MATHS

BOOKS - FULL MARKS MATHS (TAMIL ENGLISH)

THEORY OF EQUATIONS

Example Questions Solved

1. If α and β are the roots of the quadratic equation $17x^2 + 43x - 73 = 0$, construct a quadratic equation whose roots are a +2 and $\beta + 2$.

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2. If α and β are the roots of the quadratic equation $2x^2 - 7x + 13 = 0$, construct a quadratic equation whose roots are α^2 and β^2.

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3. If α, β and γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of $\sum \frac{1}{\beta\gamma}$ in terms of the coefficients .

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4. Find the sum of the squares of the roots of $ax^4 + bx^3 + cx^2 + dx + e = 0, a \neq 0.$

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5. Find the condition that the roots of cubic equation $x^3 + ax^2 + bx + c = 0$ are in the ratio p : q : r .

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6. Form the equations whose roots are reciprocals of the roots of cubic equation.

$x^3 + ax^2 + bx + c = 0$

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7. If p is real, discuss the nature of the roots of the equation $4x^2 + 4px + p + 2 = 0$, in terms of p.

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8. Find the monic polynomial equation of minimum degree with real coefficients having $2 - \sqrt{3}i$ as a root.

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9. Find a polynomial equation of minimum degree with rational coefficients , having $2 - \sqrt{3}$ as a root.

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10. Form a polynomial equation with integer coefficients with $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as a root.

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11. Show that the equation $2x^2 - 6x + 7 = 0$ cannot be satisfied by any real values of x .

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12. If $x^2 + 2(k+2)x + 9k = 0$ has equal roots then k is :

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13. Show that , if p,q,r are rational , the roots of the equation $x^2 - 2px + p^2 - q^2 + 2pr - r^2 = 0$ are rational .

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14. Prove that a line cannot intersect a circle at more two points .

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15. If $2+i$ and $3-\sqrt{2}$ are the roots of the equation $x^6-13x^5+62x^4-126x^3+65x^2+127x-140=0$ then find all the roots.

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16. Solve the equation $x^4-9x^2+20=0$

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17. Solve the equation $x^3 - 3x^2 - 33x + 35 = 0$

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18. Solve the equation $2x^3 + 11x^2 - 9x - 18 = 0$

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19. Obtain the condition that the roots of $x^3 + px^2 + qx + r = 0$ are in A.P

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20. Find the condition that the roots of $ax^3 + bx^2 + cx + d = 0$ are in geometric progression.

Assume $a, b, c, d \neq 0$.

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21. If the root of $x^3 + px^2 + qx + r = 0$ are in H.P. prove that $9pqr = 27r^2 + 2q^3$. Assume $p, q, r \neq 0$

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22. It is known that the roots of the equation $x^3 - 6x^2 - 4x + 24 = 0$ are in arithmetic progression. Find its roots.

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23. Solve the equation $2(x-3)-3(x-4)=12$.

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24. Solve the equation $2(x+2)-3(x-3)=x+7$

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25. Solve the equation $x^3-5x^2-4x+20=0$

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26. Solve the equation $x-2(x-6)=4-5x$.

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27. Solve the equation $7x^3 - 43x^2 = 43x - 7$

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28. Solve the following equation $4x + 4.9 = 6.5$

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29. Find solution , if any , of equation $2\cos^2 x - 9\cos x + 4 = 0$

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30. Show that the polynomial $9x^9 + 2x^5 - x^4 - 7x^2 + 2$ has at least six imaginary roots.

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31. Discuss the nature of the roots of the following polynomials .

(i) $x^{2018} + 1947x^{1950} + 15x^8 + 26x^6 + 2019$ (ii) $x^5 - 19x^4 + 2x^3 + 5x^2 + 11$

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Exercise 3 1

1. If the sides of a cunbic box are increased by 1, 2, 3 units respectively to form a cuboid, then the volume is increased by 52 cubic units. Find the volume of the cuboid.

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2. Construct a cubic equation roots

(i) 1,2 and 3 (ii) 1, 1 and -2 (iii) 2, 1/2 and 1

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3. If α, β and γ are the roots of the cubic equation $x^3 + 2x^2 + 3x + 4 = 0$, for a cubic equation roots are $2\alpha, 2\beta, 2\gamma$

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4. Solve the equation $3x^3 - 16x^2 + 23x - 6 = 0$ if the product of two roots is 1.

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5. Find the sum of squares of roots of the equation $2x^4 - 8x^3 + 6x^2 - 3 = 0$

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6. Solve the equation $x^3 - 9x^2 + 14x + 24 = 0$ if it is given that two of its roots are in the ratio 3:2.

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7. If α, β and γ are the roots of the polynomlal equation $ax^3 + bx^2 + cx + d = 0$, find the value of $\Sigma \frac{a}{\beta\gamma}$ in terms of the coefficients.

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8. If α, β, γ and δ are the roots of the polynomial eqauation $2x^4 + 5x^3 - 7x^2 - 8 = 0$, find a quadratic equation with integer corddicients whose roots are $\alpha + \beta + \gamma + \delta$ and $\alpha\beta\gamma\delta$.

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9. If p and q are the roots of the equation $lx^2 + nx + n = 0$, show that $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0$.

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10. If the equation $x^2 + px + q = 0$ and $x^2 + p'x + q' = 0$ have common roots, show that it must be equal to $\frac{pq' - p'q}{q - q'}$ or $\frac{q - q'}{p' - p}$.

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11. Formulate into a mathematical problem to find a number such that when its cube root is added to it, the result is 6.

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12. A 12 metre tall tree was broken into Two it was found that the height of the part which was left standing was the cube root of the length of the part that was cut away. Formulate this into a mathematical problem to find the height of the part which was cut away.

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Exercise 3 2

1. If k is real , discuss the nature of the roots of the polynomial equation $2x^2 + kx + k = 0$, in terms of k.

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2. Find a polynomial equation of minimum degree with rational coefficients, having $2 + \sqrt{3}$I as a root.

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3. Find a polynomial equation of minimum degree with rational coefficients, having 2i+3 as a root.

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4. Find a polynomial equation of minimum degree with rational coefficients, having $\sqrt{5} - \sqrt{3}$ as a root.

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5. Provet that a stralght line and parabola connot intersect at more than two points.

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Exercise 3 3

1. Solve the cubic equation : $2x^3 - x^2 - 18x + 9 = 0$ if sum of two of its roots vanishes.

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2. Solve the equation $9x^3 - 36x^2 + 44x - 16 = 0$ if the roots form an arithmetic progression.

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3. Solve the equation $3x^3 - 26x^2 + 52x - 24 = 0$ if its roots form a geometric progression.

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4. Determine k and solve the equation $2x^3 - 6x^2 + 3x + k = 0$ if one of its roots is twice the sum of the other two roots.

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5. Find all zeros of the polynomial $x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135$, if it is known

that 1 + 2i and $\sqrt{3}$ are two of its zeros.

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6. Solve the cubic equation : (i) $2x^3 - 9x^2 + 10x = 3$, (ii) $8x^3 - 2x^2 - 7x + 3 = 0$

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7. Solve the equation : $x^4 - 14x^2 + 45 = 0$

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Exercise 3 4

1. Solve : (i) $(x-5)(x-7)(x+6)(x+4)=504$, (ii) $(x-4)(x-7)(x-2)(x+1)=16$

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2. Solve : $(2x-1)(x+3)(x-2)(2x+3)+20=0$

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Exercise 3 5

1. Solve the following equation :

(i) $\sin^2 x-5\sin x+4=0$ (ii) $12x^3+8x=29x^2-4.$

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2. Examine for the rational roots of :

(i) $2x^3 - x^2 - 1 = 0$ (ii) $x^8 - 3x + 1 = 0$

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3. Solve : $8x^{\frac{3}{2n}} - 8x^{\frac{-3}{2n}} = 63$

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4. Sove : $2\sqrt{\frac{x}{a}} + 3\sqrt{\frac{a}{x}} = \frac{b}{a} + \frac{6a}{b}$

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5. Solve the equation : $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$

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6. Find all real numbers satisfying $4^x - 3\left(2^{x+2}\right) + 2^5 = 0$

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7. Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ if it is known that $\frac{1}{3}$ is a solution.

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Exercise 3 6

1. Discuss the maximum possible number of positive the negative roots of the polynomial equation $9x^9 - 4x^8 + 4x^7 - 3x^6 + 2x^5 + x^3 + 7x^2 + 7x + 2 = 0$

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2. Discuss the maximum possibel number of positive the negative zeros of the polynomials $x^2 - 5x + 6$ and $x^2 - 5x + 16$. Also draw rough sketch of the graphs.

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3. Show that the equation $x^9 - 5x^5 + 4x^4 + 2x^2 + 1 = 0$ has at least 6 imaginary solutions.

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4. Determine the number of positive and negative roots of the equation $x^9 - 5x^8 - 14x^7 = 0$.

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5. Find the exact number of real zeros and imaginary of the polynomial $x^9 + 9x^7 + 7x^5 + 5x^3 + 3x$.

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Exercise 3 7

1. A zero of $x^3 + 64$is

A. 0

B. 4

C. 4i

D. -4

Answer: D

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2. If find g are polynomials of derrees m and n respectively, and if h(x) = $(f \circ g)$(x), then the degree of h is

A. mn

B. m+ n

C. m^n

D. n^m

Answer: A

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3. A polynomial equation in x of degree n always has :

A. n distinct roots

B. n real roots

C. n imaginary roots

D. at most one root.

Answer: C

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4. If α, β and γ are the zeros of $x^3 + px^2 + qx + r$, then $\Sigma\frac{1}{\alpha}$ is

A. $-\frac{q}{r}$

B. $-\frac{q}{r}$

C. $\frac{q}{r}$

D. $-\frac{q}{p}$

Answer: A

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5. According to the rational root theorem, which number is not possible rational zero of $4x^7 + 2x^4 - 10x^3 - 5$?

A. -1

B. $\frac{5}{4}$

C. $\frac{4}{5}$

D. 5

Answer: C

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6. The polynomial $x^3 - kx^2 + 9x$ has three real zeros if and only if, k satifies

A. $|k| \leq 6$

B. k = 0

C. $|k| > 6$

D. $|k| \geq 6$

Answer: D

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7. The number of real numbers in $[0, 2\pi]$ satisfying $\sin^4 x - 2\sin^2 x + 1$ is .....

A. 2

B. 4

C. 1

D. ∞

Answer: C

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8. If $x^3 + 12x^2 + 10ax + 1999$ definitely has positive zero , if and only if .....

A. $a \geq 0$

B. $a > 0$

C. $a < 0$

D. $a \leq 0$

Answer:

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9. The polynomial $x^3 + 2x + 3$ has :

A. one negative and two imaginary zeros

B. one positive and two imaginary zeros

C. three real zeros

D. no zeros

Answer: A

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10. The number of positive zeros of the polynomial $\sum_{j=0}^{n}{}^{n}C_r(-1)^r x^r$ is

A. 0

B. n

C. $< n$

D. r

Answer: B

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Additional Questions Solved

1. Construct a cubic equation with roots 2,3,4.

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2. If α, β and γ are the cubic equation $x^3 - 6x^2 + 11x - 6 = 0$. From a cubic equation whose roots are $2\alpha, 2\beta, 2\gamma$.

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3. If the roots of $x^4 + 5x^3 - 30x^2 - 40x + 64 = 0$ are in G. P , then find the roots

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4. Determine the value of k such that the equation $(2k - 5)x^2 - 4x - 15 = 0$ and $(3k - 8)x^2 - 5x - 21 = 0$ may have a common root.

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5. If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$. Find the value of the following in

terms of coefficients.

(i) $\sum \frac{1}{\beta r}$ (ii) $\sum \frac{1}{\alpha}$ (iii) $\sum \alpha^2 \beta$

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6. α, β, γ are the roots of the equation $x^3 + 3x^2 + 2x + 1 = 0$. Find $\sum \alpha^3$ and $\sum \alpha^{-2}$.

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7. Find a polynomial equation of minimum degree with rational coefficients , having 1 - i as a root.

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8. Find a polynomial equation of minimum degree with rational co - efficients having $\sqrt{3}+\sqrt{7}$ as a root .

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9. If the roots of equation $x^3+px^2+qx+r=0$ are in A.P then show that $2p^3-9pq+27r=0$.

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10. Solve $27x^3+42x^2-28x-8=0$ given that its roots are in geometric progressive.

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11. Solve the equation $15x^3 - 23x^2 + 9x - 1 = 0$. Where roots are in H . P

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12. If one root of $x^3 + 2x^2 + 3x + k = 0$ is sum of the other two roots then find the value of k

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13. If sum of the roots of the equation $x^3 - 3x^2 - 16x + k = 0$ is zero then find the value k.

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14. Find all zeros of the polynomial $x^3 - 5x^2 - 9x - 5 = 0$,

If 2 + I is a root.

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15. Solve (x-3)(x-6)(x-1)(x+2)+54=0`

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16. Solve the equation

$(x - 4)(x - 2)(x - 1)(x + 1) + 8 = 0$

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17. Solve : $3^{2x+4} + 1 = 2.3^{x+2}$

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18. Solve : $2^{2x} - 2^{2x+3} + 2^4 = 0$

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19. Solve $(x-4)(x+2)(x+3)(x-3) + 8 = 0$

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20. Solve :

$(x+y)^{2/3} + 2(x-y)^{2/3} = 3\left(x^2 - y^2\right)^{1/3}$ and $3x - 2y = 13$

.

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21. Solve : $5^{x-1} + 5^{1-x} = 26$

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22. Solve : $12x^4 - 56x^3 + 89x^2 - 56x + 12 = 0$

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23. Solve : $\left(\sqrt{3} + \sqrt{2}\right)^x + \left(\sqrt{3} - \sqrt{2}\right)^x = 10$

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24. Solve : $x^4 + 4x^3 + 5x^2 + 4x + 1 = 0$

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25. Find the maximum possible number of real roots the equation . $x^5 - 6x^2 - 4x + 5 = 0$

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26. Find the maximum possible number of real roots the equation . $\left|x^2\right| - 5|x| + 6 = 0$

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27. Find the real roots of the equation . $x^2 + 5|x| + 6 = 0$

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28. Solve $x^4 - 4x^2 + 8x + 35 = 0$ Given $\left(2 + i\sqrt{3}\right)$ is a root.

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29. Solve $x^4 - 5x^3 + 4x^2 + 8x - 8 = 0$. Given $\left(1 - \sqrt{5}\right)$ is a root of the polynomial equation.

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30. Find a polynomial equation of the lowest degree with rational co - efficient having $\sqrt{3}, (1-2i)$ as two of its roots.

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