



MATHS

BOOKS - FULL MARKS MATHS (TAMIL ENGLISH)

APPLICATIONS OF DIFFERENTIAL CALCULUS

Example Questions Solved

1. For the functions $f(x) = x^2, x \in [0, 2]$ compute the average rate of changes in the subintervals [0, 0.5], [0.5, 1], [1, 1.5], [1.5, 2]and the instantaneous rate of changes at the points x = 0.5, 1, 1.5, 2

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2. The temperature in celsius in along rod of length 10m, insulated at both ends , is a function of length x given by T = x(10 - x).

Prove that the rate of changes of temperature

at the midpoint of the rod is zero .



3. A person learnt 100 words for an English test. The number of words the person remembers in t days after learning is given by $W(t) = 100 \times (1 - 0.1t)^2, 0 \le t \le 10$. What is the rate at which the person forgets the words 2 days after learning ?

4. A particle moves so that the distance moved is according to the law $s(t) = \frac{t^3}{3} - t^2 + 3$. At what time the velocity and acceleration are zero respectively ?

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5. A particle is fired straight up from the ground to each a height of x feet in t seconds, where $x(t) = 128t - 16t^2$. (1) Compute the maximum height of the particle reached.

(2) What is the velocity when the particle hits

the ground ?

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6. A particle moves along a horizontal line such that its position at any time t is given by $s(t) = t^3 - 6t^2 + 9t + 1$, s in meters and t in seconds.

At what time the particle is at rest?

7. If we blow air into a balloon of spherical shape at a rate of $1000cm^3$ per second. At what rate the radius of the balloon changes when the radius is 7cm? Also compute the rate at which the surface area changes .



8. The price of a product is related to the number of units available (supply) by the equation Px + 3P - 16x = 234, where P is

the price of the product per unit in Rupees (Rs) and x is the number of units. Find the rate at which the price is changing with respect to time when 90 units are available an the supply is increasing at a rate of units/week.

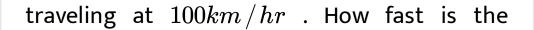
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9. Salt is poured from a conveyer belt at a rate of 30 cubic metre per minute forming a conical pile with a circular base whose beight and diameter of base are alwayes equal . How fast is the height of the pile increasing when

the pile is 10 metre high ?



10. A road running north to south acrosses a road going east to west at the point P. Car A is driving north along the first road, and car B is driving east along the second road . At a particular time car A 10 kilometers to the north of P and traveling at 80km/hr, while car B is 15 Kilometers to the east of P and



distance between the two cars changing ?



11. Find the equations of tangent and normal

to the curve $y = x^2 + 3x - 2$ at the point (1, 2). .



12. For what value of x the tangent of the curve $y = x^3 - 3x^2 + x - 2$ is parallel to the line y = x



13. Find the equation of the tangent and normal to the Lissajous curve given by

 $x=2\cos 3t \,\, ext{and}\,\, y=3\sin 2t, t\in \mathbb{R}.$

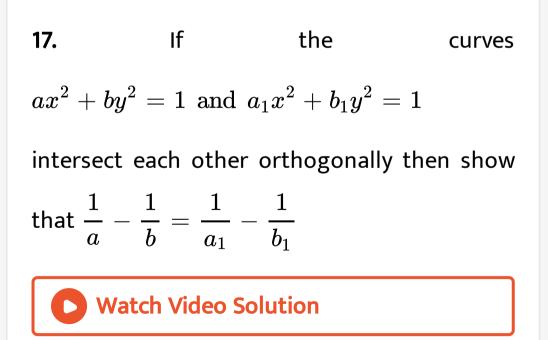
14. Find the actute angle between $y = x^2$ and $y = (x - 3)^2$. View Text Solution

15. Find the acute angle between the curves $y = x^2$ and $x = y^2$ at their points of intersection (0, 0), (1, 1).

16. Find the angle of intersection of the curve

 $y=\sin x$ with the positive x-axis .





18. Prove that the ellipse $x^2 + 4y^2 = 8$ and the hyperbola $x^2 - 2y^2 = 4$ intersect orthogonally.

19. Compute the value of 'c' satisfied by the Rolle's theorem for the function . $f(x)=x^2(1-x)^2,\,x\in[0,1].$

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20. Find the value of c in the interval $\left(\frac{1}{2}, 2\right)$

satisfied by the Roll's theorem for the

function.
$$f(x)=x+rac{1}{x}, x\in\left[rac{1}{2},2
ight]$$

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21. Compute the value of 'c' satisfied by the Rolle's theorem for the function . $f(x)=x^2(1-x)^2, x\in [0,1].$

22. Without actually solving show that the equation $x^4 + 2x^3 - 2 = 0$ has only one real root in the interval (0,1).



23. Prove, Using the Rolle's theorem that between any two distinct real zeros of the polynomial

 $a_nx^n+a_{n-1}x^{n-1}+\ldots a_1x+a_0$ there is a

zero of the polynomial.

$$na_nx^{n-1} + (n-1)a_{n-1}x^{n-2} + \ldots + a_1$$

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24. Prove that there is a zero of the polynomial $2x^3 - 9x^2 - 11x + 12$ in the interval (2,7) given that 2 and 7 are the zeros of the polynomial $x^4 - 6x^3 - 11x^2 + 24x + 28$.

25. Find the values in the interval (1,2) of the mean value theorem satisfied by the function $f(x)=x-x^2{
m for}1\leq x\leq 2.$



26. A truck travels on a toll road with a speed limit of 80km/hr. The truck completes a 164 km journey in 2 hours . At the end of the toll road the trucker is issued with a speed

violation ticket. Justify this using the Mean

Value Theorem .



27. Suppose f(x) is a differentiable function for all x with $f'(x) \leq 29$ and f(2) = 17 .

What is the maximum value of f (7)?



28. Prove , using mean value theorem, that

 $|\sinlpha-\sineta|\leq |lpha-eta|, lpha,eta\in\mathbb{R}.$

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29. A thermometer was taken from a freezer and placed in a boilling water . It took 22 seconds for the thermometer to raise from $-10^{\circ}C \rightarrow 100^{\circ}C$. Show that the rate of changes of temperature at some time t is $5^{\circ}C$ per second





30. Expand log (1+x) as a Maclaurin 's series

upto 4 non-zero terms for $-1 < x \leq 1$.

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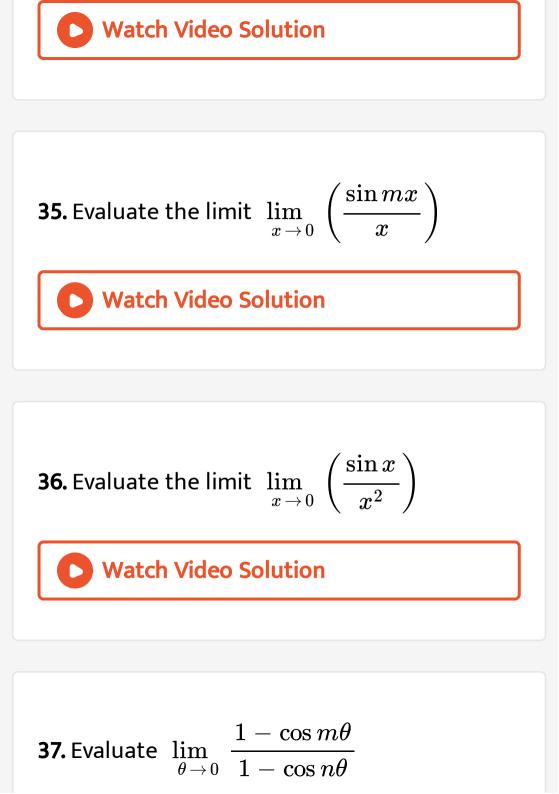
31. Expand $\tan x$ maclaurin 's series in asending powers of x upto 5^{th} power for $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

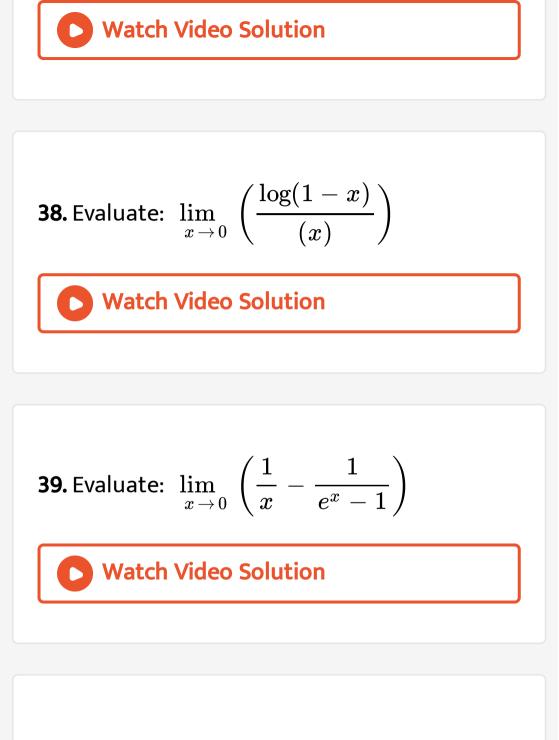
32. Write the Taylor series expansion of $\frac{1}{x}$ about x = 2 by finding the first three non-zero terms

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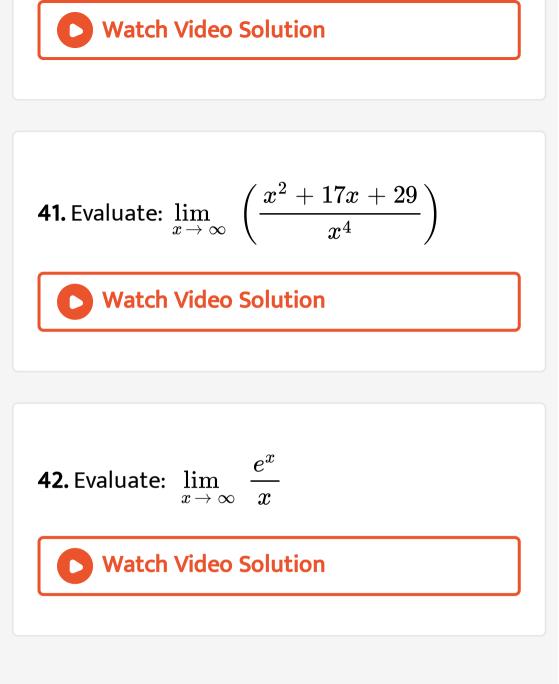
33. Evaluate:
$$\lim_{x \to 1} \left(rac{x^2 - 3x + 2}{x^2 - 4x + 3}
ight)$$

34. Compute the limit
$$\lim_{x o a} \left(rac{x^n - a^n}{x - a}
ight)$$



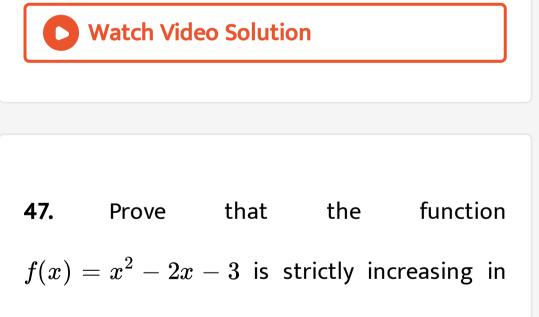


40. Evaluate: $\lim_{x \to 0} x \log x$



43. Using the I'Hopital Rule prove that, $\lim_{x\, o\,0^+}\,\,(1+x)^{rac{1}{x}}=e$ Watch Video Solution **44.** Evaluate: $\lim_{x o \infty} (1+2x)$ Watch Video Solution **45.** Evaluate: $\lim x^x$ $x \rightarrow 0$ Watch Video Solution

46. Prove that the function $f(x) = x^2 + 2$ is strictly increasing in the interval (2,7) and strictly decreasing in the interval (-2, 0).



 $(2,\infty)$

48. Find the absolute maximum and absolute minimum values of the function $f(x) = 2x^3 + 3x^2 - 12x$ on [-3, 2]

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49. Find the absolute extrema of the function

 $f(x) = 3\cos x$ on the closed interval $[0, 2\pi]$.

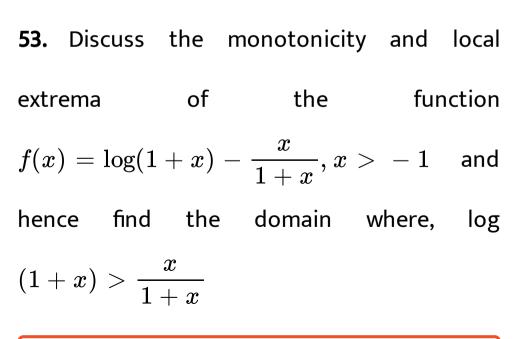
50. Find the slope at x = -1 for $f(x) = x^2 - 4x + 4$

51. Find the intervals of monotonicity and hence find the local extrema for the function $f(x)=x^{rac{2}{3}}$



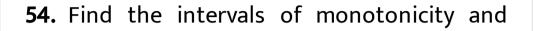
52. Prove that the function $f(x) = x - \sin x$ is increasing on the real line . Also discuss for the existence of local extrema.











local extrema of the function

$$f(x) = x \log x + 3x$$

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55. Find the slope for
$$f(x) = \frac{1}{1+x^2}$$
 at $x = 1$

56. Find the slope for $f(x) = \frac{x}{1+x^2}$ at x = 1

57. Determine the intervals of concavity of the curve $\ f(x)=(x-1)^3(x-5), x\in R$ and , points of inflection if any

58. Determine the intervals of concavity of the

curve $y = 3 + \sin x$.

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59. Find the local extremum of the function $f(x) = x^4 + 32$

60. Find the local extrema of the function $f(x) = 4x^6 - 6x^4$

61. Find the local maximum and minimum of the function x^2y^2 on the line x + y = 10

62. We have a 12 square unit piece of thin material and want to make an open box by cutting small squares from the corners of our material and folding the sides up. The question is, which cut produces the box of maximum volume ?

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63. Find the points on the unit circle $x^2 + y^2 = 1$ nearest and farthest from (1,1)

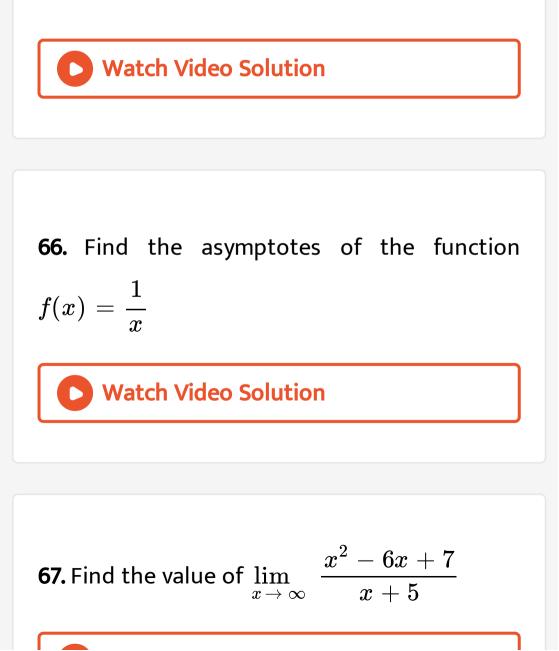


64. A steel plant is capable of producing x tonnes per day of a law-grade steel and y tonnes per day of a hight-grade steel, where $y = \frac{40 - 5x}{10 - x}$. If the fixed market price of lowgrade steel is half that of high-grade steel, then what should be optimal productions in law-grade steel and high-grade steel in order to have maximum receipts.



65. Prove that among all the rectangles of the

given area, square has the least perimeter.

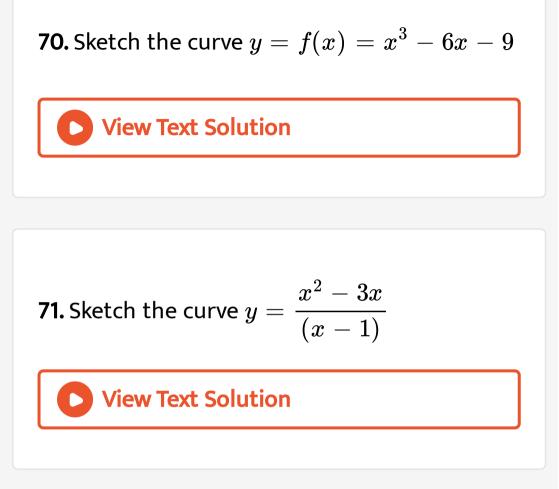




68. Find the asymptotes of the curve
$$f(x) = \frac{2x^2 - 8}{x^2 - 16}$$
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69. Sketch the curve $y = f(x)x^2 - x - 6$

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Exercise 71

1. A particle moves along a straight line in suc a way that after t second its distance from the origin is $s=2t^2+3t$ metres.

Find the instantaneous velocities at t =3 and t =6 seconds.

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2. A camera is accidentally knocked off an edge of a cliff 400 ft high. The camera falls a distance of $s=16t^2$ in t seconds. How long does the camera fall before it hits

the ground?



3. A particle moves along a horizontal line such that its equation of motion is $s(t) = 2t^3 - 15t^2 + 24t - 2$, s in meters and t in second.

Find the total distance travelled by the particle in the first 2 seconds.

4. If the volume of a cube of side length x is $V=x^3.$ Find the rate of change of the volume

with respect to x when x = 5 units.



5. If the mass m(x) (in kilograms) of a thin rod of length x(in metres) is given by, $m(x) = \sqrt{3x}$ then what is the rate of change of mass with respect to the length when it is x = 27 meters. **6.** A stone is dropped into a pond causing ripples in the form of concentric circles. The radius r of the outer ripple is increasing at a constant rate at 2 cm per second. When the radius is 5 cm find the rate of changing of the total area of the disturbed water?



7. A beacon makes one revolution every 10 seconds. It is located on a ship which is anchored 5 km from a straight shore line. How fast is the beam moving along the shore line when it makes an angle of 45° with the shore?



8. A conical water tank with vertex down of 12 meters height has a radius of 5 meters at the top. If water flows into the tank at a rate 10

cubic m/min, how fast is the depth of the water increases when the water is 8 metres deep?

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9. A ladder 17 metre long is leaning against the wall. The base of the ladder is pulled away from the wall at a rate of 5m/s. When the base of the ladder is 8 metres from the wall. How fast is the top of the ladder moving down the wall?



10. A police jeep, approaching an orthogonal intersection from the northern direction, is chasing a speeding car that has turned and moving straight east. When the jeep is 0.6 km north of the intersection and the car is 0.8 km to the east. The police deteremine with a radar that the distance between them and the cae is increasing at 20 km/hr. If the jeep is moving at 60 km/hr at the instant of measurement, what is the speed of the car?

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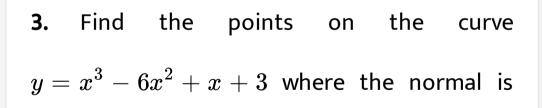
Exercise 7 2

1. Find the slope of the tangent to the following curves at the respective given points.

$$y = x^4 + 2x^2 - x \;\; ext{ at }\;\; x = 1$$

2. Find the point on the curve $y = x^2 - 5x + 4$ at which the tangent is parallel to the line 3x + y = 7.





parallel to the line x+y=1729

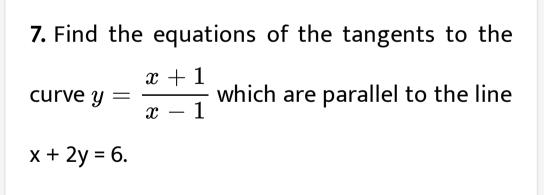
4. Find the points on the curve $y^2 - 4xy = x^2 + 5$ for which the tangent is horizontal.

5. Find the tangent and normal to the following curves at the given points on the curve.

$$y = x^2 - x^4$$
 at $(1,0)$

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6. Find the equation of the tangents to the curve $y = 1 + x^3$ for which the tangent is orthogonal with the line x + 12y = 12.



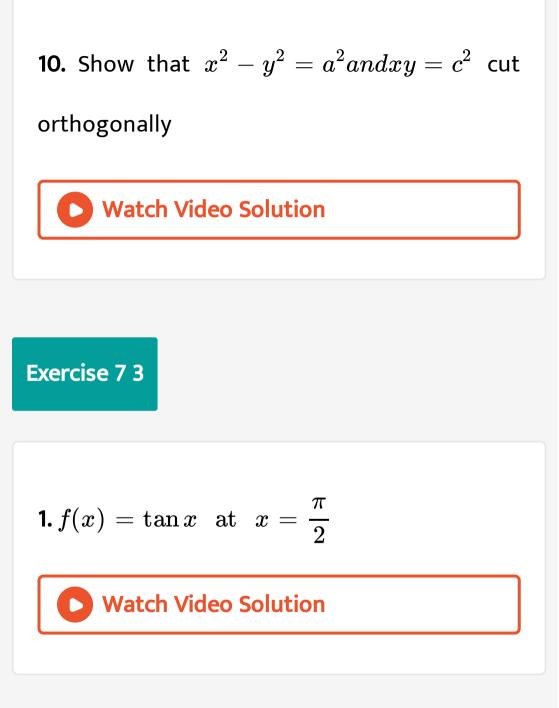
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8. Find the equation of tangent and normal to

the curve given by x = 7 $\cos t$ and $y = 2\sin t, t \in R$ at any point on the curve.

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9. Find the angle between the rectangular hyperboloa xy = 2 and the parabola $x^2 + 4y = 0.$



2. Using the Rolle's theorem, determine the values of x at which the tangent is parallel to the x-axis for the following functions :

(i) $f(x)=x^2-x, x\in [0,1]$ (ii) $f(x)=rac{x^2-2x}{x+2}, x\in [-1,6]$ (iii) $f(x)=\sqrt{x}-rac{x}{3}, x\in [0,9]$

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3. Explain why Lagrange's mean value theorem is not applicable to the following functions in

the respective intervals :

$$f(x)=rac{x+1}{x}, x\in [\,-1,2]$$

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4. Using the Lagrange's mean value theorem determine the values of x at which the tangent is parallel to the secant line at the end points of the given interval:

$$f(x) = x^3 - 3x + 2, x \in [\,-2,2]$$

5. Verify Lagrange's mean value theorem for
$$f(x) = rac{1}{x}$$
 in $[1,2]$

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6. A race car driver is racing at 20th km. If his speed never exceeds 150 km/hr, what is the maximum distance he can cover in the next two hours.

7. Suppose that for a function $f(x), \, f(x) \leq 1$ for all $1 \leq x \leq 4$. Show that $f(4) - f(1) \leq 3$

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8. Does there exist a differentiable function f(x) such that f(0) = -1, f(2) = 4 and $f'(x) \le 2$ for all x. Justify your answer.



9. Show that there lies a point on the curve $f(x)=x(x+3)e^{rac{\pi}{2}},\ -3\leq x\leq 0$ where

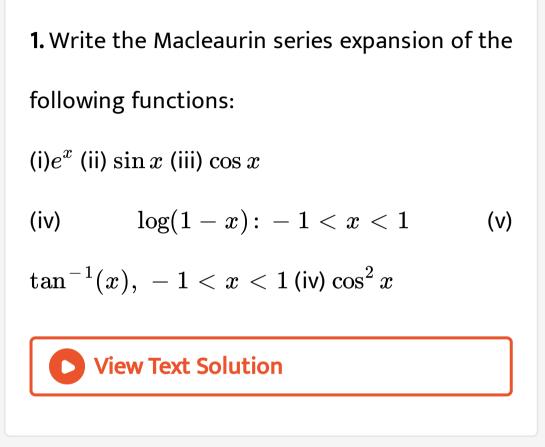
tangent drawn is parallel to the x-axis.



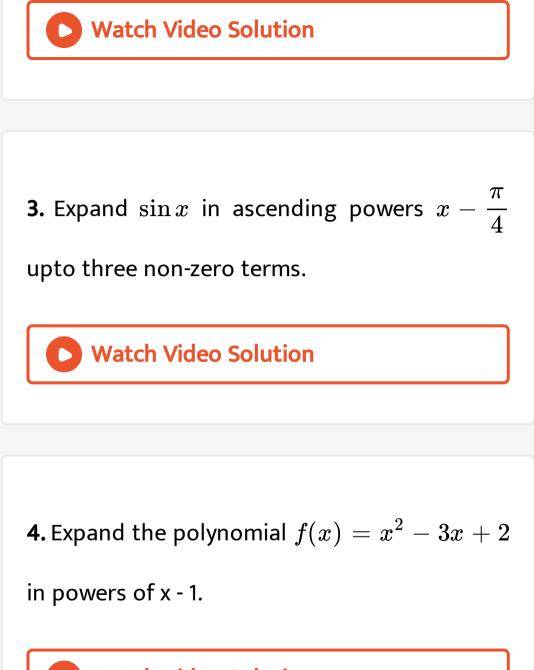
10. Using mean value theorem prove that for,

$$a>0, b>0, \left|e^{-a}-e^{-b}
ight|<|a-b|.$$





2. Write down the Taylor series expansion, of the function $\log x$ about x = 1 upto three non zero terms for x > 0.



1. Evaluate the following limits, if necessary

use 1'Hopital Rule:

 $\lim_{x
ightarrow 0} \ rac{1-\cos x}{x^2}$

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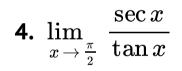
2.
$$\lim_{x
ightarrow\infty}~rac{2x^2}{x^2-5x+3}$$

3. Evaluate the following limits, if necessary

use 1'Hopital Rule:

 $\lim_{x o \infty} \; rac{x}{\log x}$

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5.
$$\lim_{x
ightarrow\infty}~e^{-x}x$$





6.
$$\lim_{x \to 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right)$$

7.
$$\lim_{x
ightarrow 1}$$
 $\left(rac{2}{x^2-1}-rac{x}{x-1}
ight)$

8.
$$\lim_{x o 0} x^x$$





9.
$$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)$$

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10.
$$\lim_{x
ightarrow rac{x}{2}}\left(\sin x
ight)^{ an x}$$

11.
$$\lim_{x o 0} rac{\cos x}{x^2}$$

12. If an initial amount A_0 of money is invested at an interest rate r compounded n times a year, the value of the investment after t years is $A = A_0 \left(1 + rac{1}{n}
ight)^{nt}$. If the interest is compounded continuously, (that is as $n o \infty$), show that the amount after t years is $A = A_0 e^{rt}.$

1. Find the local extrema of the function $f(x) = 3x^4 - 4x^3$

2. Find the intervals of monotonicities and hence find the local extremum for the following functions:

$$f(x)=rac{x}{x-5}$$

Exercise 7 7

1. Find intervals of concavity and points of inflexion for the following functions:

$$f(x) = x(x-4)^3$$

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2. Find the local extrema for the following functions using second derivative test :

$$f(x) = x \log x$$

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3. For the function $f(x) = 4x^3 + 3x^2 - 6x + 1$ find the intervals of monotonicity, local extrema, intervals of concavity and points of inflection.



Exercise 78

1. Find two positive numbers whose sum is 12

and their product is maximum.



2. Find two positive numbers whose product is

20 and their sum is minimum.



3. Find the smallest possible value of $x^2 + y^2$

given that x + y = 10.

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4. A garden is to be laid out in a rectangular area and protected by wire fence. What is the largest possible area of the fenced garden with 40 metres of wire.

5. A rectangular page is to contain $24cm^2$ of print. The margins at the top and bottom of the page are 1.5 cm and the margins at other sides of the page is 1 cm. What should be the dimensions of the page so that the area of the paper used is minimum.

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6. A farmer plans to fence a rectangular pasture adjacent to a river. The pasture must contain 1,80,000 sq. mtrs in order to provide

enough grass for herds. No fencing is needed along the river. What is the length of the minimum needed fencing material?

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7. Find the dimensions of the rectangle with maximum area that can be inscribed in a circle

of radius 10 cm.

8. Prove that among all the rectangles of the given perimeter, the square has the maximum area.



9. Find the dimensions of the largest rectangle

that can be inscribed in a semi circle of radius

r cm.



10. A manufacturer wants to design an open box having a square base and a surface area of 108 sq. cm. Determine the dimensions of the box for the maximum volume.



11. The volume of a cylinder is given by the formula $V = \pi r^2 h$. Find the greatest and least values of V if r + h = 6.

12. A hollow cone with base radius a cm and height b cm is placed on a table . Show that the volume of the largest cylinder that can be hidden underneath is $\frac{4}{9}$ times volume of the

cone.



Exercise 7 9

1. Find the asymptotes of the following curves:

$$f(x)=rac{x^2}{x+1}$$

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2. Sketch the graphs of the following

functions:

$$y=rac{x^3}{24}-\log x$$

1. The volume of a sphere is increasing in volume at the rate of $3\pi cm^3 / \sec$. The rate of change of its radius when radius is $\frac{1}{2}$ cm

- A. 3cm/s
- $\mathsf{B.}\,2cm/s$
- $\mathsf{C.}\,1cm\,/\,s$

D.
$$rac{1}{2} cm/s$$

Answer: C::D

2. A balloon rises straight up at 10m/s. An observer is 40 m away from the spot where the balloon left the ground. Find the rate of change of the balloon's angle of elevation in radian per second when the balloon is 30 metres above the ground.

A.
$$\frac{3}{25}$$
 radians/sec
B. $\frac{4}{25}$ radians/sec
C. $\frac{1}{5}$ radians/sec

D.
$$\frac{1}{3}$$
 radians/sec

Answer: A::B::C::D

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3. The position of a particle moving along a horizontal line of any time t is given by $s(t) = 3t^2 - 2t - 8$. The time at which the particle is at rest is

$$\mathsf{B}.\,t=\frac{1}{3}$$

C. t=1

D. 1=3

Answer: A::B::C

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4. A stone is thrown up vertically. The height it reaches at time t seconds is given by $x = 80t - 16t^2$. The stone reaches the maximum height in time t seconds is given by

A. 2

 $\mathsf{B}.\,2.5$

C. 3

 $\mathsf{D}.\,3.5$

Answer: B::C

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5. Find the slope for x=2 in $f(x)=x^3+2$

6. The abscissa of the point on the curve $f(x) = \sqrt{8 - 2x}$ at which the slope of the tangent is -0.25 ?

A. - 8

B. - 4

 $\mathsf{C}.-2$

D. 0

Answer: D



7. The slope of the line normal to the curve $f(x) = 2\cos 4x$ at $x = \frac{\pi}{12}$ is

A.
$$-4\sqrt{3}$$

$$B. - 4$$

$$\mathsf{C}.\,\frac{\sqrt{3}}{12}$$

D.
$$4\sqrt{3}$$

Answer: A::B::C::D

8. The tangent to the curve $y^2 - xy + 9 = 0$

is vertical when

A.
$$y=0$$

B. $y=\pm\sqrt{3}$
C. $y=rac{1}{2}$
D. $y=\pm\sqrt{3}$

Answer: C

9. Angle between $y^2 = x$ and $x^2 = y$ at the

origin is

A.
$$\frac{\tan^{-1}(3)}{4}$$

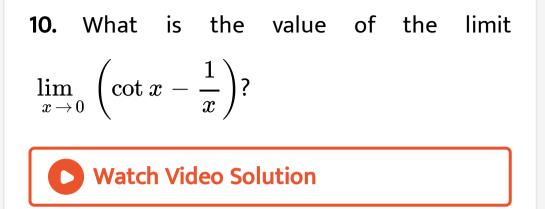
B.
$$\tan^{-1}\left(\frac{4}{3}\right)$$

C.
$$\frac{\pi}{2}$$

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D.
$$\frac{\pi}{4}$$

Answer: B



11. The function $\sin^4 x + \cos^4 x$ is increasing in the interval

A.
$$\left[\frac{5\pi}{8}, \frac{3\pi}{4}\right]$$

B. $\left[\frac{\pi}{2}, \frac{5\pi}{8}\right]$
C. $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

$\mathsf{D}.\left[0,\frac{\pi}{4}\right]$

Answer: D

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12. The number given by the Rolle's theorem for the function $x^3-3x^2, x\in [0,3]$ is

A. 1

B. s= $\sqrt{2}$ C. $\frac{3}{2}$ D. 2

Answer: B

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13. The number given by the Mean value theorem for the function $rac{1}{x}, x \in [1,9]$ is

B. 2.5

A. 2

D. 3.5

Answer: C

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14. The minimum value of the function |3-x|+9 is

A. 0

B. 3

D. 9

Answer: A::C

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15. The maximum slope of the tangent to the curve $y=e^x \sin x, x \in [0,2\pi]$ is at

A.
$$x=rac{\pi}{4}$$

B. $x=rac{\pi}{2}$

$$\mathsf{C.}\,x=\pi$$

$$\mathsf{D}.\,x=\frac{3\pi}{2}$$

Answer: B

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16. The maximum value of the function $x^2e^{-2x}, \, x > 0$ is

A.
$$\frac{1}{e}$$

B. $\frac{1}{2e}$
C. $\frac{1}{e^2}$

Answer: A::B

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17. One of the closest points on the curve $x^2-y^2=4$ to the point (6, 0) is

A. (2, 0)B. $(\sqrt{5}, 1)$ C. $(3, \sqrt{5})$

D. $(\sqrt{13}, -\sqrt{3})$

Answer: C

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18. The maximum value of the product of two positive numbers, when their sum of the squares is 200, is

A. 100

 $\mathsf{B.}\,25\sqrt{7}$

C. 28

D. $24\sqrt{14}$

Answer: A::B

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19. The curve $y = ax^4 + bx^2$ with ab > 0

A. has no horizontal tangent

B. is concave up

C. is concave down

D. has no points of inflection

Answer:



- 20. The point of inflection of the curve $y=(x-1)^3$ is
 - A. (0, 0)
 - B.(0,1)
 - C.(1,0)

D.(1,1)

Answer: A

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Additional Questions Solved

1. A water tank has the shape of an inverted circular cone with base radius 2 metres and height 4 metres. If water is being pumped into the tank at the rate of $2m^3/mm$. Find the

rate at which the water level is rising when the

water is 3m deep



2. A car A is travelling from west at 50 km/hr and car B is travelling towards north at 60 km//hr . Both are headed for the intersection of the two roads . At what rate are the cars approaching each other when car A is 0.3 kilometers and car B is 0.4 kilometers from the intersection ?



3. The distance x metres travelled by a vehicle in time t seconds after the breakes are applied is given by $x = 20t - \frac{5}{3}t^2$. Determine (i) the speed of the vehicle (in km /hr) at the instant the brakes are applied and (ii) the distance the car travelled before it stops.



4. At a particular instant ship A is 100 km west of ship B, ship A is sailing at a speed of 35 km/hr and ship B is sailing north at 25 km/hr. How fast is the distance between the ships changes after 4 hours

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5. Gravel is being duped from a conveyor belt at a rate of $30ft^3 / \min$ and its coarsened such that it from a sile in the shape of a cone whose base diameter and height are always equal . How fast is the height of the pile increasing when the pile is 10 ft high ?

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6. Prove that the sum of the intercepts on the co-ordinate axes of any tangent to the curve $x=a\cos^4 heta,y=a\sin^4 heta,0\leq heta\leq rac{\pi}{2}$ is equal to a .

7. Find the equation of normal to $y = x^3 - 3x$

that is parallel to 2x + 18y - 9 = 0



8. Prove that the curves $2x^2 + 4y^2 = 1$ and

 $6x^2 - 12y^2 = 1$ cut each other at right angles

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9. Show that the equation of the normal to the

curve $x=a\cos^3 heta, y=a\sin^3 heta$ at 'heta' is

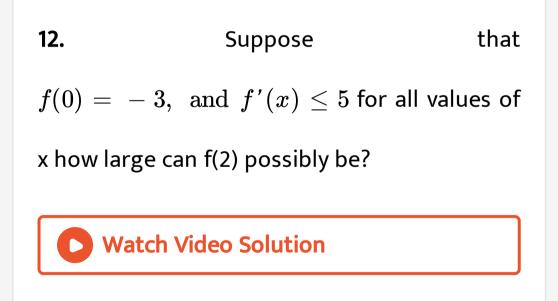
 $x\cos heta-y\sin heta=a\cos2 heta.$



10. If the curve $y^2 = x$ and xy = k are orthogonal then prove that $8k^2 = 1$

11. Verify Rolle's theorem for the following

$$f(x) = x^3 - 3x + 3 \; ext{ in } \; 0 \leq x \leq 1$$



13. Using Rolle's theorem find the point on the

curve $y=x^2+1,\;-2\leq x\leq 2$ where the

tangent is parallel to x-axis.



14. Find 'C' of Lagrange's mean value theorem

for the function

 $f(x) = 2x^3 + x^2 - x - 1, [0, 2]$

15. Find 'C' of Lagrange's mean value theorem for the function $f(x) = x^3 - 5x^2 - 3x$ in [1, 3]



16. The Taylor's series expension of
$$f(x) = \sin x$$
 about $x = rac{\pi}{2}$ is obtained by the

following way.

17. Obatin the Maclaurin's series expansion for

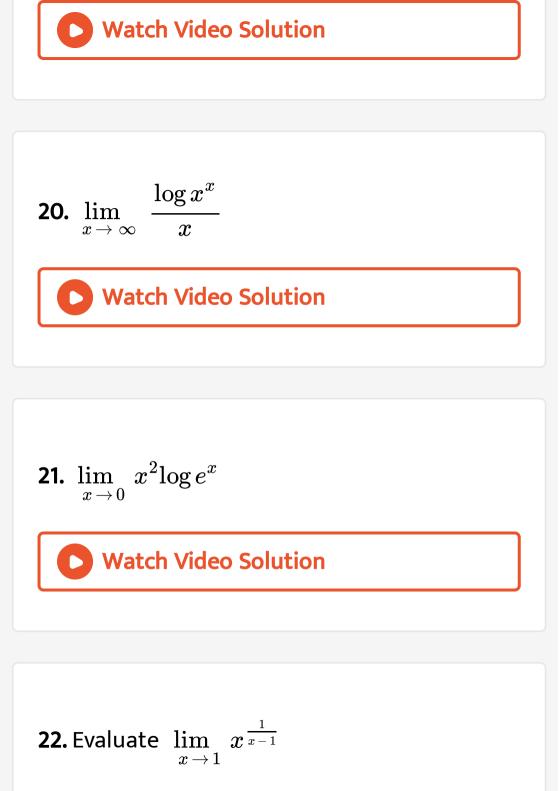
the following function .

(i)
$$e^{2x}$$
 (ii) $\sin^2 x$ (iii) $rac{1}{1+x}$ (iv) $\tan x,\ \prec x < rac{\pi}{2}$

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18. Evaluate
$$\lim_{x o rac{\pi}{2}} rac{\log \sin x}{\left(\pi - 2x
ight)^2}$$

19. Evaluate:
$$\lim_{x o 0} \ (\cot x)^{\sin x}$$





24. Find the absolute maximum and absolute

minimum values of f on the given interval

1.
$$f(x) = 1 - 2x - x^2, [\,-4,1]$$

25.
$$f(x) = x^3 - 12x + 1, [-3, 5]$$



26.
$$f(x) = rac{x}{x+1}, [1,2]$$



27. Find the absolute maximum and absolute minimum values of f on the given interval.
f(x)=sin x+ cos x, [0, pi//3]`





28.
$$f(x) = x - 2\cos x, [-\pi, \pi]$$



29. Find the local maximum and minimum

values of the following functions

$$2x^3 + 5x^2 - 4x$$

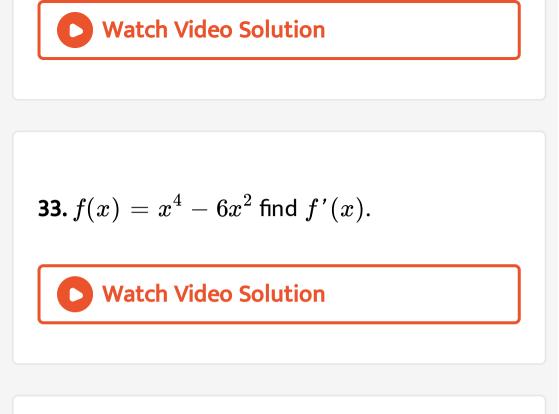
30. Find the local maximum and minimum values of the following functions $t + \cos t$ Watch Video Solution

31. Find the slope at x=2 for $y=x^4-4x^3$

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32. Find the slope at x=2

 $f(x) = 2x^3 + 5x^2 - 4x$



34.
$$f(heta) = \sin 2 heta$$
 in $(0, \pi)$

35.
$$y = 12x^2 - 2x^3 - x^4$$



36. The top and bottom margins of a poster are each 6 cms and the side margins are each 4 cms . If area of the printed material on the poster is fixed at $384cms^2$, find the dimension of the poster with the smallest area .



37. Show that the volume of the largest right

circular cone that can be inscribed in a sphere of radius a is $\frac{8}{27}$ (volume of the shpere).



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38. A closed box (cuboid) with a square base is to have a volume 2000c.c, The material for the top and bottom of the box is to cost Rs 3 per square cm and the material for the sides is to cost Rs 1.50 per square cm. If the cost of the material is to be least find the dimension of

the box.



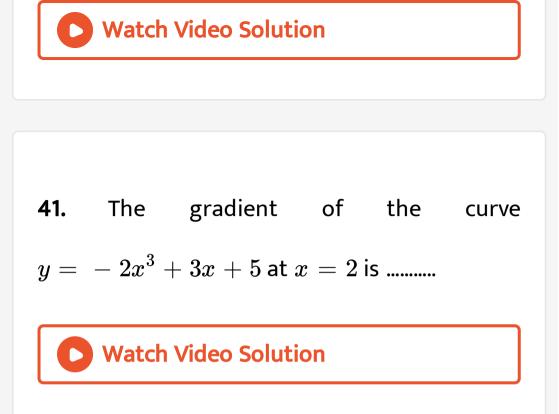
39. Find the numbers whose sum is 100 and

whose product is maximum.

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40. Find two positive numbers whose product

is 100 and whose sum is minimum.



42. The rate of change of area A of a circle of radius r is

A. $2\pi r$

B.
$$2\pi r \frac{dr}{dt}$$

C. $\pi r^2 \frac{dr}{dt}$
D. $\pi \frac{dr}{dt}$

Answer: B::D

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43. A spherical snowball is melting in such a way that its volume is decreasing at a rate of $1cm^3 / \min$. The rate at which the diameter is decreaseing when the diameter is 10 cms is ..

A.
$$\frac{-1}{50\pi}cm / \min$$

B. $\frac{1}{50\pi}cm / \min$
C. $\frac{-11}{75\pi}cm / \min$
D. $\frac{-2}{75\pi}cm / \min$

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44. The parametric equations of the curve

$$x^{2\,/\,3} + y^{2\,/\,3} = a^{2\,/\,3}$$
 are

A.
$$x=a\sin^3 heta, y=a\cos^3 heta$$

B.
$$x = a \cos^4 \theta, y = a \sin^4 \theta$$

C.
$$x=a^{3}\sin heta,y=a^{3}\cos heta$$

D.
$$x=a^3\cos heta, y=a^3\sin heta$$

Answer:

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45. If the normal to the curve
$$x^{2/3} + y^{2/3} = a^{2/3}$$
 makes an angle θ with the x-axis then the slope of the normal is

A. $-\cot \theta$

 $B.\tan\theta$

 $C. - tan \theta$

D. $\cot \theta$

Answer: A



46. What is the surface area of a sphere when

the volume is increasing at the same rate as

its radius?

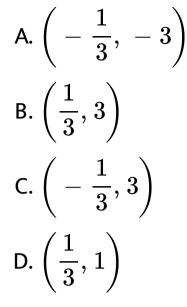
A. 1

B.
$$\frac{1}{2\pi}$$

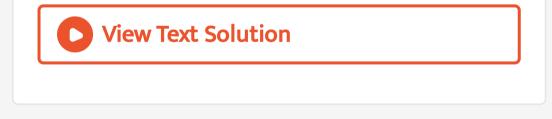
C. 4π
D. $\frac{4\pi}{3}$



47. For what values of x is the rate of increase of $x^3 - 2x^2 + 3x + 8$ is twice the rate of increase of x..



Answer: A::C



48. If the volume of an expanding cube is increasing at the rate of $4cm^3/\sec$ then the

rate of change of surface area when the volume of the cube is 8 cubic cm is

A.
$$8cm^2/\sec$$

- $\mathsf{B.}\,16cm^2\,/\,\mathrm{sec}$
- $\mathsf{C.}\,2cm^2\,/\,\mathrm{sec}$
- D. $4cm^2/\sec$

Answer: A::C::D



49. If a normal makes an angle θ with positive x-axis then the slop of the curve at the point where the normal is drawn is

A. $-\cot heta$

 $B.\tan\theta$

 $C. - tan \theta$

D. $\cot \theta$

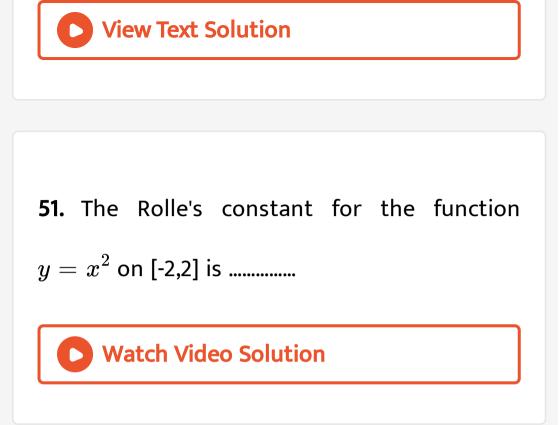
Answer: A::C



50. If the velocity of a particle moving along a straight line is directly proportional to the square of its distance from a fixed point on the line . Then its acceleration is proportional to

A. s B. *s*² C. *s*³ D. *s*⁴

Answer: B::C



52. The value 'c' of Lagranges Mean Value Theorem for $f(x) = \sqrt{x}$ when a=1 and b=4 is

A.
$$\frac{9}{4}$$

.....

B.
$$\frac{3}{2}$$

C. $\frac{1}{2}$
D. $\frac{1}{4}$

Answer: C::D

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53. In a given semi circle of diameter 4 cm a rectangle is to be inscribed . The maximum area of the rectangle is

A. 2

B. 4

C. 8

D. 16

Answer: A



54. The least possible perimeter of a rectangle

of area $100m^2$ is

A. 10

B. 20

C. 40

D. 60

Answer: A::B::C::D

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55. Which of the following curves is concave

down?

A.
$$y=\,-\,x^2$$

$$\mathsf{B.}\, y = x^2$$

$$\mathsf{C}.\, y = e^x$$

D.
$$y=x^2+2x-3$$

Answer: B

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56. The point of inflection of the curve $y=x^4$

is at:

A. x = 0

$$\mathsf{B.}\,x=3$$

 $\mathsf{C.}\,x=12$

D. nowhere

Answer:

