

## MATHS

### BOOKS - FULL MARKS MATHS (TAMIL ENGLISH)

#### COMPLEX NUMBERS

##### Example Questions Solved

1. Simplify the following :

(i)

$$i^7 \quad (ii) i^{1729} \quad (iii) i^{-1924} + i^{2018} \quad (iv) \sum_{n=1}^{102} i^n \quad (v) ii^2i^3 \dots i^{40}$$



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2. Find the values of the real numbers  $x$  and  $y$ , if the complex numbers

$(3 - i)x - (2 - i)y + 2i + 5$  and  $2x + (-1 + 2i)y + 3 + 2i$  are equal



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3. Write  $\frac{3 + 4i}{5 - 12i}$  in the  $x + iy$  form and hence find real and imaginary parts.



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4. Simplify  $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3$ .



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5. If  $\frac{z+3}{z-5i} = \frac{1+4i}{2}$  find the complex number z.



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6. If  $z_1 = 3 - 2i$  and  $z_2 = 6 + 4i$ , find  $\frac{z_1}{z_2}$  in the rectangular form.



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7. Find  $z^{-1}$ , if  $z = (2 + 3i)(1 - i)$



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8. Show that  $\left(\frac{19 + 9i}{5 - 3i}\right)^{15} - \left(\frac{8 + i}{1 + 2i}\right)^{15}$  is purely imaginary



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9. If  $z_1 = 3 + 4i$ ,  $z_2 = 5 - 12i$  and  $z_3 = 6 + 8i$ , find  $|z_1|$ ,  $|z_2|$ ,  $|z_3|$ ,  $|z_1 + z_2|$ ,  $|z_2 - z_3|$ , and  $|z_1 + z_3|$ .



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10. Find the following  $\left| \frac{2 + i}{-1 + 2i} \right|$



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11. Which one of the points  $i$ ,  $-2 + i$ ,  $2$  and  $3$  is farthest from the origin?



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12. If  $z_1$ ,  $z_2$  and  $z_3$  are complex numbers such that  $|z_1| = |z_2| = |z_3| = |z_1 + z_2 + z_3| = 1$ , find the value of  $\left| \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} \right|$ .



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13. If  $|z| = 2$ , show that  $3 \leq |z + 3 + 4i| \leq 7$ .



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14. Show that the points  $1$ ,  $\frac{-1}{2} + i\frac{\sqrt{3}}{2}$ , and  $\frac{-1}{2} - i\frac{\sqrt{3}}{2}$  are the vertices of an equilateral triangle.



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15. Let  $z_1$  and  $z_2$  be two complex numbers such that  $z_1 z_2$  and  $z_1 + z_2$  are real then



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16. Show that the equation  $z^2 = \bar{z}$  has four solutions.



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17. Find the square root of  $6 - 8i$ .



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18. Given the complex number  $z = 3 + 2i$ , represent the complex number  $z$ ,  $iz$  and  $z + iz$  in one Argand diagram. Show that these complex numbers form the vertices of an isosceles right triangle .



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19. Show that  $|3z - 5 + i| = 4$  represent a circle, and, find its centre and radius .



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20. Show that  $|z + 2 - i| < 2$  represents interior points of a circle find its centre and radius.



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**21.** Obtain the Cartesian form of the locus of  $z$  in each of the following cases.

$$(i) |z| = |z - i| \quad (ii) |2z - 3 - i| = 3$$



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**22.** Find the modulus and principal argument of the following complex numbers .

$$1 - i\sqrt{3}$$



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**23.** Find the modulus and principal argument of the following complex numbers .

$$1 + i\sqrt{3}$$



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24. Find the principal argument  $\arg z$ , when  $z = \frac{-2}{1 + i\sqrt{3}}$



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25. Find the product  $\frac{3}{2} \left( \frac{\cos(\pi)}{3} + i \frac{\sin(\pi)}{3} \right) \cdot 6 \left( \frac{\cos(5\pi)}{6} + i \frac{\sin(5\pi)}{6} \right)$

in rectangular form.



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26. Find  $\frac{2 \left( \cos \frac{9\pi}{4} + i \sin \frac{9\pi}{4} \right)}{4 \left( \cos \left( \frac{-3\pi}{2} \right) + i \sin \left( \frac{-3\pi}{2} \right) \right)}$  in rectangular form.



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27. If  $z = x + iy$  and  $\arg \left( \frac{z - 1}{z + 1} \right) = \frac{\pi}{2}$ , show that  $x^2 + y^2 - 1 = 0$ .



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**28.** If  $z = (\cos \theta + i \sin \theta)$ , show that  $z^n + (1)/(z^n) = 2 \cos n\theta$  and

$$z^{(n)} - (1)(z^{(n)}) = 2i \sin n\theta$$



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**29.** Simplify  $\left(\sin \frac{\pi}{6} + i \cos \frac{\pi}{6}\right)^{18}$ .



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**30.** Simplify  $\left(\frac{1 + \cos 2\theta + i \sin 2\theta}{1 + \cos 2\theta - i \sin 2\theta}\right)^{30}$



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**31.** Simplify (i)  $(1 + i)^{18}$       (ii)  $(-\sqrt{3} + 3i)^{31}$



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**32.** Find the cube roots of unity.



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**33.** Find the fourth roots of unity.



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**34.** Solve the equation  $z^3 + 8i = 0$ , where  $z \in \mathbb{C}$ .



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**35.** Express the following in the standard form  $a + ib$ .

$$(-3 + i)(4 - 2i)$$



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**36.** Suppose  $z_1, z_2$  and  $z_3$  are the vertices of an equilateral triangle inscribed in the circle  $|z| = 2$ . If  $z_1 = 1 + i\sqrt{3}$  then find  $z_2$  and  $z_3$ .



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### Additional Questions Solved

**1.** Evaluate the following:

$$(i) i^{135} \quad (ii) i^{19} \quad (iii) i^{-999} \quad (iv) (-\sqrt{-1})^{4n+2}, n \in N.$$



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**2.** Show that :

$$(i) \left\{ i^{19} + \left(\frac{1}{i}\right)^{25} \right\}^2 = -4 \quad (ii) \left\{ i^{17} - \left(\frac{1}{i}\right)^{34} \right\}^2 = 2i$$
$$(iii) \left\{ i^{18} + \left(\frac{1}{i}\right)^{24} \right\}^3 = 0 \quad (iv) i^n + i^{n+1} + i^{n+2} + i^{n+3} = 0, \text{ for all } n \in N.$$



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3. The value of  $\sum_{i=1}^{13} (n^n + i^{n-1})$  is



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4. Find the real values of x and y, if

$$(3x - 7) + 2iy = -5y + (5 + x)i$$



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5. For what values of x and y the numbers  $-3 + ix^2y$  and  $x^2 + y + 4i$  and complex conjugate to each other.



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**6.** Given  $x = 2 - 3i$  and  $y = 4 + i$ .

Find

$$(i) xy \quad (ii) x + y \quad (iii) x - y \quad (iv) \frac{x}{y} \quad (v) \frac{\bar{x}}{y} \quad (vi) (x - y)^2$$



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**7.** If  $z_1 = 4 - 7i$ ,  $z_2 = 2 + 3i$  and  $z_3 = 1 + i$  show that

$$z_1 + (z_2 + z_3) = (z_1 + z_2) + z_3$$



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**8.** Given  $z_1 = 1 + i$ ,  $z_2 = 4 - 3i$  and  $z_3 = 2 + 5i$  verify that.

$$z_1(z_2 - z_3) = z_1z_2 - z_1z_3$$



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9. Given  $z_1 = 4 - 7i$  and  $z_2 = 5 + 6i$  find the additive and multiplicative inverse of  $z_1 + z_2$  and  $z_1 - z_2$ .

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10. Express the following in the standard form  $a + ib$ .

$$\frac{2(i - 3)}{(1 + i)^2}$$

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11. Find the least positive integer  $n$  such that  $\left(\frac{1+i}{1-i}\right)^n = 1$ .

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12. Find  $x$  and  $y$  for which of the following is satisfied.

$$\frac{(1+i)x - 2i}{3+i} + \frac{(2-3i)y + i}{3-i} = i$$

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13. Find the modulus and argument of the following complex numbers and convert them in polar form.

$$\frac{1 + 2i}{1 - 3i}$$



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14. Find the square roots of  $-15 - 8i$



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15. Express the following in the standard form  $a + ib$ .

$$\frac{i^4 + i^9 + i^{16}}{3 - 2i^8 - i^{10} - i^5}$$



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**16.** Find the modulus or the absolute value of  $\frac{(1 + 3i)(1 - 2i)}{(3 + 4i)}$



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**17.** Find the modulus and principal argument of the following complex numbers .

$$1 - i\sqrt{3}$$



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**18.** Show that the points represented by the complex numbers  $7 + 9i$ ,  $-3 + 7i$ ,  $3 + 3i$  from a right angled triangle on the Argand diagram.



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**19.** Find the square root of  $( - 7 + 24i)$ .



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20. If the imaginary part of  $\frac{2z+1}{iz+1}$  is  $-2$ , then show that the locus of the point representing  $z$  in the argand plane is straight line.



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21. If the real part of  $\frac{\bar{z}+2}{\bar{z}-1}$  is  $4$ , then show that locus of the point representing  $z$  in the complex plane is a circle.



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22. Show that  $\left| \frac{z-3}{z+3} \right| = 2$  represent a circle



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23. If  $\arg(z-1) = \frac{\pi}{6}$  and  $\arg(z+1) = 2\frac{\pi}{3}$ , then prove that  $|z| = 1$ .



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24. P represents the variable complex number z. Find the locus of P, if

$$\operatorname{Im}\left(\frac{2z+1}{iz+1}\right) = -2.$$



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25. P represents the variable complex number z find the locus of z if :

$$\operatorname{Re}\left(\frac{z+1}{z+i}\right) = 1$$



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26. Write the following complex numbers in the polar form :

$$1 - i$$



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27. Find the modulus and principal argument of  $(1 + i)$  and hence express it in the polar form.

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28. Express the following complex numbers in the polar form.

(i)  $\frac{1+i}{1-i}$

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29. Express the following complex numbers in the polar form :  $2 + \sqrt{3}i$

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30. Find the modulus and principal argument of the following complex numbers .

$$-1 + i\sqrt{3}$$



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31. Express the following complex numbers in the polar form :  $-1 - i$



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32. Express the following complex numbers in the polar form :  $1 - i$



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33. Evaluate  $i^{50}$



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34. Evaluate  $i^{59}$



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**35.** Prove that :  $(1 + i)^{4n}$  and  $(1 + i)^{4n+2}$  are real and purely imaginary respectively.



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**36.** If  $x = \cos \alpha + i \sin \alpha$ ,  $y = \cos \beta + i \sin \beta$ , prove that  $x^m y^n + \frac{1}{x^m y^n} = 2 \cos(m\alpha + n\beta)$ .



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**37.** If

$a = \cos 2\alpha + i \sin 2\alpha$ ,  $b = \cos 2\beta + i \sin 2\beta$  and  $c = \cos 2\gamma + i \sin 2\gamma$ .

Prove that .

$$\sqrt{abc} + \frac{1}{\sqrt{abc}} = 2 \cos(\alpha + \beta + \gamma)$$



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**38.** Solve :  $x^4 + 4 = 0$



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**39.** If  $x = a + b$ ,  $y = a\omega + b\omega^2$ ,  $z = a\omega^2 + b\omega$ , show that

(i)  $xyz = a^3 + b^3$



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**40.**  $\left(\frac{1-i}{1+i}\right)^{100} = a + ib$  then (a,b) is .....

A. (2, - 1)

B. (1, 0)

C. (0, 1)

D. (- 1, 2)

**Answer:** B



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41. If  $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 = x + iy$  then  $(x, y) = \dots\dots$

A.  $(0, -2)$

B.  $(-2, 0)$

C.  $(0, 2)$

D.  $(2, 0)$

Answer: A



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42. Evaluate  $i^{60}$



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**43.** Fill in the blanks of the following :

For any two complex numbers  $z_1, z_2$  and any real number a, b,

$$|az_1 - bz_2|^2 + |bz_1| + az_2|^2 = \dots \dots \dots$$



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**44.** Multiplicative inverse of  $1 + i$  is .....



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**45.**  $\arg(z) + \arg(\bar{z})(z \neq 0)$  is .....



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**46.** If  $|z| = 4$  and  $\arg(z) = \frac{5\pi}{6}$ , then  $z = \dots \dots \dots$



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**47.**

If

$$a = \cos 2\alpha + i \sin 2\alpha, b = \cos 2\beta + i \sin 2\beta \text{ and } c = \cos 2\gamma + i \sin 2\gamma.$$

Prove that .

$$\frac{a^2b^2 + c^2}{abc} = 2 \cos 2(\alpha + \beta - \gamma)$$



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**48.** If  $p + iq = \frac{a + ib}{a - ib}$  then  $p^2 + q^2 = \dots\dots\dots$



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**49.** Evaluate  $i^{78}$



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**50.** If  $a = \cos \alpha - i \sin \alpha, b = \cos \beta - i \sin \beta, c = \cos \gamma - i \sin \gamma$ , then  
 $\left( \frac{a^2c^2 - b^2}{abc} \right)$  is  $\dots\dots\dots$



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51. If  $z_1 = 4 + 5i$  and  $z_2 = -3 + 2i$  then  $\frac{z_1}{z_2}$  is :

A.  $\frac{2}{13} - \frac{22}{13}i$

B.  $\frac{-2}{13} + \frac{22}{13}i$

C.  $\frac{-2}{13} - \frac{23}{13}i$

D.  $\frac{2}{13} + \frac{22}{13}i$

Answer: C



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52. The conjugate of  $i^{13} + i^{14} + i^{15} + i^{16}$  is .....

A. 1

B. -1

C. 0

D.  $-i$

**Answer: C**



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53. If  $-i + 2$  is one root of the equation  $ax^2 - bx + c = 0$ , then the other root is .....

A.  $-i - 2$

B.  $i - 2$

C.  $2 + i$

D.  $2i + i$

**Answer: C**



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**54.** The equation having  $4 + 3i$  and  $4 - 3i$  as root is .....

A.  $x^2 + 8x + 25 = 0$

B.  $x^2 + 8x - 25 = 0$

C.  $x^2 - 8x + 25 = 0$

D.  $x^2 - 8x - 25 = 0$

**Answer:** C



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**55.** If  $2 - i$  is one root of the equation  $ax^2 + bx + c = 0$ , and  $a, b, c$  are rational numbers, then the other root is .....



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**56.** If  $-i + 3$  is a root of  $x^2 - 6x + k = 0$ . Then the value of  $k$  is :

A. 5

B.  $\sqrt{5}$

C.  $\sqrt{10}$

D. 10

**Answer: D**



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57. If  $\omega$  is a cube root of unity, then the value of  $(1 - \omega + \omega^2)^4 + (1 + \omega - \omega^2)^4$  is .....

A. -16

B. 0

C. -32

D. 32

**Answer: A**



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58. If  $\omega$  is the cube root of unity, then the value of  $(1 - \omega)(1 - \omega^2)(1 - \omega^4)(1 - \omega^8)$  is

A. 9

B. -9

C. 16

D. 32

**Answer:** A



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59. If  $\frac{z - 1}{z + 1}$  is purely imaginary, then  $|z|$  is

A.  $|z| = 1$

B.  $|z| > 1$

C.  $|z| < 1$

D. None of these

**Answer: A**



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### Exercise 2 1

1. Simplify the following:

$$i^{1947} + i^{1950}$$



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2.  $i^{1948} - i^{-1869}$



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**3. Simplify the following:**

$$\sum_{n=1}^{12} i^n$$



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**4. Simplify the following:**

$$i^{59} + \frac{1}{i^{59}}$$



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**5. Simplify the following:**

$$i \cdot i^2 \cdot i^3 \cdot \dots \cdot i^{2000}$$



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**6. Simplify the following:**

$$\sum_{n=1}^{10} i^{n+50}$$





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## Exercise 2 2

1. Evaluate the following if  $z = 5 - 2i$  and  $w = -1 + 3i$

$$z + w$$



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2. Given the complex number  $z = 2 + 3i$ , represent the complex numbers in Argand diagram.

$$z, iz, \text{ and } z + iz$$



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3. Find the values of the real numbers  $x$  and  $y$ , if the complex numbers  $(3 - i)x - (2 - i)y + 2i + 5$  and  $2x + (-1 + 2i)y + 3 + 2i$  are equal



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### Exercise 2 3

1. If  $z_1 = 1 - 3i$ ,  $z_2 = -4i$  and  $z_3 = 5$ , show that

$$(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3)$$



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2. If  $z_1 = 3$ ,  $z_2 = 7i$ , and  $z_3 = 5 + 4i$ , show that

$$z_1(z_2 + z_3) = z_1z_2 + z_1z_3$$



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3. If  $z_1 = 2 + 5i$ ,  $z_2 = -3 - 4i$ , and  $z_3 = 1 + i$ , find the additive and multiplicative inverse of  $z_1$ ,  $z_2$  and  $z_3$ .



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## Exercise 2 4

1. Write in the rectangular form

$$\overline{(5 + 9i) + (2 - 4i)}$$



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2. If  $z = x + iy$ , find in rectangular form.

$$\operatorname{Im} (3z + 4\bar{z} - 4i)$$



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3. If  $z_1 = 2 - i$  and  $z_2 = -4 + 3i$ , find the inverse of  $z_1 z_2$  and  $\frac{z_1}{z_2}$ .



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4. The complex numbers  $u$ ,  $v$  and  $w$  are related by  $\frac{1}{u} = \frac{1}{v} + \frac{1}{w}$ . If  $v = 3 - 4i$  and  $w = 4 + 3i$ , find  $u$  in rectangular form.

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5. Prove the following properties:

$z$  is real if and only if  $z = \bar{z}$

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6. Express the following complex numbers in the polar form.

$$\frac{2 + 6\sqrt{3}i}{5 + \sqrt{3}i}$$

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7. Show that

$$(2 + i\sqrt{3})^{10} - (2 - i\sqrt{3})^{10} \text{ is purely imaginary.}$$



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## Exercise 2 5

1. Find the modulus of the complex number

$$\frac{2i}{3 + 4i}$$



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2. For any two complex number  $z_1$  and  $z_2$ , such that  $|z_1| = |z_2| = 1$  and  $z_1 z_2 \neq -1$ , then show that  $\frac{z_1 + z_2}{1 + z_1 z_2}$  is real number.



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3. Which one of the points  $10 - 8i$ ,  $11 + 6i$  is closest to  $1 + i$ .



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4. If  $|z| = 3$ , show that  $7 \leq |z + 6 - 8i| \leq 13$ .



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5. If  $|z| = 1$ , show that  $2 \leq |z^3 - 3| \leq 4$ .



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6. If  $\left|z - \frac{2}{z}\right| = 2$ . show that the greatest and least value of  $|z|$  are  $\sqrt{3} + 1$  and  $\sqrt{3} - 1$  respectively.



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7. If  $z_1, z_2$ , and  $z_3$  are three complex numbers such that  $|z_1| = 1$ ,  $|z_2| = 2$ ,  $|z_3| = 3$  and  $|z_1 + z_2 + z_3| = 1$ , show that  $|9z_1z_2 + 4z_1z_3 + z_2z_3| = 6$ .



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8. If the area of the triangle formed by the vertices  $z$ ,  $iz$ , and  $z + iz$  is 50 square units, find the value of  $|z|$ .



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9. Show that the equation  $z^3 + 2\bar{z} = 0$  has five solutions.



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10. Find the square roots of

(i)  $4 + 3i$



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Exercise 2 6

1. If  $z = x + iy$  is a complex number such that  $\left| \frac{z - 4i}{z + 4i} \right| = 1$  show that the locus of  $z$  is real axis.



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2. If  $z = x + iy$  is complex number such that  $\operatorname{Im}\left(\frac{2z + 1}{iz + 1}\right) = 0$ , show that the locus of  $z$  is

$$2x^2 + 2y^2 + x - 2y = 0.$$



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3. Obtain the Cartesian form of the locus of  $z = x + iy$  in each of cases:

$$|z + i| = |z - 1|$$



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4. Show that the equations represent a circle, and , find its centre and radius.

$$|z - 2 - i| = 3$$



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5. Obtain the Cartesian equation for the locus of  $z = x + iy$  in each of the cases:

$$|z - 4| = 16$$



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## Exercise 2 7

1. Write in polar form of the complex numbers.

$$3 - i\sqrt{3}$$



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**2.** Find the rectangular form of the complex numbers.

$$\left( \cos -\frac{\pi}{6} + i \sin -\frac{\pi}{6} \right) \left( \cos -\frac{\pi}{12} + i \sin -\frac{\pi}{12} \right)$$



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**3.** Given  $(x_1 + iy_1)(x_2 + iy_2)\dots(x_n + iy_n) = a + ib$ , show that

$$(x_1^2 + y_1^2)(x_2^2 + y_2^2)(x_3^2 + y_3^2)\dots(x_n^2 + y_n^2) = a^2 + b^2$$



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**4.** If  $\frac{1+z}{1-z} = \cos 2\theta + i \sin 2\theta$ , show that  $z = i \tan \theta$ .



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**5.** If  $\cos \alpha + \cos \beta + \cos \gamma = \sin \alpha + \sin \beta + \sin \gamma = 0$ , show that

$\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3 \cos(\alpha + \beta + \gamma)$  and



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6. If  $z = x + iy$  and  $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$ . Show that  $x^2 + y^2 + 3x - 3y + 2 = 0$ .



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## Exercise 2 8

1. If  $\omega \pm 1$  is a cube root of unity, show that  
$$\frac{a + b\omega + c\omega^2}{b + c\omega + a\omega^2} + \frac{a + b\omega + c\omega^2}{c + a\omega + b\omega^2} = -1$$



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2. Show that  $\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5 = -\sqrt{3}$



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3. Find the value of  $\left( \frac{1 + \sin \frac{\pi}{10} + i \cos \frac{\pi}{10}}{1 + \sin \frac{\pi}{10} - i \cos \frac{\pi}{10}} \right)^{10}$ .



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4. If  $2 \cos \alpha = x + \frac{1}{x}$  and  $2 \cos \beta = y + \frac{1}{y}$ , show that

$$\frac{x}{y} + \frac{y}{x} = 2 \cos(\alpha - \beta)$$



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5. Solve the equation  $z^3 + 27 = 0$



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6. If  $\omega \neq 1$  is a cube root of unity, show that the roots of the equation

$$(z - 1)^3 + 8 = 0 \text{ are } -1, 1 - 2\omega, 1 - 2\omega^2.$$



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7. Find the value of  $\sum_{k=1}^8 \left( \cos \frac{2k\pi}{9} + i \sin \frac{2k\pi}{9} \right)$



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8. If  $\omega \pm 1$  is a cube root of unity, show that

$$(1 - \omega + \omega^2)^6 + (1 + \omega - \omega^2)^6 = 128$$



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9. If  $z = 2 - 2i$ , find the rotation of  $z$  by  $\theta$  radians in the counter clockwise direction about the origin when

$$\theta = \frac{\pi}{3}$$



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10. Prove that the values of  $4\sqrt{-1}$  are  $\pm \frac{1}{\sqrt{2}}(1 \pm i)$ .



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## Exercise 29

1.  $i^n + i^{n+1} + i^{n+2} + i^{n+3}$

A. 0

B. 1

C. -1

D.  $i$

**Answer: A**



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2. The value of  $\sum_{i=1}^{13} (n^n + i^{n-1})$  is

A.  $1 + i$

B.  $i$

C. 1

D. 0

**Answer: A**



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**3.** The area of the triangle formed by the complex numbers  $z$ ,  $iz$ , and  $z + iz$  in the Argand's diagram is

A.  $\frac{1}{2}|z|^2$

B.  $|z|^2$

C.  $\frac{3}{2}|z|^2$

D.  $2|z|^2$

**Answer: A**



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4. The conjugate of a complex number is  $\frac{1}{i - 2}$ . Then, the complex number is

A.  $\frac{1}{i + 2}$

B.  $\frac{-1}{i + 2}$

C.  $\frac{-1}{i - 2}$

D.  $\frac{1}{i - 2}$

**Answer: B**



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5. If  $z = \frac{(\sqrt{3} + i)^3 (3i + 4)^2}{(8 + 6i)^2}$ , then  $|z|$  is equal to

A. 0

B. 1

C. 2

D. 3

**Answer: C**



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6. If  $z$  is non zero complex number, such that  $2i z^2 = \bar{z}$ , then  $|z|$  is

A.  $\frac{1}{2}$

B. 1

C. 2

D. 3

**Answer: A**



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7. If  $|z-2 + i| \leq 2$ , then the greatest value of  $|z|$  is

A.  $\sqrt{3} - 2$

B.  $\sqrt{3} + 2$

C.  $\sqrt{5} - 2$

D.  $\sqrt{5} + 2$

**Answer:** D



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8. If  $|z - \frac{3}{z}| = 2$ , then the least value of  $|z|$  is

A. 1

B. 2

C. 3

D. 5

**Answer: A**



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**9.** If  $|z| = 1$ , then the value of  $\frac{1+z}{1+\bar{z}}$ .

A.  $z$

B.  $\bar{z}$

C.  $\frac{1}{z}$

D. 1

**Answer: A**



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**10.** The solution of the equation  $|z| - z = 1 + 2i$  is

A.  $\frac{3}{2} - 2i$

B.  $-\frac{3}{2} + 2i$

C.  $2 - \frac{3}{2}i$

D.  $2 + \frac{3}{2}i$

**Answer: A**



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11. If  $|z_1| = 1$ ,  $|z_2| = 2$ ,  $|z_3| = 3$  and  $|9z_1z_2 + 4z_1z_3 + z_2z_3| = 12$ , then the value of  $|z_1 + z_2 + z_3|$  is

A. 1

B. 2

C. 3

D. 4

**Answer: B**



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12. If  $z$  is a complex number such that  $z \in \mathbb{C} \setminus \mathbb{R}$ , and  $z + \frac{1}{z} \in \mathbb{R}$ , then  $|z|$  is

A. 0

B. 1

C. 2

D. 3

**Answer: B**



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13.  $z_1, z_2,$  and  $z_3$  are complex numbers such that  $z_1 + z_2 + z_3 = 0$  and  $|z_1| = |z_2| = |z_3| = 1$  then  $z_1^2 + z_2^2 + z_3^2$

A. 3

B. 2

C. 1

D. 0

**Answer: A**



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14. If  $\frac{z - 1}{z + 1}$  is purely imaginary, then  $|z|$  is

A.  $\frac{1}{2}$

B. 1

C. 2

D. 3

**Answer: B**



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**15.** If  $z = x + iy$  is a complex number such that  $|z + 2| = |z - 2|$ , then the locus of  $z$  is

A. real axis

B. imaginary axis

C. ellipse

D. circle

**Answer:** B



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**16.** The principal argument of  $\frac{3}{-1+i}$  is

A.  $-\frac{5\pi}{6}$

B.  $-\frac{2\pi}{3}$

C.  $-\frac{3\pi}{4}$

D.  $\frac{-\pi}{2}$

**Answer: C**



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**17.** The principal argument of  $(\sin 40^\circ + i\cos 40^\circ)^5$  is

A.  $-110^\circ$

B.  $-70^\circ$

C.  $70^\circ$

D.  $110^\circ$

**Answer: A**



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**18.** If  $(1+i)(1+2i)(1+3i)\dots(1+ni) = x+iy$ , then  $2.5.10\dots(1+n^2)$  is

A. 1

B.  $i$

C.  $x^2 + y^2$

D.  $1 + n^2$

**Answer: D**



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**19.** If  $\omega \neq 1$  is a cubic root of unit and  $(1 + \omega)^7 = A + B\omega$ , then (A, B) equals

A. (1, 0)

B. (-1, 1)

C. (0, 1)

D. (1, 1)

**Answer: D**



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20. The principal argument of the complex number  $\frac{(1 + i\sqrt{3})^2}{4i(1 - i\sqrt{3})}$  is

A.  $\frac{2\pi}{3}$

B.  $\frac{\pi}{6}$

C.  $\frac{5\pi}{6}$

D.  $\frac{\pi}{2}$

Answer: D



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21. If  $\alpha$  and  $\beta$  are the roots of  $x^2 + x + 1 = 0$ , then  $\alpha^{2020} + \beta^{2020}$  is

A. -2

B. -1

C. 1

D. 2

**Answer: B**



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22. The product of all four values of  $\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)^{\frac{3}{4}}$  is

A. - 2

B. - 1

C. 1

D. 2

**Answer: C**



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**23.** If  $\omega \neq 1$  is a cubit root unity and  $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k$ , then k is equal to

A. 1

B. -1

C.  $\sqrt{3}i$

D.  $-\sqrt{3}i$

**Answer:** D



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**24.** The value of  $\left( \frac{1 + \sqrt{3}i}{1 - \sqrt{3}i} \right)^{10}$  is

A.  $cis \frac{2\pi}{3}$

B.  $cis \frac{4\pi}{3}$

C.  $-cis \frac{2\pi}{3}$

D.  $-cis \frac{4\pi}{3}$

**Answer: A**



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25. If  $\omega = cis \frac{2\pi}{3}$ , then number of distinct roots of

$$\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0.$$

A. 1

B. 2

C. 3

D. 4

**Answer: A**



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