



MATHS

BOOKS - FULL MARKS MATHS (TAMIL ENGLISH)

PROBABILITY DISTRIBUTIONS

Example Questions Solved

1. Suppose two coins are tossed once. If X denotes the number of talls (i) write down the

sample space (ii) find the inverse image of 1 (iii) the values of the random variable and number of elements in its inverse images .

Watch Video Solution

2. Suppose a pair of unbiased dice is rolled once if X denotes the total score of two dice write down

Sample space

3. An urn contains 2 white balls and 3 red balls . A sample of 3 balls are chosen at random from the urn. If X denotes the number of red balls chosen find the values taken by the random variable X and its number of inverse images .



4. Two balls are chosen randomly from an urn containing 6 white and 4 black balls . Suppose that we win ₹ 30 for each black ball selected

and we lose ₹ 20 for each white ball selected. If X denotes the winning amount then find the values of X and number of points in its inverse images .



5. Three fair coins are tossed simultaneously.

Find the probability mass function for number

of heads occurred.



6. A pair of fair dice is rolled once . Find the probability mass function to get the number of fours .



7. If the probability mass function f(x) of a

random variable X is

x	1	2	3	4
<i>f</i> (<i>x</i>)	$\frac{1}{12}$	$\frac{5}{12}$	$\frac{5}{12}$	$\frac{1}{12}$

find (i) its cumulative distribution hence find (ii) P (X \leq 3) and , (iii) $P(X \geq 2)$





8. A six sided die is marked 1 on one face 2 on two of its faces and 3 on remaining three faces . The die is rolled twice . If X denotes the total score in two throws .



9. Find the probability mass function f(x) of the discrete random variable X whose cumulative distribution function F(x) is given

by F (x) =
$$\begin{cases} 0 & -\infty < x < -2 \\ 0.25 & -2 \le x < -1 \\ 0.60 & -1 \le x < 0 \\ 0.90 & 0 \le x < 1 \\ 1 & 1 \le x < \infty \end{cases}$$

Also find (i) P (X < 0) and (ii) P($X \geq -1$).

Watch Video Solution

10. A random variable X has the following probability mass function.Find the value of K

$\frac{x}{f(x)}$	1	2	3	4	5	6
	k	2k	6k	5k	6k	10k



11. Find the constant C such that the function

$$f(x) = \begin{cases} Cx^2 & 1 < x < 4 \\ 0 & \text{Otherwise} \end{cases}$$

is a density function and compute (i) P

 $(1.5 < X < 3.5)(\mathrm{ii}) P(X \leq 2)(\mathrm{iii}) P(3 < X)$

12. If X is the random variable with porbability



find (i) the distribution function F(x) (ii) P



13. If X is the random variable with distribution

function F(x) given by , F(x) =
$$\begin{cases} o & x < 0 \\ x & 0 \le x < 1 \\ 1 & 1 \le x \end{cases}$$

then find (i) the probability density function

f(x) (ii) $P(0.2 \leq X \leq 0.7)$.

Watch Video Solution

14. The probability density function of random variable X is given by f(x)= $\begin{cases} k & 1 \le x < 5\\ 0 & \text{otherwise} \end{cases}$ Find (i) Distribution function (ii) P(X < 3) (iii) $P(2 < X < 4)(\text{iv})P(3 \le X)$





16. Suppose that f(x) given below represents a

probability mass function

$\frac{x}{f(x)}$	1	2	3	4	5	6
	c^2	$2c^2$	$3c^2$	$4c^2$	с	2c

Find (i) the value of c (ii) Mean and variance.



17. Two balls are chosen randomly from an urn containing 8 white and 4 black balls. Suppose that we win Rs 20 for each black ball selected and we lose Rs 10 for each white ball selected .



19. The mean and variance of a binomial variate X are respectively 2 and 1.5 . Find (i) P(X=0) (ii) P(X=1) (iii) P($X \ge 1$)



Exercise 111

1. Suppose X is the number of tails occurred when three fair coins are tossed once simultaneously. Find the values of the random variable X and number of points in its inverse

images .



2. In a pack of 52 playing cards two cards are drawn at random simultaneously if the number of black cards drawn is a ranodm variable find the valties of the random variable and number of points in its inverse images



3. An urn contains 5 mangoes and 4 apples. Three fruits are taken at random. If the number of apples taken is a random variable, then find the values of the random variable and number of points in its inverse images.

Watch Video Solution

4. Two balls are chosen randomly from an urn containing 6 red and 8 black balls. Suppose that we win Rs. 15 for each red ball selected and we lose Rs. 10 for each black ball selected. X denotes the winning amount, then find the values of X and number of points in its inverse images.

Watch Video Solution

5. A six sided die is marked '2' on one face, '3' on two of its faces, and '4' on remaining three faces. The die is thrown twice. If X denotes the total score in two throws, find the values of the random variable and number of points in its inverse images.





Exercise 11 2

 Three fair coins are tossed simultaneously.
 Find the probability mass function for number of heads occurred.

Watch Video Solution

2. A six die is marked 1 on one face 3 on two of

its faces and 5 on remaining three faces. The

die is thrown twice. If X denotes the total score in two throws find (i) the probability mass function (ii) the cumulative distribution function

(iii) $P(4 \leq X < 10)$ (iv) $P(X \geq 6)$

Watch Video Solution

3. Find the probability mass function and cumculative distribution function of number of girl child in families with 4 children,

assuming equal probabilities for boys and

girls.



4. Suppose a discrete random variable can only take the values 0, 1, and 2. The probability mass function is defined by

 $f(x) = egin{cases} rac{x^2+1}{k}, & ext{for x=0,1,2} \ 0 & ext{otherwise} \end{cases}$ Find (i) the value of k (ii) cumculative distribution function (iii) $P(X \ge 1).$

5. The cumculative distribution function of a

discrete random variable is given by

$$F(x) = egin{cases} 0 & -\infty < x < \ -1 \ 0.15 & -1 \leq x < 0 \ 0.35 & 0 \leq x < 1 \ 0.60 & 1 \leq x < 2 \ 0.85 & 2 \leq x < 3 \ 1 & 3 \leq x < \infty \end{cases}$$

Find (i) the probability mass function (ii)

P(X < 1) and (iii) $P(X \ge 2)$.

6. A random variable X has the following

probability mass function

x	1	2	3	4	5
f(x)	k^2	2 <i>k</i> ²	3k ²	2 <i>k</i>	3k

Find (i) the value of k (ii)

 $P(2 \leq X < 5)$ (iii)P(3 < X)

Watch Video Solution

7. The cumculative distribution function of a

discrete random variable is given by

$$F(x) = \begin{cases} 0 & \text{for } -\infty < x < 0\\ \frac{1}{2} & \text{for } 0 \le x < 1\\ \frac{3}{5} & \text{for } 1 \le x < 2\\ \frac{4}{5} & \text{for } 2 \le x < 3\\ \frac{9}{10} & \text{for } 3 \le x < 4\\ 1 & \text{for } 4 \le x < \infty \end{cases}$$

Find (i) the probability mass function (ii)
 $P(X < 3)$ and (iii) $P(X \ge 2)$.

Watch Video Solution

Exercise 11 3

1. The probability density function of X is given

by $f(x) = egin{cases} kxe^{-2x} & ext{for} \ x > 0 \ 0 & ext{for} \ x \le 0 \end{cases}$ Find the value of k.



2. The probability density function of X is f(x) =

$$\left\{egin{array}{ll} x & 0 < x < 1\ 2 - x & 1 \leq x < 2\ 0 & ext{otherwise} \end{array}
ight.$$
 (i) Find (i) $P(0.2 \leq X < 0.6)(ii)P(1.2 \leq X < 1.8)(iii)$

P(0.5 lt=X lt 1.5)`

3. Suppose the amount of milk sold daily at a mile booth is distributed with a minimum of 200 litres and a maximum of 600 litres with probability density function f(x)= $\left\{egin{array}{ll} k & 200 \leq x \leq 600 \\ 0 & \mathrm{otherwise} \end{array}
ight.$. Find (i) the value of k (ii) the distribution function (iii) the probability that daily sales will fall between 300 litres and 500 litres?

4. The probability density function of X is given by f(x) = $\begin{cases} ke^{-\frac{x}{3}} & \text{for } x > 0\\ 0 & \text{for } x \le 0 \end{cases}$ Find (i) the value of k (ii) the distribution function (iii) P(X < 3) (iv) $P(5 \le X)$ (v) $P(X \le 4)$.



$$f(x) = \left\{egin{array}{ccc} x+1 & -1 \leq x < 0 \ -x+1 & 0 \leq x < 1 \ 0 & ext{otherwise} \end{array}
ight.$$

then find (i) the distribution function F(x) (ii)

$$P(\ -0.5 \leq X \leq 0.5)$$

6. If X is the random variable with distribution

function F(x) given by,

$$F(x) = egin{cases} 0 & x < 0 \ rac{1}{2}ig(x^2 + xig) & 0 \leq x < 1 \ 1 & x \leq 1 \end{cases}$$

then find (i) the probability density function

f(x) (ii) $P(0.3 \leq X \leq 0.6)$

1. Two balls are drawn in succession without replacement from an urn containing four red balls and three black balls. Let X be the possible outcomes drawing red balls. Find the probability mass function and mean for X.



2. If μ and σ^2 are the mean and variance of the discrete random variable X, and E(X+3) = 10 and $E(X+3)^2 = 116$, find μ and σ^2 .

Watch Video Solution

3. Four fair coins are tossed once. Find the probability mass function, mean and variance for number of heads occurred.

4. A commuter train arrives punctually at a station every half hour. Each morning, a student leaves his house to the train station. Let x denote the amount of time, in minutes, that the student waits for the train from the time he reaches the train station. It is known that the pdf of X is $f(x) = \left\{egin{array}{ccc} rac{1}{30} & 0 < x < 30 \ 0 & ext{elsewhere} \end{array}
ight.$ Obtain interpret

the expected value of the random variable X.

5. The time to failure in thousands of hours of an electronic equipment used in a manufactured computer has the density function

 $f(x) = egin{cases} 3e^{-3x} & x > 0 \ 0 & ext{elsewhere} \end{cases}$

Find the expected life of this electronic equipment.

6. The probability density function of the random variable X is given by $f(x) = \begin{cases} 16xe^{-4x} & \text{for } x > 0 \\ 0 & \text{for } x \le 0 \end{cases} \quad \text{find} \quad \text{the} \\ \text{mean and variance of X} \end{cases}$

7. A lottery with 600 tickets gives one prize of Rs. 200, four prizes of Rs. 100, and six prizes of Rs. 50. If the ticket costs is Rs. 2, find the expected winning amount of a ticket.





Exercise 11 5

- **1.** Compute P(X=k) for the binomial distribution
- , B (n,p) where

(i) n=6 ,
$$p=rac{1}{3},$$
 k=3 (ii) n=10 , $p=rac{1}{5}$,k=4 (iii) n = 9 , $p=rac{1}{2}$, k =7

2. The probability that Mr. Q hits a target at any trial is $\frac{1}{4}$. Suppose he tries at the target 10 times. Find the probability that he hits the target (i) exactly 4 times (ii) at least one time.



3. Using binomial distribution find the mean and variance of X for the following experiments

(i) A fair coin is tossed 100 times and X denote

the number of heads .

(ii) A fair die is tossed 240 times and X denote

the number of times that four appeared .

Watch Video Solution

4. The probability that a certain kind of component will survive a electrical test is $\frac{3}{4}$. Find the probability that exactly 3 of the 5 components tested survive .

5. A retailer purchases a certain kind of electronic device from a manufacturer. The manufacturer indicates that the defective rate of the device is 5%. The inspector of the retailer randomly picks 10 items from a shipment. What is the probability that there will be (i) at least one defective item (ii) exactly two defective items.


6. If the probability that a fluorescent ligth has
a usefule life of at least 600 hours is 0.9 , find
the probabilities that amond 12 such lights.
(i) exactly 10 will have a useful life of at least
600 hours ,

(ii) at least 11 will have a useful life of at least600 hours ,

(iii) at least 2 will not have a useful life of at

least 600 hours



7. The mean and standard deviation of a binomial variate X are respectively 6 and 2. Find (i) the probability mass function (ii) P(X=3) (iii) $P(X\geq 2)$.

Watch Video Solution

8. If X ~ B (n,p) such that 4P (X=4) =P(x=2) and

n=6 . Find the distribution mean and standard deviation .



9. In a binomial distribution consisting of 5 independent trials the probability of 1 and 2 sucesses are 0.4096 and 0.2048 respectively. Find the mean and variance of the distribution

Watch Video Solution

Exercise 11 6

1. Let X be random variable with probability

density function

 $f(x)=\left\{egin{array}{cc} rac{2}{x^3} & x\geq 1\ 0 & x<1 \end{array}
ight.$

Which of the following statement is correct

A. both mean and variance exist

- B. mean exists but variance does not exist
- C. both mean and variance do not exist
- D. variance exists but mean does not exist

Answer:

2. A rod of length 2l is broken into two pieces at random. The probability density function of the shorter of the two pieces is

$$f(x) = egin{cases} rac{1}{l} & 0 < x \geq l \ 0 & l \leq x < 2l \end{cases}$$

The mean and variance of the shorter of the two pieces are respectively

A.
$$\frac{l}{2}, \frac{l^2}{3}$$

B. $\frac{l}{2}, \frac{l^2}{6}$
C. $l, \frac{l^2}{12}$
D. $\frac{l}{2}, \frac{l^2}{12}$

Answer:



3. Consider a game where the player tosses a six sided fair die. If the face that comes up is 6, the player wins Rs. 36, otherwise he loses Rs. k^2 , where k is the face that comes up k = {1, 2, 3, 4, 5}.

The expected amount to win at this game in Rs. is



Answer:

Watch Video Solution

4. A pair of dice numbered 1, 2, 3, 4, 5, 6 of a six-sided die and 1, 2, 3, 4 of a four-sided die is rolled and the sum is determined. Let the

random variable X denote this sum. Then the number of elements in the inverse image of 7

is

A. 1

B. 2

C. 3

D. 4

Answer:

5. A random variable X has binominal distribution with n = 25 and p = 0.8 then standard deviation of X is

- A. 6
- B.4
- C. 3
- D. 2



6. Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed n times. Then the possible values of X are

A. i+2n,i=0,1,2 ... n

B. 2i-n,i=0,1,2 ...n

C. n-i,i=0,1,2, ...n

D. 2i+2n, i=0,1,2, ...n

Answer:

7. If the function $f(x) = \frac{1}{12}$ for a < x < b, represents a probability density function of a continuous random variable X, then which of the following cannot be the value of a and b?

A. 0 and 12

B. 5 and 17

C. 7 and 19

D. 16 and 24

8. Four buses carrying 160 students from the same school arrive at a football stadium. The buses carry, respectively, 42, 36, 34, and 48 students. One of the students is randomly selected. Let X denote the number of students that were on the bus carrying the randomly selected student. Let Y denote the number of students on that bus. Then E[X] and E[Y] respectively are

A. 50,40

B. 40,50

C. 40.75,40

D. 41,41

Answer:



9. Two coins are to be flipped. The first coin will land on heads with probability 0.6, the second with probability 0.5. Assume that the results of

the flips are independent, and let X equal the

total number of heads that result. The value of

E[X] is

A.0.11

B. 1.1

C. 11

D. 1

Answer:

10. On a multiple-choice exam with 3 possible destructive for each of the 5 questions, the probability that a student will get 4 or more correct answers just by guessing is

A.
$$\frac{11}{243}$$

B. $\frac{3}{8}$
C. $\frac{1}{243}$
D. $\frac{5}{243}$



11. If P{X=0}=1-P{X=1}. If E{X}=3Var(X), then P{X=0}

is

A.
$$\frac{2}{3}$$

B. $\frac{2}{5}$
C. $\frac{1}{5}$
D. $\frac{1}{3}$



12. If X is a binomial random variable with expected value 6 and variance 2.4, then P(X=5) is

A.
$$\left(\frac{10}{5}\right) \left(\frac{3}{5}\right)^6 \left(\frac{2}{5}\right)^5$$

B. $\left(\frac{10}{5}\right) \left(\frac{3}{5}\right)^5$
C. $\left(\frac{10}{5}\right) \left(\frac{3}{5}\right)^4 \left(\frac{2}{5}\right)^6$
D. $\left(\frac{10}{5}\right) \left(\frac{3}{5}\right)^5 \left(\frac{2}{5}\right)^5$

Answer:

13. The random variable X has the probability

$$f(x) = egin{cases} ax+b & 0 < x < 1 \ 0 & ext{otherwise} \end{cases}$$
 and $E(X) = rac{7}{12}$, then a and b are respectively

A. 1 and
$$\frac{1}{2}$$

B. $\frac{1}{2}$ and 1

C. 2 and 1

D. 1 and 2





14. Suppose that X takes on one of the values 0, 1, and 2. If for some constant k, P(X=i)=k P(X=i-1)i=1,2 and P(X=0)= $\frac{1}{7}$ then the value of k is

A. 1

B. 2

C. 3

D. 4

Answer:



15. Which of the following is a discrete random variable?

I. The number of cars crossing a particular signal in a day.

II. The number of customers in a queue to buy train tickets at a moment.

III. The time taken to complete a telephone call.

A. I and II

B. II only

C. III only

D. II and III

Answer:

Watch Video Solution

16. If
$$f(x) = \begin{cases} 2x & 0 \le x \le a \\ 0 & \text{otherwise} \end{cases}$$
 is a probability

density function of a random variable then the

value of a is

A. 1

B. 2

C. 3

D. 4



17. The probability function of a random

variable is defined as



Then E(X) is equal to

A.
$$\frac{1}{15}$$

B. $\frac{1}{10}$
C. $\frac{1}{3}$
D. $\frac{2}{3}$



18. Let X have a Bernoulli distribution with mean 0.4, then the variance of (2X-3) is

A. 0.24

B.0.48

C. 0.6

 $D.\,0.96$

Answer:

19. If in 6 trials, X is a binomial variate which follows the relation 9P(X=4)=P(X=2), then the probability of success is

A. 0.125

 $B.\, 0.25$

 $C.\,0.375$

 $D.\,0.75$



20. A computer salesperson knows from his past experience that he sells computers to one in every twenty customers who enter the showroom. What is the probability that he will sell a computer to exactly two of the next three customers?

A.
$$\frac{57}{20^3}$$

B. $\frac{57}{20^2}$
C. $\frac{19^3}{20^3}$

D. $\frac{57}{20}$

Answer:

Watch Video Solution

Additional Problems

1. Four defective oranges are accidentally mixed with sixteen good ones . Two oranges are drawn at random from the mixed lot. If the random variable X denotes the number of

defective oranges then find the values of X

and number of points in its iverse image.



2. Find the probability mass function and the

cumulative distribution function for getting 3

s when two dice are thrown.



3. For the distribution function given by

 $F(x)= egin{cases} 0, & x < 0\ x^2, & 0 \leq x \leq 1\,.$ Find the density $1, & x>1 \end{cases}$

function.

Also evaluate (i) P(0.5 < x < 0.75) (ii)

 $P(x \leq 0.5)$ (iii) P(X > 0.75)

Watch Video Solution

4. For the probability density function f(x) =

$$\left\{egin{array}{ccc} 2e^{-2x} & x>0\ 0 & x\leq 0 \end{array}
ight.$$
 find F(2)

5. The probability density function of a random variable x is $f(x) = \begin{cases} kx^{\alpha-1}e^{-\beta x^{\alpha}} & x, \alpha, \beta, > 0\\ 0 & \text{elsewhere} \end{cases}$ Find (i) k, (ii) P(X > 10).

Watch Video Solution

6. A continuous random variable x has the p.d.f

defined by f(x) = $\begin{cases} Ce^{-ax} & 0 < x < \infty \\ 0 & \text{elsewhere} \end{cases}$ find the value of C if a > 0.



7. An urn contains 4 white and 3 Red balls. Find the probability distribution of the number of red balls in three draws when a ball is drawn at random with replacement . Also find its mean and variance.



8. Find the mean and variance of the distribution f(x) = $\begin{cases} 3e^{-3x} & 0 < x < \infty \\ 0 & \text{elsewhere} \end{cases}$

Watch Video Solution

9. Two cards are drawn with replacement from

a well shufflied deck of 52 cards . Find the

mean and variance for the number of aces .



10. In a Binomial distribution if n = 5 and P(X=3)

=2P (X=2) find p.

Watch Video Solution

11. If the sum of mean and variance of a Binomial Distribution is 4.8 for 5 trials find the distribution .

12. If on an average 1 ship out of 10 do not arrive safely to parts . Find the mean and the standard deviation of the ships returining safely out of a total of 500 ships .

Watch Video Solution

13. The overall percentage of passes in a certain examination is 80 . If six candidates appear in the examination what is the

probability that at least five pass the

examination .



14. In a hurdle race a player has to cross 10 hurdles . The probability that he will clear each hurdle is 5/6. What is the probability that he will knock down less than 2 hurdles ?

15. If $f(x) = \begin{cases} kx^2 & 0 < x < 3 \\ 0 & \text{elsewhere} \end{cases}$ is a probability

density function then the value of k is

A.
$$\frac{1}{3}$$

B. $\frac{1}{6}$
C. $\frac{1}{9}$
D. $\frac{1}{12}$

Answer:
B. 8

C. 4

D. 2

Answer:



17. A random variable X has the following

probability mass function as follows :

X-231P(X = x)
$$\frac{\lambda}{6}$$
 $\frac{\lambda}{4}$ $\frac{\lambda}{12}$

Then the value of λ is

A. 1

B. 2

C. 3



18. X is a discrete random variable which takes the values of 0,1,2 and P (X=0) = $\frac{144}{169}$ P(X=1) = $\frac{1}{169}$ then the value of P(X=2) is

A.
$$\frac{145}{169}$$

B. $\frac{24}{169}$
C. $\frac{2}{169}$
D. $\frac{143}{169}$



19. Given E(X+C)=8 and E(X-C)=12 The value of C

is

 $\mathsf{A.}-2$

B. 4

 $\mathsf{C}.-4$



20. X is a random variable taking the values 3,4 and 12 with probabilites $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{5}{12}$ then E(x) is

A. 5

B.7

C. 6



21. Variance of the random variable X is 4 . Its mean is 2 . Then $\mathsf{E}\!\left(X^2 ight)$ is

A. 2

B. 4

C. 6



22. In 5 throws of a die getting 1 or 2 is a success . Then mean number of success is

A.
$$\frac{5}{3}$$

B. $\frac{3}{5}$
C. $\frac{5}{9}$
D. $\frac{9}{5}$

•



23. The mean of a binomial distrution 5 and SD is 2, Then the value of n and p are :

A.
$$\left(\frac{4}{5}, 25\right)$$

B. $\left(25, \frac{4}{5}\right)$
C. $\left(\frac{1}{5}, 25\right)$
D. $\left(25, \frac{1}{5}\right)$



A.
$$\frac{1}{2}$$

B. $\frac{1}{3}$
C. $\frac{2}{3}$
D. $\frac{1}{4}$



25. A box contains 6 red and 4 white balls . If 3 balls are drawn at random the probability of getting 2 white balls without replacement is

A.
$$\frac{1}{20}$$

B. $\frac{18}{25}$
C. $\frac{4}{25}$

.....

D. $\frac{3}{10}$

Answer:

Watch Video Solution

26. If 2 cards are drawn from a well shuffled pack of 52 cards the probability that they are of the same colours without replacement is

A.
$$\frac{1}{2}$$

•••••

B.
$$\frac{26}{51}$$

C. $\frac{25}{51}$
D. $\frac{25}{102}$

Watch Video Solution

27. The distribution function F(x) of a random

variable X is

A. a decreasing function

B. a non -decreasing function

C. a constant function

D. increasing first and then decreasing

Answer:

Watch Video Solution