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MATHS

BOOKS - FULL MARKS MATHS (TAMIL ENGLISH)

BINOMIAL THEOREM, SEQUENCES AND SERIES

Example Questions Solved

1. Find the expansion of $(2x + 3)^5$.

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2. Evaluate 98^4.

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3. Find the middle term in the expansion of $(x+y)^6$.

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4. Find the middle terms in the expansion of $(x+y)^7$.

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5. Find the coefficient of x^6 in the expansion of $(3+2x)^{10}$.

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6. Find the coefficient of x^3 in the expansion of $(2-3x)^7$.

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7. The 2^{nd}, 3^{rd} and 4^{th} terms in the binomial expansion of $(x+a)^n$ are 240, 720 and 1080 for a suitable value of x. Find x, a and n.

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8. Expand $\left(2x-\frac{1}{2x}\right)^4$

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9. Expand $\left(x^2+\sqrt{1-x^2}\right)^5+\left(x^2-\sqrt{1-x^2}\right)^5$.

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10. Using Binomial theorem, prove that 6^n-5n always leaves remainder 1 when divided by 25 for all positive integer n.

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11. Find the last digits of the number 7^{400}.

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12. Prove that if a, b, c are in HP, if and only if $\frac{a}{c} = \frac{a-b}{b-c}$.

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13. If the 5^{th} and 9^{th} terms of a harmonic progression are $\frac{1}{19}$ and $\frac{1}{35}$ find the 12^{th} term of the sequence.

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14. Find seven numbers $A_1, A_2, \ldots\ldots A_7$ so that the sequence $4, A_1, A_2, \ldots\ldots\ldots, A_7, 7$ is in arithmetic progression and also 4 numbers G_1, G_2, G_3, G_4 so that the sequence $12, G_1, G_2, G_3, G_4, \frac{3}{8}$ is in geometric progression.

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15. If the product of the $4^{th}, 5^{th}$ and 6^{th} terms of a geometric progression is 4096 and if the product of the $5^{th}, 6^{th}$ and 7^{th} - terms of it is 32768, find the sum of first 8 terms of the geometric progression.

For any two positive numbers, the three means AM, GM and HM are in geometric progression.

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16. Find the sum up to n terms of the series : $1 + \frac{6}{7} + \frac{11}{49} + \frac{16}{343} + \ldots.$

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17. Find the sum of the first n terms of the series $\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\ldots$

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18. Find $\sum_{k=1}^{n}\frac{1}{k(k+1)}$.

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19. Find the sum : $1+\frac{4}{5}+\frac{7}{25}+\frac{10}{125}+\ldots\ldots$

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20. Find $\sum_{n=1}^{\infty} \frac{1}{n^2 + 5n + 6}$

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21. Expand $(1 + x)^{\frac{2}{3}}$ up to four terms for $|x| < 1$.

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22. Expand $\frac{1}{(1 + 3x)^2}$ in powers of x. Find a condition on x for which the expansion is valid.

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23. Expand $\frac{1}{(3+2x)^2}$ in powers of x. Find a condition on x for which the expansion is valid.

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24. Find $\sqrt[3]{65}$

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25. Prove that $\sqrt[3]{x^3+7}-\sqrt[3]{x^3+4}$ is approximately equal to $\frac{1}{x^2}$ when x is large.

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Exercise 5 1

1. Expand

(i) $\left(2x^2 - \frac{3}{x}\right)^3$

(ii) $\left(2x^2 - 3\sqrt{1-x^2}\right)^4 + \left(2x^2 + 3\sqrt{1-x^2}\right)^4$

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2. Compute

(i) 102^4

(ii) 99^4

(iii) 9^7

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3. Using binomial theorem, indicate which of the following two number is larger : $(1.01)^{1000000}$, 10000.

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4. Find the co-efficient of x^{15} in $\left(x^2+\frac{1}{x^3}\right)^{10}$

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5. Find the Co-efficient of x^6 and the co-efficient of x^2 in $\left(x^2-\frac{1}{x^3}\right)^6$

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6. Find the Co-efficient of x^4 in the expansion of $(1+x^3)^{50}\left(x^2+\frac{1}{x}\right)^5$.

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7. Find the constant term of $\left(2x^3-\frac{1}{3x^2}\right)^5$.

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8. Find the last two digits of the number 3^{600}.

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9. If n is a positive integer, show that, $9^{n+1} - 8n - 9$ is always divisible by 64.

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10. If n is an odd positive integer, prove that the coefficients of the middle terms in the expansion of $(x + y)^n$ are equal.

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11. If n is a positive integer and r is a non - negative integer, prove that the coefficients of x^r and x^{n-r} in the expansion of $(1 + x)^n$ are equal.

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12. If a and b are distinct Integers, prove that a - b is a factor of $a^n - b^n$, whenever n is a positive integer. [Hint: write $a^n = (a - b + b)^n$ and expand]

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13. In the binomial expansion of $(a + b)^n$, the coefficients of the 4^{th} and 13^{th} terms are equal to each other, find n.

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14. If the binomial coefficients of three consecutive terms in the expansion of $(a+x)^n$ are in the ratio $1:7:42$, then find n.

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15. In the binomial coefficients of $(1+x)^n$, the coefficients of the 5^{th}, 6^{th} and 7^{th} terms are in A.P. Find all values of n.

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16. Prove that $C_0^2 + C_1^2 + C_2^2 + \ldots + C_n^2 = \frac{2n!}{(n!)^2}$.

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Exercise 5 2

1. Write the first 6 terms of the sequences whose n^{th} terms are given below and classify them as arithmetic progression, geometric progression, arithmetico-geometric progression, harmonic progression and none of them.

$4\left(\frac{1}{2}\right)^n$

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2. Write the first 6 terms of the sequences whose n^{th} term a_n is given below

$$a_n = \begin{cases} \text{n}+1 & \text{if n is odd} \\ \text{n} & \text{if n is even} \end{cases}$$

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3. Write the n^{th} term of the following sequences.

2, 2, 4, 4, 6, 6, ___

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4. The product of three increasing numbers in GP is 5832. If we add 6 to the second number and 9 to the third number, then resulting numbers form an AP. Find the numbers in GP. And 9 to the third number, then resulting numbers form an AP. Find the numbers in GP.

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5. Write the n^{th} term of the sequence $\frac{3}{1^2 2^2}, \frac{5}{2(2)3^2}, \frac{7}{3^2 4^2}, \ldots\ldots$ as a difference of two terms.

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6. If t_k is the k^{th} term of a G.P, then show that t_{n-k}, t_n, t_{n+k} also form a GP for any positive integer k.

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7. If a, b, c are in geometric progression, and if $a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}}$, then prove that x, y, z are in arithmetic progression.

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8. The AM of two numbers exceeds their GM by 10 and HM by 16. Find the numbers.

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9. If the roots of the equation $(q-r)x^2+(r-p)x+p-q=0$ are equal, then show that p, q and r are in AP.

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10. If a, b, c are respectively the p^{th}, q^{th} and r^{th} terms of a G.P., show that

$(q-r)\log a+(r-p)\log b+(p-q)\log c=0.$

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Exercise 5 3

1. Find the sum of first 20 terms of the arithmetic progression having the sum of first 10 terms as 52 and the sum of the first 15 terms as 77.

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2. Find the sum up to the 17^{th} term of the series $\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+\ldots$

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3. Compute the sum of first n terms of the following series :

8 + 88 + 888 + 8888 ___

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4. Compute the sum of first n terms of $1 + (1 + 4) + (1 + 4 + 4^2) + (1 + 4 + 4^2 + 4^3) + ...$

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5. Find the general term and sum to n terms of the sequence $1, \frac{4}{3}, \frac{7}{9}, \frac{10}{27}, \ldots\ldots$

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6. Find the value of n, if the sum to n terms of the series $\sqrt{3}+\sqrt{75}+\sqrt{243}+\ldots 435\sqrt{3}$. Is

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7. Show that the sum of $(m+n)^{th}$ and $(m-n)^{th}$ term of an A.P is equal to twice the m^{th} term.

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8. A man repays an amount of Rs.3250 by paying Rs.20 in the first month and then increases the payment by Rs.15

per month. How long will it take him to clear the amount?

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9. In a race, 20 balls are placed in a line at intervals of 4 meters with the first ball 24 meters away from the starting point. A contestant in required to bring the balls back to the starting place one at a time. How far would the contestant run to bring back all balls ?

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10. The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture

originally, how many bacteria will be present at the end of 2^{nd} hour, 4^{th} hour and n^{th} hour?

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11. What will Rs. 500 amounts to in 10 years after its deposit in a bank which pays annual interest rate of 10% compounded annually?

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12. In a certain town, a viral disease caused severe health hazards upon its people distrbing their normal life. It was found that on each day, the virus which caused the disease spread in Geometric Progression. The amount of infectious

virus particle gets doubled each day, being 5 particles on the first day. Find the day when the infection virus particles just grow over 1,50,000 units ?

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Exercise 5 4

1. Expand the following in ascending powers of x and find the condition on x for which the binomial expansion is valid.

$$\frac{1}{5+x}$$

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2. Find $\sqrt[3]{1001}$ approximately. (two decimal places).

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3. Prove that $\sqrt[3]{x^3+6}-\sqrt[3]{x^3+3}$ is approximately equal to $\frac{1}{x^2}$ when x is sufficiently large.

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4. Prove that $\sqrt{\frac{1-x}{1+x}}$ is approximately equal to $1-x+\frac{x^2}{2}$ when x is very small.

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5. Write the first 6 terms of the exponential series

$e^{\frac{1}{2}x}$

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6. Write the first 4 terms of the logarithmic series

$$\log\left(\frac{1-2x}{1+2x}\right)$$

Find the intervals on which the expansions are valid.

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7. If $y = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \ldots$ then show that $x = y - \frac{y^2}{2!} + \frac{y^3}{3!} - \frac{y^4}{4!} + \ldots$

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8. If $p-q$ is small compared to either p or q, then show that $\sqrt[n]{\frac{p}{q}} \cong \frac{(n+1)p+(n-1)q}{(n-1)p+(n+1)q}$. Hence find $\sqrt[8]{\frac{15}{16}}$.

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9. Find the co-efficient of x^4 in the expansion of $\frac{3-4x+x^2}{e^{2x}}$.

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10. Find the value of $\sum_{n=1}^{\infty} \frac{1}{2n-1}\left(\frac{1}{9^{n-1}}+\frac{1}{9^{2n-1}}\right)$.

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Exercise 5 5

1. The value of 2 + 4 + 6 + ...+ 2n is

A. $\frac{n(n-1)}{2}$

B. $\frac{n(n+1)}{2}$

C. $\frac{2n(2n+1)}{2}$

D. $n(n+1)$

Answer: D

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2. The coefficient of x^6 in $(2+2x)^{10}$ is .....

A. ${}^{10}C_6$

B. 2^6

C. ${}^{10}C_6 2^6$

D. ${}^{10}C_6 2^{10}$

Answer: D

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3. The coefficient of $x^8 y^{12}$ in the expansion of $(2x + 3y)^{20}$ is

A. 0

B. $2^8 3^{12}$

C. $2^8 3^{12} + 2^{12} 3^8$

D. ${}^{20}C_8 2^8 3^{12}$

Answer: D

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4. If ${}^nC_{10} > {}^nC_r$ for all possible r, then a value of n is

A. 10

B. 21

C. 19

D. 20

Answer: D

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5. If a is the arithmetic mean and g is the geometric mean of two numbers, then

A. $a \leq g$

B. $a \geq g$

C. $a = g$

D. $a > g$

Answer: B

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6. If

$$\left(1+x^2\right)^2(1+x)^n = a_0 + a_1x + a_2x^2 + \ldots\ldots + x^{n+4}$$

and if a_0, a_1, a_2 are in AP, then n is .....

A. 1

B. 2

C. 3

D. 4

Answer: B::C

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7. If a, 8, b are in AP, a, 4, b are in GP, and if a, x, b are in HP then x is

A. 2

B. 1

C. 4

D. 16

Answer: A

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8. The sequence $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}+\sqrt{2}}, \frac{1}{\sqrt{3}+2\sqrt{2}}$ ..... form an .....

A. AP

B. GP

C. HP

D. AGP

Answer: C

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9. The HM of two positive numbers whose AM and GM are 16, 8 respectively is

A. 10

B. 6

C. 5

D. 4

Answer: D

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10. If S_n denotes the sum of n terms of an AP whose common difference is d, the value of $S_n - 2S_{n-1} + S_{n-2}$ is

A. d

B. 2d

C. 4d

D. d^2

Answer: A

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11. The remainder when 38^{15} is divided by 13 is

A. 12

B. 1

C. 11

D. 5

Answer: A

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12. The n^{th} term of the sequence 1, 2, 4, 7, 11, ... is

A. $n^3 + 3n^2 + 2n$

B. $n^3 - 3n^2 + 3n$

C. $\frac{n(n+1)(n+2)}{3}$

D. $\frac{n^2 - n + 2}{2}$

Answer: D

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13. The sum up to n terms of the series $\frac{1}{\sqrt{1}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{7}} + \ldots\ldots$ is .....

A. $\sqrt{2n+1}$

B. $\frac{\sqrt{2n+1}}{2}$

C. $\sqrt{2n+1}-1$

D. $\frac{\sqrt{2n+1}-1}{2}$

Answer: D

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14. The n^{th} term of the sequence $\frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{15}{16}, \ldots$ is

A. $2^n - n - 1$

B. $1 - 2^{-n}$

C. $2^{-n} + n - 1$

D. 2^{n-1}

Answer: B

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15. The sum upto n terms of the series

$\sqrt{3}+\sqrt{12}+\sqrt{27}+\sqrt{48}$ +... is:

A. $\frac{n(n+1)}{2}$

B. $2n(n+1)$

C. $\frac{n(n+1)}{\sqrt{2}}$

D. 1

Answer: C

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16. The value of the series $\frac{1}{2}+\frac{7}{4}+\frac{13}{8}+\frac{19}{16}+\ldots$ is

A. 14

B. 7

C. 4

D. 6

Answer: A

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17. The sum of an infinite GP is 18. If the first term is 6, the common ratio is .....

A. $\frac{1}{3}$

B. $\frac{2}{3}$

C. $\frac{1}{6}$

D. $\frac{3}{4}$

Answer: B

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18. The coefficient of x^5 in the series e^{-2x} is

A. $\frac{2}{3}$

B. $\frac{3}{2}$

C. $\frac{-4}{15}$

D. $\frac{4}{15}$

Answer: C

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19. The value of $\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+\ldots$ is

A. $\frac{e^2+1}{2e}$

B. $\frac{(e+1)^2}{2e}$

C. $\frac{(e-1)^2}{2e}$

D. $\frac{e^2+1}{2e}$

Answer: C

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20. The value of $1-\frac{1}{2}\left(\frac{2}{3}\right)+\frac{1}{3}\left(\frac{2}{3}\right)^2-\frac{1}{4}\left(\frac{2}{3}\right)^3+\ldots$ is

A. $\log\left(\frac{5}{3}\right)$

B. $\frac{3}{2}\log\left(\frac{5}{3}\right)$

C. $\frac{5}{3}\log\left(\frac{5}{3}\right)$

D. $\frac{2}{3}\log\left(\frac{2}{3}\right)$

Answer: B

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Additional Questions Solved

1. Find the coefficient of the term involving x^{32} and x^{-17} in the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$

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2. Find a positive value of m for which the coefficient of x^2 in the expansion of $(1+x)^m$ is 6.

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3. In the binomial expansion of $(1+a)^{m+n}$, prove that the coefficients of a^m and a^n are equal.

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4. The coefficient of $(r-1)^{th}$, r^{th}, and $(r+1)^{th}$ terms in the expansion of $(x+1)^n$ are in the ratio $1:3:5$. Find both n and r.

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5. Find the value of $\left(a^2+\sqrt{a^2-1}\right)^4+\left(a^2-\sqrt{a^2-1}\right)^4$

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6. Show that the coefficient of the middle term in the expansion of $(1+x)^{2n}$ is equal to the sum of the

coefficients of the two middle terms in the expansion of $(1+x)^{2n-1}$

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7. If three consecutive coefficients in the expansion of $(1+x)^n$ are in the ratio $6:33:110$, find n.

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8. If the sum of the coefficients in the expansion of $(x+y)^n$ is 4096. Then find the greatest coefficient in the expansion.

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9. Find the coefficient of $\frac{1}{x^{17}}$ in the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$.

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10. If p is a real number and if the middle term in the expansion of $\left(\frac{p}{2} + 2\right)^8$ is 1120, find p.

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11. Find the first five terms of the sequence given by

$$\begin{cases} a_1 = 1 \\ a_n = a_{n-1} + 2, n \geq 2 \end{cases}$$

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12. Find the 18^{th} and 25^{th} terms of the sequence defined by

$$a_n = \begin{cases} n(n+2), & \text{if n is even natural number} \\ \frac{4n}{n^2+1}, & \text{if n is odd natural number} \end{cases}$$

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13. Write the first six terms of the sequences given by

(i) $a_1 = a_2 = 1, a_n = a_{n-1} + a_{n-2} (n \geq 3)$

(ii) $a_1 = 4, a_{n+1} = 2na_n$

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14. An A.P. consists of 21 terms. The sum of the three terms in the middle is 129 and of the last three is 237. Find the series.

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15. Prove that the product of the 2^{nd} and 3^{rd} terms of an arithmetic progression exceeds the product of the first and fourth by twice the square of the difference between the 1^{st} and 2^{nd}.

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16. If the p^{th}, q^{th} and r^{th} terms of an A.P. are a, b, c respectively, prove that $a(q-r)+b(r-p)+c(p-q)=0$.

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17. If a, b, c are in A.P. and p is the A.M. between a and b and q is the A.M. between b and c, show that b is the A.M. between p and q.

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18. If x, y, z be respectively the p^{th}, q^{th} and r^{th} terms of a G.P. show that $x^{q-r}, y^{r-p}, z^{p-q} = 1$

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19. If the ratio of the sums of m terms and n terms of an A.P. be $m^2 : n^2$, prove that the ratio of its m^{th} and n^{th} terms is $(2m-1):(2n-1)$.

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20. Determine the number of terms of geometric progression $\{a_n\}$ if $a_1 = 3, a_n = 96, S_n = 189$.

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21. The sum of first three terms of a G.P. is to the sum of the first six terms as $125 : 152$. Find the common ratio of the G.P.

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22. Find the sum to n term the series :

$(x+y)+\left(x^2+xy+y^2\right)+\left(x^3+x^2y+xy^2+y^3\right)+\ldots..$

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23. Sum the series :

$(1+x)+\left(1+x+x^2\right)+\left(1+x+x^2+x^3\right)+\ldots...$

up to n terms

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24. Sum up to n terms the series :

$7+77+777+7777+\ldots..$

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25. If S_1, S_2, S_3 be respectively the sums of n, 2n, 3n, terms of a G.P., then prove that $S_1(S_3 - S_2) = (S_2 - S_1)^2$.

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26. If sum of the n terms of a G.P be S, their product P and the sum of their reciprocals R, then prove that $P^2 = \left(\frac{S}{R}\right)^n$

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27. Find the $\sqrt[3]{126}$ approximately to two decimal places.

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28. Write the first four terms in the expansions of the following :

(i) $\frac{1}{(2+x)^4}$ where $|x| > 2$

(ii) $\frac{1}{\sqrt[3]{6-3x}}$ where $|x| < 2$

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29. Evaluate the following :

(i) $\sqrt[3]{1003}$ correct to 4 places of decimals

(ii) $\frac{1}{\sqrt[3]{128}}$ correct to 4 places of decimals.

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30. If x so large prove that $\sqrt{x^2+25}-\sqrt{x^2+9}=\frac{8}{x}$ nearly.

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31. Show that $x^n=1+n\left(1-\frac{1}{x}\right)+\frac{n(n+1)}{1.2}\left(1-\frac{1}{x}\right)^2+\ldots.$

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