# MATHS

## BOOKS - FULL MARKS MATHS (TAMIL ENGLISH)

## DIFFERENTIAL CALCULUS - DIFFERENTIABILITY AND METHODS OF DIFFERENTIATION

**Example**

**1.** Find the slope of the tangent line to graph of $f(x) = 7x + 5$ at any point $(x_0, f(x_0))$.

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**2.** Find the slope of tangent line to the graph of $f(x) = -5x^2 + 7x$ at $(5, f(5))$.

**3.** Show that the greater integer function $f(x) = [x]$ is not differential at $x = 1$ and $x = 2$

**4.** Differentiate the following with respect to x:

(i) $y = x^3 + 5x^2 + 3x + 7$

(ii) $y = e^x + \sin x + 2$

(iii) $y = 4\cos ecx - \log x - 2e^x$

**5.** Find $F'(x)$ if $F(x) = \sqrt{x^2 - 1}$.

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**6.** Differentiate : (i) $y = \sin(x^2)$ (ii) $y = \sin^2 x$

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**7.** Differentiate : $y = (x^3 + 1)^{200}$.

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**8.** Find $f'(x)$ if $f(x) = \dfrac{1}{\sqrt[3]{x^3 + x^2 + 1}}$.

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**9.** Find the derivative of the function $g(t) = \left( \dfrac{t-2}{2t+1} \right)$.

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**10.** Differentiate $(2x+1)^5 (x^3 - x + 1)^4$.

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**11.** Differentiate : $y = e^{\sin 2x}$

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**12.** Differentiate : $2^x$ with respect to $x$.

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**13.** If $y = \tan^{-1}\left(\dfrac{1 + x^2}{1 - x^2}\right)$, find $y'$.

**14.** Find $\dfrac{dy}{dx}$ if $x^2 + y^2 = 1$.

**15.** Find the slopes of the tangent lines to the graph of $x^2 + y^2 = 4$ at the points corresponding to $x = 1$.

**16.** Find $\dfrac{dy}{dx}$ if $x^4 + x^2 y^3 - y^5 = 2x + 1$.

**17.** Find $\dfrac{dy}{dx}$ if $\sin y = y \cos 2x$

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**18.** Find the derivative of $y = \sqrt{x^2 + 4}.\ \sin^2 x.2^x$

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**19.** Differentiate $y = \dfrac{x^{\frac{3}{4}}\sqrt{x^2 + 1}}{(3x + 2)^5}$.

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**20.** Differentiate $y = x^{x^2}$ with respect to $x$.

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**21.** If $y = \tan^{-1}\left(\dfrac{1+x}{1-x}\right)$. find $y'$.

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**22.** Find $f'(x)$ if $f(x) = \cos^{-1}\left(4x^3 - 3x\right)$.

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**23.** Find $\dfrac{d^2y}{dx^2}$ if $x = at^2$, $y = 2at$, $t \neq 0$

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**24.** Find $\dfrac{dy}{dx}$ if $x = a(t - \sin t)$, $y = a(1 - \cos t)$.

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**25.** Find the derivative of $x^x$ with respect to $x \log x$.

**26.** Find the derivative of $\tan^{-1}(1 + x^2)$ with respect to $x^2 + x + 1$.

**27.** Differentiate $\sin(ax^2 + bx + c)$ with respect to $\cos(tx^2 + mx + n)$

**28.** Find $y'$, $y''$ and $y'''$ if $y = x^3 - 6x^2 - 5x + 3$.

**29.** Find $y'''$ if $y = \dfrac{1}{x}$.

**30.** Find $f''$ if $f(x) = x \cos x$.

**31.** Find $y'''$ if $x^4 + y^4 = 16$.

**32.** Find the second order derivative if x and y are given by

$x = a \cos t$

$$y = a \sin t$$

**33.** Find $\dfrac{d^2 y}{dx^2}$ if $x^2 + y^2 = 4$.

## Exercise 1

**1.** Find the derivativities of the following function using first principle.

(i) $f(x) = 6$.

(ii) $f(x) = -4x + 7$

**2.** Find the derivative from the left and from the right at $x = 1$ (if they exist) of the following functions. Are the functions differentiable at $x = 1$?

(i) $f(x) = |x - 1|$

(ii) $f(x) = \sqrt{1 - x^2}$

(iii) $f(x) = \begin{cases} x & x \le 1 \\ x^2 & x > 1 \end{cases}$

**3.** Determine whether the following function is differentiable at the indicated values.

(i) $f(x) = x|x|$ at $x = 0$

(ii) $f(x) = \}x^2 - 1|$ at $x = 1$

(iii) $f(x) = |x| + |x - 1|$ at $x = 0, 1$

(iv) $f(x) = \sin|x|$ at $x = 0$

**4.** Show that the following functions are not differentiable at the indicated value of x.

$$f(x) = \begin{cases} -x + 2 & x \leq 2 \\ x^2 & x > 2 \end{cases}$$

**5.** If $f(x) = |x + 100| + x^2$, test whether $f'(-100)$ exists.

## Exercise 2 Find The Derivative Of The Following Functions With Respect To Corresponding Independent Variables

**1.** Find the derivative of the following function $f(x) = x - 3\sin x$

**2.** If $y = \sin x - \cos x$ then find $\dfrac{dy}{dx}$

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**3.** Find derivative of $x \sin x$

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**4.** Find the derivative of the function $y = \cos x - 2\tan x$

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**5.** Find the derivative of the function $g(t) = t^3 \cos t$

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**6.** Find the derivative of the function $g(t) = 4\sec t + \tan t$

**7.** $y = e^x \sin x$ find $\dfrac{dy}{dx}$

**8.** If $y = \dfrac{\tan x}{x}$ then find $\dfrac{dy}{dx}$

**9.** If $y = \dfrac{\cos x}{1 + \sin x}$ find $\dfrac{dy}{dx}$.

**10.** Find the differential of each of the following functions

$$y = \frac{x}{\sin x + \cos x}$$

**11.** If $y = \dfrac{\tan x - 1}{\sec x}$ then $\dfrac{dy}{dx} = ?$

**12.** Find the derivative of $y = \dfrac{\sin x}{x^2}$ with respect to $x$ is

**13.** $y = \tan \theta (\sin \theta + \cos \theta)$ find $\dfrac{dy}{d\theta}$.

**14.** If $y = \cos ecx \cdot \cot x$ find $\dfrac{dy}{dx}$.

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**15.** If $y = x \sin x \cos x$ find $\dfrac{dy}{dx}$.

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**16.** If $y = e^{-x} \cdot \log x$ find $\dfrac{dy}{dx}$

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**17.** Find the derivatives of the following functions with respect to corresponding independent variables.

$y = (x^2 + 5)\log(1 + x)e^{-3x}$

**18.** Find $\dfrac{dy}{dx}$ if $y = \sin x^\circ$

**19.** $y = \log_{10} x$

**20.** Draw the graph of function $f'(x)$ if $f(x) = 2x^2 - 5x + 3$

Exercise 3 Differentiate The Following

**1.** Differentiate the following :

$$y = \left(x^2 + 4x + 6\right)^5$$

**2.** Differentiate the following :

$$y = \tan 3x$$

**3.** Differentiate the following :

$$y = \cos(\tan x)$$

**4.** Differentiate the following :

$$y = \sqrt[3]{1 + x^3}$$

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**5.** If $y = e^{\sqrt{x}} + e^{-\sqrt{x}}$ then $xy'' + \dfrac{y'}{2} =$

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**6.** Differentiate the following :

$$y = \sin(e^x)$$

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**7.** Differentiate the following :

$$f(x) = (x^2 + 4x)^7$$

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**8.** Differentiate the following :

$$h(t) = \left(t - \frac{1}{t}\right)^{3/2}$$

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**9.** Differentiate the following :

$$f(t) = \sqrt[3]{1 + \tan t}$$

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**10.** Differentiate the following :

$$y = \cos\left(a^3 + x^3\right)$$

**11.** Differentiate the following :

$$y = e^{-mx}$$

**12.** Differentiate the following :

$$y = 4\sec 5x$$

**13.** Differentiate the following :

$$y = (2x - 5)^4 \left(8x^2 - 5\right)^{-3}$$

**14.** Differentiate the following :

$$y = \left(x^2 + 1\right) \sqrt[3]{x^2 + 2}$$

**15.** Differentiate the following :

$$y = xe^{-x^2}$$

**16.** Differentiate the following :

$$s(t) = \sqrt[4]{\frac{t^3 + 1}{t^3 - 1}}$$

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**17.** Differentiate the following :

$$f(x) = \frac{x}{\sqrt{7 - 3x}}$$

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**18.** Differentiate the following :

$$y = \tan(\cos x)$$

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**19.** Differentiate the following :

$$y = \frac{\sin^2 x}{\cos x}$$

**20.** Differentiate the following :

$$y = 5^{-\frac{1}{x}}$$

**21.** Differentiate the following :

$$y = \sqrt{1 + 2\tan x}$$

**22.** If $y = \sin^3 x + \cos^3 x$ then find $\dfrac{dy}{dx}$.

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**23.** Differentiate the following :

$$y = \sin^2(\cos kx)$$

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**24.** Differentiate the following :

$$y = \left(1 + \cos^2 x\right)^6$$

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**25.** Differentiate the following :

$$y = \frac{e^{3x}}{1 + e^x}$$

**26.** Differentiate the following with respect to $x$:

$$y = \sqrt{x\sqrt{x}}$$

**27.** Differentiate the following with respect to $x$:

$$y = e^{x \cos x}$$

**28.** Differentiate the following with respect to $x$:

$$y = \sqrt{x + \sqrt{x}}$$

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**29.** Differentiate the following with respect to $x$:

$$y = \sin\left(\tan\left(\sqrt{\sin x}\right)\right)$$

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**30.** find the derivative; $y = \sin^{-1}\left(\dfrac{1 - x^2}{1 + x^2}\right)$

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**Exercise 4 Differentiate The Following**

**1.** find the derivative; $y = x^{\cos x}$

**2.** Find the derivatives of the following :

$$y = x^{\log x} + (\log x)^x$$

**3.** find the derivative; $\sqrt{xy} = e^{(x - y)}$

**4.** find the derivative; $x^x = y^x$

**5.** Differentiate the following w.r.t. x: $(\cos x)^{\log x}$

**6.** Differentiate the following w.r.t. x; $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

**7.** Find the derivatives of the following :

$$\sqrt{x^2 + y^2} = \tan^{-1}\left(\dfrac{y}{x}\right)$$

**8.** If $\tan(x + y) + \tan(x - y) = x$ then find $\dfrac{dy}{dx}$

**9.** If $\cos(xy) = x$, show that $\dfrac{dy}{dx} = -\dfrac{(1 + y\sin(xy))}{x\sin xy}$

**10.** Differentiate the following w.r.t. x; $\tan^{-1}\sqrt{\dfrac{1 - \cos x}{1 + \cos x}}$

**11.** Find the derivatives of the following :

$\tan^{-1}\left(\dfrac{6x}{1 - 9x^2}\right)$

**12.** Find the derivatives of the following : $\cos\left[2\tan^{-1}\sqrt{\dfrac{1 - x}{1 + x}}\right]$

**13.** Find the derivatives of the following : $x = a \cos^3 t$, $y = a \sin^3 t$

**14.** If $x = a(\cos t + t \sin t)$, $y = a[\sin t - t \cos t]$ then find $\dfrac{dy}{dx}$.

**15.** Find the derivatives of the following : $x = \dfrac{1 - t^2}{1 + t^2}$, $y = \dfrac{2t}{1 + t^2}$

**16.** Find the derivatives of the following : $\cos^{-1}\left(\dfrac{1 - x^2}{1 + x^2}\right)$

**17.** Find the derivatives of the following : $\sin^{-1}\left(3x - 4x^3\right)$

**18.** Find the derivatives of the following : $\tan^{-1}\left(\dfrac{\cos x + \sin x}{\cos x - \sin x}\right)$

**19.** Find the derivative of $\sin x^2$ with respect to $x^2$.

**20.** Find the derivative of $\sin^{-1}\left(\dfrac{2x}{1 + x^2}\right)$ with respect to $\tan^{-1} x$.

**21.** If $u = \tan^{-1}\dfrac{\sqrt{1+x^2}-1}{x}$ and $v = \tan^{-1}x$, find $\dfrac{du}{dv}$

**22.** Find the derivative with $\tan^{-1}\left(\dfrac{\sin x}{1+\cos x}\right)$ with respect to $\tan^{-1}\left(\dfrac{\cos x}{1+\sin x}\right)$

**23.** If $y = \sin^{-1}x$ then find $y''$.

**24.** If $y = e^{\tan^{-1} x}$. find $\dfrac{dy}{dx}$

**25.** If $y = \dfrac{\sin^{-1} x}{\sqrt{1 - x^2}}$ show that $\left(1 - x^2\right) y_2 - 3xy_1 - y = 0$

**26.** If $x = a(\theta + \sin \theta), y = a(1 - \cos \theta)$ then prove that at $\theta = \dfrac{\pi}{2}, y'' = \dfrac{1}{a}$.

**27.** If $\cos y = x \cos(a + y)$ Then prove that $\dfrac{dy}{dx} = \dfrac{\cos^2(a + y)}{\sin a}$, $\cos a \neq \pm 1$

**28.** If $y = \left(\cos^{-1} x\right)^2$, prove that $\left(1 - x^2\right) \dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - 2 = 0.$

Hence find $y_2$ when $x = 0$.

## Exercise 5 Choose The Correct Or The Most Suitable Answer From The Given Four Alternatives

**1.** $\dfrac{d}{dx}\left(\dfrac{2}{\pi}\sin x^\circ\right)$ is...................

A. $\dfrac{\pi}{180}\cos x^\circ$

B. $\dfrac{1}{90}\cos x^\circ$

C. $\dfrac{\pi}{90}\cos x^\circ$

D. $\dfrac{2}{\pi}\cos x^\circ$

**Answer: B**

**2.** If $y = f(x^2 + 2)$ and $f'(3) = 5$, then $\dfrac{dy}{dx}$ at x=1 is

A. 5

B. 25

C. 15

D. 10

**Answer: D**

**3.** If $y = \dfrac{1}{4}\mu^4$, $\mu = \dfrac{2}{3}x^3 + 5$, then $\dfrac{dy}{dx}$ is..............

A. $\dfrac{1}{27} x^2 \left(2x^3 + 15\right)^3$

B. $\dfrac{2}{27} x \left(2x^3 + 5\right)^3$

C. $\dfrac{2}{27} x^2 \left(2x^3 + 15\right)^3$

D. $-\dfrac{2}{27} x \left(2x^3 + 5\right)^3$

**Answer: C**

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**4.** If $f(x) = x^2 - 3x$, then the point at which $f'(x) = f(x)$

are................ .

A. both positive integers

B. both negative integers

C. both irrational

D. one rational and another irrational

**Answer: C**

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**5.** If $y = \dfrac{1}{a - z}$, then $\dfrac{dz}{dy}$ is ............... .

A. $(a - z)^2$

B. $-(z - a)^2$

C. $(z + a)^2$

D. $( - z + a)^2$

**Answer: A**

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**6.** If $y = \cos\left(\sin x^2\right)$, then $\dfrac{dy}{dx}$ at $x = \dfrac{\sqrt{\pi}}{2}$ is ............... .

A. $-2$

B. $2$

C. $-2\sqrt{\dfrac{\pi}{2}}$

D. $0$

**Answer: D**

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**7.** If $y = mx + c$ and $f(0) = f'(0) = 1$, then $f(2)$ is ............. .

A. $1$

B. $2$

C. $3$

D. $-3$

**Answer: C**

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**8.** If $f(x) = x \tan^{-1} x$, then $f'(1)$ is

A. $1 + \dfrac{\pi}{4}$

B. $\dfrac{1}{2} + \dfrac{\pi}{4}$

C. $\dfrac{1}{2} - \dfrac{\pi}{4}$

D. 2

**Answer: B**

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**9.** $\dfrac{d}{dx}\left(e^{x + 5 \log x}\right)$ is ............. .

A. $e^x x^4(x+5)$

B. $e^x x(x+5)$

C. $e^x + \dfrac{5}{x}$

D. $e^x - \dfrac{5}{x}$

**Answer: A**

**10.** If the derivative of $(ax - 5)e^{3x}$ at $x = 0$ is $-13$, then the value

of a is ............. .

A. 8

B. $-2$

C. 5

D. 2

**Answer: D**

**11.** $x = \dfrac{1-t^2}{1+t^2}, y = \dfrac{2t}{1+t^2}$ then $\dfrac{dy}{dx}$ is

A. $-\dfrac{y}{x}$

B. $\dfrac{y}{x}$

C. $-\dfrac{x}{y}$

D. $\dfrac{x}{y}$

**Answer: C**

**12.** If $x = a\sin\theta$ and $y = b\cos\theta$, then $\dfrac{d^2y}{dx^2}$ is............ .

A. $\dfrac{a}{b^2} \sec^2 \theta$

B. $-\dfrac{b}{a} \sec^2 \theta$

C. $-\dfrac{b}{a^2} \sec^3 \theta$

D. $-\dfrac{b^2}{a^2} \sec^3 \theta$

**Answer: C**

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**13.** The differential coefficient of $\log_m x$ with respect to $\log_x m$ is

.............. .

A. $1$

B. $-(\log_m x)^2$

C. $(\log_x m)^2$

D. $\dfrac{x^2}{m^2}$

**Answer: B**

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**14.** If $f(x) = x + 2$, then $f'(f(x))$ at $x = -4$ is............ .

A. 8

B. 1

C. 4

D. 5

**Answer: B**

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**15.** If $y = \dfrac{(1-x)^2}{x^2}$, then $\dfrac{dy}{dx}$ is............ .

A. $\dfrac{2}{x^2} + \dfrac{2}{x^3}$

B. $-\dfrac{2}{x^2} + \dfrac{2}{x^3}$

C. $-\dfrac{2}{x^2} - \dfrac{2}{x^3}$

D. $-\dfrac{2}{x^3} + \dfrac{2}{x^2}$

**Answer: D**

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**16.** If $pv = 81$, then $\dfrac{dp}{dv}$ at $v = 9$ is ............... .

A. $1$

B. $-1$

C. $2$

D. $-2$

**Answer: B**

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**17.** If $f(x) = \begin{cases} x - 5 & \text{if } x \leq 1 \\ 4x^2 - 9 & \text{if } 1 < x < 2 \\ 3x + 4 & \text{if } x \geq 2 \end{cases}$ , then the right hand

derivative of $f(x)$ at $x = 2$ is.............. .

A. 0

B. 2

C. 3

D. 4

**Answer: C**

**18.** It is given that $f'(a) \exists$, then $\displaystyle\lim_{x \to a} \frac{x f(a) - a f(x)}{x - a}$ is ............... .

A. $f(a) - a f'(a)$

B. $f'(a)$

C. $-f'(a)$

D. $f(a) + a f'(a)$

**Answer: A**

**19.** If $f(x) = \begin{cases} x + 1 & \text{when} \quad x < 2 \\ 2x - 1 & \text{when} \quad x \geq 2 \end{cases}$, then $f'(2)$ is............... .

A. 0

B. 1

C. 2

D. does not exist

**Answer: D**

**20.** If

$g(x) = (x^2 + 2x + 3) f(x)$ and $f(0) = 5$ and $\lim\limits_{x \to 0} \dfrac{f(x) - 5}{x} = 4$

then $g'(0)$ is............. .

A. 20

B. 22

C. 18

D. 12

**Answer: B**

**21.** If $f(x) = \begin{cases} x + 2 & -1 < x < 3 \\ 5 & x = 3 \\ 8 - x & x > 3 \end{cases}$ , then at x=3, $f'(x)$ is

A. 1

B. $-1$

C. 0

D. does not exist

**Answer: D**

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**22.** The derivative of $f(x) = x|x|$ at $x = -3$ is ............... .

A. 6

B. $-6$

C. does not exist

D. 0

**Answer: A**

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**23.** If $f(x) = \begin{cases} 2a - x & \text{for} & -a < x < a \\ 3x - 2a & \text{for} & x \geq a \end{cases}$ , then which one of the following is true?

A. f(x) is not differentiable at $x = a$

B. f(x) is discontinuous at $x = a$

C. f(x) is continuous for all $x \text{in} \mathbb{R}$

D. f(x) is differentiable for all $x \geq a$

**Answer: A**

**24.** If $f(x) = \begin{cases} ax^2 - b, & -1 < x < 1 \\ \frac{1}{|x|}, & \text{elsewhere} \end{cases}$ is differentiable at $x = 1$,

then .............. .

A. $a = \dfrac{1}{2}, b = \dfrac{-3}{2}$

B. $a = \dfrac{-1}{2}, b = \dfrac{3}{2}$

C. $a = -\dfrac{1}{2}, b = -\dfrac{3}{2}$

D. $a = \dfrac{1}{2}, b = \dfrac{3}{2}$

**Answer: C**

**25.** The number of points in $\mathbb{R}$ in which the function $f(x) = |x - 1| + \sin x$ is not differentiable, is.............. .

A. 3

B. 2

C. 1

D. 4

**Answer: B**

**Exercise 5 Additional Problems**

**1.** Is the function $f(x) = |x|$ differentiable at the origin. Justify your answer.

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**2.** Discuss the differentiability of the functions:

$$f(x) \begin{cases} 2x - 3, & 0 \le x \le 2 \text{at} x = 2 \\ x^2 - 3, & x > 2 \end{cases}$$

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**3.** If $y = A \cos 4x + B \sin 4x$, $A$ and $B$ are constants then Show that $y_2 + 16y = 0$

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**4.** If $y = \cos\left(m \sin^{-1} x\right)$, prove that

$$\left(1 - x^2\right) y_3 - 3xy_2 + \left(m^2 - 1\right) y_1 = 0$$

# Exercise 5 Additional Problems Find The Derivative Of Following Functions

**1.** Differentiate following as; $3 \sin x + 4 \cos x - e^x$

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**2.** Differentiate following as; $\sin 5 + \log_{10} x + 2 \sec x$

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**3.** Differentiate following as; $6 \sin x + \log_{10} x + e$

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**4.** Differentiate following as;

$$\left(x^4 - 6x^3 + 7x^2 + 4x + 2\right)\left(x^3 - 1\right)$$

**5.** Differentiate following as; $\left(3x^2 + 1\right)^2$

**6.** Differentiate following as;

$$(3\sec x - 4\cos ecx)(2\sin x + 5\cos x)$$

**7.** Differentiate ; $x^2 e^x \sin x$

**8.** Differentiate the following with respect to $x$

$$\frac{\cos x + \log x}{x^2 + e^x}$$

**9.** Differentiate the following with respect to $x$

$$\frac{\tan x + 1}{\tan x - 1}$$

**10.** Differentiate the following with respect to $x$

$$\frac{\sin x + x \cos x}{x \sin x - \cos x}$$