MATHS

NCERT - NCERT MATHEMATICS(TELUGU)

RELATIONS AND FUNCTIONS

Example

1. Let A be the set of all students of a boys school. Show that the relation R in A given by $R = \{(a, b) : a$ is sister of b} is the empty relation and $R' = \{(a, b):$ the difference between heights of a and b is less than 3 meters } is the universal relation.

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2. Let T be the set of all triangles in a plane with R a relation in T given by $R = \{(T_1, T_2) : T_1$ is congruent to $T_2\}$ Show that R is an equivalence relation.

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3. Let L be the set of all lines in a plane and R be the relation in L defined as $R = \{(L_1, L_2) : L_1$ is perpendicular to L_2 $\}$. Show that R is symmetric but neither reflexive nor transitive.

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4. Show that the relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 1), (2, 2), (3, 3), (1, 2)., (2, 3)\}$ is reflexive but neither symmetric nor transitive.

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5. Show that the relation R in the set Z of intergers given by

$R = \{(a, b) : 2$ divides a-b }

is an equivalence relation.

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6. Let R be the realtion defined in the set $A = \{1, 2, 3, 4, 5, 6, 7\}$ by $R = \{(a, b) :$ both a and b are either odd or even}. Show that R is an related to each other and all the elements of the subset $\{2, 4, 6\}$ are related to each other, but no element of the subset $\{1, 3, 5, 7\}$ is related to any element of the subset $\{2, 4, 6\}$.

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7. Let A be the set of all 50 students of Class X in a school Let $f : A \rightarrow N$ be function defined by $f(x) =$ roll number of the student x. Show that f in one-one but not onto.

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8. Show that the function $f: N \rightarrow N$, given by $f(x) = 2x$, is one-one but not onto.

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9. Prove that the function $f: R \rightarrow R$, given by $f(x) = 2x$, is one-one and onto.

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10. Show that the function $f: N \rightarrow N$, given by $f(1) = f(2) = 1$ and $f(x) = x - 1$, for every $x > 2$, is onto but not one-one.

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11. Show that $f: N \rightarrow N$, given by

$x+1$, if x is odd,

$f(x) = x - 1$, if x is even

is both one-one and onto.

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12. Show that an onto function $f: \{1, 2, 3\} \rightarrow \{1, 2, 3\}$ is always one-one.

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13. Let $f: \{2, 3, 4, 5\} \rightarrow \{3, 4, 5, 9\}$ and $g: \{3, 4, 5, 9\} \rightarrow \{7, 11, 15\}$ be function defined as

$f(2) = 3, f(3) = 4, f(4) f(5) = 5$ and $g(3) = g(4) = 7$ and $g(5) = g(9) =$

Find gof.

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14. Find gof and fog, if $f: T \rightarrow R$ and $g: R \rightarrow R$ are given by $f(x) = \cos x$ and $g(x) = 3x^2$. Show that gof $\neq$ fog.

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15. Show that if $f: R\left\{\frac{7}{5}\right\} \rightarrow R - \left\{\frac{3}{5}\right\}$ is defined by $f(x) = \frac{3x+4}{5x-7}$ and $g: R - \left\{\left(\frac{3}{5}\right\} \rightarrow R - \left\{\frac{7}{5}\right\}$ is defined by $g(x) = \frac{7x+4}{5x-3}$, then fog $= I_A$ and gof $= I_B$, where, $A = R - \left\{\frac{3}{5}\right\}, B + R - \left\{\frac{7}{5}\right\}, I_A(x) = x, \forall x \in A, I_B \mid (x) = x, \forall x \in$ are called identity functions on sets A and B, respectively.

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16. Show that if $f: A \rightarrow B$ and $g: B \rightarrow C$ are one-one, then gof $: A \rightarrow C$ is also one-one.

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17. Show that if $f: A \rightarrow B$ and $g: B \rightarrow C$ are onto, then gof: $A \rightarrow C$ is also onto.

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18. Consider functions f and g such that composite gof is defined and is one one Are f and g both necessarily one-one.

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19. Are f and g both necessarily onto, if gof is onto ?

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20. Let $f: \{1, 2, 3\} \rightarrow \{a, b, c\}$ be one-one and onto function given by $f(1) = a$, $f(2) = b$ and $f(3) = c$. Show that there exists a function

$g: \{a, b, c\} \rightarrow \{1, 2, 3\}$ such that gof $= I_x$ and fog $= I_y$, where, $X = \{1, 2, 3\}$ and $Y = \{a, b, c\}$.

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21. Let $f: N \rightarrow Y$ be a function defined as $f(x) = 4x + 3$, where, $Y = \{y \in N: y = 4x + 3 \text{ for some } x \in N\}$. Show that f is invertible. Find the inverse.

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22. Let $Y = \{n^2 : n \in N\} \subset N$. Consider $f: N \rightarrow Y as f(n) = n^2$. Show that f is invertible. Find the inverse of f.

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23. Let $f: N \rightarrow R$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f: N \rightarrow S$. where, S is the range of f, is invertible. Find the inverse of f.

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24. Consider $f: N \to N, g: N \to N$ and $h: N \to R$ defined as $f(x) = 2x, g(h) = 3y + 4$ and $h(z = \sin z, \forall x, y$ and $z$in N. Show that h(gof) = (hog) of.

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25. Consider $f: \{1, 2, 3\} \to \{a, b, c\}$ and $g: \{a, b, c\} \to$ {apple, ball, cat} defined as $f(1) = a, f(2) = b, f(3) = c, g(a)$ = apple,$g(b) =$ ball and g (c)= cat. Show that f, g and gof are invertible. Find out f^{-1}, g^{-1} and $(\text{gof})^{-1}$ and show that $(\text{gof})^{-1} = f^{-1} o g^{-1}$.

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26. Let $S = \{1, 2, 3\}$. Determine whether the functions $f: S \to S$ defined as below have inverses. Find f^{-1}, if it exists.

(a) $f = \{(1, 1), (2, 2), (3, 3)\}$

(b) $f = \{(1, 2), (2, 1), (3, 1)\}$

(c) $f\{(1, 3), (2, 3), (2, 1)\}$

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27. Show that addition, subtraction and multiplication are binary operations on R, but division is not a binary opertion on R. Further, show that division is binary opertion on the set R, of nonzero real numbers.

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28. Show that subtraction and division are not binary opertions on N.

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29. Show that $* : R \times R \rightarrow R$ given by (a,b) $\rightarrow a + 4b^2$ is a binary operation.

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30. Let P be the set of all subsets of a given set X. Show that $\cup : P \times P \rightarrow P$ given by $(A, B) \rightarrow A \cup B$ and $\cap : P \times P \rightarrow P$ given by $(A, B) \rightarrow A \cap B$ are binary operations on the set P.

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31. Show that the $\vee : R \times R$ given by $(a, b) \rightarrow$ max {a,b} and the $\wedge : R \times R \rightarrow R$ given by (a,b) $\rightarrow$ min {a,b} are binary operations.

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32. Show that $+ : R \times R \rightarrow R$ and $x : R \times R \rightarrow R$ are communtative binary opertions , but $- : R \times R \rightarrow R$ and $\div : R_* \times R_* \rightarrow R_*$ are not commutative.

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33. Show that $* : R$ x $R \rightarrow R$ defined by $a * b = a + 2b$ is not commutative.

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34. Show that addition and multiplication are associative binary opertion on R. But substraction is not associative on R. Division is not associative on R_*.

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35. Show that $* R \times R \rightarrow R$ given by $a * b \rightarrow a + 2b$ is not associative.

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36. Show that zero is the identity for addition on R and 1 is the identity for multiplication on R. But there is no identity element for the opertions $- : R \times R \rightarrow R$ and $\div : R_* \times R_* \rightarrow R_*$.

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37. Show that -a is the inverse of a for the addition opertion '+' on R and $\frac{1}{a}$ is the inverse of $a \neq 0$ for the multiplication opertion 'x' on R.

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38. Show that -a is not the inverse of $a \in N$ for the addition opertion + on N and $\frac{1}{a}$ is not the inverse of $a \neq N$ for multiplication opertion $\times$ on N , for $a \neq 1$.

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39. If R_1 and R_2 are equivalence rrelations in a set A show that $R_1 \cap R_2$ is also an equivalence relation.

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40. Let R be a relation on the set A of ordered pairs of positive integers defined by $(x, y)R(u, v)$ if and only if $xv = yu$. Show that R is an equivalence relation.

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41. Let $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Let R_1 be a relation in X given by $R_1 = \{(x, y) : \{x, y\} \subset \{1, 4, 7\}\}$ or $\{x, y\} \subset \{2, 5, 8\}$ or $\{x, y\} \subset \{3, 6, 9$

Show that $R_1 = R_2$.

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42. Let $f: X \to Y$ be a function. Define a relation R in X given by $R = \{(a, b) : f(a) = f(b)\}$. Examine whether R is an equivalence relation or not.

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43. Determine which of the following binary opertions on the set R are associative and which are commutative.

$(a) a * b = 1 \forall a, b \in R$

(b) $a * b = \dfrac{(a+b)}{2} \forall a, b \in R$

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44. Find the number of all one-one functions from set $A = \{1, 2, 3\}$ to itself.

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45. Let $A = \{1, 2, 3\}$. Then show that the number of relations containing $(1, 2)$ and $(2, 3)$ which are reflexive and transitive but not symmetric is three.

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46. Show that number of equivalence relation in the set $\{1, 2, 3\}$ containing $(1, 2)$ and $(2, 1)$ is two.

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47. Show that the number of binary opertions on $\{1, 2\}$ having 1 as identity and having 2 as the inverse of 2 is exactly one.

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48. Consider the identity function $I_N : N \to N$ defined as $I_N(x) = x \,\forall x \in N$. Show that although I_N is onto but

$I_N + I_N : N \to N$ defined as

$(I_N + I_N)(x) = I_N(x) + I_N(x)x + x = 2x$ is not onto.

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49. Consider a function $f : \left[0, \frac{\pi}{2}\right] \to R$ given by $f(x) = \sin x$ and $g : \left[0, \frac{\pi}{2}\right] \to R$ given by $g(x) = \cos x$, Show that f and g are one-one but f + g is not one-one.

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Exercise 1 1

1. Determine w hether each of the following relations are reflexive, symmetric and transitive:

(i) Relation R in the set $A = \{1, 2, 3, \ldots, 13, 14\}$ defined as

$R = \{(x, y) : 3x - y = 0\}$

(ii) Relation R in the set N of natural numbers defined as

$R = \{(x, y) : y = x + 5 \text{ and } x < 4\}$

(iii) Relation R in the set $A = \{1, 2, 3, 4, 5, 6\} as$

$R = \{(x, y) : y$ is divisible by x}

(iv) Relation R in the set Z of allintegers defined as

$R = \{(x, y) : x - y$ is an integer}

(v) Relation R in the set A of human beings in a town at a particular time

given by

(a) $R = \{(x, y) : x \text{ and }$ y work at the same place}

(b) $R = \{(x, y) : x \text{ and } y$ live in the same locality]

(c) $R = \{(x, y) : x$ is exactly 7 cm taller than y}

(d)$R = \{(x, y) : x$ is wife of y}

(e) $R = \{(x, y) : x$ is father of y}

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2. Show that the relation R in the set R of real numbers, defined as

$R = \{(a, b) : a \leq b^2\}$ is neither reflexive nor symmetric nor transitive.

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3. Check whether the realtion R defined in the set $\{1, 2, 3, 4, 5, 6\}$ as $R = \{(a, b) : b = a + 1\}$ is reflexive, symmetric or transitive.

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4. Check whether the relation R in R defined by $R = \{(a, b) : a \leq b^3\}$ is reflexive, symmetric or transitive.

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5. Show that the relation R in the set $A = \{1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : |a - b| \text{ is even}\}$, is an equivalence relation. Show that all the elements of $\{1, 3, 5\}$ are related to each other and all the elements of $\{2, 4\}$ are related to each other. But no element of $\{1, 3, 5\}$ is related to any element of $\{2, 4\}$.

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6. Show that each of the relation R in set $A = \{x \in Z : 0 \leq x \leq 12\}$, given by

(i) $R = \{(a, b) : |a - b|$ is a multiple of 4}

(ii) $R = \{(a, b) : a = b\}$

is an equivalence relation. Find the set of all elements related to 1 in each case.

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7. Give an example of a relation. Which is

(i) Symmetric but neither reflexive nor transitive.

(ii) Transitive but neither reflexive nor symmetric.

(iii) Reflexive and symmetric but not transitive.

(iv) Reflexive and transitive but not symmetric.

(v) Symmetric and transitive but not reflexive.

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8. Show that the relation R in the set A of points in a plane given by $R = \{(P, Q):$ distance of the point P from the origin is same as the distance of the point Q from the origin}, is an equivalence relation. Further, show that the set of all points related to a point $P \neq (0, 0)$ is the circle passing through P with origin as centre.

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9. Show that the relation R defined in the set A of all triangles as $R = \{T_1 \quad T_2):T_1$ is similar to $T_2\}$ is equivalence relation. Consider three right angle triangles T_1 with sides $3, 4, 5, T_2$ with sides $5, 12, 13 \text{ and } T_3$ with sides $6, 8, 10.$ Which triangles among $T_1, T_2 \text{ and } T_3$ are related ?

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10. Show that the relation R defined in the set A of all polygons as $R = \{(P_1, P_2):P_1 \text{ and } P_2$ have same number of sides}, is an equivalence relation. What is the set of all elements in A related to the right angle triangle T with sides 3,4 and 5 ?

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11. Let L be the set of all lines in XY plane and R be the relation in L defined as $R = \{(L_1, L_2) : L_1$ is parallel to $L_2\}$. Show that R is an equivalence relation. Find the set of all lines related to the line $y = 2x + 4.$

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12. Let R be the relation in the set $\{(1, 2, 3, 4\}$ given by $R = \{(1, 2), (2, 2), (1, 1)(4, 4), (1, 3), (3, 3), (3, 2)\}.$ Choose the correct answer.

A. R is reflexive symmetric but not transitive.

B. R is reflexive and transitive but not symmetric.

C. R is symmetric and transitive but not reflexive.

D. R is an equivalence relation.

Answer: B

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13. Let R be the relation in the set N given by $R = \{(a, b), a = b - 2, b > 6\}$. Choose the correct answer.

A. $(2, 4) \in R$

B. $(3, 8) \in R$

C. $(6, 8) \in R$

D. $((8, 7) \in R$

Answer: B

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Exercise 1 2

1. Show that the function $f: R_* \to R_*$ defined by $f(x) = \frac{1}{x}$ is one-one and onto, where R_* is the set of all non-zero numbes. Is the result true, if the domain R_* is replaced by N with co-domain being same as R_* ?

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2. Check the injectivty and surjectiveity of the following functions :

(i) $f: N \to N$ given by $f(x) = x^2$

(ii) $f: Z \to Z$ given by $f(x) = x^2$

(iii) $f: R \to R$ given by $f(x) = x^2$

(iv) $f: N \to N$ given by $f(x) = x^3$

(v) $f: Z \to Z$ given by $f(x) = x^3$

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3. Prove that the Greatest Integer Function $f: R \rightarrow R,$ given by $f(x) = [x],$ is neither one-one nor onto, where [x] denotes the greatest integer less than or equal to x.

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4. Show that the Modulus Functions $f: R \rightarrow R,$ given by $f(x) = |x|,$ is neither one-one nor onto, whre |x| is x, if x is positive or 0 and |x| is -x, if x is negative.

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5. Show that the Signum Function $f: R \rightarrow R,$ given by

$f(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$ is neither one-one nor onto.

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6. Let $A = \{1, 2, 3\}, B = \{4, 5, 6, 7\}$ and let $f = \{(1, 4), (2, 5), (3, 6)\}$ be function from A to B. Show that f is one-one.

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7. In each of the following cases, state whether the function is one-one onto or bijective. Justify your answwer.

(i) $f: R \rightarrow R$ defined by $f(x) = 3 - 4x$

(ii) $f: R \rightarrow R$ defined by $f(x) = 1 + x^2$

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8. Let $f: N \rightarrow N$ be defined by $f(n) = \begin{cases} \frac{n+1}{2} & \text{if } n \text{ is odd} \\ \frac{n}{2} & \text{if n is even} \end{cases}$ for all $n \in N$.

State whether the function f is bijective. Justify your answer.

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9. Let $A = R - \{3\}$ and $B = R - \{1\}$. Consider the function $f: A \to B$ defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Is f one-one and onto ? Justify your answer.

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10. Let $f: R \to R$ be defined as $f(x) = x^4$. Choose the correct answer.

A. f is one-one onto

B. f is many-one onto

C. f is one-one but not onto

D. f is neither one-one nor onto.

Answer: D

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11. Let $f: R \to R$ be defined as $f(x) = 3x$. Choose the correct answer.

A. f is one-one ,onto

B. f is many-one, onto

C. f is one-one but not onto

D. f is neither one-one nor onto.

Answer: A

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Exercise 1 3

1. Let $f: \{1, 3, 4\} \to \{1, 2, 5\}$ and $g: \{1, 2, 5\} \to \{1, 3\}$ be given by $f = \{(1, 2), (3, 5), (4, 1)$ and $g = \{(1, 3), (2, 3), (5, 1)\}$. Write down gof.

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2. Let f, g and h be functions from R to R. Show that

$(f+g)oh = foh + goh$

$(f.\,g)oh = (fog).$ (goh) `

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3. Find gof and fog, if

(i) $f(x) = |x|$ and $g(x) = |5x - 2|$

(ii) $f(x) = 8x^3$ and $g(x) = x^{\frac{1}{3}}$.

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4. If $f(x) = \dfrac{(4x-3)}{(6x-4)}, x \neq \dfrac{2}{3},$ show that fof $(x) = x,$ for all $x \neq \dfrac{2}{3}$.

What is the inverse of f ?

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5. State with reason wheher following functins have inverse

(i) $f:\{1, 2, 3, 4\} \rightarrow \{10\}$ with

$f = \{(1, 10), (2, 10), (3, 10), (4, 10)\}$

(ii)$g:\{5, 6, 7, 8\} \rightarrow \{1, 2, 3, 4\}$ with

$g = \{(5, 4), (6, 3), (7, 4), (8, 2)\}$

(iii) $h:\{2, 3, 4, 5\} \rightarrow \{7, 9, 11, 13\}$ with

$h = \{(2, 7), (3, 9), (4, 11), (5, 13)\}$

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6. Show that $f:[-1, 1] \rightarrow R$, given by $f(x) = \dfrac{x}{(x+2)}$ is one-one. Find the inverse of the function $f:[-1, 1] \rightarrow$ R.

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7. Consider $f: R \rightarrow R$ given by $f(x) = 4x + 3.$ Show that f is invertible. Find the inverse of f.

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8. Consider f: $R_{+\rightarrow[4,\infty)}$ given by $f(x)=x^2+4$. Show that f is invertible with the inverse f^{-1} given by $f^{-1}(y)=\sqrt{y-4}$, where R_+ is the set of all non-negative real numbers.

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9. Consider $f: R_+ \rightarrow [-5, \infty)$ given by $f(x)=9x^2+6x-5$. Show that f is invertible with $f^{-1}(y)=\left(\frac{\sqrt{y+6}-1}{3}\right)$.

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10. Let $f: X \rightarrow Y$ be an invertible function. Show that f has uniques inverse.

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11. Consider $f:\{1, 2, 3\} \to \{a, b, c\}$ given by $f(1) = a$, $f(2) = b$ and $f(3) = c$. Find f^{-1} and show that $\left(f^{-1}\right)^{-1} = f$.

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12. Let $f: X \to Y$ be an invertible function. Show that the inverse of f^{-1} is f, i.e., $\left(f^{-1}\right)^{-1} = f$.

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13. If $f: R \to R$ be given by $f(x) = \left(3 - x^3\right)^{\frac{1}{3}}$, then fof (x) is

A. $x^{\frac{1}{3}}$

B. x^3

C. x

D. $\left(3 - x^3\right)$.

Answer: C

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14. Let $f: R - \left\{-\frac{4}{3}\right\} \rightarrow R$ be a function defined as $f(x) = \frac{4x}{3x+4}$. The inverse of f is the map g: Range $f \rightarrow R\left\{-\frac{4}{3}\right\}$ given by

A. $g(y) = \frac{3y}{3-4y}$

B. $g(y) = \frac{4y}{4-3y}$

C. $g(y) = \frac{4y}{3-4y}$

D. $g(g) = \frac{3y}{4-3y}$

Answer: B

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Exercise 1 4

1. Determine whether or not each of the defination of $*$ given below gives a binary opertion. In the even that $*$ is not a binary opertion, give justification for this.

(i) On Z^{+}, define $*$ by $a * b = a - b$

(ii) On Z^{+}, define $*$ by$a * b = ab$

(iii) On R, define $*$ by $a * b = ab^2$

(iv) On Z^{+}, define $*$ by $a * b = |a - b|$

(v) On Z^{+}, define $*$ by $a * b = a$

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2. Determine whether or not each of the defination of $*$ given below gives a binary opertion. In the even that $*$ is not a binary opertion, give justification for this.

(i) On Z^{+}, define $*$ by $a * b = a - b$

(ii) On Z^{+}, define $*$ by$a * b = ab$

(iii) On R, define $*$ by $a * b = ab^2$

(iv) On Z^{+}, define $*$ by $a*b=|a-b|$

(v) On Z^{+}, define $*$ by $a*b=a$

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3. Consider the binary opertion $\wedge$ on the set $\{1, 2, 3, 4, 5\}$ defined by $a \wedge b =$ min $\{a, b\}$. Write the opertion table of the opertion $\wedge$.

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4. Consider a binary opertion $*$ on the set $\{1, 2, 3, 4, 5\}$ given by the following multiplication table (Table 1.2)

(i) Compute $(2*3)*4$ and $2*(3*4)$

(ii) Is $*$ commutative ?

(iii) Compute $(2 * 3) * (4 * 5)$.

*	1	2	3	4	5
1	1	1	1	1	1
2	1	2	1	2	1
3	1	1	3	1	1
4	1	2	1	4	1
5	1	1	1	1	5

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5. Let $*'$ be the binary opertion on the set $\{1, 2, 3, 4, 5\}$ defined by $a\&'$ b = H.C.F. of a and b. Is the opertion $*'$ same as the opertion $*$ defined in Q.4 above ? Your answer

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6. Let $*$ be the binary opertion on N given by $a * b = L.C.M.$ of a and b. Find

(i) $5 * 7, 20 * 16$

(ii) Is $*$ commutative ?

(iii) Is $*$ associative ?

(iv) Find the identity of $*$?

(v) Which elements of N are invertible for the opertion $*$?

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7. Let A ={ 1, 2, 3,4, 5}, a*b = L.C.M of a and b is a binary operation on A, explain ?

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8. Let $*$ be the binary opertion on N defined by $a * b = H.C.F.$ of a and b. Is $*$ commutative ? Is $*$ associative ? Does there exist identity for this binary opertion on N ?

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9. Let $*$ be a binary opertion on the set Q of rational numbers as follows:

(i) $a \star b = a - b$

(ii) $a * b = a^2 + b^2$

(iii) $a * b = a + ab$

(iv) $a * b = (a - b)^2$

(v) $a * b = \frac{ab}{4}$

(vi) $a * b = ab^2$

Find which of the binary opertions are commutative and which are associative.

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10. Find which of the opertions given above has identity.

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11. Let $A = N \times N$ and $*$ be the binary opertion on A defined by

$(a, b) * (c, d) = (a + c, b + d)$

Show that $*$ is commutative and associative. Find the identity element for $*$ on A, if any.

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12. State whether the following statements are true or false, Justify.

(i) For an arbitraty binary opertion $*$ on a set $N, a * a = a \forall a \in N$.

(ii) If $*$ is a commutative binary opertion on N, then $a * (b * c) = (c * b) * a$

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13. Consider a binary opertion $*$ on N defined as $a * b = a^3 + b^3$. Choose the correct answer.

A. Is $*$ both associative and commutative ?

B. Is $*$ commutative but not associative ?

C. Is $*$ associative but not commutative ?

D. Is $*$ neither comutative nor associative ?

Answer: B

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Miscellaneous Exercise On Chapter 1

1. Let $f: R \rightarrow R$ be defined as $f(x) = 10x + 7$. Find the function $g: R \rightarrow R$ such that $gof = f0g = I_R$.

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2. Let $f: W \rightarrow W$ be defined as $f(n) = n - 1$, if n is odd and $f(n) = n + 1$, if n is even. Show that f is invertible. Find the inverse of f. Here, W is the set of all whole numbers.

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3. If $f: R \to R$ is defined by $f(x) = x^2 - 3x + 2$, find $f(f(x))$.

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4. Show that the function $f: R \to R\{x \in R: -1 < x < 1\}$ defined by $f(x) = \dfrac{x}{1+|x|}, x \in R$ is one one and onto function.

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5. Show that the function $f: R \to R$ given by $f(x) = x^3$ is injective.

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6. Give examples of two functions $f: N \to Z$ and $g: Z \to Z$ such that g o f is injective but g is not injective.

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7. Give examples of two functions $f: N \rightarrow N$ and $g: N \rightarrow N$ such g o f is onto but f is not onto.

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8. Given a non empty set X, consider P (X) which is the set of all subsets of X.

Define the relation R in P(X) as follows :

For subsets A, B in P(X), ARB if and only if $A \subset B$. Is R an equivalence relation on $P(X)$? Justify your answer.

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9. Find the number of all onto functins from the set $\{1, 2, 3. . , n\}$ to itself.

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10. Let $S = \{a, b, c\}$ and $T = \{1, 2, 3\}$. Find F^{-1} of the following F from S to T, if exists.

(i) $F = \{(a, 3), (b, 2), (c, 1)\}$

(ii) $F = \{(a, 2), (b, 1), (c, 1)\}$

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11. Consider the binary opertions $* : R \times R \rightarrow R$ and $o : R \times R \rightarrow R$ defined as $a * b = |a - b|$ and $aob = a, \forall a, b \in R$. Show that $*$ is commutative but not associative, o is associative but not commutative. Further, show that $\forall a, b, c \in R, a * (boc) = (a * b)o(a * c)$. [If it is so, we say that the opertion $*$ distributes over the opertion 0]. Does o distribute over $*$? Justify your answer.

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12. Given a non-empty set X, let $* : P(X) \times P(X) \rightarrow P(X)$ be defined as $A * B = (A - B) \cup (B - A), \forall A, B \in P(X)$. Show that the empty set ϕ is the identity for the opertion $*$ and all the elements A of $P(X)$ are invertible with $A^{-1} = A$.

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13. If A = {0,1,2,3,4,5} Define a binary operation * on A as a*b = $\begin{cases} a+b & \text{if} \quad a+b<6 \\ a+b-6 & \text{if} \quad a+b \geq 6 \end{cases}$, Show that 0 is the identity and each element $a \neq 0$ is invertible and find the inverse of a.

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14. Let $A = \{-1, 0, 1, 2\}, B = \{-4, -2, 0, 2\}$ and $f, g, A \to B$ be functions defined by $f(x) = x^2 - x, x \in A$ and $g(x) = 2\left|x - \frac{1}{2}\right| - 1, x \in A$ Are f and g equal ? Justify your answer.

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15. Let $A = \{1, 2, 3\}$. Then number of relations containing $(1, 2)$ and $(1, 3)$ which are reflexive ans symmetric but not transitive is

A. 1

B. 2

C. 3

D. 4

Answer: A

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16. Let $A = \{1, 2, 3\}$. Then number of equivalence relations containing $(1, 2)$ is

A. 1

B. 2

C. 3

D. 4

Answer: B

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17. Let $f: R \to R$ be the Signumb Function defined as

$$f(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$$

and $g: Ro \to R$ be the Greatest Integer Function given by $g(x) = [x]$, where $[x]$ is greatest integer less than or equal to x. Then does fog and gof coincide in $(0, 1]$?

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18. Number of binary opertions on the set $\{a, b\}$ are

A. 10

B. 16

C. 20

D. 8

Answer: B

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