MATHS

BOOKS - ARIHANT MATHS (HINGLISH)

RELATIONS AND FUNCTIONS

Illustrative Examples

1. Let R be the relation in the set Z of integers given by R={(a,b):2 divides a-b}. Show that the relation R transitive ? Write the equivalence class [0].

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2. Check whether the relationR in the set R of real numbers defined by : $R = \{(a, b) : 1 + ab > 0\}$ is reflexive, symmetric or transitive.

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3. Let N be the set of all natural numbers and let R be relation in N. Defined by

$R = \{(a, b) : \text{a is a multiple of b}\}.$

show that R is reflexive transitive but not symmetric .

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4. Let A be the set of all students of a boys school. Show that the relation R in A given by R = {(a, b) : a is sister of b} is the empty relation and R'= {(a, b) : the difference between heights of a and b is less than 3 meters} is the un

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5. Show that the relation S on the set : $A = \{x \in Z : 0 \leq x \leq 12\}$ given by:

S = $\{(a, b) : a, b \in Z, |a - b|$ is divisible by 3} is an equivalence relation.

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6. Let L be the set of all lines in a plane and R be the relation in L defined as $R = \{(L_1, L_2) : L_1(\text{ is perpendicular to L})_2\}$. Show that R is symmetric but neither reflexive nor transitive.

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7. Let T be the set of all triangles in a plane with R a relation in T given by $R = \{(T_1, T_2) : T_1 \text{ is congruent to } T_2\}$. Show that R is an equivalence relation.

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8. Let $A = \{1, 2, 3, , 9\}$ and R be the relation on $A \times A$ defined by $(a, b) R (c, d)$ if $a + d = b + c$ for all $(a, b), (c, d) \in A \times A$. Prove that R is an equivalence relation and also obtain the equivalence class [(2, 5)].

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9. Let N denote the set of all natural numbers and R be the relation on $N \times N$ defined by (a,b)R (c,d) if ad(b+c)=bc(a+d) then R is

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10. Let R be the relation defined on the set $A = \{1, 2, 3, 4, 5, 6, 7\}$ by $R = \{(a, b):$ both a and b are either odd or even}. Show that R is an equivalence relation. Further, show that all the elements of the subset {1, 3, 5, 7} are related to each other and all the elements of the subset {2, 4, 6} are related to each other, but no element of the subset {1, 3, 5, 7} is related to any element of the subset {2, 4, 6}.

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11. Let $A = \{x \in Z : 0 \leq x \leq 12\}$. Show that $R = \{(a, b) : a, b \in A, |a - b| is \div isib \leq by 4\}$ is an equivalence relation. Find the set of all elements related to 1. Also write the equivalence class [2]

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12. Let $f: A \rightarrow B$ be a function defined as $f(x) = \frac{2x+3}{x-3}$, where A=R-{3} and B=R-{2}. Is the function f one-one and onto ? Is f invertible ? If yes, then find its inverse.

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13. Prove that the function f : $[0, \infty) \rightarrow R$, given by $f(x) = 9x^2 + 6x - 5$ is not invertible.

Modify the co-domain of the function f to make it invertible, and hence, find f^{-1}.

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14. Consider $f: \overrightarrow{R-5, \infty}$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible with $f^{-1}(y) = \left(\frac{\sqrt{y+6}-1}{3}\right)$.

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15. Let $A = R - \{3\}$ and $B = R - [1]$. Consider the function $f: \overrightarrow{AB}$ defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Show that f is one-one and onto and hence find f^{-1}

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16. Let $A = R - \{2\}$ and $B = R - \{1\}$ if $f: A \rightarrow B$ is a function defined by $f(x) = \frac{x-1}{x-2}$ show that f is one-one and onto. Hence find f^{-1}.

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17. Prove that function $f: N \to N$, defined by $f(x) = x^2 + x + 1$ is one-one but not onto. Find inverse of $f: N \to S$, where S is range of f.

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18. Let $Y = \{n^2 : n \in N\} \in N$. Consider $f: N \to Y$ as $f(n) = n^2$. Show that f is invertible. Find the inverse of f.

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19. Let $f: N \to R$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f: N \to S$, where, S is the range of f, is invertible. Find the inverse of f.

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20. Let $f: N \rightarrow N$ be defined by: $f(n) = \{n+1, \quad \text{if} \quad n \, is \, oddn - 1, \quad \text{if} \quad n \, is \, even$ Show that f is a bijection.

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21. If $a \cdot b$, denoted the larger of 'a' and 'b' and if $aob = a \cdot b + 3$, then write the value of $(5)o(10)$, where * and o are binary operations.

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22. Let * be a binary operation, on the set of all-zero real numbers, given by a $a \cdot b = \frac{ab}{5}$ for all $a, bR - \{0\}$. Find the value of x given that $2 \cdot (x \cdot 5) = 10.$

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23. Examine whether the operation $' * '$ defined on R by $a * b = ab + 1$ is :

(i) a binary or not

(ii) if a binary operation, is it associative or not?

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24. Show that addition, subtraction and multiplication are binary operations on R, but division is not a binary operation on R. Further, show that division is a binary operation on the set R of nonzero real numbers.

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25. Show that subtraction and division are not binary operations on N.

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26. Show that the $\vee : R \to R$ given by $(a, b) \to max\{a, b\}$and the $\wedge : R \to R \to$ given by $(a, b) \to m \in \{a, b)$are binary operations.

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27. Show that $* . R \times R \to R$ given by $a * b \to a + 2b$ is not associative.

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28. Determine whether the binary operation $' * '$on the set N of natural numbers defined by $a * b = 2^{ab}$ is associative or not.

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29. Let $' * '$ be a binary operation on Q defined by :

$a * b = \dfrac{2ab}{3}$.

Show that $' * '$ is commutative as well as associative.

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30. Examine which of the following is a binary operation :

(i) $a * b = \frac{a+b}{2}, a, b \in N$ For binary operation, check the commutative and associative property.

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31. Discuss the commutativity and associativity of binary operation * defined on Q by the rule $a \cdot b = a - b + ab$ for all $a, b \in Q$

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32. Let X be a non-empty set, and $P(X)$ be its power set. If * is an operation defined on the elements of $P(X)$ by $A \cdot B = A \cap B \forall A, B \in P(x)$, then prove that is a binary operation in $P(x)$ which is commutative as well as associative. Find its identity element. If '0' is another binary operation defined an $P(X)$ on $AoB = A \cup B,$ then verify that '0' distributes itself over.

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33. Let A = R R and * be the binary operation on A defined by (a, b) * (c, d) = (a + c, b + d). Show that * is commutative and associative. Find the identity element for * on A.

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34. Let $A = Z \times Z$ and $' * '$ be a binary operation on A defined by :

$(a, b) * (c, d) = (ad + bc, bd).$

Find the identity element for $' * '$ in A.

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35. Consider the binary operation* on the set {1, 2, 3, 4, 5} defined by a * b=min. {a, b}. Write the operation table of the operation *.

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Frequently Asked Questions

1. Show that a one-one function $f: \{1, 2, 3\} \rightarrow \{1, 2, 3\}$must be onto.

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2. Let A be the set of all 50 students of class XII in a central school. Let $f: A \rightarrow N$ be a function defined by $f(x) = Roll\nu mberofstudentx$ Show that f is one-one but not onto.

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3. Prove that the function $f: R \to R$ given by:

f(x) = 5x is one-one and onto.

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4. Show that the function $f: N \to N$ given by $f(1) = f(2) = 1$ and $f(x) = x - 1$ for every $x \geq 2$, is onto but not one-one.

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5. Show that the function $f: R \to R$ defined by $f(x) = x^2$ is neither one-one nor onto.

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6. Show that $f: N \to N$ given by

$$f(x) = \begin{cases} x + 1, \text{ if x is odd} \\ x - 1, \text{ if x is even} \end{cases}$$

is both one-one and onto.

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7. If $f: R \rightarrow R$ is defined by $f(x) = 5x + 2$, find $f(f(x))$.

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8. If $f : R \div R$ be defined by $f(x) = \left(3 - x^3\right)^{1/3}$, then find $fof(x)$

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9. If $f: R \rightarrow R$ and $g: R \rightarrow R$ are given by by f(x)=cos x and $g(x) = 3x^2$,

then shown that $gof \neq fog$.

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10. Let $f: \{2, 3, 4, 5\} \to \{3, 4, 5, 9\}$ and $g: \{3, 4, 5, 9\} \to \{7, 11, 15\}$ be functions defined as $f(2) = 3, f(3) = 4, f(4) = f(5) = 5, g(3) = g(4) = 7,$ and $g(5) = g(9)$

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11. $f(x) = \frac{4x + 3}{6x - 4}$; $x \neq \frac{2}{3}$ Show that $fof(x) = x$ for all x except $x = \frac{2}{3}$

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12. Show that the function $f: R \to R$ defined by $f(x) = \frac{x}{x^2 + 1}, \forall x \in R$ is neither one-one nor onto. Also, if $g: R \to R$ is defined as $g(x) = 2x - 1$, find fog(x).

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13. If the function $f: R \rightarrow R$ be given by $f(x) = x^2 + 2$ and $g: R \rightarrow R$ be given by $g(x) = \frac{x}{x-1}, x \neq 1$, find fog and gof and hence find fog (2) and gof (-3).

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14. Let $f: \{1, 2, 3\} \rightarrow \{a, b, c\}$be one-one and onto function given by $f(1) = a, \ f(2) = b$and $f(3) = c.$ Show that there exists a function $g: \{a, b, c\} \rightarrow \{1, 2, 3\}$such that $gof = I_x$and `fog=

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15. If $f: R - \left\{\frac{7}{5}\right\} \rightarrow R - \left\{\frac{3}{5}\right\}$ be defined as $f(x) = \frac{3x+4}{5x-7}$ and $g: R - \left\{\frac{3}{5}\right\} \rightarrow R - \left\{\frac{7}{5}\right\}$ be defined as $g(x) = \frac{7x+4}{5x-3}$. Show that $gof = I_A$ and $fog = I_B$, where $B = R - \left\{\frac{3}{5}\right\}$ and $A = R - \left\{\frac{7}{5}\right\}$.

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Questions From Ncert Exemplar

1. For the set $A = \{1,\ 2,\ 3\}$, define a relation R on the set A as follows: $R = \{(1,\ 1),\ (2,\ 2),\ (3,\ 3),\ (1,\ 3)\}$ Write the ordered pairs to be added to R to make the smallest equivalence relation.

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2. Let R be the relation in the set Z of integers given by R={(a,b):2 divides a-b}. Show that the relation R transitive ? Write the equivalence class [0].

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3. $A = \{1, 2, 3\}$ पर निम्नलिखित सम्बन्धो में से कौन-सा सम्बन्ध एक फलन है?

$f = \{(1, 3), (2, 3), (3, 2)\}, g = \{(1, 2), (1, 3), (3, 1)\}$

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4. On the set N of all natural numbers, a relation R is defined as follows: nRm Each of the natural numbers n and m leaves the same remainder less than 5 when divided by 5. Show that R is an equivalence relation. Also, obtain the pairwise disjoint subsets determined by R .

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5. Show that the function $f : R \to R$ defined by $f(x) = \frac{x}{x^2+1} \forall x \in R$ is neither one-one nor onto. Also if $g : R \to R$ is defined by $g(x) = 2x - 1$ find fog(x)

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Exercise 1 A Short Answer Type Questions

1. Let N be the set of all natural niumbers and let R be a relation in N , defined by

$R = \{(a, b) : a \quad \text{is a factor of b}\}.$

then , show that R is reflexive and transitive but not symmetric .

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2. Determine whether Relation R on the set $A = \{1, \ 2, \ 3, \ , \ 13, \ 14\}$ defined as $R = \{(x, \ y) : 3x - y = 0\}$

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3. Determine whether Relation R on the set N of all natural numbers defined as $R = \{(x, \ y) : y = x + 5 \text{ and } x < 4\}$

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4. Determine whether Relation R on the set Z of all integer defined as $R = \{(x, \ y) : y \text{ is divisible by } x\}$

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5. Determine whether each of the following relations are reflexive, symmetric and transitive:(i) Relation R in the set $A = \{1, 2, 3, \ldots, 13, 14\}$ defined as $R = \{(x, y) : 3xy = 0\}$(ii) Relation R in the set N o

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6. Show that the relation R in R (set of real numbers) is defined as R= $\{(a, b), a \leq b\}$is reflexive and transitive but not symmetric.

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7. Let R be a relation defined by $R = \{(a, b) : a \geq b, a, b \in \mathbb{R}\}$. The relation R is (a) reflexive, symmetric and transitive (b) reflexive, transitive but not symmetric (c) symmetric, transitive but not reflexive (d) neither transitive nor reflexive but symmetric

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8. Show that the relation R in the set of real numbers, defined as $R = \{a, b) : a \leq b^2\}$ is neither reflexive, nor symmetric, nor transitive.

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9. Relation R defines on the set of natural numbers N such that $R = \{(a, b) : a$ is divisible by $b\}$, then show that R is reflexive and transitive but not symmetric.

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10. Check whether the relation R in real numbers defined by $R = \left\{(a, b) : a < b^3\right\}$ is reflexive, symmetric or transitive.

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11. Is the relation R in the set $A = \{1, 2, 3, 4, 5\}$ defined as

$R = \{(a, b) : b = a + 1\}$ reflexive ?

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12. Prove that on the set of integers, Z, the relation R defined as

$aRb \Leftrightarrow a = \pm b$ is an equivalence relation.

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13. Show that the relation R in the set {1, 2, 3} defined as:

(a) $R = \{(1, 2), (2, 1)\}$

is symmetric, but neither reflexive nor transitive

(b) $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}$

is reflexive, but neither symmetric nor transitive

(c) $R = \{(1, 3), (3, 2), (1, 2)\}$

is transitive, but neither reflexive nor symmetric.

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Exercise 1 A Long Answer Type Questions I

1. If 'R' is relation 'less than' from :

Set A = {(1, 2, 3, 4, 5}` to Set B = {1, 4, 6},

write down the Cartesian Product corresponding to 'R'.

Also, find the inverse relation to 'R'.

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2. Prove that the relation R in the set of integers Z defined by R = {(x,y) : x - y. is an integer) is an equivalence relation.

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3. Prove that the relation R on Z defined by $(a, b) \in R \Leftrightarrow \quad a-b$ is divisible by 5 is an equivalence relation on Z .

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4. Show that the relation R in the set $A = \{1, 2, 3, 4, 5\}$given by $R = \{(a, b) : |ab| is even\}$, is an equivalence relation. Show that all the elements of $\{1, 3, 5\}$are related to each other and all the e

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5. Let Z be the set of all integers and R be the relation on Z defined as $R = \{(a, b); a, \ b \ \in Z, \text{ and } (a - b) \text{ is divisible by } 5\}$. Prove that R is an equivalence relation.

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6. Show that the relation R in the set $A = \{x \in Z : 0 \leq x \leq 12\}$ given by $R = \{a, b) : |a - b|$ is a multiple of 4} is an equivalence relation.

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7. Let R be a relation on the set of all lines in a plane defined by $(l_1,\ l_2) \in R$ line l_1 is parallel to line l_2 . Show that R is an equivalence relation.

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Exercise 1 A Long Answer Type Questions Ii

1. The relation $'R'$ in $N \times N$ such that $(a,\ b)\ R\ (c,\ d) \Leftrightarrow a + d = b + c$ is reflexive but not symmetric reflexive and transitive but not symmetric an equivalence relation (d) none of these

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2. Let R be a relation on the set A of ordered pairs of positive integers defined by $(x, y)R(u, v)$if and only if $xv = yu$. Show that R is an equivalence relation.

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3. Show that the relation R defined on the set A of all triangles in a plane as $R = \{(T_1, T_2) : T_1$ is similar to $T_2)$ is an equivalence relation. Consider three right angle triangle T_1 with sides $3, 4, 5$; T_2 with sides $5, 12, 13$ and T_3 with sides 6, 8, 10. Which triangles among T_1, T_2 and T_3 are related?

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4. Let L be the set of all lines in XY -plane and R be the relation in L defined as $R = \{(L_1, L_2) : L_1$ is parallel to $L_2\}$. Show that R is an equivalence relation. Find the set of all lines related to the line $y = 2x + 4$.

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5. Let L be the set of all lines in the plane and R be the relation in L, defined as :

R = $\{(l_i, l_j) = l_i$ is parallel to $l_j, \forall i, j\}$.

Show that R is an equivalence relation. Find the set of all lines related to the line $y = 7x + 5$.

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6. Show that the relation R in the set $A = \{x \in z, 0 \leq x \leq 12\}$ given by $R = \{(a, b): |a - b|$ is a multiple of 4} is an equivalence relation.

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Exercise 1 B Long Answer Type Questions I

1. Show that the function $f: R \rightarrow R$ given by :

$f(x) = ax + b$, where $a, b \in R$, $a \neq 0$ one one and onto.

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2. A function $f: R \rightarrow R$ be defined by f(x) = 5x + 6, prove that f is one-one and onto.

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3. State whether the following function is one-one onto or bijective:

$f: R \rightarrow R$ defined by $f(x) = 1 + x^2$.

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4. Show that $f: R \rightarrow R$ given by $f(x) = x^3$ is bijective.

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5. A function $f: R \rightarrow R$ is defined by $f(x) = 4x^3 + 5, x \in R$.

Examine if f is one-one and onto.

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6. Show that the function $f: R \rightarrow R$ defined by :

(i) $f(x) = \frac{4x - 3}{5}, x \in R$

(ii) $f(x) = \frac{3x - 1}{2}, x \in R$

is one-one.

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7. Consider the function $f(x) = \frac{x - 3}{x + 1}$ defined from $R - \{-1\}$ to $R - \{1\}$.

Prove that f is one-one .

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8. Show that function $f: R \rightarrow \{x \in R: -1 < x < 1\}$defined by $f(x) = \frac{x}{1 + |x|}, x \in R$ is one one and onto function

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9. Check the injectivity and surjectivity of the following functions:(i) $f: N \rightarrow N$given by $f(x) = x^2$(ii) $f: Z \rightarrow Z$given by $f(x) = x^2$(iii) $f: R \rightarrow R$given by $f(x) = x^2$(iv) $f: N \rightarrow N$given by $f(x) = x^3$(v) `f : Z -

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10. Check the injectivity and surjectivity of the following functions:(i) $f: N \rightarrow N$given by $f(x) = x^2$(ii) $f: Z \rightarrow Z$given by $f(x) = x^2$(iii) $f: R \rightarrow R$given by $f(x) = x^2$(iv) $f: N \rightarrow N$given by $f(x) = x^3$(v) `f : Z -

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11. Check the injectivity and surjectivity of the following functions:

$f: R \rightarrow R$, given by $f(x) = x^2$

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12. Let A and B be two sets. Show that $f: A \times B \to B \times A$ defined by $f(a,\ b) = (b,\ a)$ is a bijection.

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13. Show that the Modulus Function $f: R \to R$, given by $f(x) = |x|$, is neither one-one nor onto, where $|x|$is x, if x is positive or 0 and $|x|$is x, if x is negative.

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14. Show that the signup function $f: R \to R$ given by

$f(x) = \begin{cases} 1 & \text{if} & x > 0 \\ 0 & \text{if} & x = 0 \\ -1 & \text{if} & x < 0 \end{cases}$ is neither one-one nor onto.

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15. Let $A = \{a_1, a_2, a_3, a_4, a_5\}$ and $B = \{b_1, b_2, b_3, b_4\}$, when $a_l's$ and $b_l's$ are school going students. Define a relation from a set A to set B by x R y iff y is a true friend of x.

If $R = \{(a_1, b_1), (a_2, b_1), (a_3, b_3), (a_4, b_2), (a_5, b_2)\}$

Is R a bijective function?

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Exercise 1 C Short Answer Type Questions

1. If $f: R \to R$ defined by $f(x) = x^2 - 2x + 3$, then find $f(f(x))$.

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2. If $f(x) = \frac{x}{x-1}, x \neq 1$, then show that fof (x) = x.

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3. If $f(x) = \log\left(\frac{1-x}{1+x}\right), \ -1 < x < 1,$ then show that : $f(-x) = -f(x)$.

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4. If $f(x) = \frac{3x-1}{x+1}, x \neq -1,$ then find fof (x).

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5. If $f(x) = \frac{2x+3}{3x-2}, x \neq \frac{2}{3},$ then find fof (x).

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6. Consider a function $f(x) = \frac{3x+4}{x-2}, x \neq 2.$ Find a function g(x) on a suitable domain such that :

(gof) (x) = x = (fog) (x).

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7. Let f, g and h be functions from R to R. Show that

$$(f+g)oh = foh + goh\left(f\dot{g}\right)oh = (foh)g\dot{o}h$$

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Exercise 1 C Long Answer Type Questions I

1. Find fog and gof, if :

$$f(x) = x^2, g(x) = x+1$$

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2. Find fog and gof, if :

$$f(x) = 4x - 1, g(x) = x^2 + 2$$

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3. Find fog and gof, if :

$f(x) = |x+1|, g(x) = 2x - 1$

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4. Describes fog and gof, where :

$f(x) = \sqrt{1-x^2}, g(x) = \log x$

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5. Let $f(x) = 2x^2$ and $g(x) = 3x - 4, x \in R$. Find the following :

(i) fof (x) (ii) gog (x)

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6. If $f(x) = \frac{x-1}{x+1}, x \neq -1,$. then show that $f(f(x)) = -\frac{1}{x}$, prove that $x \neq 0$.

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7. Let $f: R \rightarrow R$ and $g: R \rightarrow R$ be two functions defined by $f(x) = |x|$ and $g(x) = [x]$, where [x] denotes the greatest integer less than or equal to x. Find (fog) (5.75) and (gof) - $(-\sqrt{5})$.

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8. Let $f: R \rightarrow R$be the Signum Function defined as $f(x) = \{1, x > 00, x = 0 - 1, x < 1$ and $g: R \rightarrow R$be the Greatest Integer Function given by $g(x) = [x]$, where [x] is greatest integer less than or equal to x. Then does fo

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9. Find gof and fog, if $f: R \rightarrow R$and $g: R \rightarrow R$are given by $f(x) = \cos x$ and $g(x) = 3x^2$. Show that $gof \neq fog$.

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10. Consider $f: N \rightarrow N, g: N \rightarrow N$ and $h: N \rightarrow R$ defined as $f(x) = 2x, g(y) = 3y + 4$ and $h(z) = \sin z, \forall x, y$ and z in N. Show that $ho(gof) = (hog)of$.

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Exercise 1 D Short Answer Type Questions

1. Let $S = \{1, 2, 3\}$. Determine whether the functions $f: S \rightarrow S$defined as below have inverses. Find f^{-1}, if it exists.(a) $f = \{(1, 1), (2, 2), (3, 3)\}$ (b) $f = \{(1, 2), (2, 1), (3, 1)\}$(c) `f =

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2. Let $S = \{a, b, c\} and T = \{1, 2, 3\}$. Find $F^{-1}$of the following functions F from S to T, if it exists.(i $)F = \{(a, 3), (b, 2), (c, 1)\}$(ii)

$F = \{(a, 2), (b, 1), (c, 1)\}$

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3. Are the following functions invertible in their respective domains ? If so, find the inverse in each case :

(i) f(x) = x + 1

(ii) $f(x) = \frac{x-1}{x+1}, x \neq -1.$

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4. Let $f: \overrightarrow{NY}$ be a function defined as $f(x) = 4x + 3$, where $Y = \{y \in N : y = 4x + 3 \text{ for some } x \in N\}$. Show that f is invertible and its inverse is (1) $g(y) = \frac{3y+4}{3}$ (2) $g(y) = 4 + \frac{y+3}{4}$ (3) $g(y) = \frac{y+3}{4}$ (4) $g(y) = \frac{y-3}{4}$

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5. Consider $f: R \rightarrow R$given by $f(x) = 4x + 3$. Show that f is invertible.

Find the inverse of f.

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6. Consider $f: R \rightarrow R$ given by the following. Show that 'f'is invertible.

Find the inverse of 'f'.

$f(x) = 5x + 2$

Watch Video Solution

7. (a) If $f: R \rightarrow R$ defined by $f(x) = \dfrac{3x+5}{2}$ is an invertible function, find f^{-1}.

Show that $f: R \rightarrow R$ defined by $f(x) = \dfrac{4x-3}{5}, x \in R$ is invertible function and find f^{-1}.

Watch Video Solution

8. If $f : R \to R :$

$$f(x) = \frac{3x+6}{8}$$

is an invertible function and find f^{-1}.

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Exercise 1 D Long Answer Type Questions I

1. If $f(x) = \frac{4x+3}{6x-4}, x \neq \frac{2}{3}$, show that $fof(x) = x$ for all $x \neq \frac{2}{3}$.

What is the inverse of f?

Watch Video Solution

2. Show that the function f in $A = \mid R - \left\{\frac{2}{3}\right\}$ defined as $f(x) = \frac{4x+3}{6x-4}$ is one-one and onto. Hence find f^{-1}.

Watch Video Solution

3. Show that $f:[-1, 1] \rightarrow R$, given by $f(x) = \frac{x}{(x+2)}$ is one- one . Find the inverse of the function $f:[-1, 1]$

Watch Video Solution

4. Consider $f:\{1, 2, 3\} \rightarrow \{a, b, c\}$ given by $f(1) = a$, $f(2) = b$ and $f(3) = c$. Find f^{-1} and show that $\left(f^{-1}\right)^{-1} = f$.

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5. Consider as $f(1) = a, f(2) = b, f(3) = c,$

$g:[a, b, c] \rightarrow$ {apple, ball, cat}

Defined as $f(1) = a, f(2) = b, f(3) = c,$

g(a) = apple, g(b) = ball, g(c) = cat

Show that f, g and gof are invertible.

Find f^{-1}, g^{-1} and $(gof)^{-1}$ and show that :

$(gof)^{-1} = f^{-1} o g^{-1}$.

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Watch Video Solution

6. Prove that function $f: R \rightarrow R, f(x) = \frac{3-2x}{7}$ is one-one onto. Also, find f^{-1}.

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7. Let $f: N^{\rightarrow}$ be a function defined as $f(x) = 9x^2 + 6x - 5$. Show that $f: N\overrightarrow{S}$, where S is the range of f, is invertible. Find the inverse of f and hence $f^{-1}(43)$ and $f^{-1}(163)$.

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8. Consider f $R \rightarrow \left\{-\frac{4}{3}\right\} \rightarrow R - \left\{\frac{4}{2}\right\}$ given by $f(x) = \frac{4x+3}{3x+4}$

Show that f is bijective. Find the inverse of f and hence find f^–1(0) and x such that f–1(x) = 2.

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9. If the function $f: R \rightarrow R$ be defined by f(x) = $4x - 3$ and $g: R \rightarrow R$ by $g(x) = x^3 + 5$, then find fog`.

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Exercise 1 E Short Answer Type Questions

1. If the binary operation $*$ on the set Z of integers is defined by $a * b = a + b - 5$, then write the identity element for the operation $' * '$ in Z.

Watch Video Solution

2. Check $* : R \times R \rightarrow R$ given by :

$a * b \rightarrow a + 3b^2$ is commutative.

Watch Video Solution

3. Let 'x' be an operation defined as $x: R \times R \rightarrow R$ Such that

$a * b = 2a + b, a, b \in R$

Check if $' * '$ is a binary operation

If yes, find if it is associative too.

Watch Video Solution

4. Let $* : N \times N \rightarrow N$ be an operation defined as

$a * b = a + ab, \forall a, b \in N$

Check if $' * '$ is a binary operation.

If yes, find if it is associative too.

Watch Video Solution

5. Let P be the set of all subsets of a given set X. Show that

$\cup : P \times P \rightarrow P$given by $(A, B) \rightarrow A \cup B$and $\cap : P \times P \rightarrow P$given by

$(A, B) \rightarrow A \cap B$are binary operations on the set P.

Watch Video Solution

6. Determine whether or not each of the definition of $' * '$ given below gives a binary operation. In the event that $' * '$ is not a binary operation, given justification for this:

(i) On Z^+, define $' * '$ by $a * b = a - b$

(ii) On Z^+, define $' * '$ by $a * b = ab$

(iii) On R, define $' * '$ by $a * b = ab^2$

(iv) On Z^+, define $' * '$ by $a * b = |a - b|$

(v) On Z^(+), define $' * '$ by $a * b = a$.

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7. Show that the binary operation $' * '$ defined from $N \times N \to N$ and given by $a * b = 2a + 3b$ is not commutative.

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8. For each binary operation * defined below, determine whether * is commutative or associative.(i) $OnZ, def \in ea \cdot b = a - b$(ii) $OnQ, def \in ea \cdot b = ab + 1$(iii) $OnQ, def \in ea \cdot b = \frac{ab}{2}$(iv) `O

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9. For each binary operation $' * '$ defined below, determine whether $' * '$ is commutative and whether $' * '$ is associative :

(ii) On Q, define $' * '$ by $a * b = ab - 1$

Watch Video Solution

10. For each binary operation $' * '$ defined below, determine whether $' * '$ is commutative and whether $' * '$ is associative :

(iii) On Q, define $' * '$ by $a * b = \frac{ab}{4}$

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11. For each binary operation $' * '$ defined below, determine whether $' * '$ is commutative and whether $' * '$ is associative :

(iv) On Q, define $' * '$ by $a * b = \frac{ab}{3}$

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12. For each binary operation $' * '$ defined below, determine whether $' * '$ is commutative and whether $' * '$ is associative :

(v) On Z^{+}, define $' * '$ by $a * b = 2^{ab}$

Watch Video Solution

13. For each binary operation $' * '$ defined below, determine whether $' * '$ is commutative and whether $' * '$ is associative :

(vi) On Z^{+}, define $' * '$ by $a * b = a^{b}$

Watch Video Solution

14. For each binary operation $' * '$ defined below, determine whether $' * '$ is commutative and whether $' * '$ is associative :

(vii) On $R - \{-1\}$, define $' * '$ by $a * b = \frac{a}{b+1}$

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15. Is $\cdot$ defined on the set $\{1, 2, 3, 4, 5\} by a \cdot b = L\dot{C}\dot{M}\cdot$of a and b a binary operation? Justify your answer.

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16. Let $\cdot$ be the binary operation on N given by $a \cdot b = L\dot{C}\dot{M}\cdot$of a and b. Find (i) $5 \cdot 7, 20 \cdot 16$(ii) Is $\cdot$ commutative? (iii) Is $\cdot$ associative? (iv) Find the identity of $\cdot$ in N (v) Which elements of N are invert

Watch Video Solution

17. Let * be a binary operation on N defined by $a * b = HCF$ of a and b.

Show that * is both commutative and associative.

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18. If $n(A) = p$ and $n(B) = q$, then the number of relations from set A to set B = ________.

Watch Video Solution

19. (a) Let $' * '$ be a binary operation defined on Q, the set of rational numbers, as follows :

(i) $a * b = a - b,$ for $a, b \in Q$

(ii) $a * b = a^2 + b^2$, for $a, b \in Q$

(iii) $a * b = a + ab,$ for $a, b \in Q$

(iv) $a * b = (a - b)^2$, for $a, b \in Q$

(v) $a * b = \frac{ab}{4}$, for $a, b \in Q$

(vi) $a * b = ab^2$, for $a, b \in Q$.

Find which of the binary operations are commutative and which are associative.

View Text Solution

20. If $A = \{1, 2, 3\}$, then the relation $R = \{(1, 2), (2, 3), (1, 3)\} \in A$ is ___.

A. transitive only

B. reflexive only

C. symmetric only

D. symmetric and transitive only

Answer:

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21. In the binary operation $*: Q \times Q \rightarrow Q$ is defined as :

(i) $a * b = a + b - ab, a, b \in Q$

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22. The binary operation $*$ defined on $\mathbb{N}$ by $a * b = a + b + ab$ for all $a, b \in \mathbb{N}$ is--

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23. Discuss the commutativity and associativity of the binary operation * on R defined by $a \cdot b = \frac{ab}{4} f \text{ or } alla, b \in R.$

Watch Video Solution

24. Find the domain and range of the real function $f(x) = \frac{x}{1 - x^2}$

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25. Show that the operation $' * '$ on

Q - {1} defined by $a * b = a + b - ab$

for all $a, b \in Q - \{1\}$, satisfies the commutative law.

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Exercise 1 E Long Answer Type Questions I

1. Consider the infimum binary operation $\wedge$ on the set $S = \{1, 2, 3, 4, 5\}$ defined by $a \wedge b =$ Minimum of $a \, and \, b$. Write the composition table of the operation $\wedge \cdot$

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2. State whether the following statements are true or false. Justify. (i) For an arbitrary binary operation $\cdot$ on a set

$N, \quad a \cdot a = a \forall a \in N$. (ii) If $\cdot$ is a commutative binary operation on N, then `a" "*" "(b" "*" "c)"

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3. Let $' * '$ be a binary operation defined on $N \times N$ by :

$(a, b) * (c, d) = (a + c, b + d)$.

Find $(1, 2) * (2, 3)$.

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4. Let $' * '$ be a binary operation defined on $N \times N$ by :

$(a, b) * (c, d) = (a + c, b + d)$.

Prove that $' * '$ is commutative and associative.

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5. Let $' * '$ be a binary operation defined on $N \times N$ by :

$(a * b) = \frac{ab}{2}$.

Find the identity element for $' * '$, if it exists.

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6. Let $A = Q \times Q$ and let * be a binary operation on A defined by $(a, b) \cdot (c, d) = (ac, b + ad)$ for $(a, b), (c, d) \in A$. Then, with respect to * on A. Find the invertible elements of A .

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7. Let $A = Q \times Q$, where Q is the set of all rational numbers and $' * '$ be the operation on A defined by :

$(a, b) * (c, d) = (ac, b + ad)$ for $(a, b), (c, d) \in A$.

Then, find : (i) The identity element of $' * '$ in A

(ii) Invertible elements of A and hence write the inverse of elements (5, 3) and $\left(\frac{1}{2}, 4\right)$.

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8. A binary operation $*$ is defined on the set R of real numbers by $a*b=\{a, \quad \text{if} \quad b=0, |a|+b, \quad \text{if} \quad b\neq 0$ If atleast one of a and b is 0, then prove that $a*b=b*a$. Check whether $*$ is commutative. Find the identity element for $*$,if it exists

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Objective Type Questions A Multiple Choice Questions

1. Let R be the relation on the set $A=\{1,\ 2,\ 3,\ 4\}$ given by $R=\{(1,\ 2),\ (2,\ 2),\ (1,\ 1),\ (4,\ 4),\ (1,\ 3),\ (3,\ 3),\ (3,\ 2)\}$. Then, R is reflexive and symmetric but not transitive (b) R is reflexive and transitive but not symmetric (c) R is symmetric and transitive but not reflexive (d) R is an equivalence relation

A. R is reflexive and symmetric but not transitive

B. R is reflexive and transitive but not symmetric

C. R is symmetric and transitive but not reflexive

D. R is an equivalence relation

Answer: B

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2. Let R be a relation on the set N given by $R = \{(a, b) : a = b - 2, b > 6\}$. Then,

A. $(2, 4) \in R$

B. $(3, 8) \in R$

C. $(6, 8) \in R$

D. $(8, 7) \in R$

Answer: C

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3. Let$A = \{1, 2, 3\}$Then number of relations containing $(1, 2) and (1, 3)$ which are reflexive and symmetric but not transitive is (A) 1 (B) 2 (C) 3 (D) 4

A. 1

B. 2

C. 3

D. 4

Answer: A

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4. Let $A = \{1, 2, 3\}$. Then number of equivalence relations containing (1, 2) is (A) 1 (B) 2 (C) 3 (D) 4

A. 1

B. 2

C. 3

D. 4

Answer: B

Watch Video Solution

5. Let $f: R \rightarrow R$be defined as $f(x) = x^4$. Choose the correct answer. (A) f is one-one onto (B) f is many-one onto (C) f is one-one but not onto (D) f is neither one-one nor onto

A. f is one-one onto

B. f is many-one onto

C. f is one-one but not onto

D. f is neither one-one nor onto

Answer: D

Watch Video Solution

6. Let $f: R \to R$ be defined as $f(x) = 3x$. Then :

A. f is one-one onto

B. f is many-one onto

C. f is one-one but not onto

D. f is neither one-one nor onto

Answer: A

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7. If $f: R \to R$ be given by $f(x) = \left(3 - x^3\right)^{1/3}$, then fof(x) is :

A. $x^{1/3}$

B. x^3

C. x

D. $3 - x^3$

Answer: C

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8. Let $f: R - \left\{\frac{5}{4}\right\} \to R$ be a function defines $f(x) = \frac{5x}{4x+5}$. The inverse of f is the map g: Range $f \to R - \left\{\frac{5}{4}\right\}$ given by

A. $g(y) = \frac{3y}{3 - 4y}$

B. $g(y) = \frac{4y}{4 - 3y}$

C. $g(y) = \frac{5y}{5 - 4y}$

D. $g(y) = \frac{3y}{4 - 3y}$

Answer: C

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9. Consider a binary operation $' * '$ on N defined as : $a * b = a^3 + b^3$.

Then :

A. is $' * '$ both associative and commutative?

B. is $' * '$ commutative but not associative?

C. is $' * '$ associative but not commutative?

D. Is $' * '$ neither commutative nor associative?

Answer: B

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10. Number of binary operations on the set {a, b} are (A) 10 (B) 16 (C) 20 (D) 8

A. 10

B. 16

C. 20

D. 8

Answer: B

Watch Video Solution

11. Let R be a relation on the set N of natural numbers defined by $n\ R\ m$ iff n divides m. Then, R is (a) Reflexive and symmetric (b) Transitive and symmetric (c) Equivalence (d) Reflexive, transitive but not symmetric

A. Reflexive and symmetric

B. Transitive and symmetric

C. Equivalence

D. Reflexive, transitive but not symmetric

Answer: D

Watch Video Solution

12. Set A has 3 elements and the set B has 4 elements. Then, the number of injective mappings that can be defined from A to B is :

A. 144

B. 12

C. 24

D. 64

Answer: C

Watch Video Solution

13. Let $f: R \rightarrow R$ be defined by $f(x) = \sin x$ and $g: R \rightarrow R$ be defined by $g(x) = x^2$, then fog is :

A. $x^2 \sin x$

B. $(\sin x)^2$

C. $\sin x^2$

D. $\frac{\sin x}{x^2}$

Answer: C

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14. Let $f: R \rightarrow R$ be defined by $f(x) = x^2 + 1$. Then pre-images of 17 and - 3 respectively, are:

A. $\phi, \{4, -4\}$

B. $\{3, -3\}, \phi$

C. $\{4, -4\}, \phi$

D. $\{4, -4\}, \{2, -2\}$

Answer: C

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15. Let $f : R \to R$ be defined by :

$$f(x) = \begin{cases} 2x & x > 3 \\ x^2 & 1 < x < 3 \\ 3x & x \leq 1 \end{cases}$$

Then, $f(-1) + f(2) + f(4)$ is :

A. 9

B. 14

C. 5

D. None of these

Answer: A

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16. If $f(x) = \log x$ and $g(x) = e^x$, then (fog) (x) is:

A. e^x

B. x

C. log x

D. 1

Answer: B

Watch Video Solution

17. If $f(x) = |x|$ and $g(x) = x - 2$, then gof is equal to:

Watch Video Solution

18. Consider the set Q with binary operation $' * '$ as:

$a * b = \frac{ab}{4}$. Then, the identity element is:

A. $\frac{1}{4}$

B. 1

C. 4

D. 16

Answer: C

Watch Video Solution

19. If $f: R \to R$ be given by $f(x) = \left(3 - x^3\right)^{1/3}$, then $f^{-1}(x)$ equals:

A. x^3

B. $x^{1/3}$

C. $3 - x^3$

D. $\left(3 - x^3\right)^{1/3}$

Answer: D

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20. The number of one-one functions from a set containing 2 elements to a set containing 3 elements is:

A. 2

B. 3

C. 6

D. 4

Answer: C

Watch Video Solution

21. Let R be a relation on the set N given by $R=\{(a,\ b): a=b-2,\ b>6\}$. Then, $(2,\ 4)\in R$ (b) $(3,\ 8)\in R$ (c) $(6,\ 8)\in R$ (d) $(8,\ 7)\in R$

A. $(2, 4)\in R$

B. $(3, 8)\in R$

C. $(6, 8)\in R$

D. $(8, 7)\in R$

Answer: C

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22. Let $f: R \rightarrow R$ be defined as $f(x) = 2x$.

A. f is one-one onto

B. f is many-one onto

C. f is one-one but not onto

D. f is neither one-one nor onto

Answer: A

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23. If $f(x) = \log(1+x) \text{ and } g(x) = e^x$, then the value of (gof) (x) is :

A. e^{1+x}

B. $1+x$

C. $\log x$

D. None of these

Answer: B

Watch Video Solution

24. Let $A=\{(a,b)\}\forall a, b\in N$. Then the relation R is :

A. Reflexive

B. Symmetric

C. Transitive

D. None of these

Answer: D

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25. The domain of the function $f(x) = \frac{x}{|x|}$ is :

A. $R - \{0\}$

B. R

C. Z

D. W

Answer: A

Watch Video Solution

26. If a binary operation is defined by $a * b = a^b$, then $3 * 2$ is equal to :

A. 4

B. 2

C. 9

D. 8

Answer: C

Watch Video Solution

27. Let $f: R \rightarrow R$ be defined as $f(x) = x^4$. Then :

A. f is one-one onto

B. f is many-one onto

C. f is one-one but not onto

D. f is neither one-one nor onto

Answer: D

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28. Consider the set $A = \{1, 2, 3, 4\}$. Which of the following relations R form a reflexive relation?

A. R = {(1,1),(1,2),(2,2),(3,4)}

B. R = {(1,1),(2,2),(2,3),(3,3),(3,4)}

C. R = {(1,1),(2,2),(2,3),(3,3),(3,4),(4,4)}

D. R = {(1,1),(2,1),(2,3),(3,3),(3,4),(4,4)}

Answer: C

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29. Let

$A = \{1, 2, 3\}$ and $Let R = \{(1, 1), (2, 2), (3, 3), (1, 3), (3, 2), (1, 2)\}$

then R is

A. Reflexive and symmetric but not transitive

B. Reflexive and transitive but not symmetric

C. Symmetric and transitive but not reflexive

D. An equivalence relation.

Answer: B

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30. Let R be a relation defined on $A = \{1, 2, 3\}$ by :

$R = \{(1, 3), (3, 1), (2, 2)\}$. R is :

A. Reflexive

B. Symmetric

C. Transitive

D. Reflexive but not Transitive

Answer: B

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Objective Type Questions B Fill In The Blanks

1. If f be greatest integer function defined asf(x)=[x] and g be the mdoulus function defined as g(x)=|x|, then the value of g of $\left(-\frac{5}{4}\right)$ is _____

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2. If $A = \{0, 1, 3\}$, then the number of relations on A is__________.

Watch Video Solution

3. If $f : R - R$ is defined by $f(x) = 3x + 4$, then $f(f(x))$ is__________.

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4. If $f(x) = e^x$ and $g(x) = \log x$, then gof is__________.

Watch Video Solution

5. If $f(x) = \frac{3x-1}{x+1}, x \neq$ __________, then fof (x) is__________

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Objective Type Questions C True False Questions

1. Given $A = \{1, 2, 3\}$, then the relation $R = \{(1, 1), (2, 2), (3, 3)\}$ is reflexive.

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2. Given a set A = {a, b, c, d}, then the relation:

$R = \{(a, a), (b, b), (c, c), (d, d)\}$ is reflexive?.

Watch Video Solution

3. A bijection function is both one-one and onto?.

Watch Video Solution

4. $f: R \rightarrow R$ given by f (x) = 2x is one-one function.

Watch Video Solution

5. If $f(x) = \log\left(\frac{1-x}{1+x}\right), \; -1 < x < 1$, then $f(-x) = f(x)$.

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Objective Type Questions D Very Short Answer Types Questions

1. Let $A = \{1, 2, 3, 4\}$. Let R be the equivalence relation on $A \times A$ defined by $(a, b)R(c, d)$ iff $a + d = b + c$. Find $\{(1, 3)\}$.

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2. If $R = \{(1, -1), (2, -2), (3, -1)\}$ is a relation, then find the range of R.

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3. If $R = \{(x, y) : x + 2y = 10\}$ is a relation in N, write the range of R.

Watch Video Solution

4. Give an example of a relation which is reflexive and symmetric but not transitive.

Watch Video Solution

5. Give an example of a relation which is transitive but neither reflexive nor symmetric.

Watch Video Solution

6. Let R be the relation "greater than" from $A = \{1, 4, 5\}$ to $B = \{1, 2, 4, 5, 6, 7\}$. Write down the elements corresponding to R.

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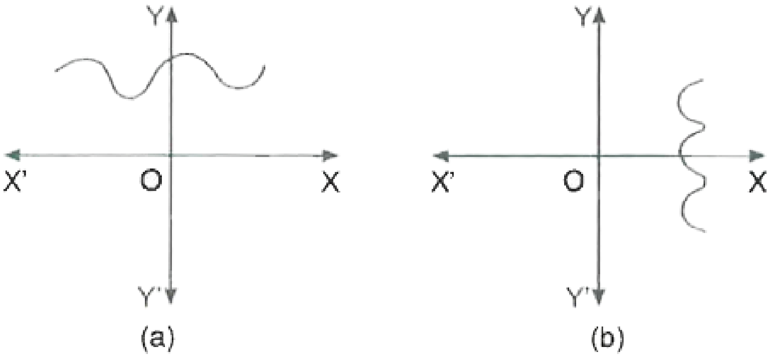
7. Which one of the following graphs represents the function of x? why?



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8. Show that $f(x) = 3x + 5$ for all $\xi n Q$, is one-one.

Watch Video Solution

9. Show that the function $f: N \rightarrow N$, given by $f(x) = 2x$, is one-one but not onto.

Watch Video Solution

10. If f is a function from $R \rightarrow R$ such that $f(x) = x^2 \forall x \in R$, then show that 'f' is not one-one.

Watch Video Solution

11. Let $A = \{1, 2, 3\}, B = \{4, 5, 6, 7\}$and let $f = \{(1, 4), (2, 5), (3, 6)\}$ be a function from A to B. Show that f is one-one.

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12. The function P is defined as:

"To each person on the earth is assigned a date of birth". Is this function one-one? Give reason.

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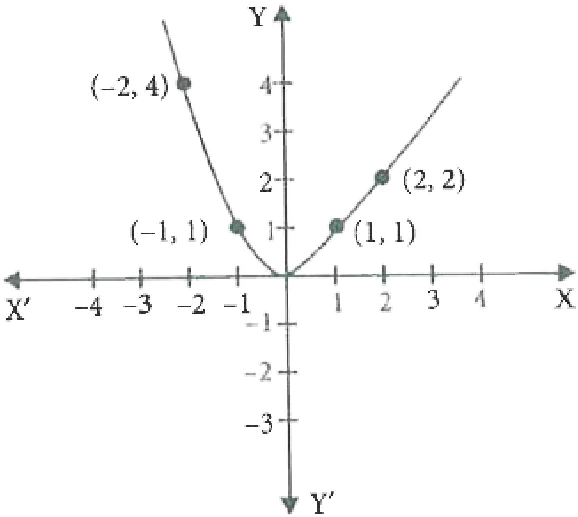
13. Write the function whose graph is shown below:



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14. Consider functions f and g such that composite gof is defined and is one-one. Are f and g both necessarily one-one.

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15. Are f and g both necessarily onto, if gof is onto?

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16. Give examples of two functions $f: N \to Z$ $and g: Z \to Z$ such that o f is injective but is not injective. (Hint: Consider $f(x) = x$ $and g(x) = |x|$)

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17. Give examples of function:

$f: N \to N$ and $g: N \to N$

such that gof is onto but f is not onto.

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18. Find fog, if $f: R \rightarrow R$ and $g: R \rightarrow R$ are given by:

$f(x) = \cos x \text{ and } g(x) = x^2.$

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19. If f (x) = sin x, g(x) = x^2, if $x \in R$, then find [(fog)(x)].

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20. If $f: R \rightarrow R$ is defined by $f(x) = 3x + 1$, find $f(f(x))$.

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21. Let $' * '$ be a binary operation on N given by:

$a * b = LCM(a, b)$ for all $a, b \in N$.

Find $6 * 7$

Watch Video Solution

22. Let $' * '$ be a binary operation on N given by:

$a * b = LCM(a, b)$ for all $a, b \in N$.

Find $20 * 16$

Watch Video Solution

23. The binary operation $* : R \times R \to R$ is defined as:

$a * b = 2a + b.$

Find $(2 * 3) * 5.$

Watch Video Solution

24. Show that $+ : R \times R \to R$and $\times : R \times R \to R$are commutative binary operations, but $: R \times R \to R$and $\div : R_{.} \times R_{.} \to R_{.}$are not commutative.

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25. Show that $+: R \times R \to R$ and $\times : R \times R \to R$ are commutative binary operations, but $: R \times R \to R$ and $\div : R_{.} \times R_{.} \to R_{.}$ are not commutative.

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26. Show that addition and multiplication are associative binary operation on R. But subtraction is not associative on R. Division is not associative on R*.

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27. Show that subtraction and division are not binary operations on N.

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28. Show that a is not the inverse of $a \in N$ for the addition operation + on N and $\frac{1}{a}$ is not the inverse of $a \in N$ for multiplication operation $\times$ on N, for $a \neq 1$.

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29. Show that a is not the inverse of $a \in N$ for the addition operation + on N and $\frac{1}{a}$ is not the inverse of $a \in N$ for multiplication operation $\times$ on N, for $a \neq 1$.

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Ncert File Question From Ncert Book Exercise 1 1

1. Determine whether each of the following relations are reflexive, symmetric and transitive:(i) Relation R in the set $A = \{1, 2, 3, ..., 13, 14\}$ defined as $R = \{(x, y) : 3xy = 0\}$(ii) Relation R in the set N o

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2. Show that the relation R in the set R of real numbers, defined as $R = \{(a, b) : a \leq b^2\}$is neither reflexive nor symmetric nor transitive.

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3. Check whether the relation R defined in the set $\{1, 2, 3, 4, 5, 6\}$as $R = \{(a, b) : b = a + 1\}$is reflexive, symmetric or transitive.

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4. Show that the relation R in R defined as $R = \{(a, b) : a \leq b\}$, is reflexive and transitive but not symmetric.

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5. Check whether the relation R in R defined by $R = \{(a, b) : a \leq b^3\}$is reflexive, symmetric or transitive.

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6. Show that the relation R in the set $\{1, 2, 3\}$given by $R = \{(1, 2), (2, 1)\}$is symmetric but neither reflexive nor transitive.

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7. Show that the relation R in the set A of all the books in a library of a college, given by R = {(x, y) : x and y have same number of pages} is an equivalence relation.

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8. Show that the relation R on the set $A = \{1,\ 2,\ 3,\ 4,\ 5\}$, given by $R = \{(a,\ b) : |a - b|$ is even $\}$, is an equivalence relation. Show that all the elements of {1, 3, 5} are related to each other and all the elements of {2, 4} are related to each other. But, no element of {1, 3, 5} is related to any element of {2, 4}.

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9. Show that the relation R on the set $A = \{x \in Z; 0 \leq x \leq 12\}$, given by $R = \{(a,\ b) : a = b\}$, is an equivalence relation. Find the set of all elements related to 1.

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10. Give an example of a relation. Which is (i) Symmetric but neither reflexive nor transitive. (ii) Transitive but neither reflexive nor symmetric. (iii) Reflexive and symmetric but not transitive. (iv) Reflexive and transitive but not symmetric. (v) Symm

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11. Show that the relation R on the set A of points in a plane, given by $R = \{(P, Q):$ Distance of the point P from the origin is same as the distance of the point Q from the origin}, is an equivalence relation. Further show that the set of all points related to a point $P \neq (0, 0)$ is the circle passing through P with origin as centre.

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12. Show that the relation R defined on the set A of all triangles in a plane as $R = \{(T_1, T_2) : T_1$ is similar to $T_2)$ is an equivalence relation. Consider three right angle triangle T_1 with sides $3, 4, 5$; T_2 with sides $5, 12, 13$ and T_3 with sides 6, 8, 10. Which triangles among T_1, T_2 and T_3 are related?

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13. Show that the relation R defined on the set A of a polygons as R = $\{P_1, P_2 : P_1 \text{and } P_2$ have same number of sides } is an equivalence relation. What is the set of all elements in A related to the right angle triangle T with sides 3,4 and 5 ?

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14. Let L be the set of all lines in XY -plane and R be the relation in L defined as $R = \{(L_1, L_2) : L_1$ is parallel to $L_2\}$. Show that R is an equivalence relation. Find the set of all lines related to the line $y = 2x + 4$.

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15. Let R be the relation on the set $A = \{1, 2, 3, 4\}$ given by $R = \{(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)\}$. Then, R is reflexive and symmetric but not transitive (b) R is reflexive and transitive but not symmetric (c) R is symmetric and transitive but not reflexive (d) R is an equivalence relation

A. R is reflexive and symmetric but not transitive

B. R is reflexive and transitive but not symmetric

C. R is symmetric and transitive but not reflexive

D. R is an equivalence relation

Answer: B

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16. Let R be the relation in the set N given by $R = \{(a, b) : a = b2, b > 6\}$. Choose the correct answer.(A) $(2, 4) \in R$ (B) $(3, 8) \in R$ (C) $(6, 8) \in R$ (D) $(8, 7)R$

A. $(2, 4) \in R$

B. $(3, 8) \in R$

C. $(6, 8) \in R$

D. $(8, 7) \in R$

Answer: C

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Ncert File Question From Ncert Book Exercise 1 2

1. Show that the function $f: R_0 \to R_0$, defined as $f(x) = \frac{1}{x}$, is one-one onto, where R_0 is the set of all non-zero real numbers. Is the result true, if the domain R_0 is replaced by N with co-domain being same as R_0 ?

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2. Check the injectivity and surjectivity of the following functions:(i) $f: N \to N$given by $f(x) = x^2$(ii) $f: Z \to Z$given by $f(x) = x^2$(iii) $f: R \to R$given by $f(x) = x^2$(iv) $f: N \to N$given by $f(x) = x^3$(v) `f : Z -

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3. Prove that the Greatest Integer Function $f: R \to R$, given by $f(x) = [x]$, is neither one-one nor onto, where [x] denotes the greatest integer less than or equal to x.

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4. Show that the Modulus Function $f: R \to R$, given by $f(x) = |x|$, is neither one-one nor onto, where $|x|$is x, if x is positive or 0 and $|x|$is x, if x is negative.

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5. Show that the Signum Function $f: R \to R$, given by $f(x) = \{1, \quad \text{if} \quad x > 00, \quad \text{if} \quad x = 0 - 1, \quad \text{if} \quad x < 0$is neither one-one nor onto.

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6. Let $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$and let $f = \{(1, 4), (2, 5), (3, 6)\}$be a function from A to B. Show that f is one-one.

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7. Show that the function $f: R \rightarrow R: f(x) = 3 - 4x$ is one-one onto and hence bijective.

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8. Let A and B be two sets. Show that $f: A \times B \rightarrow B \times A$ defined by $f(a, \ b) = (b, \ a)$ is a bijection.

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9. Let $f \ : N \rightarrow N$ be defined by $f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } nisodd \frac{n}{2}, & \text{if } niseven \end{cases}$ for all $n \in N$. State whether the function f is bijective. Justify your answer.

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10. Let $A = R - \{3\}$and $B = R - \{1\}$. Consider the function $f: A \to B$ defined by $(x) = \left(\frac{x-2}{x-3}\right)$. Is f one-one and onto? Justify your answer.

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11. Let $f: R \to R$be defined as $f(x) = x^4$. Choose the correct answer. (A) f is one-one onto (B) f is many-one onto (C) f is one-one but not onto (D) f is neither one-one nor onto

A. f is one-one onto

B. f is many-one onto

C. f is one-one but not onto

D. f is neither one-one nor onto

Answer: D

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12. Let $f: R \rightarrow R$be defined as $f(x) = 3x$. Choose the correct answer.(A) f is one-one onto (B) f is many-one onto(C) f is one-one but not onto (D) f is neither one-one nor onto.

A. f is one-one onto

B. f is many-one onto

C. f is one-one but not onto

D. f is neither one-one nor onto

Answer: A

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Ncert File Question From Ncert Book Exercise 1 3

1. Let $f:\{1, 3, 4\} \to \{1, 2, 5\}$ and $g:\{1, 2, 5\} \to \{1, 3\}$ be given by $f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(1, 3), (2, 3), (5, 1)\}$. Write down gof.

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2. Let f, g and h be functions from R to R. Show that $(f+g)oh = foh + goh$

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3. Find gof and fog, if $f(x) = |x|$ and $g(x) = |5x - 2|$

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4. Find gof and fog, if :

$f(x) = 8x^3$ and $g(x) = x^{\frac{1}{3}}$.

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5. If $f(x) = \frac{4x+3}{6x-4}, \ x \neq \frac{2}{3}$, show that $fof(x) = x$ for all $x \neq \frac{2}{3}$. What is the inverse of f?

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6. State with reason whether following functions have inverse (i) $f \colon \{1, 2, 3, 4\} \to \{10\} with f = \{(1, 10), (2, 10), (3, 10), (4, 10)\}$(ii) $g \colon \{5, 6, 7, 8\} \to \{1, 2, 3, 4\} with g = \{(5, 4), (6, 3), (7, 4), (8, 2)\}$(iii) `h : {2,3,4,5} ->{7,9

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7. Show that $f \colon [-1, 1] \to R$, given by $f(x) = \frac{x}{(x+2)}$ is one- one . Find the inverse of the function $f \colon [-1, 1]$

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8. Consider $f: R \rightarrow R$given by $f(x) = 4x + 3$. Show that f is invertible.

Find the inverse of f.

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9. Consider $f: R_{+} \overrightarrow{4, \infty}$ given by $f(x) = x^2 + 4$. Show that f is invertible with the inverse (f^{-1}) of f given by $f^{-1}(y) = \sqrt{y - 4}$, where R_{+} is the set of all non-negative real numbers.

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10. Consider $f: \mathbb{R}_{+} \rightarrow [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$.

Show that f is invertible with $f^{-1}(y) = \dfrac{\sqrt{y+6} - 1}{3}$

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11. Let $f: X \rightarrow Y$be an invertible function. Show that f has unique inverse. (Hint: suppose $g_1(\text{ and } g)_2$are two inverses of f. Then for all $y \in Y, fog_1(y) = I_Y(y) = fog_2(y)$Use one oneness of f).

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12. Consider $f: \{1, 2, 3\} \rightarrow \{a, b, c\}$given by $f(1) = a, \ f(2) = b$and $f(3) = c$. Find $f^{-1}$and show that $\left(f^{-1}\right)^{-1} = f$.

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13. Let f be an invertible function. Show that the inverse of f^{-1} is f

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14. If $f: R \rightarrow R$be given by $f(x) = \left(3 - x^3\right)^{1/3}$, then $fof(x)$is(a) $\frac{1}{x^3}$ (b) x^3 (c) x (d) $\left(3 - x^3\right)$

A. $x^{1/3}$

B. x^3

C. x

D. $(3 - x^3)$

Answer: C

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15. Let $f: R - \left\{\frac{5}{4}\right\} \to R$ be a function defines $f(x) = \frac{5x}{4x+5}$. The inverse of f is the map g: Range $f \to R - \left\{\frac{5}{4}\right\}$ given by

A. $g(y) = \frac{3y}{3-4y}$

B. $g(y) = \frac{4y}{4-3y}$

C. $g(y) = \frac{4y}{3-4y}$

D. $g(y) = \frac{3y}{4-3y}$

Answer: B

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Ncert File Question From Ncert Book Exercise 1 4

1. Determine whether or not each of the definition of given below gives a binary operation. In the event that * is not a binary operation, give justification for this. (i) $OnZ^+, def \in e \cdot bya \cdot b = a - b$(ii) `O n Z^+,

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2. For each binary operation * defined below, determine whether * is commutative or associative.(i) $OnZ, def \in ea \cdot b = a - b$(ii) $OnQ, def \in ea \cdot b = ab + 1$(iii) $OnQ, def \in ea \cdot b = \frac{ab}{2}$(iv) `O

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3. Consider the binary operation* on the set {1, 2, 3, 4, 5} defined by a * b=min. {a, b}. Write the operation table of the operation *.

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4. Consider a binary operation * on the set {1, 2, 3, 4, 5} given by the following multiplication table (FIGURE) Compute (2*3) *4 and 2* (3*4) Is * commutative? (iii) Compute (2*3)*(4*5)

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5. Let $\cdot\,'$be the binary operation on the set $\{1, 2, 3, 4, 5\}$defined by $a \cdot\,' b = H\dot{C}\dot{F}\cdot$of a and b. Is the operation $\cdot\,'$same as the operation $\cdot$ defined in Exercise 4 above? Justify your answer.

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6. Let * be the binary operation on N given by $a \cdot b = LCM of a$ and $b, a \forall a, bN.$ Find 5 * 7.

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7. Is $\cdot$ defined on the set $\{1, 2, 3, 4, 5\} by a \cdot b = L\dot{C}\dot{M}$.of a and b a binary operation? Justify your answer.

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8. Let * be a binary operation on N defined by $a * b = HCF$ of a and b. Show that * is both commutative and associative.

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9. Let $\cdot$ be a binary operation on the set Q of rational numbers as follows: (i) $a \cdot b = a - b$ (ii) $a \cdot b = a^2 + b^2$ (iii)

$a \cdot b = a + ab$ (iv) $a \cdot b = (a - b)^2$ (v) $a \cdot b = \frac{ab}{4}$ (vi) $a \cdot b = ab^2$. Find wh

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10. Let $f(x) = x^2$ and $g(x) = 2x + 1$ be two real functions. Find $(f + g)(x), (f - g)(x), (fg)(x), \left(\frac{f}{g}\right)(x)$

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11. Let A = R R and * be the binary operation on A defined by (a, b) * (c, d) = (a + c, b + d). Show that * is commutative and associative. Find the identity element for * on A.

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12. State whether the following statements are true or false. Justify. (i) For an arbitrary binary operation $\cdot$ on a set

N, $a \cdot a = a \forall a \in N$. (ii) If $\cdot$ is a commutative binary operation on N, then `a" "*" "(b" "*" "c)"

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13. Consider a binary operation $\cdot$ on N defined as $a \cdot b = a^3 + b^3$. Choose the correct answer. (A) Is $\cdot$ both associative and commutative? (B) Is $\cdot$ commutative but not associative? (C) Is $\cdot$ associative but not commutative? (D) Is

A. Is $*$ both associative and commutative ?

B. Is $*$ commutative but not associative ?

C. Is $*$ associative but not commutative ?

D. Is $*$ neither commutative nor associative ?

Answer: B

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Misellaneous Exercise On Chapter 1

1. Let $f: R \rightarrow R$be defined as $f(x) = 10x + 7$. Find the function $g: R \rightarrow R$such that$gof = fog = 1_R$

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2. Let $f: W \rightarrow W$be defined as $f(n) = n - 1$, if is odd and $f(n) = n + 1$, if n is even. Show that f is invertible. Find the inverse of f. Here, W is the set of all whole numbers.

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3. If $f: R \rightarrow R$is defined by $f(x) = x^2 - 3x + 2$, find $f(f(x))$.

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4. Show that function $f: R \rightarrow \{x \in R: -1 < x < 1\}$defined by $f(x) = \frac{x}{1 + |x|}, x \in R$ is one one and onto function

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5. Show that the function $f: R \rightarrow R$given by $f(x) = x^3$is injective.

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6. Give examples of two functions $f: \quad N \rightarrow Z \quad andg: \quad Z \rightarrow Z$ such that o f is injective but is not injective. (Hint: Consider $f(x) \quad = \quad x \quad andg(x) \quad = \quad |x|$)

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7. Given examples of two functions $f: \quad N \rightarrow N \quad andg: \quad N \rightarrow N$ such that of is onto but f is not onto. (Hint: Consider

$f(x) = x \quad and g(x) = |x|$).

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8. Given a non-empty set X , consider $P(X)$ which is the set of all subjects of X . Define a relation in $P(X)$ as follows: For subjects A, B in $P(X), \quad A R B$ if $A \subset B$. Is R an equivalence relation on $P(X)$? Justify your answer.

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9. Given a non-empty set X, consider the binary operation $\cdot : P(X) \times P(X) \to P(X)$given by $A \cdot B = A \cap B \forall A, B \in P(X)$is the power set of X. Show that X is the identity element for this operation and X is the only invertible element i

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10. Find the number of all onto functions from the set $A = \{1,\ 2,\ 3,\ ,\ n\}$ to itself.

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11. Let $S = \{a, b, c\} and T = \{1, 2, 3\}$. Find $F^{-1}$of the following functions F from S to T, if it exists.(i $)F = \{(a, 3), (b, 2), (c, 1)\}$(ii) $F = \{(a, 2), (b, 1), (c, 1)\}$

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12. Consider the binary operations $\cdot : R \times R \to R$ and $o : R \times R \to R$ defined as $a \cdot b|a - b|$ and $a\ o\ b = a, \forall a, b \in R$. Show that * is commutative but not associative, o is associative but not commutative. Further, show that `AAa

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13. Given a non -empty set X, let $\cdot : P(X) \times P(X) \rightarrow P(X)$ be defined as A * B = (A B) ∪ (B A), ∀ A, B ∈ P(X)

$A \cdot B = (A - B) \cup (B - A), \forall A, B \in P(X)$.

Show that the empty set φ is the identity for the

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14. Define a binary operation *on the set $\{0, 1, 2, 3, 4, 5\}$ as $a \cdot b = \{a + b \text{ if } a + b < 6a + b - 6, \text{ if } a + b \geq 6$ Show that zero is the identity for this operation and each element $a \neq 0$ of the set is invertible with 6 a being t

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15. Let $A = \{-1, 0, 1, 2\}$, $B = \{-4, -2, 0, 2\}$ and $f, g: A \rightarrow B$ be functions defined by $f(x) = x^2 - x, x \in A$ and

$g(x) = 2\left|x - \left(\frac{1}{2}\right)\right| - 1, x \in A$. Are f and g equal? Justify your answer.

(Hint: One may note that two functio

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16. Let A={1,2,3}. Then the number of relations containing (1,2) and (1,3) which are reflexive and symmetric but not transitive is

A. 1

B. 2

C. 3

D. 4

Answer: A

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17. Let $A = \{1, 2, 3\}$. Then number of equivalence relations containing (1, 2) is (A) 1 (B) 2 (C) 3 (D) 4

A. 1

B. 2

C. 3

D. 4

Answer: B

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18. Let $f: R \rightarrow R$be the Signum Function defined as $f(x) = \{1, x > 00, x = 0 - 1, x < 1$ and $g: R \rightarrow R$be the Greatest Integer Function given by $g(x) = [x]$, where [x] is greatest integer less than or equal to x. Then does fo

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19. समुच्चय {a,b} में द्विआधारी संक्रियाओं की संख्या है :

A. 10

B. 16

C. 20

D. 8

Answer: B

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Exercise

1. Let $A = \{(0, 1, 2, 3\}$ and define a relation R on A as follows:

$R = \{(0, 0), (0, 1), (0, 3), (1, 0), (1, 1), (2, 2), (3, 0), (3, 3)\}$, Is R reflexive? Symmetive? Transitive?

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2. Let $A = \{1, 2, 3, ;\cdot 9\}$ and R be the relation in AxA defined by $(a, b)R(c, d)$ if $a + d = b + c$ for $(a, b), (c, d)$ in AxA. Prove that R is an equivalence relation. Also obtain the equivalence class [(2,5)].

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3. If $f = \{(5, 2), (6, 3)\}, g = \{(2, 5), (3, 6)\}$, write $f \circ g$.

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4. Let $f: R \to R$ be defined by $f(x) = x^2 + 1$.

Find the pre-image of (i) 17 (ii) - 3.

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5. Let the function $f: R \to R$ be defined by $f(x) = \cos x, \forall x \in R$.

Show that f is neither one-one nor onto.

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6. Let $A = R - \{3\}, B = R - \{1\}$, and let $f: A\overrightarrow{B}$ be defined by $f(x) = \frac{x-2}{x-3}$ is f invertible? Explain.

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7. If the mappings f and g are given by : $f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(2, 3), (5, 1), (1, 3)\}$, write fog.

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8. Let * be the binary operation defined on Q. Find which of the following binary operations are commutative

(i) $a * b = a - b, \forall a, b \in Q$ (ii) $a * b = a^2 + b^2, \forall a, b \in Q$

(iii) $a * b = a + ab, \forall a, b \in Q$ (iv) $a * b = (a - b)^2, \forall a, b \in Q$

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9. Let * be a binary operation on R defined by $a \cdot b = ab + 1$. Then, * is commutative but not associative associative but not commutative neither commutative nor associative (d) both commutative and associative

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10. An operation $*$ on $\mathbb{Z}$, the set of integers, is defined as, $a * b = a - b + ab$ for all $a, b \in \mathbb{Z}$. Prove that $*$ is a binary operation on $\mathbb{Z}$ which is neither commutative nor associative.

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Revision Exercise

1. Let $f : X \rightarrow Y$ be a function. Define a relation R in X given by $R = \{(a, b) : f(a) = f(b)\}$. Examine if R is an equivalence relation.

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2. If R_1 and R_2 are equivalence relations in a set A, show that $R_1 \cap R_2$ is also an equivalence relation.

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3. Let $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, Let R_1 be a relation on X given by $R_1 = \{(x, y) : x - y$ is divisible by 3$\}$ and R_2 be another relation on X given by $R_2 = \{(x, y) : \{x, y\} \subset \{1, 4, 7\}$ or $\{x, y\} \subset \{2, 5, 8\}$ or $\{x, y\} \subset \{3, 6, 9\}\}$. Show that $R_1 = R_2$.

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4. Show that the number of equivalence relations on the set {1, 2, 3} containing (1, 2) and (2, 1) is two.

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5. Let $A = \{1, 2, 3\}$. Then, show that the number of relations containing (1, 2) and (2, 3) which are reflexive and transitive but not symmetric is three.

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6. Find the number of all one-one functions from set $A = \{1, 2, 3\}$to itself.

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7. Find the number of all onto functions from the set $A = \{1, 2, 3, , n\}$ to itself.

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8. Give examples of two one-one functions f_1 and f_2 from R to R such that $f_1 + f_2 : RrarR$ defined by :

$(f_1 + f_2)(x) = f_1(x) + f_2(x)$

is not one-one.

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9. Show that if f_1 and f_2 are one-one maps from R to R , then the product $f_1 \times f_2 : R \to R$ defined by $(f_1 \times f_2)(x) = f_1(x) f_2(x)$ need not be one-one.

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10. Let $f : A \to A$ be a function such that $fof = f$. Show that f is onto if and only if f is one-one. Describe f in this case.

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11. Consider the identity function $I_N : N \to N$ defined as :

$I_N(x) = x \, \forall x \in N.$

Show that although I_N is onto but $I_N + I_N : N \to N$ defined as :

$(I_N + I_N)(x) = I_N(x) + I_N(x) = x + x = 2x$ is not onto.

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12. Consider a function $f: \left[0, \frac{\pi}{2}\right] \to R$ given by $f(x) = \sin x$ and $g: \left[0, \frac{\pi}{2}\right] \to R$ given by $g(x) = \cos x$. Show that f and g are one-one, but $f + g$ is not one-one.

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13. Find fof^{-1} and f^{-1} of for the function :

$f(x) = \frac{1}{x}, x \neq 0$. Also prove that $fof^{-1} = f^{-1}of$.

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14. Show that the number of binary operations on $\{1, 2\}$having 1 as identity and having 2 as the inverse of 2 is exactly one.

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15. Determine whether the following binary operation on the set N is associative and commutative :

$a * b = 1 \, \forall a, b \in N.$

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16. Determine which of the following binary operations on the set N are associative and which are commutative.(a) $(b)(c)a \cdot b = 1 \, \forall a, b \in N(d)$ (e) (b) $(f)(g)a \cdot b = (h)\left((i)\frac{a+b}{j}2(k)(l)\, \forall a, b \in N(m)\right.$(n)

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17. Consider the binary operations $\cdot : R \times R \to R$ and $o: R \times R \to R$ defined as $a \cdot b|a - b|$ and $a\ o\ b = a, \forall a, b \in R$. Show that * is commutative but not associative, o is associative but not commutative. Further, show that `AAa

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18. Define a binary operation * on the set $A = \{0, 1, 2, 3, 4, 5\}$ given by $a \cdot b = ab$ (mod 6). Show that 1 is the identity for *. 1 and 5 are the only invertible elements with $1^{-1} = 1$ and $5^{-1} = 5$

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Check Your Understanding

1. $IfA = (1, 2, 3)$, then the relation $R = \{(1, 1)(2, 2), (3, 1), (1, 3)\}$ is

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2. Give an example of a relation which is symmetric but neither reflexive nor transitive.

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3. Which of the following functions is (are) even, odd or neither:

$f(x) = \sqrt{1 + x + x^2} - \sqrt{1 - x + x^2}$

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4. What is the domain of the function $f(x) = \frac{1}{x - 2}$?

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5. If $f(x) = \begin{cases} x - 2 & x < 2 \\ 3 & x = 2 \\ x + 2 & x > 3 \end{cases}$,then find f(8).

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6. If $a * b = 3a + 4b$, then the value of $3 * 4$ is..............

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7. If $a * b = \frac{a}{2} + \frac{b}{3}$, then the value of $2 * 3$ is.............

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8. Let A = {1,2,3}. For $x, y \in A$, let xRy if and only if $x > y$. Write down R as subset of $A \times A$.

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9. Show that a is not the inverse of $a \in N$ for the addition operation + on N and $\frac{1}{a}$ is not the inverse of $a \in N$ for multiplication operation $\times$ on N, for $a \neq 1$.

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10. Show that a is not the inverse of $a \in N$ for the addition operation + on N and $\frac{1}{a}$ is not the inverse of $a \in N$ for multiplication operation $\times$ on N, for $a \ne 1$.

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Competition File Questions From Jee Main

1. Consider the following relations: R = {(x, y) | x, y are real numbers and x = wy for some rational number w}; $S = \left\{\left(\frac{m}{n}, \frac{p}{q}\right)\text{m , n , pandqa r ei n t e g e r ss u c ht h a tn ,q} \ne \text{0andq m}\right.$

. Then (1) neither R nor S is an equivalence relation (2) S is an equivalence relation but R is not an equivalence relation (3) R and S both are equivalence relations (4) R is an equivalence relation but S is not an equivalence relation

A. R is an equivalence relation but S is not an equivalence relation

B. neither R nor S is an equivalence relation

C. S is an equivalence relation but R is not an equivalence relation

D. R and S both are equivalence relations.

Answer: C

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2. The domain of the function

$f(x) = \frac{1}{\sqrt{|x| - x}}$, is

A. $(-\infty, \infty)$

B. $(0, \infty)$

C. $(-\infty, 0)$

D. $(-\infty, \infty) - \{0\}$

Answer: C

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3. Let $f(x)=x^2$ and $g(X)=\sin x$ for all $x \varepsilon R$. Then the set of all x satisfying $(fogogof)(x)=(gogof)(x)$, where $(fog)(x)=f(g(x))$ is

A. $\pm\sqrt{n\pi}, n \in \{0, 1, 2, \ldots\ldots.\}$

B. $\pm\sqrt{n\pi}, n \in \{1, 2, \ldots\ldots..\}$

C. $\frac{\pi}{2}+2n\pi, n \in \{\ldots\ldots-2, -1, 0, 1, 2, \ldots\ldots\}$

D. $2n\pi, n \in \{\ldots\ldots-2, -1, 0, 1, 2, \ldots\ldots..)$

Answer: A

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4. The function $f:[0, 3]\overrightarrow{1, 29}$, defined by $f(x)=2x^3-15x^2+36x+1$, is one-one and onto onto but not one-one one-one but not onto neither one-one nor onto

A. one-one and onto

B. onto but not one-one

C. one-one but not onto

D. neither one-one nor onto.

Answer: B

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5. If $a \in R$ and the equation $-3(x-[x])^2 + 2(x-[x]) + a^2 = 0$ (where [x] denotes the greatest integer $\leq x$) has no integral solution, then all possible values of a lie in the interval: (1) (-2,-1) (2) $(\infty, -2) \cup (2, \infty)$ (3) $(-1, 0) \cup (0, 1)$ (4) (1,2)

A. $(1, 2)$

B. $(-2, -1)$

C. $(-\infty, -2) \cup (2, \infty)$

D. $(-1, 0) \cup (0, 1)$

Answer: D

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6. The function $f : R \rightarrow \left[-\frac{1}{2}, \frac{1}{2}\right]$ defined as $f(x) = \frac{x}{1+x^2}$ is

A. Surjective but not injective

B. Neither injective nor surjective

C. Invertible

D. Injective but not surjective

Answer: A

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7. Let the function $f(x)$ defined on $f : R - (-1, 1) \rightarrow A$ and $f(x)$-

(x^(2))/(1-x^(2))$F \in dAsucht$^f(x)` is subjective.

A. $R-[-1,0)$

B. $R-[-1,1)$

C. $R-[-1,2)$

D. $R-[0,1)$

Answer: A

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Chapter Test 1

1. R is a relation is defined on the set {1,2,3,4} as follow

R={(1,2), (2,2), (1,1), (4,4), (1,3), (3,3), (3,2)}

Then choose the coR Rect option of the following

A. R is reflexive and symmetric but not transitive

B. R is reflexive and transitive but not symmetric

C. R is symmetric and transitive but not reflexive

D. R is an equivalence relation

Answer: B

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2. If f be greatest integer function defined asf(x)=[x] and g be the mdoulus function defined as g(x)=|x|, then the value of g of $\left(-\frac{5}{4}\right)$ is _____

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3. Give an example of a relation which is symmetric but neither reflexive nor transitive.

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4. Are f and g both necessarily onto, if $gof$is onto?

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5. Show that $+ : R \times R \rightarrow R$and $\times : R \times R \rightarrow R$are commutative binary operations, but $: R \times R \rightarrow R$and $\div : R_{.} \times R_{.} \rightarrow R_{.}$are not commutative.

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6. Prove that the relation R on the set $N \times N$ defined by $(a, b) R (c, d) a + d = b + c$ for all $(a, b), (c, d) \in N \times N$ is an equivalence relation. Also, find the equivalence classes [(2, 3)] and [(1, 3)].

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7. Show that the Signum Function $f : R \rightarrow R$, given by :

$f(x) = \begin{cases} 1, \text{ if } & x > 0 \\ 0, \text{ if } & x = 0 \\ \text{-1, if } & x < 0 \end{cases}$ is neither one-one nor onto.

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8. If $f(x) = \frac{x-1}{x+1}, x \neq -1,$. then show that $f(f(x)) = -\frac{1}{x}$, prove that $x \neq 0$.

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9. Let $Y = \{n^2 : n \in N\} \in N$. Consider $f: N \rightarrow Y$as $f(n) = n^2$. Show that f is invertible. Find the inverse of f.

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10. Is $\cdot$ defined on the set $\{1, 2, 3, 4, 5\} by a \cdot b = L\dot{C}\dot{M}$.of a and b a binary operation? Justify your answer.

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11. If R_1 and R_2 are equivalence relations in a set A, show that $R_1 \cap R_2$ is also an equivalence relation.

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12. Consider the binary operations $\cdot : R \times R \to R$ and o: $R \times R \to R$ defined as $a \cdot b|a-b|$ and $a \; o \; b = a, \forall a, \; b \in R$. Show that * is commutative but not associative, o is associative but not commutative. Further, show that `AAa

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