



## MATHS

### BOOKS - FULL MARKS MATHS (TAMIL ENGLISH)

### COMBINATORICS AND MATHEMATICAL INDUCTION

#### Example

1. Suppose one girl or one boy has to be selected for a competition from a class comprising 17 boys and 29 girls. In how many different ways can this selection be made?

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2. Consider the 3 cities Chennai, Trichy and Tirunelveli. In order to reach Tirunelveli from Chennai, one has to pass through Trichy. There are 2

roads connecting chennai with Trichy and there are 3 roads connecting Trichy with Tirunelveli. What are the total number of ways of going chennai to Tirunelveli?

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3. A school library has 75 books on Mathematics, 35 books on physics. A student can choose only one book , In how many ways a student can choose a book on Mathematics or physics?

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4. If an electricity consumer has the consumer number say 238:110:29, then describe the linking and count of house connections upto the 29th consumer "connection" linked to the larger capacity transformer number 238 subject to the condition that each smaller capacity transformer can have a maximal consumer link of say 100.

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5. A person wants to buy a car. There are two brands of car available in the market and each brand has 3 variant models and each model comes in five different colours. In how many ways she can choose a car to buy?

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6. A Women wants to select one silk saree and one sungudi saree from a textile shop located at kancheepuram. In that shop, there are 20 different varieties of silk saree and 8 different varieties of sungudi saree, In how many ways she can select her sarees?

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7. In a village , out of the total number of people , 80 percentage of the people own coconut groves and 65 percent of the people own paddy fields. What is the minimum percentage of people own both?

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8. (i) Find the number of strings of length  $n$ , which can be formed using the letters of the word BIRD, without repetition of the letters.

(ii) How many strings of length 5 can be formed out of the letters of the word "PRIME" taking all the letters at a time without repetition.



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9. How many strings of length 6 can be formed using letters of the word FLOWER if (i) either starts with F or ends with R? (ii) neither starts with F nor ends with R?



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10. How many licence plates may be made using either two distinct letters followed by four digits or two digits followed by 4 distinct letters where all digits and letters are distinct?



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11. Count the number of positive integers greater than 7000 and less than 8000 which are divisible by 5, provided that no digits are repeated.



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12. How many 4 - digit even numbers can be forms using the digits 0,1,2,3 and 4,if repetition of digits are not permitted?



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13. Find the total number of outcomes when 5 coins are tossed once.



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14. In how many ways (i) 5 diffrents balls be distributed among 3 boxes?  
(ii) 3 different balls be distributed among 5 boxes?



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15. There are 10 bulbs in a room . Each one of them can be operated independently. Find the number of ways in which the room can be illuminated.



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16. Find the value of (i)  $5!$  (ii)  $6! - 5!$  (iii)  $\frac{8!}{5! \times 2!}$



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17. Simplify  $\frac{7!}{2!}$



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18. Evaluate  $\frac{n!}{r!(n-r)!}$  when (i)  $n=7, r=5$  (ii)  $n=50, r=47$  (iii) For any  $n$  with  $r = 3$

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19. Let  $N$  denote the number of days. If the value of  $N!$  is equal to the total number of hours in  $N$  days then find the value of  $N$ ?

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20. If  $\frac{6!}{n!} = 6$ , then find the value of  $n$ .

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21. If  $n! + (n-1)! = 3$ , then find the value of  $n$ .

A. 5

B. 4

C. 3

D. 2

**Answer:** *D*



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**22.** What is the unit digit of the sum  $2! + 3! + 4! + \dots + 22!$ ?



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**23.** If  $\frac{1}{7!} + \frac{1}{8!} = \frac{A}{9!}$  then the value of A is



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**24.** Show that  $\frac{(2n)!}{n!} = 2^n \{1, 3, 5, \dots, (2n - 1)\}$



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25. Evaluate: (i)  ${}^4P_4$  (ii)  ${}^5P_2$  (iii)  ${}^8P_4$  (iv)  ${}^6P_4$



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26. If  ${}^{(n+2)}P_4 = 42 \times {}^n P_2$ , find n.



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27. If  ${}^{10}P_r = {}^7P_{r+2}$  find r.



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28. How many 'letter strings' together can be formed with the letters of the "VOWELS" so that

(i) the strings begin with E

(ii) the strings begin with E and end with W.



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**29.** A number of four different digits is formed with the use of the digits 1,2,3,4 and 5 in all possible ways. Find the following

- (i) How many such numbers can be formed?
- (ii) How many of these are even?
- (iii) How many of these are exactly divisible by 4?



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**30.** How many different strings can be formed together using the letters of the word "EQUATION" so that

- (i) the vowels always come together?
- (ii) the vowels never come together?



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**31.** There are 15 candidates for an examination. 7 candidates are appearing for mathematics examination while the remaining 8 are appearing for different subjects . In how many ways can they be seated in a row so that no two mathematics candidates are together ?

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**32.** In how many ways 5 boys and 4 girls can be seated in a row so that no two girls are together.

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**33.** 4 boys and 4 girls from a line with the boys and girls alternating. Find the number of ways of marking this line.

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- 34.** A van has 8 seats. It has two seats in the front with two rows of three seats behind. The van belongs to a family, consisting of seven members,  $F, M, S_1, S_2, S_3, D_1, D_2$ . How many ways can the family sit in the van if
- There are no restrictions?
  - Either  $F$  or  $M$  drives the van?
  - $D_1, D_2$  sits next to a window and is driving?



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- 35.** If the letters of the word TABLE are permuted in all possible ways and the words thus formed are arranged in the dictionary order (alphabetical order), find the ranks of the word TABLE



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- 36.** Find the number of ways of arranging the letters of the word BANANA.



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37. Find the number of ways of arranging the letters of the word RAMANUJAN so that the relative positions of vowels and consonants are not changed.

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38. Three twins pose for a photograph standing in a line. How many arrangements are there (i) when there are no restrictions. (ii) when each person is standing next to his or her twin?

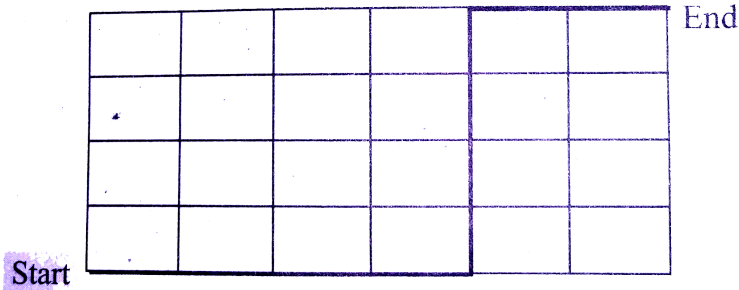
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39. How many numbers can be formed using the digits 1,2,3,4,2,1 such that even digits occupies even place?

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40. How many paths are there from start to end on a  $6 \times 4$  grid as shown in the picture?

### How Many Paths?



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41. If the different permutations of all letters of the word BHASKARA are listed as in a dictionary, how many strings are there in the list before the first word starting with B?

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42. If the letters of the word IITJEE are permuted in all possible ways and the strings thus are arranged in the dictionary order find the rank of the

word IITJEE.

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43. Evaluate the following (i)  $^{10}C_3$  (ii)  $^{15}C_{13}$  (iii)  $^{100}C_{99}$  (iv)  $^{50}C_{50}$

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44. Find the value of  $^5C_2$  and  $^7C_3$  using the property 5

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45. If  ${}^nC_4 = 495$ , what is n?

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46. If  ${}^nP_r = 11880$  and  ${}^nC_r = 495$  find n and r .

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47. Prove that  ${}^{24}C_4 + \sum_{r=0}^4 {}^{(28-r)}C_3 = {}^{29}C_4$

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48. Prove that  ${}^{10}C_2 + 2 \times {}^{10}C_3 + {}^{10}C_4 = {}^{12}C_4$

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49. If  ${}^{(n+2)}C_7 : {}^{(n-1)}P_4 = 13 : 24$  find n.

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50. A salad at a certain restaurant consists of 4 of the following fruits: apple, banana, guava, pomegranate, grapes, papaya and pineapple. Find the total possible number of fruit salads.

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51. A Mathematics club has 15 members. In that 8 are girls. 6 of the members are to be selected for a competition and half of them should be girls. How many ways of these selections are possible?

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52. In rating 20 brands of cars, a car magazine picks a first, second, third, fourth and fifth best brand and then 7 more as acceptable, In how many ways can it be done?

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53. From a class of 25 students, 10 students are to be chosen for an excursion party, There are 4 students who decide that either all of them will join or none of them will join. In how many ways can the excursion party be chosen?



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54. A box of one dozen apple contains a rotten apple. If we are choosing 3 apples simultaneously, in how many ways , one can get only good apples.



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55. An exam paper contains 8 questions, 4 in Part A and 4 in Part B. Examiners are required to answer 5 questions. In how many ways can this be done if (i) There are no restrictions of choosing a number of questions in either parts.(ii) At least two questions from Part A must be answered



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56. Out of 6 consonants and 4 vowels, how many strings of 3 consonants and 2 vowels can be formed ?



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57. If a set of  $m$  parallel lines intersect another set of  $n$  parallel lines (not parallel to the lines in the first set), then find the number of parallelograms formed in this lattice structure.

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58. What is the formula to find how many diagonals are there in a polygon with  $n$  sides?

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59. By the principle of mathematical induction, prove that, for all integers  $n \geq 1$ ,

$$1+2+3+\dots+n = \frac{n(n+1)}{2}$$

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60. Prove that the sum of first  $n$  positive odd numbers is  $n^2$

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61. Using the principle of Mathematical Induction ,  $\forall n \in \mathbb{N}$ , prove that

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

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62. Prove by mathematical induction

$$\frac{1}{1.2} + \frac{1}{2.3} + \dots + \frac{1}{(n)(n+1)} = \frac{n}{(n+1)}$$

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63. Prove that  $x^n - y^n$  is divisible by  $x - y$  for all positive integers  $n$ .

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64. Prove by mathematical induction that for every natural number  $n$ ,  $3^{2n+2} - 8n - 9$  is divisible by 8 for all  $n \geq 1$ .



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65. Use the Principle of Mathematical Induction to verify that, for  $n$  any positive integer,  $6^n - 1$  is divisible by 5.



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66. Evaluate :  $\frac{(n + 2)!}{n!}$



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67. By the principle of mathematical induction , prove that , for  $n \in \mathbb{N}$

$$\cos \alpha + \cos(\alpha + \beta) + \cos(\alpha + 2\beta) + \dots + \cos(\alpha + (n - 1)\beta) = \cos \left( \alpha - \right.$$



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68. Using the mathematical induction, show that for any natural number

$n$ , with the assumption  $i^2 = -1$ ,

$$(r(\cos \theta + i \sin \theta))^n = r^n(\cos n\theta + i \sin n\theta)$$



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### Exercise 4 1

1. (i) A person went to a restaurant for dinner. In the menu card, the person saw 10 Indian and 7 Chinese food items. In how many ways can the person select either an Indian or a Chinese food?

(ii) There are 3 types of toy car and 2 types of toy train available in a shop.

Find the number of ways a baby can buy a toy car and a toy train?

(iii) How many of two-digit numbers can be formed using 1,2,3,4,5 without repetition of digits?

(iv) There are 10 persons enter in to a conference hall in which there are 10

seats, In how many ways they can take their place?

(v) In how many ways 5 persons can be seated in a row?

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2. (i) A mobile phone has a passcode of 6 digits. What is the maximum number of attempts one makes to retrieve the passed.

(ii) Given four flags of different colours, how many different signals can be generated if each signal requires the use three flags, One below the other?

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3. Four children are running a race .

(i) In how many ways can the first two places be filled ?

(ii) In how many different ways could they finish the race ?

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4. Count the number of three - digit numbers which can be formed from the digits 2,4,6,8, if

(i) repetitions of digits is allowed ?

(ii) repetitions of digits is not allowed ?



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5. How many three - digit numbers are there with 3 in the unit place ?

( i) With repetition (ii) without repetition



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6. How many numbers are there between 100 and 500 with the digits 0,2,3,4,5 if

(i) repetition of digits allowed (ii) the repetition of digits is not allowed



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7. How many three-digit odd numbers can be formed by using the digits 0,1,2,3,4,5 ? If

- (i) the repetition of digits is not allowed
- (ii) the repetition of digits is allowed

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8. Count the numbers between 999 and 10,000 subject to the condition that there are .

- (i) no restriction .
- (ii) no digit is repeated .
- (iii) at least one of the digits is repeated .

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9. How many three- digit numbers, which are divisible by 5, can be formed using the digits 0,1,2,3,4, 5 if

(i) repetition of digits are not allowed ?

(ii) repetition of digits are allowed ?

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**10.** To travel from a place A to place B, there are two different bus routes  $B_1, B_2$ , two different train routes  $T_1, T_2$  and one air route  $A_1$ . From place B to place C there is one bus route say  $B_1$  two different train routes say  $T_1, T_2$  and one air route  $A_1$ . Find the number of routes of commuting from place A to place C via place B without using similar mode of transportation.

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**11.** How many numbers are there between 1 and 1000 (both inclusive) which are divisible neither by 2 nor by 5 ?

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**12.** How many strings can be formed using the letters of the word LOTUS if the word .

(i) either starts with L or ends with S.

(ii) neither starts with L nor ends with S ?



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**13.** (i) Count the total number of ways of answering 6 objective type questions , each question having 4 choices.

(ii) In how many ways 10 pigeons can be placed in 3 different pigeon holes ?

(iii) Find the number of ways of distributing 12 distance prizes to 10 students ?



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**14.** Find the value of

$6!$

(ii)  $4! + 5!$

(iii)  $3! - 2!$

(iv)  $3! \times 4!$

(v)  $\frac{12!}{9! \times 3!}$  (vi)  $\frac{(n+3)!}{(n+1)!}$



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15. Evaluate  $\frac{n!}{r!(n-r)!}$  when

(i)  $n = 6, r = 2$

(ii)  $n = 10, r = 3$

(iii) for any  $n$  with  $r = 2$



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16. Find the value of  $n$  if

(i)  $(n+1)! = 20(n-1)!$

(ii)  $\frac{1}{8!} + \frac{1}{9!} = \frac{n}{10!}$



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## Exercise 4 2

1. If  ${}^{n-1}P_3 : P_4 = 1, 10$  find  $n$ .



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2. If  ${}^{10}P_{r-1} = 2 \times 6P_{r^2}$  find  $r$



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3. Suppose 8 people enter an event in a swimming meet. In how many ways could the gold, silver and bronze prizes be awarded?



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4. Determine the number of permutations of the letters of the word SIMPLE if all are taken at a time ?

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5. A test consists of 10 multiple choice questions . In how many ways can the test be answered if

(i) Each question has four choices ?

(ii) The first four questions have three choices and the remaining have five choices ?

(iii) Question number  $n$  has  $n + 1$  choices ?

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6. A student appears in an objective test which contain 5 multiple choice questions. Each question has 4 choices out of which one correct answer.

(i) What is the maximum number of different answers can the students give ?

(ii) How will the answer change if each question may have more than one correct answers ?

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7. How many strings can be formed from the letters of the word ARTICLE, so that vowels occupy the even places ?

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8. 8 woman and 6 man are standing in a line.

(i) How many arrangements are possible if any individual can stand in any position ?

(ii) In how many arrangements will 6 men be standing next to one another ?

(iii) In how many arrangements will no two men be standing next to one another ?

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9. Find the distinct permutations of the letters or the word MISSISSIPPI ?

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10. How many ways can the product  $a^2b^3c^4$  be expressed without exponents ?

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11. In how many ways 4 mathematics books , 3 physics books, 2 chemistry books and 1 biology book can be arranged on a shelf so that all books of the same subjects are together ?

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12. In how many ways can the letters of the word SUCCESS be arranged so that all S's are together ?





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**13.** A coin is tossed 8 times.

(i) How many different sequences of heads and tails are possible ?

(ii) How many different sequences containing six heads and two tails are possible ?



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**14.** How many strings are there using the letters of the word INTERMEDIATE, if

Q The vowels and consonants are alternative.



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**15.** If the letters of the word GARDEN are permuted in all possible ways and the strings thus formed are arranged in the dictionary order, then

find the ranks of the words

Q GARDEN



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16. Find the number of strings that can be made using all letters of the word THING. If these words are written as in a dictionary , what will be the  $85^{th}$  string ?



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17. if the letters of the word FUNNY are permuted in all possible ways and the strings thus formed are arranged in the dictionary order, find the rank of the word FUNNY .



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18. Find the sum of all 4-digit numbers that can be formed using digits 1, 2, 3, 4, and 5 repetitions not allowed?

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19. Find the sum of all 4-digit numbers that can be formed using digits 0, 2, 5, 7, 8 without repetition?

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### Exercise 4 3

1. If  ${}^n C_{12} = {}^n C_9$  find  ${}^{21} C_n$

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2. If  ${}^{15} C_{2r-1} = {}^{15} C_{2r+4}$  find  $r$

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3. If  ${}^n P_r = 720$ . If  ${}^n C_r = 120$  find  $n, r$

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4. Prove that  ${}^{15} C_3 + 2 \times {}^{15} C_4 + {}^{15} C_5 = {}^{17} C_5$

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5. Prove that  ${}^{35} C_5 + \sum_{r=0}^{4(39-r)} C_4 = {}^{40} C_5$

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6. If  ${}^{n+2} C_8 : {}^{(n-2)} P_4 = 57 : 16$ , find the value of  $n$ .

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7. Prove that  ${}^{2n}C_n = \frac{2^n \times 1 \times 3 \times \dots (2n - 1)}{n!}$



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8. Prove that if  $1 \leq r \leq n$  then  $n \times {}^{(n-1)}C_{r-1} = (n - r + 1) \cdot {}^n C_{r-1}$



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9. (i) A kabaddi coach has 14 players ready to play. How many different teams of 7 players could the coach put on the court?

(ii) There are 15 persons in a party and if , each 2 of them shakes hands wit each with each other. How many handshakes happen in the party?

(iii) How many chords can be drawn through 20 points on a circle?

(iv) In a parking lot one hunderd, one year old cars are parked. Out of them five are to be chosen at random for to check its pollution devices. How many different set of five cars are possible?

(v) How many ways can a team of 3 boys, 2 girls and 1 transgender be selected from 5 boys, 4 girls and 2 transgender?



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**10.** Find the total number of subsets of a set with .

(i) 4 elements (ii) 5 elements

(iii)  $n$  elements



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**11.** A trust has 25 members .

(i) How many ways 3 officers can be selected ?

(ii) In how many ways can a president, Vice president and a Secretary be selected ?



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**12.** How many ways a committee of six persons from 10 persons can be chosen along with a chair person and a secretary ?



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**13.** How many different selections of 5 books can be made from 12 different books if,

(i) Two particular books are always selected ?

(ii) Two particular books are never selected ?

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**14.** There are 5 teachers and 20 students. Out of them a committee of 2 teachers and 3 students is to be formed. Find the number of ways in which this can be done. Further find in how many of these committees.

Q a particular teacher is included?

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**15.** In an examination a student has to answer 5 questions, out of 9 questions in which 2 are compulsory . In how many ways a student can

answer the questions ?

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**16.** Determine the number of 5 card combinations out of a deck of 52 cards if there is exactly three aces in each combination .

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**17.** Find the number of ways of forming a committee of 5 members out of 7 Indians and 5 Americans so that always Indians will be majority in the committee.

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**18.** A committee of 7 peoples has to be formed from 9 men and 4 women .  
In how many can this be done when then committee consists of  
(i) exactly 3 women ?



(ii) at least 3 woman ?

(iii) at most 3 women ?

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**19.** 7 relatives of a man comprises 4 ladies and 3 gentlemen, his wife also has 7 relatives , 3 of them are ladies and 4 gentlemen. In how many ways can they invite a dinner party of 3 ladies and 3 gentlemen , so that there are 3 of man's relative and 3 of the wife ,s relatives ?

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**20.** A box contains two white balls, three black balls and four balls. In how many ways can three balls be drawn from the box, if atleast one black ball is to be included in the draw ?

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**21.** Find the number of strings of 4 letters that can be formed with the letters of the word EXAMINATION.

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**22.** How many triangle can be formed by joining 15 points on the plane, in which on line joining any three points?

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**23.** How many triangles can be formed by 15 points , in which 7 of them lie on one line and the remaining 8 on another parallel line ?

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**24.** There are 11 points in a plane. No three of these lies in the same straight line except 4 points, which are collinear. Find ,

(i) the number of straight lines that can be obtained from the pairs of these points ?

(ii) the number of triangles that can be formed for which the points are their vertices ?

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25. A polygon has 90 diagonals. Find the number of its sides ?

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#### Exercise 4 4

1. By the principle of mathematical induction, prove that, for  $n \geq 1$

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \left( \frac{n(n+1)}{2} \right)^2$$

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2. By the principle of mathematical induction, prove that, for  $n \geq 1$

$$1^2 + 3^2 + 5^2 + \dots + (2n - 1)^2 = \left( \frac{n(2n - 1)(2n + 1)}{3} \right)$$

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3. Prove that the sum of first  $n$  non-zero even numbers is  $n^2 + n$

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4. By the principle of mathematical induction, prove that, for  $n \geq 1$

$$1.2 + 2.3 + 3.4 + \dots + n.(n + 1) = \frac{n(n + 1)(n + 2)}{3}$$

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5. Using the mathematical induction, show that for any natural number,

$$n \geq 2, \left(1 - \frac{1}{2^2}\right) \left(1 - \frac{1}{3^2}\right) \left(1 - \frac{1}{4^2}\right) \dots \left(1 - \frac{1}{n^2}\right) = \frac{n + 1}{2n}$$

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6. Using the mathematical induction, show that for any natural number

$$n \geq 2, \frac{1}{1+2} + \frac{1}{1+2+3} + \frac{1}{1+2+3+4} + \dots + \frac{1}{1+2+3+\dots+n}$$



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7. Using the mathematical induction, show that for any natural number  $n$ ,

$$\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \dots + \frac{1}{n.(n+1).(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$$



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8. Using the mathematical induction, show that for any natural number  $n$ ,

$$\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + \dots + \frac{1}{(3n-1)(3n+2)} = \frac{n}{6n+4}$$



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9. Prove by the mathematical induction that

$$1! + (2 \times 2!) + (3 \times 3!) + \dots + (n \times n!) = (n + 1)! - 1$$

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10. Using the mathematical induction, show that for any natural numbers

$$x^{2n} - y^{2n} \text{ is divisible by } (x+y).$$

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11. By the principle of mathematic induction, prove that, for  $n \geq 1$ ,

$$1^2 + 2^2 + 3^2 + \dots + n^2 > \frac{n^3}{3}$$

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12. Use induction to prove that  $n^3 - n + 3$ , is divisible by 3, for all natural numbers n

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13. Use induction to prove that  $5^{n+1} + 4 \times 6^n$  when divided by 20 leaves a remainder 9 for all natural numbers n .

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14. Use induction to prove that  $10^n + 3 \times 4^{n+2} + 5$ , is divisible by 9, for all natural no. n.

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15. Prove that using the Mathematical induction

$$\sin(\alpha)\sin\left(\alpha + \frac{\pi}{6}\right) + \sin\left(\alpha + \frac{2\pi}{6}\right) + \dots + \sin\left(\alpha + \frac{(n-1)\pi}{6}\right) = \dots$$

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## Exercise 4 5

1. The sum of the digits at the 10th place of all numbers formed with the help of 2,4,5,7 taken all at a time is

A. 432

B. 108

C. 36

D. 18

**Answer: A**



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2. In an examination there are three multiple choice questions and each question has 5 choices Number of ways in which a student can fail to get all answer correct is

A. 125



B. 124

C. 64

D. 63

**Answer: A::B::D**



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3. The number of ways in which of following prize be given to a class of 30 boys first and second in mathematics, first and second in physics, first in chemistry and first in English is

A.  $30^4 \times 29^2$

B.  $30^3 \times 29^3$

C.  $30^2 \times 29^4$

D.  $30 \times 29^5$

**Answer: B::C::D**



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4. The number of 5 digit numbers all digits of which are odd is

A. 25

B.  $5^5$

C.  $5^6$

D. 625

**Answer:**



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5. In 3 fingers , the number of ways four rings can be worn is .....ways

A.  $4^3 - 1$

B.  $3^4$

C. 68

D. 64

**Answer: C::D**



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6. If  ${}^{n+5}P_{n+1} = \left(\frac{11(n-1)}{2}\right) \cdot {}^{n+3}P_n$  then the value of n are

A. 7 and 11

B. 6 and 7

C. 2 and 11

D. 2 and 6

**Answer: A::D**



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7. The product of r consecutive positive integers is divisible by

A.  $r!$

B.  $(r - 1)!$

C.  $(r + 1)!$

D.  $r!$

**Answer:**



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8. The number of five digit telephone numbers having at least one of their digits repeated is

A. 90000

B. 10000

C. 30240

D. 69760

**Answer:**

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9. If  ${}^{a^2-a}C_2 = {}^{a^2-a}C_4$  then the value of  $a$  is

A. 2

B. 3

C. 4

D. 5

**Answer: C**

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10. There are 10 points in a plane and 3 of them are collinear. The number of straight line joining any two points is .....

A. 45

B. 43

C. 39

D. 38

**Answer:**



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11. The number of ways in which a host lady invite 8 people for a party of 8 out of 12 people of whom two do not want to attend the party together is

A.  $2 \times {}^{11}C_7 + {}^{10}C_8$

B.  ${}^{11}C_7 + {}^{10}C_8$

C.  ${}^{12}C_8 - {}^{10}C_6$

D.  ${}^{10}C_6 + (2!)$

**Answer: A::B::C**



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12. The number of parallelograms that can be formed from a set of four parallel lines intersecting another set of three parallel lines..... .

A. 6

B. 9

C. 12

D. 18

**Answer: A**



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13. Everybody in a room shakes hands with everybody else. The total number of shake hands is 66. The number of persons in the room is

A. 11

B. 12

C. 10

D. 6

**Answer: A::B**



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**14.** Number of sides of a polygon having 44 diagonals is

A. 4

B.  $4!$

C. 11

D. 22

**Answer: A**



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15. If 10 lines are drawn in a plane such that no two of them are parallel and no three are concurrent, then the total number of points of intersection are

A. 45

B. 40

C.  $10!$

D.  $2^{10}$

**Answer: D**



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16. In a plane are 10 points are there out of which 4 points are collinear, then the number of triangles formed is

A. 110

B.  ${}^{10}C_3$

C. 120

D. 116

**Answer: A**



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17. In  ${}^{2n}C_3 : {}^nC_3 = 11:1$  then n is

A. 5

B. 6

C. 11

D. 7

**Answer:**



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18.  ${}^{(n-1)}C_r + {}^{(n-1)}C_{(r-1)}$  is

A.  ${}^{(n+1)}C_r$

B.  ${}^{(n-1)}C_r$

C.  ${}^nC_r$

D.  ${}^nC_{(r-1)}$

**Answer: C**



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19. The number of ways of choosing 5 cards out of a deck of 52 which include at least one king is

A.  ${}^{52}C_5$

B.  ${}^{48}C_5$

C.  ${}^{52}C_5 + {}^{48}C_5$

D.  ${}^{52}C_5 - {}^{48}C_5$

**Answer: B::C::D**



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**20.** The number of rectangles that a chessboard has

A. 81

B.  $9^9$

C. 1296

D. 6561

**Answer: A::B**



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**21.** The number of 10 digit number that can be written by using the digits

2 and 3 is

A.  ${}^{10}C_2 + {}^9C_2$

B.  $2^{10}$

C.  $2^{10} - 2$

D.  $2!$

**Answer: A::B**



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22. If  $P_r$  stands for  ${}^rP_r$  then the sum of the series

$1 + P_1 + 2 \times P_2 + 3 \times P_3 + \dots + n \times P_n$  is:

A.  $P_{n+1}$

B.  $P_{n+1} - 1$

C.  $P_{n-1} + 1$

D.  ${}^{(n+1)}P_{(n-1)}$

**Answer: A**

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23. The product of first  $n$  odd natural numbers equals:

A.  ${}^{2n}C_n \times {}^n P_n$

B.  $\left(\frac{1}{2}\right)^n \times {}^{2n}C_n \times {}^n P_n$

C.  $\left(\frac{1}{4}\right)^n \times {}^{2n}C_n \times {}^{2n}P_n$

D.  ${}^n C_n \times {}^n P_n$

Answer: A::B::C

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24. If  ${}''C_4, {}''C_5, {}''C_6$  are in AP then value of  $n$  is

A. 14

B. 11

C. 9

D. 5

**Answer: A::D**



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25.  $1 + 3 + 5 + 7 + \dots + 17$  is equal to

A. 101

B. 81

C. 71

D. 61

**Answer: A**



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[Additional Questions](#)

1. If the letter of the word 'RACHIT' are arranged in all possible ways as listed in dictionary , then what is the rank of the word 'RACHIT'?

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2. Count the number of positive integers greater than 6000 and less than 7000 which are divisible by 5, provided that no digits are repeated?

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3. Find the number of integers greater than 7000 that can be formed with the digits 3,5,7,8 and 9 where no digits are repeated.

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4. How many words (with or without dictionary meaning ) can be made from the letters of the word MONDAY, assuming that no letter is repeated



, if

	$C_1$		$C_2$
(a)	4 letters are used at a time	(i)	720
(b)	All letters are used at a time	(ii)	240
(c)	All letters are used but the first is a vowel	(iii)	360

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5. How many automobile license plates can be made, if each plate contains two different letters followed by three different digits?

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6. If  $\frac{n!}{3!(n-4)!}$  and  $\frac{n!}{5!(n-5)!}$  are in the ratio 5:3 find the value of  $n$ .

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7. How many 3-digit even numbers can be made using the digits 1,2,3,4,6,7 if no digit is repeated?





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8. If  ${}^{n-1}P_3 : {}^n P_4 = 1 : 9$  find  $n$ .



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9. Out of 18 points in a plane, no three are in the same line except five points which are collinear. Find the number of lines that can be formed joining the points.



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10. We wish to select 6 person from 8 but , if the person A is chosen , then B must be chosen. In how many ways can selections be made?



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11. How many 3-digit numbers can be made using digit 1,2,3,4,6,7 if no digit is repeated ?

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12. Find the number of 4-digit numbers that can be formed using the digits 1,2,3,4,5 if no digit is repeated. How many of these will be even ?

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13. Iff  ${}^5P_r = {}^6P_{r-1}$  find r.

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14. How many words, with or without meaning , can be made from the letters of the word MONDAY, assuming that no letters is repeated , if  
(i) 4 letters are used at a time

(ii) all letters are used at a time

I II III IV V VI

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15. A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected, if the team has

(i) no girls (ii) atleast one boy and one girl (iii) at least three girls

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16. A committee of 6 is to be chosen from 10 men and 7 women so as to contain atleast 3 men and 2 women. In how many different ways can this be done?

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17. Using the digits 1,2,3,4,5,6,7 a number of 4 different digits is formed.

Find

$C_1$		$C_2$	
(a)	How many numbers are formed?	(i)	840
(b)	How many number are exactly divisible by 2?	(ii)	200
(c)	How many numbers are exactly divisible by 25?	(iii)	360
(d)	How many of these are exactly divisible by 4?	(iv)	40



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18. If  ${}^{22}P_{r+1} : {}^{20}P_{r+2} = 11 : 52$ , find r.



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19. If  ${}^nP_r = {}^nP_{r+1}$  and  ${}^nC_r = {}^nC_{r-1}$  find n and r.



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20. A committee of 7 peoples has to be formed from 9 men and 4 women .

In how many can this be done when then committee consists of

(i) exactly 3 women ?

(ii) at least 3 woman ?

(iii) at most 3 women ?



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21. In  ${}^{2n}C_3 : {}^n C_3 = 11:1$  then n is



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22. Find the number of ways of selecting 9 balls from 6 red balls, 5 white balls and 5 blue balls if each selection consists of 3 balls of each colour.



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23. If  $\frac{1}{7!} + \frac{1}{9!} = \frac{x}{10!}$ , find x.



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24. If  ${}^nC_4$ ,  ${}^nC_5$  and  ${}^nC_6$  are in A.P. then find n.

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25. Prove by induction the inequality  $(1 + x)^n \geq 1 + nx$ . whenever x is positive and n is a positive interger.

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26. Prove that  $3^{2n} - 1$  is divisible by 8.

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27. Prove that  $x^n - y^n$  is divisible by x - y for all positive integers n.

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**28.** Prove by mathematical induction that for every natural number  $n$ ,  $3^{2n+2} - 8n - 9$  is divisible by 8.

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**29.** Use the principle of mathematical induction to prove that for every natural number  $n$ .

$$\left(1 + \frac{3}{1}\right) \left(1 + \frac{5}{4}\right) \left(1 + \frac{7}{9}\right) \dots \left(1 + \frac{(2n+1)}{n^2}\right) = (n+1)^2$$

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**30.**  $n^3 - n$  is divisible by 6, for each natural number  $n \geq 2$

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**31.** Prove that For any natural number  $n$ ,  $7^n - 2^n$  is divisible by 5.

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32. Find  $\frac{10!}{5! \times 2!}$



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33. Compute  ${}_{10}C_1$



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