MATHS

BOOKS - RD SHARMA MATHS (ENGLISH)

BINARY OPERATIONS

Others

1. Discuss the commutativity and associativity of the binary operation . on R defined by $a \cdot b = a - b + ab$ for all $a, b \in R$, where on RHS we have usual addition, subtraction and multiplication of real numbers.

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2. If $a \cdot b = a^2 + b^2$, then the value of $(4 \cdot 5) \cdot 3$ is

A. $41^2 + 3^2$

B. $3^2 + 9^2$

C. $4^2 + 5^2 + 3^2$

D. None Of These

Answer: A

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3. Let * be a binary operation on set of integers I, defined by a*b=2a+b-3. Find the value of 3*4.

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4. Discuss the commutativity and associativity of the binary operation * on R defined by $a \cdot b = \frac{ab}{4} f \text{ or } alla, b \in R \cdot$

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5. Define a binary operation * on the set $A = \{0,\ 1,\ 2,\ 3,\ 4,\ 5\}$ as a * b=a+b(mod 6) Show that zero is the identity for this operation and each element a of the set is invertible with $6 - a$ being the inverse of $a \cdot$

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6. Consider the set $S = \{1, -1, i, -i\}$ for fourth roots of unity. Construct the composition table for multiplication on S and deduce its various properties.

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7. On $R - [1]$, a binary operation * is defined by $a \cdot b = a + b - ab$. Prove that * is commutative and associative. Find the identity element for * on $R - [1]\cdot$ Also, prove that every element of $R - [1]$ is invertible.

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8. Let * be a binary operation on Q_0 (set of non-zero rational numbers) defined by $a \cdot b = \frac{3ab}{5}$ for all

$a, b \in Q_0$.Find the identity element.

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9. If the binary operation * on the set Z of integers is defined by $a \cdot b = a + 3b^2$, find the value of $2 \cdot 4$.

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10. Show that the operation $\vee$ and $\wedge$ on R defined as $a \vee b =$ Maximum of a and b ; $a \wedge b =$ Minimum of a and b are binary operations of R.

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11. Let n be a positive integer. Prove that the relation R on the set Z of all integers numbers defined by $(x, y) \in R \Leftrightarrow x - y$ is divisible by n, is an equivalence relation on Z.

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12. Define a binary operation $*$ on the set $A = \{1, 2, 3, 4\}$ as $a * b = ab \pmod{5}$. Show that 1 is the identity for $*$ and all elements of the set A are invertible with $2^{-1} = 3$ and $4^{-1} = 4$.

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13. On the set $R - \{-1\}$ a binary operation $\cdot$ is defined by $a \cdot b = a + b + ab$ for all $a, b \in R - 1\{-1\}$. Prove that * is commutative as well as associative on $R - \{-1\}$. Find the identity element and prove that every element of $R - \{-1\}$ is invertible.

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14. Q^{+} denote the set of all positive rational numbers. If the binary operation . on Q^{+} is defined as $a.\,b = \frac{ab}{2}$, then the inverse of 3 is (a) $\frac{4}{3}$ (b) 2 (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

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15. Let $' \cdot '$ be a binary operation on Q_0 (set of all non-zero rational numbers) defined by $a \cdot b = \frac{ab}{4}$ for all $a, b \in Q_0$. Then, find the identity element in Q_0 inverse of an element in Q_0.

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16. On the power set P of a non-empty set A, we define an operation $*$ by $X \cdot Y = \left(X^{-\cap}Y\right) \cup \left(X \cap Y^{-}\right)$ Then which are the following statements is true about 1) commutative and associative without an identity 2)commutative but not associative without an identity 3)associative but not commutative without an identity 4) commutative and associative with an identity

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17. If the binary operation * on Z is defined by $a \cdot b = a^2 - b^2 + ab + 4$, then value of $(2 \cdot 3) \cdot 4$ is`

A. 21

B. 33

C. 37

D. 41

Answer: B

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18. Is * defined by $a^*b = \frac{a+b}{2}$ is binary operation on Z.

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19. Let $' \cdot '$ be a binary operation on N given by $a \cdot b = L\dot{C}\dot{M}a\dot{,} b$ for all $a, b \in N$. Find $5 \cdot 7, 20 \cdot 16$ (ii) Is * commutative? Is * associative? Find the identity element in N Which element of N are invertible? Find them.

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20. On the set $M = A(x) = \{[\times \quad \times] : x \in R\} of 2x2$ matrices, find the identity element for the multiplication of matrices as a binary operation. Also, find the inverse of an element of M.

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21. Let $+_6$ (addition modulo 6) be a binary operation on $S = \{0, 1, 2, 3, 4, 5\}$. Write the value of $2 +_6 4^{-1} +_6 3^{-1}$.

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22. Let $A = QxQ$ and let * be a binary operation on A defined by $(a, b) \cdot (c, d) = (ac, b + ad)$ for $(a, b), (c, d) \in A$. Then, with respect to * on A Find the identity element in A Find the invertible elements of A.

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23. Let $A = NxN,$ and let * be a binary operation on A defined by $(a, b) \cdot (c, d) = (ad + bc, bd)$ for all $(a, b), c, d) \in NxN.$ Show that : $' \cdot '$ is commutative on A $' \cdot'$ is associative on$A$ A has no identity element.

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24. Discuss the commutativity and associativity of binary operation * defined on Q by the rule $a \cdot b = a - b + ab$ for all $a, b \in Q$

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25. Let * be a binary operation on N, the set of natural numbers, defined by $a \cdot b = \ a^b$ for all $a, b \in N$. Is $' \cdot '$ associative or commutative on N?

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26. Let *, be a binary operation on N, the set of natural numbers defined by $a \cdot b = a^b$, for all $a, b \in N$. is * associative or commutative on N?

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27. On Q, the set of all rational numbers, a binary operation * is defined by $a \cdot b = \dfrac{ab}{5}$ for all $a, b \in Q$. Find the identity element for * in Q. Also, prove that every non-zero element of Q is invertible.

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28. Let * be a binary operation on set $Q-[1]$ defined by $a \cdot b = a + b - ab$ for all$a, b \in Q-[1]$. Find the identity element with respect to $\cdot onQ$. Also, prove that every element of $Q-[1]$ is invertible.

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29. Find $\frac{dy}{dx}$ if $3x - 4y = \sin x$

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30. Let $S = \{0, 1, 2, 3, 4\}$ and * be an operation on S defined by $a \cdot b = r$, where r is the last non-negative remainder when $a + b$ is divided by 5. Prove that * is a binary operation on S.

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31. Let $S = (0, 1, 2, 3, 4,)$ and * be an operation on S defined by $a \cdot b = r$, where$r$ is the least non-negative remainder when $a + b$ is divided by 5. Prove that * is a binary operation on S.

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32. Define a binary operation * on the set $A = \{0, 1, 2, 3, 4, 5\}$ given by $a * b = ab(\mod 6)$. Show that 1 is the identity for *. 1 and 5 are the only invertible elements with $1^{-1} = 1$ and $5^{-1} = 5$

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33. Let $S = \{a + \sqrt{2}b : a, b \in Z\}$. Then prove that an operation * on S defined by $(a_1 + \sqrt{2}b_1) \cdot (a_2 + \sqrt{2}b_2) = (a_1 + b_2) + \sqrt{2}(b_1 + b_2)$ for all $b_1, a_2 \in Z$ is binary operation ofn S.

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34. Let A be a set having more than one element. Let $' \cdot '$ be a binary operation on A defined by $a \cdot b = \sqrt{a^2 + b^2}$ for all $a, b, \in A$. Is $' \cdot '$ commutative or associative on A?

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35. Let $A = NxNand' \cdot '$ be a binaryoperation on A defined by $(a, b) \cdot (C, d) = (ac, bd)$ for all $a, b, c, d, \in N$. Show that $' \cdot '$ is commutative and associative binary operation on A.

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36. Let S be the set of all rational numbers except 1 and * be defined on S by $a \cdot b = a + b - ab$, for all $a, b \in S$. Find its identity element

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37. Q, the set of all rational number, * is defined by $a \cdot b = \frac{a - b}{2}$, show that * is no associative.

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38. Find the identity element in set Q^{+} of all positive rational numbers for the operation * defined by $a \cdot b = \frac{ab}{2}$ for all $a, b \in Q^{+}$.

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39. If * defined on the set R of real numbers by $a \cdot b = \frac{3ab}{7}$, find the identity element in R for the binary operation *.

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40. Let S be a non-empty set and $P(s)$ be the power set of set S. Find the identity element for all union U as a binary operation on $P(S)$.

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41. If * is defined on the set R of all real numbers by $a \cdot b = \sqrt{a^2 + b^2}$, find the identity element in R with respect to *.

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42. If the binary operation . on the set Z is defined by a.b $= a + b - 5,$ then find the identity element with respect to ..

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43. Let * be a binary operation o Q defined by a*b= $\frac{ab}{4}$ for all a,b $\in$ Q ,find identity element in Q

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44. If the binary operation o is defined on the set Q^+ of all positive rational numbers by $aob = \frac{ab}{4}$. Then, $3o\left(\frac{1}{5}o\frac{1}{2}\right)$ is equal to $\frac{3}{160}$ (b) $\frac{5}{160}$ (c) $\frac{3}{10}$ (d) $\frac{3}{40}$

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45. Let $S = \{a + \sqrt{2}\, b : a,\ b \in Z\}$. Then, prove that an operation $*$ on S defined by $(a_1 + \sqrt{2} b_1) \cdot (a_2 + \sqrt{2} b_2) = (a_1 + a_2) + \sqrt{2}(b_1 + b_2)$ for all $a_1,\ a_2,\ b_1,\ b_2 \in Z$ is a binary operation on S.

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46. Let $S = \{1,\ 2,\ 3,\ 4\}$ and $\cdot$ be an operation on S defined by $a \cdot b = r,$ where r is the least non-negative remainder when product is divided by 5. Prove that $\cdot$ is a binary operation on S.

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47. Let $S = (0, 1, 2, 3, 4,)$ and * be an operation on S defined by $a \cdot b = r$, where$r$ is the least non-negative remainder when $a + b$ is divided by 5. Prove that * is a binary operation on S.

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48. Show that the operation $\vee$ and $\wedge$ on R defined as $a \vee b =$ Maximum of a and b ; $a \wedge b =$ Minimum of a and b are binary operations of R.

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49. On the set Q of all rational numbers an operation * is defined by $a \cdot b = 1 + ab$. Show that * is a binary operation on Q.

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50. On the set W of all non-negative integers $\cdot$ is defined by $a \cdot b = a^b$. Prove that $\cdot$ is not a binary operation on

W.

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51. On the set C of all complex numbers an operation $'o'$ is defined by $z_1 \, o \, z_2 = \sqrt{z_1 z_2}$ for all $z_1, \; z_2 \in C$. Is o a binary operation on C ?

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52. Let M be the set of all 2X2 real singular matrices . On M , let $\cdot$ be an operation defined by, $A \cdot B = AB$ for all $A, \; B \in M$. Prove that $\cdot$ is a binary operation on M.

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53. Determine whether * on N defined by $a \cdot b = a^b$ for all $a,\ b \in N$ define a binary operation on the given set or not:

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54. Determine whether O on Z defined by $a\,O\,b = a^b$ for all $a,\ b \in Z$ define a binary operation on the given set or not:

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55. Determine whether * on N defined by $a \cdot b = a + b - 2$ for all $a,\ b \in N$ define a binary operation on the given set or not:

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56. Determine whether $' \times_6 '$ on $S = \{1, 2, 3, 4, 5\}$ defined by $a \times_6 b =$ Remainder when ab is divided by 6 define a binary operation on the given set or not:

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57. Determine whether $' +_6 '$ on $S = \{0, 1, 2, 3, 4, 5\}$ defined by

$a +_6 b = \{a + b, \quad \text{if} \quad a + b < 6 a + b - 6, \quad \text{if} \quad a + b \geq 6$

define a binary operation on the given set or not:

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58. 'o' on N defined by a o b= a b + b a for all a, b∈N define a binary operation on the given set or not:

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59. $' \cdot '$ on Q defined by $a \cdot b = \dfrac{a-1}{b+1}$ for all $a, \ b \in Q$ define a binary operation on the given set or not:

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60. Determine whether or not the definition of * On Z^{+} , defined * by $a \cdot b = a - b$ gives a binary operation. In the event that * is not a binary operation give justification of this. Here, Z^+ denotes the set of all non-negative integers.

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61. Determine whether or not the definition of * On Z^+, defined * by $a \cdot b = ab$ gives a binary operation. In the event that * is not a binary operation give justification of this. Here, Z^+ denotes the set of all non-negative integers.

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62. Determine whether or not the definition of * On R, define by $a \cdot b = ab^2$ gives a binary operation. In the event that * is not a binary operation give justification of this. Here, Z^+ denotes the set of all non-negative integers.

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63. Determine whether or not the definition of * On Z^+ , define * by $a \cdot b = |a - b|$ gives a binary operation. In the event that * is not a binary operation give justification of this. Here, Z^+ denotes the set of all non-negative integers.

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64. Determine whether or not the definition of * On Z^+ , define * by $a \cdot b = a$ gives a binary operation. In the event that * is not a binary operation give justification of this. Here, Z^{+} denotes the set of all non-negative integers.

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65. Determine whether or not the definition of * On R, define * by $a \cdot b = a + 4b^2$ gives a binary operation. In the event that * is not a binary operation give justification of this. Here, R denotes the set of all real numbers.

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66. Let $\cdot$ be a binary operation on set of integers I, defined by $a \cdot b = 2a + b - 3$. Find the value of $3 \cdot 4$.

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67. Is $\cdot$ defined on the $set\{1, 2,3, 4,5\}$ by $a^*b = L\dot{C}\dot{M}\cdot$ of a and b a binary operation? Justify your answer.

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68. Let $S = \{a,\ b,\ c\}$. Find the total number of binary operations on $S\cdot$

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69. Find the total number of binary operations on $\{a,\ b\}$.

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70. Prove that the operation * on the set $M = \{[a00b] : a,\ b \in R - \{0\}\}$ defined by $A \cdot B = AB$ is a binary operation.

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71. Let S be the set of all rational numbers of the form $\frac{m}{n}$, where $m \in Z$ and $n = 1,\ 2,\ 3.$ Prove that * on S defined by $a \cdot b = ab$ is not a binary operation.

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72. The binary operation $* : R \times R \to R$ is defined as $a * b = 2a + b.$ Find $(2 * 3) * 4.$

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73. Let * be a binary operation of N given by a $a \cdot b = LCM\,(a, b)$ for all $a, b \in N$. Find $5 \cdot 7$.

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74. Let * be a binary operation on $Q - \{0\}$ defined by $a \cdot b = \frac{ab}{2}$ for all $a, b \in Q - \{0\}$. Prove that * is commutative on $Q - \{0\}$.

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75. Let A be a set having more than one element. Let * be a binary operation on A defined by $a \cdot b = \sqrt{a^2 + b^2}$ for all $a, b, \in A$. Is * commutative or associative on A?

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76. If the operation * is defined on the set $Q - \{0\}$ of all rational numbers by the rule $a^*b = \frac{ab}{4}$ for all $a, b \in Q - \{0\}$. Show that * is commutative on $Q - \{0\}$

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77. Examine whether the binary operation $*$ defined on R by $a * b = ab + 1$ is commutative or not.

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78. Let A be a set having more than one element. Let * be a binary operation on A defined by $a \cdot b = \sqrt{a^2 + b^2}$ for all $a, b, \in A$. Is * commutative or associative on A?

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79. Discuss the commutativity of the binary operation * on R defined by $a^*b = a - b + ab$ for all a ,$b \in R$, where on RHS we have usual addition, subtraction and multiplication of real numbers.

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80. Discuss the commutativity of the binary operation * on R defined by $a \cdot b = a^2 b$ for all $a, b \in R$.

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81. Discuss the commutativity and associativity of binary operation * defined on Q by the rule $a \cdot b = a - b + ab$ for all $a, b \in Q$

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82. Let $\cdot$ be a binary operation on N, the set of natural numbers, defined by $a \cdot b = a^b$ for all $a, b \in N$. Is $' \cdot '$ commutative on N?

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83. Let * be a binary operation on N given by $a \cdot b = 2a + 3b$ for all $a, \ b \in N$, then find: $5 \cdot 4, \ 1 \cdot 4,$

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84. Let $*$ be the binary operation on N defined by $a \ * \ b \ = \ H\dot{C}\dot{F}.$ of a and b. Is $*$ commutative? Is $*$ associative? Does there exist identity for this binary operation on N?

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85. Consider the binary operations $\cdot : R \times R \to R$ and $o: R \times R \to R$ defined as $a \cdot b = |a - b|$ and $aob = a$ for all $a, b \in R$. Show that $\cdot$ is commutative but not associative, o is associative but not commutative.

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86. Let A be a non-empty set and S be the set of all functions from A to itself. Prove that the composition of functions $'o'$ is a non-commutative binary operation on S. Also, prove that $'o'$ is an associative binary operation on S.

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87. Let $A = NxNand' \cdot '$ be a binaryoperation on A defined by $(a, b) \cdot (C, d) = (ac, bd)$ for all $a, b, c, d, \in N$. Show that $' \cdot '$ is commutative and associative binary operation on A.

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88. Let A be a set having more than one element. Let * be a binary operation on A defined by $a \cdot b = a$ for all $a, \ b \in A$. Is * commutative or associative on A ?

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89. Let $\cdot$ be a binary operation on N defined by $a \cdot b = LCM(a,\ b)$ for all $a,\ b \in N$. Find $2 \cdot 4,\ 3 \cdot 5,\ 1 \cdot 6$.

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90. Let $' \cdot '$ be a binary operation on N given by $a \cdot b = L\dot{C}\dot{M}(a, b)$ for all $a, b \in N$. Find $5 \cdot 7, 20 \cdot 16$

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91. Determine whether * on N defined by a * b=1 for all $a,\ b \in N$ is associative or commutative?

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92. Determine whether $\cdot$ on Q defined by $a \cdot b = \frac{a+b}{2}$ for all $a,\ b \in Q$ is commutative?

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93. Let A be any set containing more than one element. Let $\cdot$ be a binary operation on A defined by $a \cdot b = b$ for all $a,\ b \in A$. Is $\cdot$ commutative A ?

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94. Check the commutativity of * on Z defined by $a \cdot b = a + b + ab$ for all $a,\ b \in Z$.

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95. Check the commutativity of * on N defined by $a \cdot b = 2^{ab}$ for all $a, \ b \in N$.

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96. Check the commutativity of * on Q defined by $a \cdot b = a - b$ for all $a, \ b \in Q$.

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97. Check the commutativity of o on Q defined by $a \ \ b = a^2 + b^2$ for all $a, \ b \in Q$.

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98. Check the commutativity and associativity of * on Q defined by $a \cdot b = \frac{ab}{2}$ for all $a, \ b \in Q$.

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99. Check the commutativity of * on Q defined by $a \cdot b = ab^2$ for all $a, \ b \in Q$.

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100. Check the commutativity of * on Q defined by $a \cdot b = a + ab$ for all $a, \ b \in Q$.

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101. Check the commutativity and associativity of * on R defined by $a \cdot b = a + b - 7$ for all $a,\ b \in Q$.

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102. Check the commutativity of * on Q defined by $a \cdot b = (a - b)^2$ for all $a,\ b \in Q$.

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103. Check the commutativity and associativity of * on Q defined by $a \cdot b = ab + 1$ for all $a,\ b \in Q$.

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104. Let *, be a binary operation on N, the set of natural numbers defined by $a \cdot b = a^b$, for all $a, b \in N$. Is * associative or commutative on N?

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105. Check the commutativity and associativity of * on Z defined by a*b=$a - b$ for all $a, \ b \in Z$.

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106. Check the commutativity and associativity of * on Q defined bya*b= $\frac{ab}{4}$ for all $a,\ b \in Q$.

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107. Check the commutativity and associativity of * on Z defined bya*b= $a+b-ab$ for all $a,\ b \in Z$.

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108. Check the commutativity and associativity of * on N defined by a*b=$\gcd(a,\ b)$ for all $a,\ b \in N$.

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109. Let S be the set of all rational number except 1 and * be defined on S by $a \cdot b = a + b - ab,$ for all $a, bS \cdot$ Prove that (i) * is a binary operation on `(i i) * is commutative as well as associative.

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110. Show that the binary operation * on Z defined by a*b= $3a + 7b$ is not commutative.

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111. On the set Z of integers a binary operation * is defined by a*b=$ab + 1$ for all $a, b \in Z$. Prove that * is not associative on Z.

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112. Q, the set of all rational number, * is defined by a*b= $\frac{a-b}{2}$, show that * is not associative.

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113. On Z, the set of all integers, a binary operation * is defined by a*b= a+3b-4 . Prove that * is neither commutative nor associative on Z.

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114. On the set Q of all rational numbers if a binary operation * is defined by a*b= $\frac{ab}{5}$, prove that * is associative on Q.

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115. The binary operation * is defined by a*b=$\frac{ab}{7}$ on the set Q of all rational numbers. Show that * is associative.

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116. On Q , the set of all rational numbers a binary operation * is defined by a*b= $\frac{a+b}{2}$. Show that * is not associative on Q.

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117. Let S be the set of all rational number except 1 and * be defined on S by $a \cdot b = a + b - ab,$ for all $a, bS\cdot$ Prove that (i) * is a binary operation on `(i i) * is commutative as well as associative.

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118. Let S be the set of all rational number except 1 and * be defined on S by $a \cdot b = a + b - ab,$ for all $a, bS\cdot$ Prove that (i) * is a binary operation on `(i i) * is commutative as well as associative.

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119. If * defined on the set R of real numbers by a*b= $\frac{3ab}{7}$,

find the identity element in R for the binary operation *.

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120. Find the identity element in set Q^{+} of all positive

rational numbers for the operation * defined by a*b=$\frac{ab}{2}$

for all $a, b \in Q^{+}$.

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121. If * is defined on the set R of all real numbers by a*b=

$\sqrt{a^2 + b^2}$, find the identity element in R with respect to *.

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122. Let S be a non-empty set and $P(s)$ be the power set of set S. Find the identity element for all union () as a binary operation on $P(S)\cdot$

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123. Let S be a non- empty set and P (s) be the power set of set S .Find the identity element for all union ⋃ as a binary operation on P(S) .

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124. Find the identity element in the set I^+ of all positive integers defined by a*b=$a + b$ for all $a, \ b \in I^+$.

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125. Find the identity element in the set of all rational numbers except -1 with respect to * defined by a*b= $a + b + ab$.

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126. If the binary operation * on the set Z is defined by a*b $= a + b - 5,$ then find the identity element with respect to *.

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127. On the set Z of integers, if the binary operation * is defined by a*b=$a+b+2$, then find the identity element.

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128. On Q, the set of all rational numbers, a binary operation * is defined by $a \cdot b = \frac{ab}{5}$ for all $a, b \in Q$. Find the identity element for * in Q. Also, prove that every non-zero element of Q is invertible.

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129. Let * be a binary operation on set $Q-[1]$ defined by $a\cdot b=a+b-ab$ for all$a, b\in Q-[1]\cdot$ Find the identity element with respect to $\cdot onQ\cdot$ Also, prove that every element of $Q-[1]$ is invertible.

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130. Show that the binary operation * on $A=R-\{-1\}$ defined as $a\cdot b=a+b+ab$ for all a, bA is commutative and associative on $A\cdot$ Also find the identity element of $\cdot$ in A and prove that every element of A is invertible.

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131. Let $' \cdot '$ be a binary operation on Q_0 (set of all non-zero rational numbers) defined by $a \cdot b = \frac{ab}{4}$ for all $a, b \in Q_0$. Then, find the identity element in Q_0 inverse of an element in Q_0.

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132. Let $' \cdot '$ be a binary operation on Q_0 (set of all non-zero rational numbers) defined by $a \cdot b = \frac{ab}{4}$ for all $a, b \in Q_0$. Then, find the identity element in Q_0 inverse of an element in Q_0.

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133. Let * be a binary operation on N given by a*b= $L\dot{C}\dot{M}(a, b)$ for all $a, b \in N$. (i) Find 5*7, 20*16 (ii) Is * commutative?

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134. Let $' \cdot '$ be a binary operation on N given by $a \cdot b = L\dot{C}\dot{M}a\dot{,} b$ for all $a, b \in N\cdot$ Find $5 \cdot 7, 20 \cdot 16$ (ii) Is * commutative? Is * associative? Find the identity element in N Which element of N are invertible? Find them.

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135. Define a binary operation * on the set $A = \{0, 1, 2, 3, 4, 5\}$ given by $a \cdot b = ab$ (mod 6). Show that 1 is the identity for *. 1 and 5 are the only invertible elements with $1^{-1} = 1$ and $5^{-1} = 5$

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136. On the set $M = A(x) = \{[\times \quad \times] : x \in R\} of 2x2$ matrices, find the identity element for the multiplication of matrices as a binary operation. Also, find the inverse of an element of M.

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137. Let X be a non-empty set and let * be a binary operation on $P(X)$ (the power set of set X) defined by $A \cdot B = A \cup B$ for all $A,\ B \in P(X)$. Prove that * is both commutative and associative on $P(X)$. Find the identity element with respect to * on $P(X)$. Also, show that $\varphi \in P(X)$ is the only invertible element of $P(X)$.

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138. Let X be a nonempty set and *be a binary operation on $P(X)$, the power set of X, defined by $A \cdot B = A \cap B$ for all $A, B \in P(X)$. (

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139. Let X be a non-empty set and let * be a binary operation on $P(X)$ (the power set of set X) defined by $A \cdot B = (A - B) \cup (B - A)$ for all $A, \ B \in P(X)$. Show that φ is the identity element for * on $P(X)$.

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140. Let X be a non-empty set and let * be a binary operation on $P(X)$ (the power set of set X) defined by $A \cdot B = (A - B) \cup (B - A)$ for all $A, \ B \in P(X)$. Show that φ is the identity element for * on $P(X)$.

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141. Let $A = Q \times Q$ and let $*$ be a binary operation on A defined by $(a, b) \cdot (c, d) = (ac, b + ad)$ for $(a, b), (c, d) \in A$. Then, with respect to $*$ on A. Find the identity element in A.

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142. Let $A = Q \times Q$ and let * be a binary operation on A defined by $(a, b) \cdot (c, d) = (ac, b + ad)$ for $(a, b), (c, d) \in A$. Then, with respect to * on A. Find the invertible elements of A.

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143. Let $A = N \cup \{0\} \times N \cup \{0\}$ and let * be a binary operation on A defined by $(a, b)*(c, d) = (a + c, b + d)$ for all $(a, b), (c, d) \in A$. Show that * is commutative on A.

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144. Let $A = N \cup \{0\} \times N \cup \{0\}$ and let * be a binary operation on A defined by $(a, b) \cdot (c, d) = (a + c, b + d)$ for all $(a, b), (c, d) \in A$. Show that * is associative on A.

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145. Let $A = N \times N$, and let * be a binary operation on A defined by $(a, b) \cdot (c, d) = (ad + bc, bd)$ for all $(a, b), (c, d) \in N \times N$. Show that: * is commutative on A. (ii) * is associative on A.

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146. Let $A = N \times N$, and let * be a binary operation on A defined by $(a, b) \cdot (c, d) = (ad + bc, bd)$ for all $(a, b), (c, d) \in N \times N$. Show that A has no identity element.

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147. Show that the number of binary operations on $\{1, \quad 2\}$ having 1 as identity and having 2 as the inverse of 2 is exactly one.

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148. Determine the total number of binary operations on the set $S = \{1, \ 2\}$ having 1 as the identity element.

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149. Let * be a binary operation on Z defined by a*b= $a + b - 4$ for all $a, \ b \in Z$. Show that * is both commutative and associative.

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150. Let * be a binary operation on Z defined by a*b= $a+b-4$ for all $a,\ b \in Z$. (i)Find the identity element in Z. (ii) Find the invertible elements in Z.

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151. Let * be a binary operation on Q_0 (set of non-zero rational numbers) defined by a*b=$\frac{3ab}{5}$ for all $a,\ b \in Q_0$. Show that * is commutative as well as associative. Also, find the identity element, if it exists.

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152. Let * be a binary operation on $Q - \{-1\}$ defined by $a^{*}b = a + b + ab$ for all $a,\ b \in Q - \{-1\}$. Then, Show that * is both commutative and associative on $Q - \{-1\}$. (ii) Find the identity element in $Q - \{-1\}$

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153. Let * be a binary operation on $Q - \{-1\}$ defined by $a \cdot b = a + b + ab$ for all $a,\ b \in Q - \{-1\}$. Then, Show that every element of $Q - \{-1\}$ is invertible. Also, find the inverse of an arbitrary element.

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154. Let R_0 denote the set of all non-zero real numbers and let $A = R_0 \times R_0$. If * is a binary operation on A defined by $(a, b) \cdot (c, d) = (ac, bd)$ for all $(a, b), (c, d) \in A$. Show that * is both commutative and associative on A (ii) Find the identity element in A

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155. Let * be a binary operation on the set Q_0 of all non-zero rational numbers defined by a*b=$\frac{ab}{2}$, for all $a, b \in Q_0$. Show that (i) * is both commutative and associative (ii) Find the identity element in Q_0 (iii) Find the invertible elements of Q_0 .

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156. On $R-[1]$, a binary operation * is defined by $a \cdot b = a + b - ab$. Prove that * is commutative and associative. Find the identity element for * on $R-[1]$. Also, prove that every element of $R-[1]$ is invertible.

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157. Let R_0 denote the set of all non-zero real numbers and let $A = R_0 \times R_0$. If . is a binary operation on A defined by $(a,\ b) \cdot (c,\ d) = (ac,\ bd)$ for all $(a,\ b),\ (c,\ d) \in A$. Show that . is both commutative and associative on A

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158. Let R_0 denote the set of all non-zero real numbers and let $A = R_0 \times R_0$. If * is a binary operation on A defined by $(a, b) \cdot (c, d) = (ac, bd)$ for all $(a, b), (c, d) \in A$. Find the identity element in A .

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159. Let * be the binary operation on N defined by a*b= HCF of a and b . Does there exist identity for this binary operation on N ?

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160. Consider the set $S = \{1, -1\}$ of square roots of unity and multiplication $(\times)$ as a binary operation on S Construct the composition table for multiplication $(\times)$ on S Also, find the identity element for multiplication on S and the inverses of various elements.

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161. Consider the set $S = \{1, \omega, \omega^2\}$ of all cube roots of unity. Construct the composition table for multiplication $(\times)$ on S. Also, find the identity element for multiplication on S. Also, check its commutativity and find the identity element. Prove that every element of S is invertible.

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162. Consider the set $S = \{1, \ -1, \ i, \ -i\}$ of fourth roots of unity. Construct the composition table for multiplication on S and deduce its various properties.

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163. Consider the set $S = \{1, \ 2, \ 3, \ 4\}$. Define a binary operation * on S as follows: $a \cdot b = r$, where r is the least non-negative remainder when ab is divided by 5. Construct the composition table for * on S.

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164. Consider the infimum binary operation $\wedge$ on the set $S = \{1, 2, 3, 4, 5\}$ defined by $a \wedge b =$ Minimum of $a\ and\ b$. Write the composition table of the operation $\wedge$.

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165. Consider a binary operation * on the set {1, 2, 3, 4, 5} given by the following multiplication table (FIGURE) Compute (2*3) *4 and 2* (3*4) Is * commutative? (iii) Compute (2*3)*(4*5)

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166. Define a binary operation * on the set $A = \{0, 1, 2, 3, 4, 5\}$ as a * b=a+b(mod 6) Show that zero is the identity for this operation and each element a of the set is invertible with $6-a$ being the inverse of a.

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167. Define a binary operation * on the set $A=\{0, 1, 2, 3, 4, 5\}$ as $a \cdot b=a+b$ (mod 6). Show that zero is the identity for this operation and each element a of the set is invertible with $6-a$ being the inverse of a.

OR A binary operation * on the set $\{0, 1, 2, 3, 4, 5\}$ is defined as

$$a \cdot b = \{a+b, \quad \text{if} \quad a+b<6a+b-6, \quad \text{if} \quad a+b \geq 6$$

Show that zero is the identity for this operation and each element a of set is invertible with $6-a,$ being the inverse of a.

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168. Define a binary operation $*$ on the set $A = \{1,\ 2,\ 3, 4\}$ as $a * b = ab \pmod 5$. Show that 1 is the identity for $*$ and all elements of the set A are invertible with $2^{-1} = 3$ and $4^{-1} = 4$.

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169. Construct the composition table for the composition of functions (o) defined on the $S = \{f_1,\ f_2,\ f_3,\ f_4\}$ of four functions from C (the set of all complex numbers) to itself, defined by

$$f_1(z) = z,\ \ f_2(z) = -z,\ \ f_3(z) = \frac{1}{z},\ \ f_4(z) = -\frac{1}{z}$$

for all $z \in C$.

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170. Construct the composition table for $\times_4$ on set $S = \{0, 1, 2, 3\}$.

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171. Construct the composition table for $+_5$ on set $S = \{0, 1, 2, 3, 4\}$.

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172. Construct the composition table for $\times_6$ on set $S = \{0, 1, 2, 3, 4, 5\}$.

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173. Construct the composition table for $\times_5$ on $Z_5 = \{0, 1, 2, 3, 4\}$.

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174. For the binary operation $\times_{10}$ on set $S = \{1, 3, 7, 9\}$, find the inverse of 3.

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175. For the binary operation $\times_7$ on the set $S = \{1, 2, 3, 4, 5, 6\}$, compute $3^{-1} \times_7 4$.

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176. Find the inverse of 5 under multiplication modulo 11 on Z_{11} .

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177. Write the multiplication table for the set of integers modulo 5.

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178. Consider the binary operation * and o defined by the following tables on set $S = \{a, b, c, d\}$. (FIGURE) Show that both the binary operations are commutative and associative. Write down the identities and list the inverse of elements.

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179. Define a binary operation * on the set $A = \{0, 1, 2, 3, 4, 5\}$ as $a \cdot b = a + b$ (mod 6). Show that zero is the identity for this operation and each element a of the set is invertible with $6 - a$ being the inverse of $a \cdot$

OR A binary operation * on the set $\{0, 1, 2, 3, 4, 5\}$ is defined as

$a \cdot b = \{a + b, \quad \text{if} \quad a + b < 6a + b - 6, \quad \text{if} \quad a + b \geq 6$

Show that zero is the identity for this operation and each element a of set is invertible with $6-a$, being the inverse of a.

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180. Write the identity element for the binary operations * on the set R_0 of all non-zero real numbers by the rule $a^{*} b=\frac{ab}{2}$ for all $a, b \in R_0$

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181. On the set Z of all integers a binary operation $*$ is defined by $a * b=a+b+2$ for all $a, b \in Z$. Write the inverse of 4.

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182. Define a binary operation on a set.

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183. Define a commutative binary operation on a set.

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184. Define an associative binary operation on a set.

Watch Video Solution

185. Write the total number of binary operations on a set consisting of two elements.

Watch Video Solution

186. Write the identity element for the binary operation * defined on the set R of all real numbers by the rule a*b= $\frac{3ab}{7}$ for all $a,\ b \in R$.

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187. Let * be a binary operation, on the set of all non-zero real numbers, given by $a \cdot b = \frac{ab}{5}$ for all $a,\ b \in R - \{0\}$. Write the value of x given by $2 \cdot (x \cdot 5) = 10$.

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188. Write the inverse of 5 under multiplication modulo 11 on the set {1, 2, ..., 10}.

Watch Video Solution

189. Define identity element for a binary operation defined on a set.

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190. Write the composition table for the binary operation multiplication modulo $10 \left(\times_{10} \right)$ on the set

$S = \{2, \ 4, \ 6, \ 8\}$.

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191. Write the composition table for the binary operation multiplication modulo $10\,(\times_{10})$ defined on the set $S = \{1, \ 3, \ 7, \ 9\}$.

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192. For the binary operation multiplication modulo $5\,(\times_5)$ defined on the set $S = \{1, \ 2, \ 3, \ 4\}$. Write the value of $\left(3 \times_5 4^{-1}\right)^{-1}$.

Watch Video Solution

193. Write the composition table for the binary operation $\times_5$ (multiplication modulo 5) on the set $S = \{0, 1, 2, 3, 4\}$.

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194. A binary operation * is defined on the set R of all real numbers by the rule a*b= $\sqrt{a^2 + b^2}$ for all $a, b \in R$. Write the identity element for * on R.

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195. Let $+_6$ (addition modulo 6) be a binary operation on $S = \{0, 1, 2, 3, 4, 5\}$. Write the value of

$2 +_6 4^{-1} +_6 3^{-1}$.

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196. Let * be a binary operation defined by $a^* b = 3a + 4b - 2$. Find 4*5.

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197. If the binary operation * on the set Z of integers is defined by $a^*b = a + 3b^2$, find the value of 2^*4.

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198. Let * be a binary operation on N given by $a^{*} b = HCF(a, b), \ a, b \in N$. Write the value of $22^{*}4$.

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199. Let * be a binary operation on set of integers I, defined by $a^{*}b = 2a + b - 3$. Find the value of $3^{*}4$.

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200. If $a^{*}b = a^2 + b^2$, then the value of $(4^{*}5)^{*}3$ is (i) $(4^2 + 5^2) + 3^2$ (ii) $(4 + 5)^2 + 3^2$ (iii) $41^2 + 3^2$ (iv) $(4 + 5 + 3)^2$

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201. If $a*b$ denote the bigger among a and b and if $ab = (a*b) + 3$, then $4*7 =$ (a) 14 (b) 31 (c) 10 (d) 8

A. 31

B. 14

C. 10

D. 7

Answer: C

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202. If the binary operation * on Z is defined by $a^{*} b=a^{2}-b^{2}+a b+4$, then value of $(2^{*}3)^{*}4$ is (a) 233 (b) 33 (c) 55 (d) -55

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203. For the binary operation * on Z defined by a*b= $a+b+1$ the identity element is

A. -2

B. 0

C. 1

D. -1

Answer: D

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204. If a binary operation * is defined on the set Z of integers as $a^*b = 3a - b$, then the value of $(2^*3)^*4$ is

A. 2

B. 4

C. 5

D. 6

Answer: C

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205. Q^+ denote the set of all positive rational numbers. If the binary operation $\odot$ on Q^+ is defined as $a \odot b = \frac{ab}{2}$, then the inverse of 3 is

A. A. $\frac{4}{3}$

B. B. $\frac{1}{3}$

C. C. $\frac{2}{3}$

D. D. $\frac{5}{3}$

Answer: A

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206. If G is the set of all matrices of the form $[\times \quad \times]$, where $x \in R - \{0\}$, then the identity element with respect to the multiplication of matrices as binary operation, is $[1111]$ (b) $[-1/2-1/2-1/2-1/2]$ (c) $[1/21/21/21/2]$ (d) $[-1-1-1-1]$

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207. Q^+ is the set of all positive rational numbers with the binary operation * defined by a*b= $\frac{ab}{2}$ for all $a,\ b \in Q^+$. The inverse of an element $a \in Q^+$ is a (b) $\frac{1}{a}$ (c) $\frac{2}{a}$ (d) $\frac{4}{a}$

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208. If the binary operation $\odot$ is defined on the set Q^+ of all positive rational numbers by $a \odot b = \frac{ab}{4}$. Then, $3 \odot \left(\frac{1}{5} \odot \frac{1}{2}\right)$ is equal to

A. $\frac{3}{160}$

B. $\frac{5}{160}$

C. $\frac{3}{10}$

D. $\frac{3}{40}$

Answer: A

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209. Let * be a binary operation defined on set $Q - \{1\}$ by the rule $a^*b = a + b - ab$. Then, the identity element for * is

A. 1

B. $\frac{a-1}{a}$

C. $\frac{a}{a-1}$

D. 0

Answer: D

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210. Which of the following is true? * defined by (a) $a \cdot b = \frac{a+b}{2}$ is a binary operation on Z (b) * defined by $a \cdot b = \frac{a+b}{2}$ is a binary operation on Q (c) all binary commutative operations are associative (d) subtraction is a binary operation on N

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211. The binary operation * defined on N by a*b= $a+b+ab$ for all $a,\ b \in N$ is (a) commutative only (b) associative only (c) commutative and associative both (d) none of these

A. commutative and associative both

B. associative only

C. commutative and associative both

D. None of these

Answer: C

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212. If a binary operation * is defined by $a^{*} b = a^2 + b^2 + ab + 1$,then $(2^{*}3)^{*}2$ is equal to (a) 20 (b) 40 (c) 400 (d) 445

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213. Let * be a binary operation on R defined by a*b= $ab+1$. Then, * is (a)commutative but not associative (b)associative but not commutative (c)neither commutative nor associative (d) both commutative and associative

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214. Subtraction of integers is (a)commutative but not associative (b)commutative and associative (c)associative but not commutative (d) neither commutative nor associative

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215. The law $a+b=b+a$ is called

A. closure law

B. associative law

C. commutative law

D. distributive law

Answer: C

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216. An operation * is defined on the set Z of non-zero integers by a*b=$\frac{a}{b}$ for all $a,\ b \in Z$. Then the property satisfied is (a) closure (b) commutative (c) associative (d) none of these

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217. On Z an operation * is defined by a*b=a^2+b^2 for all $a,\ b\in Z$. The operation * on Z is (a)commutative and associative (b)associative but not commutative (c) not associative (d) not a binary operation

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218. A binary operation * on Z defined by a*b= $3a+b$ for all $a,\ b\in Z$, is (a) commutative (b) associative (c) not commutative (d) commutative and associative

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219. Let * be a binary operation on N defined by a*b= $a+b+10$ for all $a,\ b \in N$. The identity element for * in N is (a) -10 (b) 0 (c) 10 (d) non-existent

A. -10

B. 0

C. 10

D. Does not Exist

Answer: A

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220. Consider the binary operation * defined on $Q-\{1\}$ by the rule a*b= $a+b-ab$ for all $a,\ b \in Q-\{1\}$. The identity element in $Q-\{1\}$ is (a) 0 (b) 1 (c) $\frac{1}{2}$ (d) -1

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221. For the binary operation * defined on $R-\{1\}$ by the rule $a^*b=a+b+ab$ for all $a,\ b \in R-\{1\}$, the inverse of a is

A. a

B. $-\frac{a}{a+1}$

C. $\frac{1}{a}$

D. a^2

Answer: B

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222. For the multiplication of matrices as a binary operation on the set of all matrices of the form $[ab-ba], \ a, \ b \in R$ the inverse of $[23-32]$ is $[-23-3-2]$ (b) $[23-32]$ (c) $[2/13-3/133/132/13]$ (d) $[1001]$

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223. On the set Q^+ of all positive rational numbers a binary operation * is defined by $a^*b=\frac{ab}{2}$ for all

$a, \ b \in Q^+$. The inverse of 8 is (a) $\frac{1}{8}$ (b) $\frac{1}{2}$ (c) 2 (d) 4

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224. Let * be a binary operation defined on Q^+ by the rule $a*b = \frac{ab}{3}$ for all $a, \ b \in Q^+$. The inverse of $4*6$ is (a) $\frac{9}{8}$ (b) $\frac{2}{3}$ (c) $\frac{3}{2}$ (d) none of these

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225. The number of binary operations that can be defined on a set of 2 elements is (a) 8 (b) 4 (c) 16 (d) 64

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226. The number of commutative binary operations that can be defined on a set of 2 elements is

A. 1

B. 2

C. 4

D. 16

Answer: B

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