MATHS

BOOKS - FULL MARKS MATHS (TAMIL ENGLISH)

APPLICATIONS OF DIFFERENTIAL CALCULUS

Example Questions Solved

1. For the functions $f(x) = x^2, x \in [0, 2]$ compute the average rate of changes in the subintervals $[0, 0.5], [0.5, 1], [1, 1.5], [1.5, 2]$ and the instantaneous rate of changes at the points x = 0.5 , 1, 1.5, 2

Watch Video Solution

2. The temperature in celsius in along rod of length 10m, insulated at both ends , is a function of length x given by $T = x(10 - x)$.

Prove that the rate of changes of temperature at the midpoint of the rod is zero .

Watch Video Solution

3. A person learnt 100 words for an English test. The number of words the person remembers in t days after learning is given by $W(t) = 100 \times (1 - 0.1t)^2, 0 \leq t \leq 10.$ What is the rate at which the person forgets the words 2 days after learning ?

Watch Video Solution

4. A particle moves so that the distance moved is according to the law $s(t) = \frac{t^3}{3} - t^2 + 3$. At what time the velocity and acceleration are zero respectively ?

Watch Video Solution

5. A particle is fired straight up from the ground to each a height of x feet in t seconds, where x $(t) = 128t - 16t^2$.

(1) Compute the maximum height of the

particle reached.

(2) What is the velocity when the particle hits the ground ?

Watch Video Solution

6. A particle moves along a horizontal line such that its position at any time t is given by $s(t) = t^3 - 6t^2 + 9t + 1$, s in meters and t in seconds.

At what time the particle is at rest?

Watch Video Solution

7. If we blow air into a balloon of spherical shape at a rate of $1000cm^3$ per second. At what rate the radius of the balloon changes when the radius is 7cm? Also compute the rate at which the surface area changes .

Watch Video Solution

8. The price of a product is related to the number of units available (supply) by the equation $Px + 3P - 16x = 234$, where P is the price of the product per unit in Rupees (Rs) and x is the number of units. Find the rate at which the price is changing with respect to time when 90 units are available an the supply is increasing at a rate of units/week .

Watch Video Solution

9. Salt is poured from a conveyer belt at a rate of 30 cubic metre per minute forming a conical pile with a circular base whose beight and diameter of base are alwayes equal . How fast is the height of the pile increasing when the pile is 10 metre high ?

Watch Video Solution

10. A road running north to south acrosses a road going east to west at the point P. Car A is driving north along the first road, and car B is driving east along the second road . At a particular time car A 10 kilometers to the north of P and traveling at $80km/hr$, while car B is 15 Kilometers to the east of P and traveling at $100km/hr$. How fast is the distance between the two cars changing ?

Watch Video Solution

11. Find the equations of tangent and normal to the curve $y = x^2 + 3x - 2$ at the point (1, 2). .

Watch Video Solution

12. For what value of x the tangent of the curve $y = x^3 - 3x^2 + x - 2$ is parallel to the line $y = x$

Watch Video Solution

13. Find the equation of the tangent and normal to the Lissajous curve given by $x = 2\cos 3t$ and $y = 3\sin 2t, t \in \mathbb{R}$.

Watch Video Solution

14. Find the actute angle between $y = x^2$ and $y = (x - 3)^2$.

View Text Solution

15. Find the acute angle between the curves $y = x^2$ and $x = y^2$ at their points of intersection $(0, 0), (1, 1)$.

Watch Video Solution

16. Find the angle of intersection of the curve $y = \sin x$ with the positive x-axis .

Watch Video Solution

17. If the curves $ax^2 + by^2 = 1$ and $a_1x^2 + b_1y^2 = 1$ intersect each other orthogonally then show that $\frac{1}{a} - \frac{1}{b} = \frac{1}{a_1} - \frac{1}{b_1}$

Watch Video Solution

18. Prove that the ellipse $x^2 + 4y^2 = 8$ and the hyperbola $x^2 - 2y^2 = 4$ intersect orthogonally .

Watch Video Solution

19. Compute the value of 'c' satisfied by the Rolle's theorem for the function . $f(x) = x^2(1 - x)^2, x \in [0, 1]$.

Watch Video Solution

20. Find the value of c in the interval $\left(\frac{1}{2}, 2\right)$ satisfied by the Roll's theorem for the function. $f(x) = x + \frac{1}{x}, x \in \left[\frac{1}{2}, 2\right]$

Watch Video Solution

21. Compute the value of 'c' satisfied by the Rolle's theorem for the function .

$f(x) = x^2(1-x)^2, x \in [0, 1]$.

Watch Video Solution

22. Without actually solving show that the equation $x^4 + 2x^3 - 2 = 0$ has only one real root in the interval (0,1) .

Watch Video Solution

23. Prove, Using the Rolle's theorem that between any two distinct real zeros of the polynomial

$a_n x^n + a_{n-1} x^{n-1} + \ldots \ldots a_1 x + a_0$ there is a zero of the polynomial .

$$na_nx^{n-1} + (n-1)a_{n-1}x^{n-2} + \ldots. + a_1$$

Watch Video Solution

24. Prove that there is a zero of the polynomial $2x^3 - 9x^2 - 11x + 12$ in the interval (2,7) given that 2 and 7 are the zeros of the polynomial $x^4 - 6x^3 - 11x^2 + 24x + 28.$

Watch Video Solution

25. Find the values in the interval (1,2) of the mean value theorem satisfied by the function $f(x) = x - x^2 \text{for} 1 \leq x \leq 2.$

Watch Video Solution

26. A truck travels on a toll road with a speed limit of $80 km / hr$. The truck completes a 164 km journey in 2 hours . At the end of the toll road the trucker is issued with a speed violation ticket. Justify this using the Mean Value Theorem .

Watch Video Solution

27. Suppose $f(x)$ is a differentiable function for all x with $f'(x) \leq 29$ and $f(2) = 17$. What is the maximum value of f (7) ?

Watch Video Solution

28. Prove , using mean value theorem, that $|\sin \alpha - \sin \beta| \leq |\alpha - \beta|, \alpha, \beta \in \mathbb{R}.$

Watch Video Solution

29. A thermometer was taken from a freezer and placed in a boilling water . It took 22 seconds for the thermometer to raise from $-10^\circ C \rightarrow 100^\circ C$. Show that the rate of changes of temperature at some time t is $5^\circ C$ per second

Watch Video Solution

30. Expand log (1+x) as a Maclaurin 's series upto 4 non-zero terms for $-1 < x \leq 1$.

Watch Video Solution

31. Expand $\tan x$ maclaurin 's series in asending powers of x upto 5^{th} power for $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

Watch Video Solution

32. Write the Taylor series expansion of $\frac{1}{x}$ about $x = 2$ by finding the first three non-zero terms

Watch Video Solution

33. Evaluate: $\operatorname*{Lim}_{x-1}\left(\frac{x^2-3x+2}{x^2-4x+3}\right)$

Watch Video Solution

34. Compute the limit $\lim_{x \to a}\left(\frac{x^n-a^n}{x-a}\right)$

Watch Video Solution

35. Evaluate the limit $\lim_{x \to 0}\left(\frac{\sin mx}{x}\right)$

Watch Video Solution

36. Evaluate the limit $\lim_{x \to 0}\left(\frac{\sin x}{x^2}\right)$

Watch Video Solution

37. Evaluate $\lim_{\theta \to 0}\frac{1 - \cos m\theta}{1 - \cos n\theta}$

Watch Video Solution

38. Evaluate: $\lim_{x \to 0} \left(\frac{\log(1-x)}{(x)}\right)$

Watch Video Solution

39. Evaluate: $\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{e^x - 1}\right)$

Watch Video Solution

40. Evaluate: $\lim_{x \to 0} x \log x$

Watch Video Solution

41. Evaluate: $\lim_{x \to \infty} \left(\frac{x^2 + 17x + 29}{x^4}\right)$

Watch Video Solution

42. Evaluate: $\lim_{x \to \infty} \frac{e^x}{x}$

Watch Video Solution

43. Using the l'Hopital Rule prove that,

$$\lim_{x \to 0^{+}} (1+x)^{\frac{1}{x}} = e$$

Watch Video Solution

44. Evaluate: $\lim_{x \to \infty} (1+2x)$

Watch Video Solution

45. Evaluate: $\lim_{x \to 0} x^{x}$

Watch Video Solution

46. Prove that the function $f(x)=x^2+2$ is strictly increasing in the interval (2,7) and strictly decreasing in the interval (-2, 0).

Watch Video Solution

47. Prove that the function $f(x)=x^2-2x-3$ is strictly increasing in $(2, \infty)$

Watch Video Solution

48. Find the absolute maximum and absolute minimum values of the function $f(x) = 2x^3 + 3x^2 - 12x$ on [-3, 2]

Watch Video Solution

49. Find the absolute extrema of the function $f(x) = 3\cos x$ on the closed interval $[0, 2\pi]$.

Watch Video Solution

50. Find the slope at $x = -1$ for $f(x) = x^2 - 4x + 4$

Watch Video Solution

51. Find the intervals of monotonicity and hence find the local extrema for the function $f(x) = x^{\frac{2}{3}}$

Watch Video Solution

52. Prove that the function $f(x) = x - \sin x$ is increasing on the real line . Also discuss for the existence of local extrema.

Watch Video Solution

53. Discuss the monotonicity and local extrema of the function $f(x) = \log(1+x) - \frac{x}{1+x}, x > -1$ and hence find the domain where, log $(1+x) > \frac{x}{1+x}$

Watch Video Solution

54. Find the intervals of monotonicity and local extrema of the function $f(x) = x \log x + 3x$

Watch Video Solution

55. Find the slope for $f(x) = \frac{1}{1+x^2}$ at $x = 1$

Watch Video Solution

56. Find the slope for $f(x) = \frac{x}{1+x^2}$ at $x = 1$

Watch Video Solution

57. Determine the intervals of concavity of the curve $f(x) = (x-1)^3(x-5), x \in R$ and , points of inflection if any

Watch Video Solution

58. Determine the intervals of concavity of the curve $y = 3 + \sin x$.

Watch Video Solution

59. Find the local extremum of the function $f(x) = x^4 + 32$

Watch Video Solution

60. Find the local extrema of the function $f(x) = 4x^6 - 6x^4$

Watch Video Solution

61. Find the local maximum and minimum of the function x^2y^2 on the line $x + y = 10$

Watch Video Solution

62. We have a 12 square unit piece of thin material and want to make an open box by cutting small squares from the corners of our material and folding the sides up. The question is, which cut produces the box of maximum volume ?

Watch Video Solution

63. Find the points on the unit circle $x^2 + y^2 = 1$ nearest and farthest from (1,1)

64. A steel plant is capable of producing x tonnes per day of a law-grade steel and y tonnes per day of a hight-grade steel, where $y = \frac{40 - 5x}{10 - x}$. If the fixed market price of low-grade steel is half that of high-grade steel, then what should be optimal productions in law-grade steel and high-grade steel in order to have maximum receipts.

65. Prove that among all the rectangles of the given area, square has the least perimeter.

Watch Video Solution

66. Find the asymptotes of the function $f(x) = \frac{1}{x}$

Watch Video Solution

67. Find the value of $\lim_{x \to \infty} \frac{x^2 - 6x + 7}{x + 5}$

Watch Video Solution

68. Find the asymptotes of the curve $f(x)=\frac{2x^2-8}{x^2-16}$

Watch Video Solution

69. Sketch the curve $y=f(x)x^2-x-6$

View Text Solution

70. Sketch the curve $y = f(x) = x^3 - 6x - 9$

View Text Solution

71. Sketch the curve $y = \dfrac{x^2 - 3x}{(x - 1)}$

View Text Solution

Exercise 7 1

1. A particle moves along a straight line in suc a way that after t second its distance from the origin is $s = 2t^2 + 3t$ metres.

Find the instantaneous velocities at t =3 and t =6 seconds.

Watch Video Solution

2. A camera is accidentally knocked off an edge of a cliff 400 ft high. The camera falls a distance of $s = 16t^2$ in t seconds.

How long does the camera fall before it hits the ground?

Watch Video Solution

3. A particle moves along a horizontal line such that its equation of motion is $s(t) = 2t^3 - 15t^2 + 24t - 2$, s in meters and t in second.

Find the total distance travelled by the particle in the first 2 seconds.

Watch Video Solution

4. If the volume of a cube of side length x is $V = x^3$. Find the rate of change of the volume with respect to x when x = 5 units.

Watch Video Solution

5. If the mass m(x) (in kilograms) of a thin rod of length x(in metres) is given by, $m(x) = \sqrt{3x}$ then what is the rate of change of mass with respect to the length when it is x = 27 meters.

Watch Video Solution

6. A stone is dropped into a pond causing ripples in the form of concentric circles. The radius r of the outer ripple is increasing at a constant rate at 2 cm per second. When the radius is 5 cm find the rate of changing of the total area of the disturbed water?

Watch Video Solution

7. A beacon makes one revolution every 10 seconds. It is located on a ship which is anchored 5 km from a straight shore line. How fast is the beam moving along the shore line when it makes an angle of 45° with the shore?

Watch Video Solution

8. A conical water tank with vertex down of 12 meters height has a radius of 5 meters at the top. If water flows into the tank at a rate 10 cubic m/min, how fast is the depth of the water increases when the water is 8 metres deep?

Watch Video Solution

9. A ladder 17 metre long is leaning against the wall. The base of the ladder is pulled away from the wall at a rate of 5m/s. When the base of the ladder is 8 metres from the wall.

How fast is the top of the ladder moving down the wall?

Watch Video Solution

10. A police jeep, approaching an orthogonal intersection from the northern direction, is chasing a speeding car that has turned and moving straight east. When the jeep is 0.6 km north of the intersection and the car is 0.8 km to the east. The police deteremine with a radar that the distance between them and the cae is increasing at 20 km/hr. If the jeep is moving at 60 km/hr at the instant of measurement, what is the speed of the car?

Watch Video Solution

Exercise 7 2

1. Find the slope of the tangent to the following curves at the respective given points.

$y = x^4 + 2x^2 - x$ at $x = 1$

Watch Video Solution

2. Find the point on the curve $y = x^2 - 5x + 4$ at which the tangent is parallel to the line 3x + y = 7.

Watch Video Solution

3. Find the points on the curve $y = x^3 - 6x^2 + x + 3$ where the normal is parallel to the line $x + y = 1729$

Watch Video Solution

4. Find the points on the curve $y^2 - 4xy = x^2 + 5$ for which the tangent is horizontal.

Watch Video Solution

5. Find the tangent and normal to the following curves at the given points on the curve.

$y = x^2 - x^4$ at $(1, 0)$

Watch Video Solution

6. Find the equation of the tangents to the curve $y = 1 + x^3$ for which the tangent is orthogonal with the line $x + 12y = 12$.

Watch Video Solution

7. Find the equations of the tangents to the curve $y = \frac{x+1}{x-1}$ which are parallel to the line x + 2y = 6.

Watch Video Solution

8. Find the equation of tangent and normal to the curve given by x = 7 $\cos t$ and $y = 2\sin t, t \in R$ at any point on the curve.

Watch Video Solution

9. Find the angle between the rectangular hyperboloa xy = 2 and the parabola $x^2 + 4y = 0$.

Watch Video Solution

10. Show that $x^2 - y^2 = a^2 and xy = c^2$ cut orthogonally

Watch Video Solution

Exercise 7 3

1. $f(x) = \tan x$ at $x = \frac{\pi}{2}$

Watch Video Solution

2. Using the Rolle's theorem, determine the values of x at which the tangent is parallel to the x-axis for the following functions :

(i) $f(x) = x^2 - x, x \in [0, 1]$

(ii) $f(x) = \dfrac{x^2 - 2x}{x + 2}, x \in [-1, 6]$

(iii) $f(x) = \sqrt{x} - \dfrac{x}{3}, x \in [0, 9]$

Watch Video Solution

3. Explain why Lagrange's mean value theorem is not applicable to the following functions in the respective intervals :

$$f(x) = \frac{x+1}{x}, x \in [-1, 2]$$

Watch Video Solution

4. Using the Lagrange's mean value theorem determine the values of x at which the tangent is parallel to the secant line at the end points of the given interval:

$$f(x) = x^3 - 3x + 2, x \in [-2, 2]$$

Watch Video Solution

5. Verify Lagrange's mean value theorem for

$$f(x) = \frac{1}{x} \text{ in } [1, 2]$$

Watch Video Solution

6. A race car driver is racing at 20^{th} km. If his speed never exceeds 150 km/hr, what is the maximum distance he can cover in the next two hours.

Watch Video Solution

7. Suppose that for a function $f(x)$, $f(x) \leq 1$ for all $1 \leq x \leq 4$. Show that $f(4) - f(1) \leq 3$

Watch Video Solution

8. Does there exist a differentiable function f(x) such that f(0) = -1, f(2) = 4 and $f'(x) \leq 2$ for all x. Justify your answer.

Watch Video Solution

9. Show that there lies a point on the curve $f(x) = x(x+3)e^{\frac{\pi}{2}}, \; -3 \le x \le 0$ where tangent drawn is parallel to the x-axis.

Watch Video Solution

10. Using mean value theorem prove that for, $a > 0, b > 0, \left|e^{-a} - e^{-b}\right| < |a - b|$.

Watch Video Solution

Exercise 7 4

1. Write the Macleaurin series expansion of the following functions:

(i)e^{x} (ii) $\sin x$ (iii) $\cos x$

(iv) $\log(1-x): -1 < x < 1$ (v) $\tan^{-1}(x), -1 < x < 1$ (iv) $\cos^{2} x$

View Text Solution

2. Write down the Taylor series expansion, of the function $\log x$ about x = 1 upto three non zero terms for $x > 0$.

Watch Video Solution

3. Expand $\sin x$ in ascending powers $x - \frac{\pi}{4}$ upto three non-zero terms.

Watch Video Solution

4. Expand the polynomial $f(x) = x^2 - 3x + 2$ in powers of x - 1.

Watch Video Solution

Exercise 7 5

1. Evaluate the following limits, if necessary use 1'Hopital Rule:

$$\lim_{x \to 0} \frac{1-\cos x}{x^2}$$

Watch Video Solution

2. $\lim_{x \to \infty} \frac{2x^2}{x^2 - 5x + 3}$

Watch Video Solution

3. Evaluate the following limits, if necessary use 1'Hopital Rule:

$$\lim_{x \to \infty} \frac{x}{\log x}$$

Watch Video Solution

4. $\lim_{x \to \frac{\pi}{2}} \frac{\sec x}{\tan x}$

Watch Video Solution

5. $\lim_{x \to \infty} e^{-x} x$

Watch Video Solution

6. $\lim_{x \to 0} \left(\frac{1}{\sin x} - \frac{1}{x}\right)$

Watch Video Solution

7. $\lim_{x \to 1} \left(\frac{2}{x^2 - 1} - \frac{x}{x - 1}\right)$

Watch Video Solution

8. $\lim_{x \to 0} x^x$

Watch Video Solution

9. $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)$

Watch Video Solution

10. $\lim_{x \to \frac{x}{2}} (\sin x)^{\tan x}$

Watch Video Solution

11. $\lim_{x \to 0} \frac{\cos x}{x^2}$

Watch Video Solution

12. If an initial amount A_0 of money is invested at an interest rate r compounded n times a year, the value of the investment after t years is $A = A_0\left(1+\frac{1}{n}\right)^{nt}$. If the interest is compounded continuously, (that is as $n \to \infty$), show that the amount after t years is $A = A_0 e^{rt}$.

Watch Video Solution

Exercise 7 6

1. Find the local extrema of the function

$f(x) = 3x^4 - 4x^3$

Watch Video Solution

2. Find the intervals of monotonicities and hence find the local extremum for the following functions:

$f(x) = \frac{x}{x-5}$

Watch Video Solution

Exercise 7 7

1. Find intervals of concavity and points of inflexion for the following functions:

$f(x) = x(x-4)^3$

Watch Video Solution

2. Find the local extrema for the following functions using second derivative test :

$f(x) = x \log x$

Watch Video Solution

3. For the function $f(x) = 4x^3 + 3x^2 - 6x + 1$ find the intervals of monotonicity, local extrema, intervals of concavity and points of inflection.

Watch Video Solution

Exercise 7 8

1. Find two positive numbers whose sum is 12 and their product is maximum.

Watch Video Solution

2. Find two positive numbers whose product is 20 and their sum is minimum.

Watch Video Solution

3. Find the smallest possible value of $x^2 + y^2$ given that x + y = 10.

Watch Video Solution

4. A garden is to be laid out in a rectangular area and protected by wire fence. What is the largest possible area of the fenced garden with 40 metres of wire.

Watch Video Solution

5. A rectangular page is to contain $24cm^2$ of print. The margins at the top and bottom of the page are 1.5 cm and the margins at other sides of the page is 1 cm. What should be the dimensions of the page so that the area of the paper used is minimum.

Watch Video Solution

6. A farmer plans to fence a rectangular pasture adjacent to a river. The pasture must contain 1,80,000 sq. mtrs in order to provide enough grass for herds. No fencing is needed along the river. What is the length of the minimum needed fencing material?

Watch Video Solution

7. Find the dimensions of the rectangle with maximum area that can be inscribed in a circle of radius 10 cm.

Watch Video Solution

8. Prove that among all the rectangles of the given perimeter, the square has the maximum area.

Watch Video Solution

9. Find the dimensions of the largest rectangle that can be inscribed in a semi circle of radius r cm.

Watch Video Solution

10. A manufacturer wants to design an open box having a square base and a surface area of 108 sq. cm. Determine the dimensions of the box for the maximum volume.

Watch Video Solution

11. The volume of a cylinder is given by the formula $V = \pi r^2 h$. Find the greatest and least values of V if r + h = 6.

Watch Video Solution

12. A hollow cone with base radius a cm and height b cm is placed on a table . Show that the volume of the largest cylinder that can be hidden underneath is $\frac{4}{9}$ times volume of the cone .

Watch Video Solution

Exercise 7 9

1. Find the asymptotes of the following curves:

$$f(x) = \frac{x^2}{x+1}$$

Watch Video Solution

2. Sketch the graphs of the following functions:

$$y = \frac{x^3}{24} - \log x$$

Watch Video Solution

Exercise 7 10

1. The volume of a sphere is increasing in volume at the rate of $3\pi cm^3 / \sec$. The rate of change of its radius when radius is $\frac{1}{2}$ cm

A. $3cm / s$

B. $2cm / s$

C. $1cm / s$

D. $\frac{1}{2}cm / s$

Answer: C::D

Watch Video Solution

2. A balloon rises straight up at 10m/s. An observer is 40 m away from the spot where the balloon left the ground. Find the rate of change of the balloon's angle of elevation in radian per second when the balloon is 30 metres above the ground.

A. $\frac{3}{25}$radians/sec

B. $\frac{4}{25}$ radians/sec

C. $\frac{1}{5}$ radians/sec

D. $\frac{1}{3}$ radians/sec

Answer: A::B::C::D

Watch Video Solution

3. The position of a particle moving along a horizontal line of any time t is given by $s(t) = 3t^2 - 2t - 8$. The time at which the particle is at rest is

A. t=0

B. $t = \frac{1}{3}$

C. t=1

D. 1=3

Answer: A::B::C

Watch Video Solution

4. A stone is thrown up vertically. The height it reaches at time t seconds is given by $x = 80t - 16t^2$. The stone reaches the maximum height in time t seconds is given by

A. 2

B. 2.5

C. 3

D. 3.5

Answer: B::C

Watch Video Solution

5. Find the slope for x=2 in $f(x) = x^3 + 2$

Watch Video Solution

6. The abscissa of the point on the curve $f(x) = \sqrt{8 - 2x}$ at which the slope of the tangent is -0.25 ?

A. -8

B. -4

C. -2

D. 0

Answer: D

Watch Video Solution

7. The slope of the line normal to the curve $f(x) = 2\cos 4x$ at $x = \frac{\pi}{12}$ is

A. $-4\sqrt{3}$

B. -4

C. $\frac{\sqrt{3}}{12}$

D. $4\sqrt{3}$

Answer: A::B::C::D

Watch Video Solution

8. The tangent to the curve $y^2 - xy + 9 = 0$ is vertical when ....

A. $y = 0$

B. $y = \pm\sqrt{3}$

C. $y = \frac{1}{2}$

D. $y = \pm\sqrt{3}$

Answer: C

Watch Video Solution

9. Angle between $y^2 = x$ and $x^2 = y$ at the origin is

A. $\dfrac{\tan^{-1}(3)}{4}$

B. $\tan^{-1}\left(\dfrac{4}{3}\right)$

C. $\dfrac{\pi}{2}$

D. $\dfrac{\pi}{4}$

Answer: B

Watch Video Solution

10. What is the value of the limit $\lim_{x \to 0}\left(\cot x - \frac{1}{x}\right)$?

Watch Video Solution

11. The function $\sin^4 x + \cos^4 x$ is increasing in the interval

A. $\left[\frac{5\pi}{8}, \frac{3\pi}{4}\right]$

B. $\left[\frac{\pi}{2}, \frac{5\pi}{8}\right]$

C. $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

D. $\left[0, \frac{\pi}{4}\right]$

Answer: D

Watch Video Solution

12. The number given by the Rolle's theorem for the function $x^3 - 3x^2, x \in [0, 3]$ is

A. 1

B. s=$\sqrt{2}$

C. $\frac{3}{2}$

D. 2

Answer: B

Watch Video Solution

13. The number given by the Mean value theorem for the function $\frac{1}{x}, x \in [1, 9]$ is

A. 2

B. 2.5

C. 3

D. 3.5

Answer: C

Watch Video Solution

14. The minimum value of the function $|3 - x| + 9$ is

A. 0

B. 3

C. 6

D. 9

Answer: A::C

Watch Video Solution

15. The maximum slope of the tangent to the curve $y = e^x \sin x, x \in [0, 2\pi]$ is at

A. $x = \frac{\pi}{4}$

B. $x = \frac{\pi}{2}$

C. $x = \pi$

D. $x = \frac{3\pi}{2}$

Answer: B

Watch Video Solution

16. The maximum value of the function $x^2 e^{-2x}, x > 0$ is

A. $\frac{1}{e}$

B. $\frac{1}{2e}$

C. $\frac{1}{e^2}$

D. $\frac{4}{e^4}$

Answer: A::B

Watch Video Solution

17. One of the closest points on the curve $x^2 - y^2 = 4$ to the point (6, 0) is

A. $(2, 0)$

B. $(\sqrt{5}, 1)$

C. $(3, \sqrt{5})$

D. $(\sqrt{13}, \; -\sqrt{3})$

Answer: C

Watch Video Solution

18. The maximum value of the product of two positive numbers, when their sum of the squares is 200, is

A. 100

B. $25\sqrt{7}$

C. 28

D. $24\sqrt{14}$

Answer: A::B

Watch Video Solution

19. The curve $y = ax^4 + bx^2$ with $ab > 0$

A. has no horizontal tangent

B. is concave up

C. is concave down

D. has no points of inflection

Answer:

Watch Video Solution

20. The point of inflection of the curve $y = (x - 1)^3$ is

A. $(0, 0)$

B. $(0, 1)$

C. $(1, 0)$

D. $(1, 1)$

Answer: A

Watch Video Solution

Additional Questions Solved

1. A water tank has the shape of an invertd circular cone with base radius 2 metres and height 4 metres. If water is being pumped into the tank at the rate of $2m^3 / mm$. Find the rate at which the water level is rising when the water is 3m deep

Watch Video Solution

2. A car A is travelling from west at 50 km/hr and car B is travelling towards north at 60 km//hr . Both are headed for the intersection of the two roads . At what rate are the cars approaching each other when car A is 0.3 kilometers and car B is 0.4 kilometers from the intersection ?

Watch Video Solution

3. The distance x metres travelled by a vehicle in time t seconds after the breakes are applied is given by $x = 20t - \frac{5}{3}t^2$. Determine (i) the speed of the vehicle (in km /hr) at the instant the brakes are applied and (ii) the distance the car travelled before it stops.

Watch Video Solution

4. At a particular instant ship A is 100 km west of ship B, ship A is sailing at a speed of 35 km/hr and ship B is sailing north at 25 km/hr. How fast is the distance between the ships changes after 4 hours

Watch Video Solution

5. Gravel is being duped from a conveyor belt at a rate of $30ft^3 / \min$ and its coarsened such that it from a sile in the shape of a cone whose base diameter and height are always equal . How fast is the height of the pile increasing when the pile is 10 ft high ?

Watch Video Solution

6. Prove that the sum of the intercepts on the co-ordinate axes of any tangent to the curve $x = a\cos^4\theta, y = a\sin^4\theta, 0 \leq \theta \leq \frac{\pi}{2}$ is equal to a .

Watch Video Solution

7. Find the equation of normal to $y = x^3 - 3x$ that is parallel to $2x + 18y - 9 = 0$

Watch Video Solution

8. Prove that the curves $2x^2 + 4y^2 = 1$ and $6x^2 - 12y^2 = 1$ cut each other at right angles

.

View Text Solution

9. Show that the equation of the normal to the curve $x = a\cos^3\theta, y = a\sin^3\theta$ at $'\theta'$ is $x\cos\theta - y\sin\theta = a\cos 2\theta.$

Watch Video Solution

10. If the curve $y^2 = x$ and $xy = k$ are orthogonal then prove that $8k^2 = 1$

Watch Video Solution

11. Verify Rolle's theorem for the following

$f(x) = x^3 - 3x + 3 \text{ in } 0 \leq x \leq 1$

Watch Video Solution

12. Suppose that

$f(0) = -3, \text{ and } f'(x) \leq 5$ for all values of

x how large can f(2) possibly be?

Watch Video Solution

13. Using Rolle's theorem find the point on the curve $y = x^2 + 1, \; -2 \le x \le 2$ where the tangent is parallel to x-axis.

Watch Video Solution

14. Find 'C' of Lagrange's mean value theorem for the function $f(x) = 2x^3 + x^2 - x - 1, [0, 2]$

Watch Video Solution

15. Find 'C' of Lagrange's mean value theorem for the function $f(x) = x^3 - 5x^2 - 3x$ in [1, 3]

Watch Video Solution

16. The Taylor's series expension of $f(x) = \sin x \text{about} x = \frac{\pi}{2}$ is obtained by the following way .

Watch Video Solution

17. Obatin the Maclaurin's series expansion for the following function .

(i) e^{2x} (ii) $\sin^2 x$ (iii) $\frac{1}{1+x}$ (iv) $\tan x, \prec x < \frac{\pi}{2}$

View Text Solution

18. Evaluate $\lim_{x \to \frac{\pi}{2}} \frac{\log \sin x}{(\pi - 2x)^2}$

Watch Video Solution

19. Evaluate: $\lim_{x \to 0} (\cot x)^{\sin x}$

Watch Video Solution

20. $\lim_{x \to \infty} \frac{\log x^x}{x}$

Watch Video Solution

21. $\lim_{x \to 0} x^2 \log e^x$

Watch Video Solution

22. Evaluate $\lim_{x \to 1} x^{\frac{1}{x-1}}$

Watch Video Solution

23. $\lim_{x \to 0} (\cos x)^{x}$

View Text Solution

24. Find the absolute maximum and absolute minimum values of f on the given interval

1. $f(x) = 1 - 2x - x^2, [-4, 1]$

Watch Video Solution

25. $f(x) = x^3 - 12x + 1, [-3, 5]$

Watch Video Solution

26. $f(x) = \frac{x}{x+1}, [1, 2]$

Watch Video Solution

27. Find the absolute maximum and absolute minimum values of f on the given interval. f(x)=sin x+ cos x, [0, pi//3]`

Watch Video Solution

28. $f(x) = x - 2\cos x, [-\pi, \pi]$

View Text Solution

29. Find the local maximum and minimum values of the following functions

$2x^3 + 5x^2 - 4x$

Watch Video Solution

30. Find the local maximum and minimum values of the following functions

$t + \cos t$

Watch Video Solution

31. Find the slope at x=2 for $y = x^4 - 4x^3$

Watch Video Solution

32. Find the slope at x=2

$f(x) = 2x^3 + 5x^2 - 4x$

Watch Video Solution

33. $f(x) = x^4 - 6x^2$ find $f'(x)$.

Watch Video Solution

34. $f(\theta) = \sin 2\theta$ in $(0, \pi)$

View Text Solution

35. $y = 12x^2 - 2x^3 - x^4$

View Text Solution

36. The top and bottom margins of a poster are each 6 cms and the side margins are each 4 cms . If area of the printed material on the poster is fixed at $384 cms^2$, find the dimension of the poster with the smallest area .

View Text Solution

37. Show that the volume of the largest right circular cone that can be inscribed in a sphere of radius a is $\frac{8}{27}$ (volume of the shpere).

View Text Solution

38. A closed box (cuboid) with a square base is to have a volume 2000c.c, The material for the top and bottom of the box is to cost Rs 3 per square cm and the material for the sides is to cost Rs 1.50 per square cm. If the cost of the material is to be least find the dimension of the box.

Watch Video Solution

39. Find the numbers whose sum is 100 and whose product is maximum.

Watch Video Solution

40. Find two positive numbers whose product is 100 and whose sum is minimum.

Watch Video Solution

41. The gradient of the curve $y = -2x^3 + 3x + 5$ at $x = 2$ is ....

Watch Video Solution

42. The rate of change of area A of a circle of radius r is ....

A. $2\pi r$

B. $2\pi r \frac{dr}{dt}$

C. $\pi r^2 \frac{dr}{dt}$

D. $\pi \frac{dr}{dt}$

Answer: B::D

View Text Solution

43. A spherical snowball is melting in such a way that its volume is decreasing at a rate of $1cm^3 / \min$. The rate at which the diameter is decreaseing when the diameter is 10 cms is ..

A. $\dfrac{-1}{50\pi} cm / \min$

B. $\dfrac{1}{50\pi} cm / \min$

C. $\dfrac{-11}{75\pi} cm / \min$

D. $\dfrac{-2}{75\pi} cm / \min$

Answer: A::D

Watch Video Solution

44. The parametric equations of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ are ....

A. $x = a\sin^3\theta, y = a\cos^3\theta$

B. $x = a\cos^4\theta, y = a\sin^4\theta$

C. $x = a^3\sin\theta, y = a^3\cos\theta$

D. $x = a^3\cos\theta, y = a^3\sin\theta$

Answer:

Watch Video Solution

45. If the normal to the curve $x^{2/3} + y^{2/3} = a^{2/3}$ makes an angle θ with the x -axis then the slope of the normal is ....

A. $-\cot\theta$

B. $\tan\theta$

C. $-\tan\theta$

D. $\cot\theta$

Answer: A

Watch Video Solution

46. What is the surface area of a sphere when the volume is increasing at the same rate as its radius?

A. 1

B. $\dfrac{1}{2\pi}$

C. 4π

D. $\dfrac{4\pi}{3}$

Answer: A::B::D

Watch Video Solution

47. For what values of x is the rate of increase of $x^3 - 2x^2 + 3x + 8$ is twice the rate of increase of x..

A. $\left(-\frac{1}{3}, -3\right)$

B. $\left(\frac{1}{3}, 3\right)$

C. $\left(-\frac{1}{3}, 3\right)$

D. $\left(\frac{1}{3}, 1\right)$

Answer: A::C

View Text Solution

48. If the volume of an expanding cube is increasing at the rate of $4cm^3/\sec$ then the rate of change of surface area when the volume of the cube is 8 cubic cm is ....

A. $8cm^2 / \sec$

B. $16cm^2 / \sec$

C. $2cm^2 / \sec$

D. $4cm^2 / \sec$

Answer: A::C::D

View Text Solution

49. If a normal makes an angle θ with positive x-axis then the slop of the curve at the point where the normal is drawn is ....

A. $-\cot\theta$

B. $\tan\theta$

C. $-\tan\theta$

D. $\cot\theta$

Answer: A::C

View Text Solution

50. If the velocity of a particle moving along a straight line is directly proportional to the square of its distance from a fixed point on the line . Then its acceleration is proportional to ....

A. s

B. s^2

C. s^3

D. s^4

Answer: B::C

View Text Solution

51. The Rolle's constant for the function $y = x^2$ on [-2,2] is ....

Watch Video Solution

52. The value 'c' of Lagranges Mean Value Theorem for $f(x) = \sqrt{x}$ when a=1 and b=4 is ....

A. $\frac{9}{4}$

B. $\frac{3}{2}$

C. $\frac{1}{2}$

D. $\frac{1}{4}$

Answer: C::D

Watch Video Solution

53. In a given semi circle of diameter 4 cm a rectangle is to be inscribed . The maximum area of the rectangle is ....

A. 2

B. 4

C. 8

D. 16

Answer: A

Watch Video Solution

54. The least possible perimeter of a rectangle of area $100m^2$ is ....

A. 10

B. 20

C. 40

D. 60

Answer: A::B::C::D

Watch Video Solution

55. Which of the following curves is concave down ?

A. $y = -x^2$

B. $y = x^2$

C. $y = e^x$

D. $y = x^2 + 2x - 3$

Answer: B

View Text Solution

56. The point of inflection of the curve $y = x^4$ is at:

A. $x = 0$

B. $x = 3$

C. $x = 12$

D. nowhere

Answer:

Watch Video Solution