MATHS

BOOKS - RD SHARMA MATHS (ENGLISH)

MATHEMATICAL INDUCTION

Others

1. Prove that for $n \in N, 10^n + 3.\ 4^{n+2} + 5$ is divisible by 9 .

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2. Prove by induction that the sum of the cubes of three consecutive natural numbers is divisible by 9.

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3. A sequence $a_1, a_2, a_3, \ldots$ is defined by letting $a_1 = 3 \; and \; a_k = 7a_{k-1}$ for natural numbers $k, k \geq 2.$ Show that $a_n = 3.7_n - 1$ for all $n \in N$.

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4. A sequence $x_1, x_2, x_3, \ldots.$ is defined by letting $x_1 = 2$ and $x_k = \frac{x_{k-1}}{k}$ for all natural numbers $k, k \geq 2$ Show that $x_n = \frac{2}{n!}$ for all $n \in N.$

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5. Show by the Principle of Mathematical induction that the sum S_n, of the nterms of the series $1^2 + 2 \times 2^2 + 3^2 + 2 \times 4^2 + 5^2 + 2 \times 6^2 + 7^2 + \ldots.$ is given by $S_n = \begin{cases} \frac{n(n+1)^2}{2} \end{cases}$, if n is even , then $\frac{n^2(n+1)}{2}$, if n is odd

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6. Prove that the number of subsets of a set containing n distinct elements is 2^n for all $n \in N$.

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7. Using the principle of mathematical induction prove that :

$1.3 + 2.3^2 + 3.3^3 + \quad + n.3^n = \frac{(2n-1)3^{n+1} + 3}{4}$ for all $n \in N$.

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8. Prove by the principle of mathematical induction that for all $n \in N$, $n^2 + n$ is even natural number.

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9. Using the principle of mathematical induction prove that

$1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \frac{1}{1+2+3+4} + \quad + \frac{1}{1+2+3+ \quad + n} = \frac{2n}{n+}$

for all $n \in N$

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10. Prove by induction that the sum $S_n = n^3 + 3n^2 + 5n + 3$ is divisible by 3 for all $n \in N$.

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11. Using the principle of mathematical induction prove that

$$\frac{1}{1.\,2.\,3} + \frac{1}{2.\,3.\,4} + \frac{1}{3.\,4.\,5} + \; + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$$

for all $n \in N$

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12. Using the principle of mathematical induction. Prove that $(x^n - y^n)$ is divisible by $(x - y)$ for all $n \in N$.

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13. Using the principle of mathematical induction prove that $41^n - 14^n$ is a multiple of 27 .

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14. Using the principle of mathematical induction, prove that $\left(2^{3n} - 1\right)$ is divisible by 7 for all $n \in N$

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15. Using principle of mathematical induction prove that $\sqrt{n} < \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots\ldots + \frac{1}{\sqrt{n}}$ for all natural numbers $n \geq 2.$

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16. Prove that: $1 + 2 + 3 + \ldots. + n < \frac{(2n+1)^2}{8}$ for all $n \in N.$

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17. Prove that: $1^2 + 2^2 + 3^2 \ldots . + n^2 > \frac{n^3}{3}, n \in N$

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18. A sequence $x_0, x_1, x_2, x_3, \cdot\cdot$ is defined by letting $x_0 = 5$ and $x_k = 4 + x_{k-1}$ for all natural number k. Show that $x_n = 5 + 4n$ for all $n \in N$ using mathematical induction..

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19. Prove by the principle of mathematical induction that $n < 2^n$ for all $n \in N$.

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20. Prove by the principle of mathematical induction that for all $n \in N, 3^{2n}$ when divided by 8 , the remainder is always 1.

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21. Using the principle of mathematical induction, prove that : $1.\ 2.\ 3 + 2.\ 3.\ 4 + \ + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$ for all $n \in N$.

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22. Prove the following by using the principle of mathematical induction for all $n \in N : 1^3 + 2^3 + 3^3 + \cdot\cdot\dot{} + n^3 = \left(\frac{n(n+1)}{2}\right)^2$

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23. Prove that: $\left(1+\frac{1}{1}\right)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{n}\right)=(n+1)$ for all $n \in N$.

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24. Using principle of MI prove that $2.7^n + 3.5^n - 5$ is divisible by 24

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25. Prove by the principle of mathematical induction that $\frac{n^5}{5}+\frac{n^3}{3}+\frac{7n}{15}$ is a natural number for all $n \in N$.

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26. For all positive integer n, prove that $\frac{n^7}{7}+\frac{n^5}{5}+\frac{2}{3}n^3-\frac{n}{105}$ is an integer

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27. If $P(n)$ is the statement $2^{3n} - 1$. Is an integral multiple $7,$ and if $P(r)$ is true, prove that $P(r+1)$ is true.

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28. Let $P(n)$ be the statement $3^\wedge \mathrm{n} > \mathrm{n}$. If $P(n)$ is true, $P(n+1)$ is also true.

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29. If P(n) is the statement $n^2 > 100$, prove that whenever $P(r)$ is true, $P(r+1)$ is also true.

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30. Prove by the principle of mathematical induction that for all $n \in N$:

$$1^2 + 2^2 + 3^2 + \quad + n^2 = \frac{1}{6}n(n+1)(2n+1)$$

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31. Prove by the principle of mathematical induction that for all $n \in N$:

$$1 + 4 + 7 + \quad + (3n - 2) = \frac{1}{2}n(3n - 1)$$

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32. Prove by the principle of mathematical induction that: $n(n+1)(2n+1)$ is divisible by 6 for all $n \in N$.

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33. Prove by the principle of mathematical induction that for all $n \in N$, $n^2 + n$ is even natural number.

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34. Prove that : $\cos^2\alpha+\cos^2(\alpha+\beta)-2\cos\alpha\cos\beta\cos(\alpha+\beta)=\sin^2\beta$

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35. Prove that $\frac{1}{n+1}+\frac{1}{n+2}+\ldots+\frac{1}{2n}>\frac{13}{24}$,for all natural number $n>1.$

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36. Prove the following by using the principle of mathematical induction for all $n\in N:(2n+7)<(n+3)^2$.

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37. Prove by induction the inequality $(1+x)^n \geq 1+nx$ whenever x is positive and n is a positive integer.

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38. If $P(n)$ is the statement n^3+n is divisible 3 is the statement $P(3)$ true ? Is the statement $P(4)$ true?

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39. If $P(n)$ is the statement $n(n+1)(n+2)$ is divisible is 12 prove that the statements $P(3)$ and $P(4)$ are true, but that $P(5)$ is not true.

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40. Let $P(n)$ be the statement 7 divides $\left(2^{3n}-1\right)$. What is $P(n+1)$?

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41. If $P(n)$ is the statement $n(n+1)$ is even, then what is $P(3)$?

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42. If $P(n)$ is the statement n^3+n is divisible by 3, prove that $P(3)$ is true but $P(4)$ is not true.

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43. If $P(n)$ is the statement n^2+n is even, and if $P(r)$ is true then $P(r+1)$ is true.

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44. If $P(n)$ is the statement $2^n \geq 3n$, and if $P(r)$ is true, prove that $P(r+1)$ is true.

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45. Given an example of a statement $P(n)$ such that it is true of all $n \epsilon N$.

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46. If $P(n)$ is the statement $n^2 - n + 41$ is prime. Prove that $P(1),\ P(2) and\ P(3)$ are true. Prove also that $P(41)$ is not true.

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47. Given an example of a statement $P(n)$ which is true for all $n \geq 4$ but $P(1),\ P(2) and\ P(3)$ are not true. Justify your answer.

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48. Prove by the principle of mathematical induction that for all $n \in N$:

$$\frac{1}{1.\ 3} + \frac{1}{3.5} + \frac{1}{5.7} + + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$$

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49. Using principle of mathematical induction prove that $x^{2n} - y^{2n}$ is divisible by $x + y$ for all $nbelongs \to N$.

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50. Prove by the principle of induction that for all nN, $\left(10^{2n-1} + 1\right)$ is divisible by 11.

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51. Prove by induction that $4 + 8 + 12 + \quad + 4n = 2n(n + 1)$ for all nN.

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52. prove that

$\cos\alpha\cos 2\alpha\cos 4\alpha......\cos\left(2^{n-1}\alpha\right)=\dfrac{\sin 2^n\alpha}{2^n\sin\alpha}f\text{ or }\ all\,n\in N$

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53. Let $U_1=1,\ U_2=1\,and\,U_{n+2}=U_{n+1}+U_nf\text{ or }\ n\geq 1.$ use mathematical induction to show that:

$U_n=\dfrac{1}{\sqrt{5}}\left\{\left(\dfrac{1+\sqrt{5}}{2}\right)^n-\left(\dfrac{1-\sqrt{5}}{2}\right)^n\right\}f\text{ or }\ all\,n\geq 1.$

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54. Prove the following by the principle of mathematical induction: $1+2+3+\ +n=\dfrac{n(n+1)}{2}i\dot{e}$, the sum o the first n natural numbers is $\dfrac{n(n+1)}{2}.$

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55. Prove the following by the principle of mathematical induction:

$$1^2 + 2^2 + 3^2 + \quad + n^2 = \frac{n(n+1)(2n+1)}{6}$$

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56. Prove the following by the principle of mathematical induction:

$$1 + 3 + 3^2 + \quad + 3^{n-1} = \frac{3^n - 1}{2}$$

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57. Using the principle of mathematical induction, prove that

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \ldots + \frac{1}{n(n+1)} = \frac{n}{(n+1)}.$$

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58. Prove the following by the principle of mathematical induction:

$1 + 3 + 5 + \quad + (2n-1) = n^2 i.e.$ the sum of first n odd natural numbers is n^2.

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59. Prove the following by the principle of mathematical induction:

$$\frac{1}{2.5}+\frac{1}{5.8}+\frac{1}{8.11}+ \; + \frac{1}{(3n-1)(3n+2)}=\frac{n}{6n+4}$$

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60. Prove the following by the principle of mathematical induction:

$$\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+\ldots.+\frac{1}{(3n-2)(3n+1)}=\frac{n}{3n+1}$$

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61. Prove the following by the principle of mathematical induction:

$$\frac{1}{3.5}+\frac{1}{5.7}+\frac{1}{7.9}+\frac{1}{(2n+1)(2n+3)}=\frac{n}{3(2n+3)}$$

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62. Prove the following by the principle of mathematical induction: $\frac{1}{3.7}+\frac{1}{7.11}+\frac{1}{11.15}+\dots.+\frac{1}{(4n-1)(4n+3)}=\frac{n}{3(4n+3)}$

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63. Prove the following by the principle of mathematical induction: $1.2+2.2^2+3.2^3+\ \ +n.2^n=(n-1)2^{n+1}+2$

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64. Prove the following by the principle of mathematical induction: $2+5+8+11+\ \ +(3n-1)=\frac{1}{2}n(3n+1)$

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65. Prove the following by the principle of mathematical induction: $1.3+3.5+\ \ +(2n-1)(2n+1)=\frac{n(4n^2+6n-1)}{3}$

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66. Prove the following by the principle of mathematical induction:

$$1.\ 2 + 2.\ 3 + 3.\ 4 + \ + n(n+1) = \frac{n(n+1)(n+2)}{3}$$

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67. Prove the following by the principle of mathematical induction:

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ + \frac{1}{2^n} = 1 - \frac{1}{2^n}$$

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68. Prove the following by the principle of mathematical induction:

$$1^2 + 3^2 + 5^2 + \ + (2n-1)^2 = \frac{1}{3}n(4n^2 - 1)$$

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69. Prove the following by the principle of mathematical induction:

$a + (a + d) + (a + 2d) + \; + (a + (n - 1)d) = \frac{n}{2}[2a + (n - 1)d]$

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70. Prove the following by the principle of mathematical induction:

$5^{2n} - 1$ is divisible by 24 for all $n \in N$.

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71. Prove the following by the principle of mathematical induction:

$3^{2n} + 7$ is divisible by 8 for all $n \in N$.

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72. Prove the following by the principle of mathematical induction:

$5^{2n+2} - 24n - 25$ is divisible 576 for all n

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73. Prove the following by the principle of mathematical induction: $3^{2n+2} - 8n - 9$ is divisible 8 for all $n \in N$.

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74. Prove the following by the principle of mathematical induction: $(ab)^n = a^n b^n$ for all $n \in N$.

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75. Prove the following by using the principle of mathematical induction for all $n \in N$:$n(n+1)(n+5)$is a multiple of 3.

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76. Prove the following by the principle of mathematical induction:

$7^{2n} + 2^{3n-3}.\,3^{n-1}$ is divisible 25 for all $n \in N$.

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77. Prove the following by the principle of mathematical induction:

$2.\,7^n + 3.\,5^n - 5$ is divisible 24 for all $n \in N$.

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78. Prove the following by the principle of mathematical induction:

$11^{n+2} + 12^{2n+1}$ is divisible 133 for all $n \in N$.

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79. Prove the following by the principle of mathematical induction:

$n^3 - 7n + 3$ is divisible 3 for all $n \in N$.

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80. Prove the following by the principle of mathematical induction: $1+2+2^2 \ldots\ldots + 2^n = 2^{n+1} - 1$ for all $n \in N$.

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81. Prove the following by the principle of mathematical induction: $7+77+777+ \quad + 777 + \quad + \ddot{n} - digits7 = \frac{7}{81}\left(10^{n+1} - 9n - 10\right)$ for all $n \in N$

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82. Prove the following by the principle of mathematical induction: $\frac{n^7}{7} + \frac{n^5}{5} + 2\frac{n^3}{3} - \frac{n}{105}$ is a positive integer for all $n \in N$.

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83. Prove the following by the principle of mathematical induction:

$\frac{n^{11}}{11} + \frac{n^5}{5} + \frac{n^3}{3} + \frac{62}{165}n$ is a positive integer for $n \in \mathbb{N}$.

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84. Prove the following by the principle of mathematical induction:

$$\frac{1}{2}\tan\left(\frac{x}{2}\right) + \frac{1}{4}\tan\left(\frac{x}{4}\right) + \ldots + \left(\frac{1}{2^n}\right)\tan\left(\frac{x}{2^n}\right) = \left(\frac{1}{2^n}\right)\cot\left(\frac{x}{2^n}\right) - \text{co}$$

for all n belongs to N.

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85. Prove the following by the principle of mathematical induction:

$$\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\left(1 - \frac{1}{4^2}\right) - - - - - - \left(1 - \frac{1}{n^2}\right) = \frac{n+1}{2n}$$

for all natural numbers $n \geq 2$.

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86. Prove the following by the principle of mathematical induction: $\frac{(2n)!}{2^{2n}(n!)^2} \leq \frac{1}{\sqrt{3n+1}}$ for all $n \in N$.

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87. Prove the following by the principle of mathematical induction: $x^{2n-1} + y^{2n-1}$ is divisible by $x + y$ for all $n \in \mathbb{N}$.

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88. Prove that: $\sin x + \sin 3x + + \sin(2n-1)x = \frac{\sin^2 nx}{\sin x}$ for all $n \in \mathbb{N}$

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89. Given $a_1 = \frac{1}{2}\left(a_0 + \frac{A}{a_0}\right)$, $a_2 = \frac{1}{2}\left(a_1 + \frac{A}{a_1}\right)$ and $a_{n+1} = \frac{1}{2}\left(a_n + \frac{A}{a_n}\right)$ for $n \geq 2$, where $a > 0, A > 0$. prove that

$$\frac{a_n - \sqrt{A}}{a_n + \sqrt{A}} = \left(\frac{a_1 - \sqrt{A}}{a_1 + \sqrt{A}}\right)2^{n-1}.$$

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90. Let $P(n)$ be the statement: $2^n \geq 3n$. If $P(r)$ is true, show that $P(r+1)$ is true. Do you conclude that $P(n)$ is true for all $n \in N$

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91. The distributive law from algebra states that for all real numbers c,a1 and a2,we have $c(a_1 + a_2) = ca_1 + ca_2$ Use this law and mathematical induction to prove that,for all natural numbers,$n \geq 2$,if $c, a_1, a_2, \ldots, $ an are any real numbers,then $c(a_1 + a_2 + \ldots. + a_n) = ca_1 + ca_2 + \ldots + ca_n$

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92. State First principle of mathematical induction

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93. Write the set of values of n for which the statement $P(n): 2n < n!$ is true.

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94. Let us prove the following equality using the second principle: For any natural number n , 1 + 3 + ... + (2n + 1) = (n + 1)2.

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95. If $P(n): 2 \times 4^{2n+1} - 3^{3n+1}$ is divisible byλ for all $n \in N$ is true, then find the value of λ.

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96. If $x^n - 1$ is divisible by $x - \lambda$, then the least prositive integral value of λ is 1 b. 3 c. 4 d. 2

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97. For all $n \in N, 3 \times 5^{2n+1} + 2^{3n+1}$ is divisible by a.19 b. 17 c. 23 d. 25

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98. If $10^n + 3 \times 4^{n+2} + \lambda$ is divisible by 9 or all natural numbers, then the least positive integral value of λ is

a. 5 b. 3 c. 7 d. 1

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99. Let $P(n): 2^n < (1 \times 2 \times 3 \times \quad \times n)$. Then the smallest positive integer for which P(n) is true is a.1 b. 2 c. 3 d. 4

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100. A student was asked to prove a statement by induction. He proved (i) P(5) is true and (ii) truth of P(n) => truth of P(n+1), n $\in$ N . On the basis of this, he could conclude that P(n) is true

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101. If $P(n): 49^n + 16^n + \lambda$ is divisible by 64 for $n \in N$ is true, then the least negative integral value of λ is a.-3 b. -2 c. -1 d. -4

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