$ is not differentiable at t} Then the set S is equal to: (1) ϕ (2) {0} (3) $\{\pi\}$ (4) $\{0, \pi\}$

A. ϕ (empty set)

B. {1}

C. $\{\pi\}$

D. $\{0, \pi\}$

Answer: A

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12. Let $y = y(x)$ be the solution of the differential equation, $x\left(\frac{dy}{dx}\right) + y = x\log_e x$, $(x > 1)$ if $2y(2) = \log_e 4 - 1$, then $y(e)$ is equal to: (a) - $\left(\frac{e}{2}\right)$ (b) - $\left(\frac{e^2}{2}\right)$ (c) $\frac{e}{4}$ (d) $\frac{e^2}{4}$

A. $\frac{e^2}{4}$

B. - $\frac{e}{2}$

C. $-\frac{e^2}{2}$

D. $\frac{e}{4}$

Answer: D

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13. for $x > 1$ if $(2x)^{2y} = 4e^{2x-2y}$ then $\left(1 + \log_e 2x\right)^2 \frac{dy}{dx}$

A. $\log_e 2x$

B. $x\log_e 2x$

C. $\frac{x\log_e 2x + \log_e 2}{x}$

D. $\frac{x\log_e 2x - \log_e 2}{x}$

Answer: D

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14. let S be the set of all points in $(-\pi, \pi)$ at which the function , f(x) =min {sinx , cos x} is not differentiable Then S is a subset of which of the following ?

A. $\left\{-\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{4}\right\}$

B. 1

C. 2

D. 3

Answer: A

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15. let $f(x) = \begin{cases} \dfrac{\sqrt{2}\cos x - 1}{\cot x - 1} & x \neq \dfrac{\pi}{4} \\ k & x = \dfrac{\pi}{4} \end{cases}$

Find k for which f(x) is continuous

A. $-\frac{1}{2}$

B. $\frac{1}{3}$

C. $\frac{1}{2}$

D. 1

Answer: C

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16. Let $f(x) = 15 - |x - 10|$ and $g(x) = f(f(x))$ then $g(x)$ is non differentiable at

A. {5,10,15}

B. {5,10,15,20}

C. {10}

D. {5,15}

Answer: A

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CHAPTER TEST

1. A function $f: R \rightarrow R$ is defined as follows : f(x)=$\begin{cases} x, & \text{if} \quad x \leq 1 \\ 5, & \text{if} \quad x > 1 \end{cases}$

Which one of the following is true ?

A. f is continuous at 0 and 1

B. f is continuous at 1 and 2

C. f is continuous at 0 and 2

D. f is continuous at 0,1 and 2

Answer: C

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2. if $\sqrt{x} + \sqrt{y} = 4$, then $\frac{dy}{dx}$ is :

A. $-\sqrt{\frac{x}{y}}$

B. $-\sqrt{\frac{y}{x}}$

C. $\sqrt{\frac{x}{y}}$

D. $\sqrt{\frac{y}{x}}$

Answer: B

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3. If f (x)=x+7 , and g (x)=x-7 , x $\in$ R , then $\frac{d}{dx}$ (gof)(x)=_____

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4. Find $\frac{dy}{dx}$ If $\sin^2 x + \cos^2 y = 1$

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5. Find $\frac{dy}{dx}$ when x=4t , y=$\frac{4}{t}$

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6. The function f(x) Is defined as follows :

$$f(x) = \begin{cases} x^2 + ax + b \ , \ 0 \le x < 2 \\ 3x + 2 \ , \ 2 \le x \le 4 \\ 2ax + 5b \ , \ 4 < x \le 8 \end{cases}.$$

If f(x) is continuous on [0,8], find the values of 'a' and 'b'.

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7. If $y = \tan^{-1}\left(\frac{\sqrt{1 + x^2} + \sqrt{1 - x^2}}{\sqrt{1 + x^2} - \sqrt{1 - x^2}}\right), x^2 \le 1$, then find $\frac{dy}{dx}$

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8. If $x^y = e^{x-y}$, then show that $\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$

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9. If $y = e^{ax}\cos bx$, Show that $\frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + \left(a^2 + b^2\right)y = 0$

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10. Verify Rolle's Theorem for the function : f(x) = sin x + cos x in the interval $[0, 2\pi]$

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11. Differentiate $\tan^{-1}\left(\frac{\sqrt{1 + x^2} - \sqrt{1 - x^2}}{\sqrt{1 + x^2} + \sqrt{1 - x^2}}\right) w.r.t. \sin^{-1}\left(\frac{2x}{1 + x^2}\right)$

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12. Find $\frac{dy}{dx}$, if $y^x + x^y + x^x = a^b$.

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