MATHS

NCERT - NCERT MATHEMATICS(TELUGU)

VECTOR ALGEBRA

Example

1. Represent graphically a displacement of 40 km , 30° west of south .

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2. Classify the following measures as scalars and vectors.

(i) 5 seconds

(ii) $1000cm^3$

(iii) 10 Newton

(iv) 30km/hr

(v) $10g/cm^3$

(vi) 20m/s towards north

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3. Find the values of x,y and z so that the vectors $\vec{a} = x\hat{i} + 2\hat{j} + z\hat{k}$ and $\vec{b} = 2\hat{i} + y\hat{j} + \hat{k}$ are equal .

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4. Let $\vec{a} = \hat{i} + 2\hat{j}$ and $\vec{b} = 2\hat{i} + \hat{j}$. Is $|\vec{a}| = |\vec{b}|$? Are the vectors $\vec{a}$ and $\vec{b}$ equal?

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5. Find unit vector in the direction of vector $\bar{a} = (2\bar{i} + 3\bar{j} + \bar{k})$

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6. Find a vector in the direction of vector $\bar{a} = \bar{i} - 2\bar{j}$ has magnitude 7 units.

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7. Find the unit vector in the direction of the sum of the vectors

$\bar{a} = 2\bar{i} + 2\bar{j} - 5\bar{k}$ and $\bar{b} = 2\bar{i} + \bar{j} + 3\bar{k}$.

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8. Write direction ratios of the vector $\bar{r} = \bar{i} + \bar{j} - 2\bar{k}$ and hence calculate its direction cosines.

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9. Find the vector joining the points P(2,3,0) and Q(-1,-2,-4) directed from P to Q.

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10. Consider two points P and Q with position vectors $\overline{OP} = 3\bar{a} - 2\bar{b}$ and $\overline{OQ} = \bar{a} + \bar{b}$. Find the position vector of a point R which divides the line joining P and Q in the ratio 2:1

(i) internally.

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11. Show that the points $A\left(2\hat{i} - \hat{j} + \hat{k}\right), B\left(\hat{i} - 3\hat{j} - 5\hat{k}\right), C\left(3\hat{i} - 4j - 4\hat{k}\right)$ are vertices of a right angled triangle.

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12. Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes 1 and 2 respectively and when $\vec{a} \cdot \vec{b} = 1$.

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13. Find angle θ between the vectors $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$.

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14. If $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - 5\hat{k}$, then show that the vectors $\vec{a} + \vec{b}$ and $\vec{a} - b$ are perpendicular.

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15. Find the projection of the vector $\vec{a} = 2\hat{i} + 3\hat{j} + 2k$ on the vector $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$.

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16. Find $\left|\vec{a} - \vec{b}\right|$, if two vectors $\vec{a}$ and $\vec{b}$ are such that $\left|\vec{a}\right| = 2, \left|\vec{b}\right| = 3$ and $\vec{a} \cdot \vec{b} = 4$.

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17. If $\vec{a}$ is a unit vector and $\left(\vec{x} - \vec{a}\right) \cdot \left(\vec{x} + \vec{a}\right) = 8$, then find $\left|\vec{x}\right|$.

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18. For any two vectors $\vec{a}$ and $\vec{b}$, we always have $\left|\vec{a} \cdot \vec{b}\right| \leq \left|\vec{a}\right|\left|\vec{b}\right|$ (Cauchy- Schwartz inequality).

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19. Show that the points $A\left(-2\hat{i}+3\hat{j}+5\hat{k}\right), B\left(\hat{i}+2\hat{j}+3\hat{k}\right)$ and $C\left(7\hat{i}-\hat{k}\right)$ are collinear.

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20. Find $\left|\vec{a}\times\vec{b}\right|$, if $\vec{a}=2\hat{i}+\hat{j}+3\hat{k}$ and $\vec{b}=3\hat{i}+5\hat{j}-2\hat{k}$

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21. Find a unit vector perpendicular to each of the vector $\left(\vec{a}+\vec{b}\right)$ and $\left(\vec{a}-\vec{b}\right)$, where $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=\hat{i}+2\hat{j}+3\hat{k}$

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22. Find the area of atriangle having the points A(1,1,1), B (1,2,3) and C(2,3,1) as its vertices.

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23. Find the area of a parallelogram whose adjacent sides are given by the vectors $\vec{a} = 3\hat{i} + \hat{j} + 4\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$

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24. Write all the unit vectors in XY - plane.

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25. If $\hat{i}+\hat{j}+\hat{k}, 2\hat{i}+5\hat{j}, 3\hat{i}+2\hat{j}-3\hat{k}$ and $\hat{i}-6\hat{j}-\hat{k}$ are position vectors of points A,B,C and D respectively , then find the angle between $\overrightarrow{AB}$ and $\overrightarrow{CD}$. Deduce that $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear.

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26. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $\left|\vec{a}\right|=3, \left|\vec{b}\right|=4, \left|\vec{c}\right|=5$and each one of them being perpendicular to the sum of the other two , find $\left|\vec{a}+\vec{b}+\vec{c}\right|$.

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27. Three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ satisfy the condition $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. Evaluate the quantity $\mu = \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$, if $\left|\vec{a}\right| = 3, \left|\vec{b}\right| = 4$ and $\left|\vec{c}\right| = 2.$

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Exercise 10 1

1. Represent graphically a displacement of 40 km , 30° east of north.

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2. Classify the following measures as scalars and vectors .

(i) 10 kg

(ii) 2 meters north

(iii)40°

(iv) 40 watt

(v) 10^{19} coulomb

(vi) $20m/s^2$

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3. Classify the following as scalar and vector quantities.

(i) time period

(ii) distance

(iii) force

(iv) velocity

(v) work done

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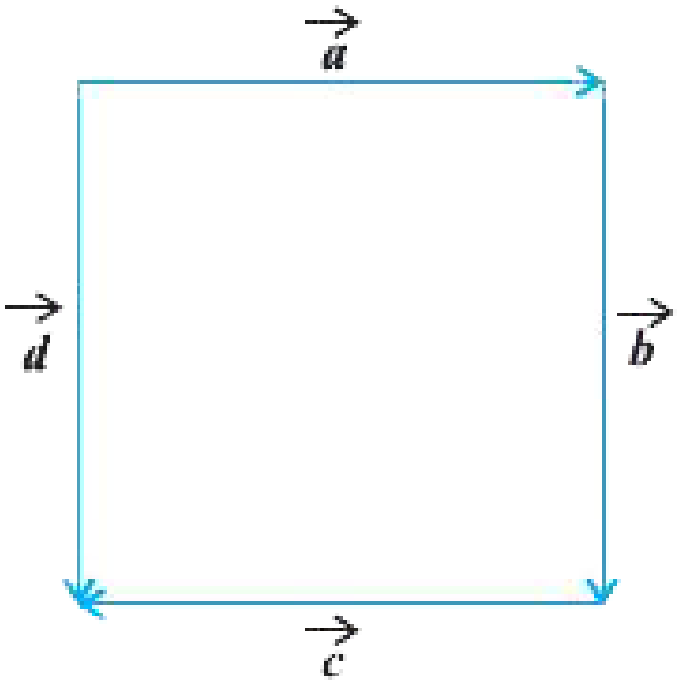
4. In Fig 10.6 (a square), identify the following vectors

(i) Coinitial

(ii) Equal

(iii) collinear but not equal



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5. Answer the followings true or false.

(i) $\vec{a}$ and $-\vec{a}$ are collinear.

(ii) Two collinear vectors are always equal in magnitude.

(iii) Two vectors having same magnitude are collinear.

(iv) Two collinear vectors having the same magnitude are equal.

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Exercise 10 2

1. Compute the magnitude of the following vectors :

$\vec{a} = \hat{i} + \hat{j} + k$, $\vec{b} = 2\hat{i} - 7\hat{j} - 3\hat{k}$, $\vec{c} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$

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2. Write two different vectors having same magnitude.

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3. Write two different vectors having same direction.

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4. Find the values of x and y so that the vectors $2\hat{i}+3\hat{j}$ and $x\hat{i}+y\hat{j}$ are equal.

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5. Find the scalar and vector components of the vector with initial point (2,1) and terminal point (-5,7).

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6. Find the sum of the vectors $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$.

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7. Find the unit vector in the direction of the vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$.

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8. Find the unit vector in the direction of vector $\overrightarrow{PQ}$, where P and Q are the points (1,2,3) and (4,5,6), respectively.

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9. For given vectors $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$, find the unit vector in the direction of the vector $\vec{a} + \vec{b}$.

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10. Find a vector in the direction of vector $5\hat{i} - \hat{j} + 2\hat{k}$ which has magnitude 8 units.

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11. Show that the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $-4\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear.

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12. Find the direction cosines of the vector $\hat{i} + 2\hat{j} + 3\hat{k}$.

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13. Find the direction cosines of the vector joining the points A(1,2,-3) and B(-1,-2,1), directed from A to B.

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14. Show that the vector $\hat{i} + \hat{j} + \hat{k}$ is equally inclined to the axes OX, OY and OZ.

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15. Find the position vector of a point R which divides the line joing two points P and Q whose position vectors are $\hat{i}+2\hat{j}-\hat{k}$ and $-\hat{i}+\hat{j}+\hat{k}$ repectively , in the ratio $2:1$

(i) nternally

(ii) externally

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16. Find the position vector of the mid point of the vector joining the points P(2,3,4) and Q(4,1,-2).

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17. Show that the points A,B and C with position vectors , $\vec{a}=3\hat{i}-4\hat{j}-4\hat{k}$, $\vec{b}=2\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}-3\hat{j}=5\hat{k}$

,respectively form the vertices of a right angled triangle.

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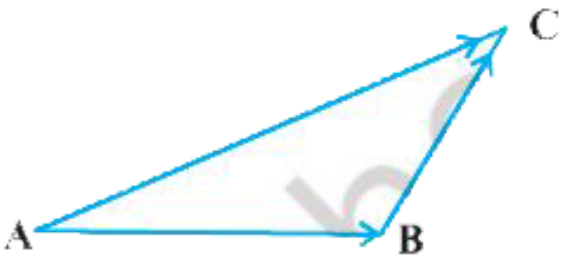
18. In triangle ABC (Fig 10.18), which of the following is not true :



A. $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \overrightarrow{0}$

B. $\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC} = \overrightarrow{0}$

C. $\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC} = \overrightarrow{0}$

D. $\overrightarrow{AB} - \overrightarrow{CB} + \overrightarrow{CA} = \overrightarrow{0}$

Answer: C

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19. If $\vec{a}$ and $\vec{b}$ are two collinear vectors , then which of the following are incorrect :

A. $\vec{b} = \lambda \vec{a}$, for some scalar λ

B. $\vec{a} = \pm \vec{b}$

C. the respective components of $\vec{a}$ and $\vec{b}$ are not proportional

D. both the vectors $\vec{a}$ and $\vec{b}$ have same direction , but different magnitudes.

Answer: B::C::D

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Exercise 10 3

1. Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 2, respectively having $\vec{a} \cdot \vec{b} = \sqrt{6}$.

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2. Find the angle between the vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$

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3. Find the projection of the vector $\hat{i} - \hat{j}$ on the vector $\hat{i} + \hat{j}$.

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4. Find the projection of the vector $\hat{i}+3\hat{j}+7\hat{k}$ on the vector $7\hat{i}-\hat{j}+8\hat{k}$.

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5. Show that each of the given three vectors is a unit vector.

$$\frac{1}{7}\left(2\hat{i}+3\hat{j}+6\hat{k}\right), \frac{1}{7}\left(3\hat{i}-6\hat{j}+2\hat{k}\right), \frac{1}{7}\left(6\hat{i}+2\hat{j}-3\hat{k}\right)$$

Also , show that they are mutually perpendicular to each other.

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6. Find $\left|\vec{a}\right|$ and $\left|\vec{b}\right|$, if $\left(\vec{a}+\vec{b}\right)\cdot\left(\vec{a}-\vec{b}\right)=8$ and $\left|\vec{a}\right|=8\left|\vec{b}\right|$.

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7. Evaluate the product $\left(3\vec{a} - 5\vec{b}\right) \cdot \left(2\vec{a} + 7\vec{b}\right)$.

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8. Find the magnitude of two vectors $\vec{a}$ and $\vec{b}$, having the same magnitude and such that the angle between them is 60° and their scalar product is $\frac{1}{2}$.

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9. Find $\left|\vec{x}\right|$, if for a unit vector $\vec{a}$, $\left(\vec{x} - \vec{a}\right) \cdot \left(\vec{x} + \vec{a}\right) = 12$.

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10. If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$, then find the value of λ.

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11. Show that $|\vec{a}|\vec{b} + |\vec{b}|\vec{a}$ is perpendicular to $|\vec{a}|\vec{b} - |\vec{b}|\vec{a}$, for any two nonzero vectors $\vec{a}$ and $\vec{b}$.

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12. If $\vec{a} \cdot \vec{a} = 0$ and $\vec{a} \cdot \vec{b} = 0$, then what can be concluded about the vector $\vec{b}$?

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13. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$.

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14. If either vector $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$, , then $\vec{a} \cdot \vec{b} = 0$. But the converes need not be true . Justify your answer with an example.

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15. If either vector A,B,C of a triangle ABC are (1,2,3),(-1,0,0), (0,1,2), respectively , then find $\angle ABC$. [$\angle ABC$ is the angle

between the vectors $\overrightarrow{BA}$ and $\overrightarrow{BC}$].

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16. Show that the points A(1,2,7),B(2,6,3) and C(3,10,-1) are collinear.

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17. Show that the vectors $2\hat{i}-\hat{j}+\hat{k}, \hat{i}-3\hat{j}-5\hat{k}$ and $3\hat{i}-4\hat{k}$ form the vertices of a right angled triangle.

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18. If $\vec{a}$ is a nonzero vector of mangitude 'a' and λ a nonzero scalar , then $\lambda\vec{a}$ is unit vector if

A. $\lambda = 1$

B. $\lambda = -1$

C. $a = |\lambda|$

D. $a = 1/|\lambda|$

Answer: D

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Exercise 10 4

1. Find $\left|\vec{a} \times \vec{b}\right|$, if $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$.

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2. Find a unit vector perpendicular to each of the vector $\vec{a} + \vec{b}$ and $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$.

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3. If a unit vector $\vec{a}$ makes angles $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$ and an acute angle θ with $\hat{k}$ then find θ and hence , the components of $\vec{a}$.

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4. Show that $\left(\vec{a}-\vec{b}\right)\times\left(\vec{a}+\vec{b}\right)=2\left(\vec{a}\times\vec{b}\right)$

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5. Find λ and μ if $\left(2\hat{i}+6\hat{j}+27\hat{k}\right)\times\left(\hat{i}+\lambda\hat{j}+\mu\hat{k}\right)=\vec{0}$.

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6. Given that $\vec{a}\cdot\vec{b}=0$ and $\vec{a}\times\vec{b}=\vec{0}$. What can you conclude about the vectors $\vec{a}$ and $\vec{b}$?

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7. Let the vectors $\vec{a},\vec{b}\vec{c}$ be given as $a_1\hat{i}+a_2\hat{j}+a_3\hat{k},b_1\hat{i}+b_2\hat{j}+b_3\hat{k}c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$. Then show

that $\vec{a} \times \left(\vec{b} + \vec{c}\right) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$

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8. If either $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$, then $\vec{a} \times \vec{b} = \vec{0}$. Is the converse true ? Justify your answer with an example.

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9. Find the area of the tirangle with vertices A (1,1,2),B (2,3,5) and C(1,5,5).

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10. Find the area the parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$.

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11. Let the vectors $\vec{a}$ and $\vec{b}$ be such that $\left|\vec{a}\right| = 3$ and $\left|\vec{b}\right| = \frac{\sqrt{2}}{3}$, then $\vec{a} \times \vec{b}$ is a unit vector , if the angle between $\vec{a}$ and $\vec{b}$ is

A. $\pi/6$

B. $\pi/4$

C. $\pi/3$

D. $\pi/2$

Answer: B

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12. Area of a rectangle having vertices A,B C and D with position vectors $-\hat{i}+\frac{1}{2}\hat{j}+4\hat{k}, \hat{i}+\frac{1}{2}\hat{j}+4\hat{k}$ and $-\hat{i}-\frac{1}{2}\hat{j}+4\hat{k}$, respectively is

A. $\frac{1}{2}$

B. 1

C. 2

D. 4

Answer: C

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Miscellaneous Exercise On Chapter 10

1. Write down a unit vector in XY-plane, making an angle of 30° with the positive direction of x-axis.

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2. Find the scalar components and magnitude of the vector joining the points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$.

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3. A girl walks 4 km towards west, then she walks 3 km in a direction 30° east of north and stops. Determine the girl's displacement from her initial point of departure.

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4. If $\vec{a} = \vec{b} + \vec{c}$, then is it true that $\left|\vec{a}\right| = \left|\vec{b}\right| + \left|\vec{c}\right|$? Justify your answer .

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5. Find the value of x for which $x\left(\hat{i} + \hat{j} + \hat{k}\right)$ is a unit vector.

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6. Find a vector of magnitude 5 units and parallel to the resultant of the vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$.

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7. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{j} - \hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$, find a unit vector parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$.

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8. Show that the points A(1,-2,-8),B(5,0,-2)andC(11,3,7) are collinear , and find the ratio in which B divides AC.

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9. Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $\left(2\vec{a} + \vec{b}\right)$ and $\left(\vec{a} - 3\vec{b}\right)$ externally in the ratio $1:2$ Also , show that P is the mid point of the line segment RQ.

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10. The two adjacent sides of a parallelogram are $2\hat{i} - 4\hat{j} + 5k$ and $\hat{i} - 2\hat{j} - 3\hat{k}$. Find the unit vector parallel to its diagonal Also , find its area.

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11. Show that the direction cosines of a vector equally inclined to the axes OX,OY and OZ are $\pm\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$.

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12. Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$

Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c} \cdot \vec{d} = 15$.

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13. The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one . Find the value of λ.

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14. If θ is the angle between two vectors $\vec{a}$ and $\vec{b}$ then $\vec{a} \cdot \vec{b} \geq 0$ only when

A. $0 < \theta < \frac{\pi}{2}$

B. $0 \leq \theta \leq \frac{\pi}{2}$

C. $0 < \theta < \pi$

D. $0 \leq \theta \leq \pi$

Answer: B

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15. Let $\vec{a}$ and $\vec{b}$ be two unit vectors and θ is the angle between them Then $\vec{a} + \vec{b}$ is a unit vector if

A. $\theta = \frac{\pi}{4}$

B. $\theta = \frac{\pi}{3}$

C. $\theta = \frac{\pi}{2}$

D. $\theta = \frac{2\pi}{3}$

Answer: D

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16. The value of $\hat{i}.\left(\hat{j}\times\hat{k}\right)+\hat{j}\cdot\left(\hat{i}\times\hat{k}\right)+\hat{k}\cdot\left(\hat{i}\times\hat{j}\right)$ is

A. 0

B. -1

C. 1

D. 3

Answer: C

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17. If θ is the angle between any two vectors $\vec{a}$ and $\vec{b}$, then $\left|\vec{a} \cdot \vec{b}\right| = \left|\vec{a} \times \vec{b}\right|$ when θ is equal to

A. 0

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2}$

D. π

Answer: B

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