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MATHS

BOOKS - FULL MARKS MATHS (TAMIL ENGLISH)

DIFFERENTIALS AND PARTIAL DERIVATIVES

Example Questions Solved

1. Find the linear approximation for $f(x) = \sqrt{1+x}, x \geq -1$, at $x_0 = 3$. Use the linear approximation to estimate f (3.2).

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2. The approximate value of $\sqrt{9.2}$ is:

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3. Let us assume that the the the shape of a soap bubble is a sphere . Use linear approximation to approximate the increase in the surface area of a soap bubble as its radius increases from 5 cm to 5.2 cm also calculate the percentage error.

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4. A right circular cylinder has radius r = 10 cm and height h = 20 cm suppose that the radius of the cylinder is increased from 10 cm to 10.1 cm and the height does not change. Estimate the change in the volume of the cylinder . Also calculate the relative error and percentage error .

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5. Let f , g : $(a, b) \to R$ be differentiable functions. Show that d(fg) = fdg + gdf.

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6. Let $g(x) = x^2 + \sin x$. Calculate the differential dg.

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7. If the radius of a sphere , with radius 10 cm , has to decrease by 0.1 cm approximately how much will its volume decrease ?

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8. Consider $f(x, y) = \dfrac{xy}{x^2 + y^2}$ if $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$. Show that f is not continuous at (0,0) and continuous at all other points of R^2.

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9. Consider $g(x, y) = \frac{2x^2y}{x^2 + y^2}$. If $(x, y) \neq (0, 0)$ and $g(0, 0) = 0$

Show that g is continuous on R^2.

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10. Let f(x,y) = 0 if $xy \neq 0$ and $f(x, y)$ = 1 if xy = 0 .

(i) Calculate : $\frac{\partial f}{\partial x}(0, 0), \frac{\partial f}{\partial y}(0, 0)$

(ii) Show that f is not continuous at (0,0)

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11. Let F (x,y) = $x^3y + y^2x + 7$ for all $(x, y) \in R^2$. Calculate $\frac{\partial F}{\partial x}(-1, 3)$ and $\frac{\partial F}{\partial y}(-2, 1)$

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12. If w(x,y,z) = $x^2y + y^2z + z^2x, x, y, z \in R,$ find the differential dw.

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13. Let W(x,y,z) $= x^2 - xy + 3\sin z, x, y, z \in R,$ Find the linear approximation at (2,-1,0).

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14. Verify the above theorem for

$F(x, y) = x^2 - 2y^2 + 2xy$ and $x(t) = \cos t, y(t) = \sin t, t \in [0, 2\pi]$

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15. Let g

$(x, y) = x^2 - yx + \sin(x + y), x(t) = e^{3t}, y(t) = t^2, t \in R.$

Find $\frac{dg}{dt}$.

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16. Let $g(x, y) = 2y + x^2, x = 2r - s, y = r^2 + 2s, r, s \in \mathbb{R}$. Find $\frac{\partial g}{\partial r}, \frac{\partial g}{\partial s}$

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17. Show that $F(x, y) = \frac{x^2 + 5xy - 10y^2}{3x + 7y}$ is a homogeneous function of degree 1.

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18. If $u = \sin^{-1}\left(\frac{x + y}{\sqrt{x} + \sqrt{y}}\right)$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u.$

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Exercise 8 1

1. Let $f(x) = \sqrt[3]{x}$. Find the linear approximation at x = 27. Use the linear approximation to approximate $\sqrt[3]{27.2}$

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2. Using the approximation to find approximate value of

$(123)^{\frac{2}{3}}$

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3. Find a linear approximation for the following functions at the indicated points.

$f(x) = x^3 - 5x + 12, x_0 = 2$

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4. The radius of a circular plate is measured as 12.65 cm instead of the actual length 12.5 cm. Find the following is calculating the area of the circular plate:

(i) Absolute error

(ii) Relative error

(iii) Percentage error

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5. A sphere is made of ice having radius 10 cm. Its radius decreases from 10 cm to 9.8 cm. Find approximations for the following:

(i) change in the volume

(ii) change in the surface area

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6. The time T, taken for a complete oscillation of a single pendulam with length l, is given by the equation $T = 2\pi\sqrt{\frac{l}{g}}$, where g is a constant. Find the approximate percentage error in the calculated value of T corresponding to an error of 2 percent in the value of l.

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7. Show that the percentage error in the nth root of a number is approximately $\frac{1}{n}$ times the percentage error in the number.

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Exercise 8 2

1. Find differential dy for $y = \frac{(1-2x)^3}{3-4x}$.

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2. Find df for $f(x) = x^2 + 3x$ and evalaute it for

(i) x=2 and dx = 0.1

(ii) x=3 and dx= 0.02

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3. Find Δf and df for the function f for the indicated values of x, Δx and compare

$f(x) = x^3 - 2x^2, x = 2, \Delta x = 0.5$

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4. Assuming $\log_{10} e = 0.4343$, find an approximate value of $\log_{10} 1003$.

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5. The trunk of a tree has diameter 30 cm. During the following year, the circumference grew 6 cm.

(i) Approximately, how much did the tree's diameter grow?

(ii) What is the percentage increase in area of the tree's cross-section?

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6. An egg of a particular bird is very nearly spherical. If the radius to the inside of the shell is 5mm and radius to the outside of the shell is 5.3 mm, find the volume of the shell approximately.

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7. Assume that the cross section of the artery of human is circular. A drug is given to a patient to dilate his arteries. If the radius of an artery is increased from 2 mm to 2.1 mm, how much is cross-sectional area increased approximately?

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8. In a newly developed city, it is estimated that the voting population (in thousands) will increase according to $V(t) = 30 + 12t^2 - t^3, 0 \le t \le 8$ where t is the time in years. Find the approximate change in voters for the time change from 4 to $4\left(\frac{1}{6}\right)$ year.

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9. The relation between the number of words y a person learns in x hours is given by $y = 52\sqrt{x}, 0 \le x \le 9$. What si the approximate number of words learned when x changes from

(i) 1 to 1.1 hour?

(ii) 4 to 4.1 hour?

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10. A circular plate expands uniformly under the influence of heat. If it's radius increases from 10.5 cm to 10.75 cm, then find an approximate change in the area and the approximate percentage change in the area.

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11. A coat of paint of thickness 0.2 cm is applied to the faces of a cube whose edge is 10 cm. Use the differentials to find approximately how many cubic centimeters of paint is used to paint this cube. Also calculate the exact amount of pain used to pain this cube.

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Exercise 8 3

1. Evaluate $\lim_{(x,y)\to(1,2)}$, g(x,y) , if the limit exist where g(x,y) $= \dfrac{3x^2 - xy}{x^2 + y^2 + 3}$

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2. Evaluate $\lim_{(x,y)\to(0,0)} \cos\left(\dfrac{x^3 + y^3}{x + y + 2}\right)$. If the limit exists.

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3. Let $f(x, y) = \dfrac{y^2 - xy}{\sqrt{x} - \sqrt{y}}$ for $(x, y) \neq (0, 0)$. Show that $\lim_{(x,y)\to(0,0)} f(x, y) = 0$

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4. Evaluate $\lim_{(x,y)\to(0,0)} \cos\left(\dfrac{e^x \sin y}{y}\right)$, if the limit exists.

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5. Let $g(x, y) = \frac{x^2 y}{x^4 + y^2}$ for (x,y) $\neq$ (0,0) and f(0,0)=0.

(i) Show that $\lim_{(x,y) \to (0,0)} g(x, y) = 0$ along every line $y = mx, m \in R$.

(ii) Show that $\lim_{(x,y) \to (0,0)} g(x, y) = \frac{k}{1 + k^2}$, along every parabola $y = kx^2, k \in R\{0\}$.

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6. Show that $f(x, y) = \frac{x^2 - y^2}{y^2 + 1}$ is continous at every, $(x, y) \in R^2$

.

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7. Let $g(x, y) = \frac{e^y \sin x}{x}$, for $x \neq 0$ and g(0,0) =1. Show that g is continous at (0,0).

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Exercise 8 4

1. Find the partial derivatives of the functions at the indicated point

$f(x, y) = 3x^2 - 2xy + y^2 + 5x + 2, (2, -5)$

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2. For each of the functions find the $f_x, f_y,$ and show that $f_{xy} = f_{yx}.$

$f(x, y) = \tan^{-1}\left(\frac{x}{y}\right)$

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3. If U(x,y,z) $= \frac{x^2 + y^2}{xy} + 3z^2y$, find $\frac{\partial U}{dx} + \frac{\partial U}{dy} + \frac{\partial U}{dz}$

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4. If $U(x, y, z) = \log(x^3 + y^3 + z^3)$ find $\frac{\partial U}{dx} + \frac{\partial U}{dy} + \frac{\partial U}{dz}$

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5. For each of the function find the g_{xy}, g_{yy} and g_{yx},

$g(x, y) = xe^y + 3x^2y$

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6. If w(x,y,z) $=x^2(y-z)+y^2(z-x)+z^2(x-y)$, then $\frac{\partial w}{\partial x}+\frac{\partial w}{\partial y}+\frac{\partial w}{\partial z}$ is

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7. If $V(x, y)=e^x(x\cos y-y\sin y)$, then prove that $\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}=0$

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8. If $w(x, y)=xy+\sin(xy)$, then prove that $\frac{\partial^2 w}{\partial y\,\partial x}=\frac{\partial^2 w}{\partial x\,\partial y}$

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9. If $v(x, y, z) = x^3 + y^3 + z^3 + 3xyz$, show that $\frac{\partial^2 v}{\partial y \partial z} = \frac{\partial^2 v}{\partial z \partial y}$

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10. A firm produces two types of calculators each week, x number of type A and y number of type B. The weekly revenue and cost functions (in rupees) are R(x,y) $= 80x + 90y + 0.04xy - 0.05x^2 - 0.05y^2$ and $C(x, y) = 8x + 6y + 2000$ respectively.

(i) Find the profit function P(x,y).

(ii) Find $\frac{\partial P}{\partial x}(1200, 1800)$ and $\frac{\partial P}{\partial y}(1200, 1800)$ and interpret these results.

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Exercise 8 5

1. If $w(x, y) = x^3 - 3xy + 2y^2, x, y \in R,$ find the linear approximation for w at (1,-1).

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2. Let z(x,y) $= x^2y + 3xy^4, x, y \in R.$ Find the linear approximation for z at (2,-1).

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3. If $v(x, y) = x^2 - xy + \frac{1}{4}y^2 + 7, x, y \in R,$ find the differential dv.

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4. Let W(x,y,z) $= x^2 - xy + 3\sin z, x, y, z \in R,$ Find the linear approximation at (2,-1,0).

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5. Let V(x,y,z) =xy + yz + zx, $x, y, z \in R$. Find the differential dV.

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Exercise 8 6

1. If $u(x, y) = x^2y + 3xy^4, x = e^t$ and y= sin t, find $\frac{du}{dx}$ and evaluate it at t=0.

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2. If $u(x, y, z) = xy^2z^3, x = \sin t, y = \cos t, z = 1 + e^{2t}$, find $\frac{du}{dx}$.

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3. If $w(x, y, z) = x^2 + y^2 + z^2, x = e^t, y = e^t \sin t$ and $z = e^t \cos t$, find $\frac{dw}{dt}$.

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4. Let $U(x, y, z) = xyz, x = e^{-t}, y = e^{-t} \cos t, z = \sin t, t \in R$. Find $\frac{dU}{dt}$.

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5. If w(x,y) $= 6x^3 - 3xy + 2y^2, x = e^s, y = \cos s \in R$, find $\frac{dw}{ds}$, and evaluate at s=0.

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6. If z(x,y) $= x\tan^{-1}(xy), x = t^2, y = se^t, s, t \in R$, Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ at s=t=1.

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7. Let $z(x, y) = xe^y + ye^{-x}, x = e^{-t}, y = st^2, s, t \in \mathbb{R}$. Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$.

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8. W (x,y,z) = xy + yz , x = u - v ,y = uv , z = u +v,u,v in R. Find $\frac{\partial w}{\partial u}, \frac{\partial w}{\partial v}$ and evaluate them at $\left(\frac{1}{2}, 1\right)$

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Exercise 8 7

1. In each of the following cases , determine whether the following function is homogeneous or not. If it is so , find the degree. (i) $f(x, y) = x^2y + 6x^3 + 7$ (ii) $h(x, y) = \frac{6x^2y^3 - \pi y^5 + 9x^4y}{2020x^2 + 2019y^2}$

(iii) $g(x, y, z) = \frac{\sqrt{3x^2 + 5y^2 + z^2}}{4x + 7y}$ (iv) $U(x, y, z) = xy + \sin\left(\frac{y^2 - 2z^2}{xy}\right)$

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2. Prove that $f(x, y) = x^3 - 2x^2y + 3xy^2 + y^3$ is homogenous, what is the degree? Verify Euler's Theorem for f.

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3. Prove that $g(x, y) = x\log\left(\frac{y}{x}\right)$ is homogenous, what is the degree? Verify Euler's Theorem for g.

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4. If $u(x, y) = \frac{x^2 + y^2}{\sqrt{x + y}}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{3}{2}u.$

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5. If $v(x, y) = \log\left(\frac{x^2 + y^2}{x + y}\right)$, prove that

$$x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = 1.$$

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6. If $w(x, y, z) = \log\left(\frac{5x^3y^4 + 7y^2xz^4 - 75y^3z^4}{x^2 + y^2}\right)$, find $x\frac{\partial w}{\partial x} + y\frac{\partial w}{\partial y} + z\frac{\partial w}{\partial z}$,

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Additional Questions Solved

1. Using differentials, find the approximate value of each of the following upto 3 places of the following upto 3 places of decimal. $(255)^{\frac{1}{4}}$.

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2. Using differentials, find the approximate value of each of the following upto 3 places of the following upto 3 places of decimal. $(401)^{\frac{1}{2}}$.

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3. Find approximate value of f (5.001) where $f(x) = x^3 - 7x^2 + 15$

.

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4. If the radius of a sphere , is measured as 7m with an error of 0.02 m then find the approximate error in calculating its volume .

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5. Find the differential dy and evaluate dy for the given values of x and dx.

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6. The edge of a cube was found to be 30 cm with a possible error in measurement of 0.1 cm .Use differentials to estimate the maximum possible error in computing (i) the volume of the cube and (ii) the surface area of cube .

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7. The radius of a circular disc is given as 24 cm with a maximum error in measurement of 0.02 cm . (i) Use differentials to estimate the maximum error in the calculated area of the disc. (ii) Compute the relative error.

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8. If u = log (tan x + tan y + tanz) , prove that $\sum \sin 2x \frac{\partial u}{\partial x} = 2$

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9. If U = (x-y) (y-z) (z-x) then show that $U_x + U_y + U_z$ = 0

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10. If $u = x^2 + 3xy + y^2$ Verify $\frac{\partial^2 u}{\partial x \, \partial y} = \frac{\partial^2 u}{\partial y \, \partial x}$.

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11. If $u = \frac{x}{y^2} - \frac{y}{x^2}$, show that $\frac{\partial^2 u}{\partial x \, \partial y} = \frac{\partial^2 u}{\partial y \, \partial x}$.

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12. Suppose that $z = ye^{x^2}$ where x = 2t and y = 1 then find $\frac{dz}{dt}$

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13. If $w = x + 2y + z^2$ and $x = \cos t, y = \sin t, z = t$ Find $\frac{dw}{dt}$.

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14. If f(x,y) = $\frac{1}{\sqrt{x^2 + y^2}}$ then show that $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = -f$

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15. Using Euler's theorem , prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$ if $u = \sin^{-1}.\left(\frac{x-y}{\sqrt{x}+\sqrt{y}}\right)$

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16. If $u = x^y$ then is equal to ....

A. yx^{y-1}

B. u log x

C. u log y

D. xy^{x-1}

Answer: A

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17. If $u=\sin^{-1}\left(\frac{x^4+y^4}{x^2+y^2}\right)$ and f= sin u then f is a homogenous function of degree ________

A. 0

B. 1

C. 2

D. 4

Answer: A

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18. If $u = \frac{1}{\sqrt{x^2+y^2}}$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is equal to ....

A. $\frac{1}{2}u$

B. u

C. $\frac{3}{2}u$

D. $-u$

Answer:

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19. The curve $y^2(x-2) = x^2(1+x)$ has ....

A. an asymptote parallel to x - axis

B. an asymptote parallel to y - axis

C. asymptotes parallel to both axis

D. no asymptotes

Answer: A::B::C::D

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20. If $x = r\cos\theta, y = r\sin\theta$, then $\frac{\partial r}{\partial x}$=________

A. $\sec\theta$

B. $\sin\theta$

C. $\cos\theta$

D. $\operatorname{coses}\theta$

Answer: A::C

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21. If $u = \log\left(\frac{x^2 + y^2}{xy}\right)$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is ....

A. 0

B. u

C. 2u

D. u - 1

Answer:

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22. The percentage error in the 11th root of the number 28 is approximately _______ times the percentage error in 28.

A. $\frac{1}{28}$

B. $\frac{1}{11}$

C. 11

D. 28

Answer: B

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23. The curve $a^2y^2 = x^2(a^2 - x^2)$ has ....

A. only one loop between x = 0 and x = a

B. two loops between x = 0 and x = a

C. two loops between x = - a and x = a

D. no loop

Answer: A::B::D

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24. An asymptote to the curve $y^2(a+2x)=x^2(3a-x)$ is ....

A. x = 3a

B. $x=\ -a/2$

C. x = a/2

D. x = 0

Answer: A::B

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25. In which region the curve $y^2(a+x)=x^2(3a-x)$ does not lie ?

A. $x > 0$

B. $0 < x < 3a$

C. $x \le -a$ and $x > 3a$

D. $-a < x < 3a$

Answer: A::C

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26. If u= y sin x then $\frac{\partial^2 u}{\partial x \, \partial y}$=__________

A. $\cos x$

B. $\cos y$

C. sin x

D. 0

Answer: C

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27. If $u = f\left(\frac{y}{x}\right)$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$=________

A. 0

B. 1

C. 2u

D. u

Answer:

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28. The curve $9y^2 = x^2(4 - x^2)$ is symmetrical about ....

A. y - axis

B. x - axis

C. y = x

D. both the axes

Answer: A::B

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29. The curve $ay^2 = x^2(3a - x)$ cuts the y - axis at ....

A. $x = -3a, x = 0$

B. $x = 0, x = 3a$

C. $x = 0, x = a$

D. $x = 0$

Answer:

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