



## MATHS

# **BOOKS - RD SHARMA MATHS (ENGLISH)**

# DIFFERENTIABILITY

#### Others

1. Find the values of a and b so that the function  $f(x) = \begin{cases} x^2 + 3x + a & ext{if } x \leq 1 \\ bx + 2 & ext{if } x > 1 \end{cases}$  is differentiable at each  $x \in R$ 

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**2.** Show that  $f(x)=\{12x-13,$  if  $x\leq 3, 2x^2+5$  if x>3 is

differentiable at x=3 . Also, find  $f^{\,\prime}(3)_{\cdot}$ 

**3.** Show that  $f(x) = (x)^{\frac{1}{3}}$  is not differentiable at x = 0.

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4. Show that f(x) = |x - 3| is continuous but not differentiable at x = 3.

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5. Show that the function  $f(x) = \left\{x^m \sin\left(rac{1}{x}
ight), 0, x 
eq 0, x = 0 ext{ is } 
ight.$ 

differentiable at x = 0,if m > 1 and continuous but not differentiable at x = 0,if 0 < m < 1.

6. Discuss the continuity and differentiability of the function f(x) = |x| + |x-1| in the interval (-1,2).

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7. Show that the function 
$$f$$
 defined as follows  $f(x) = \begin{cases} 3x-2 & 0 < x \leq 1 \\ 2x^2-x & 1 < x \leq 2 \end{cases}$  is continuous at x=2 but not  $5x-4 & x>2 \end{cases}$ 

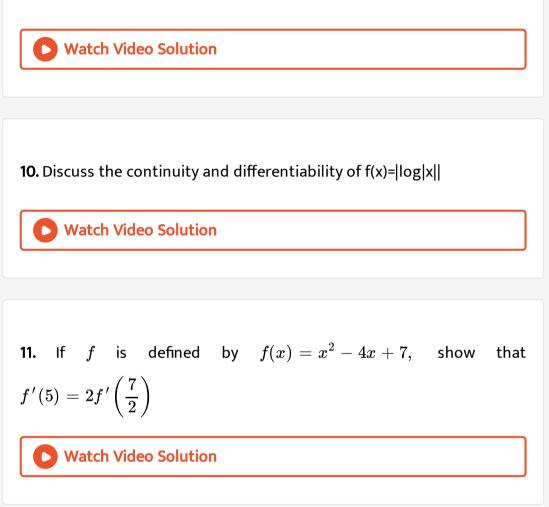
differentiable.

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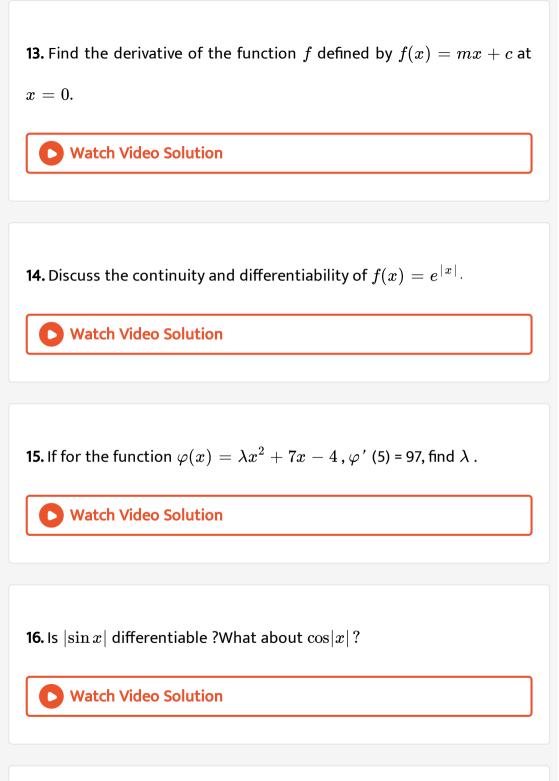
8. Discuss the continuity and differentiability of 
$$f(x) = egin{cases} (x-c)\cos\left(rac{1}{x-c}
ight) & x 
eq c \ 0 & x = c \end{cases}$$

9. Write an example of a function which is everywhere continuous but

fails to be differentiable exactly at five points.



12. If f is defined by  $f(x)=x^2, ext{ find } f'(2).$ 



17. The set of points where the function f(x) given by  $f(x)=|x-3|\cos x$  is differentiable, is (a) R (b)  $R-\{3\}$  (c)  $(0,\infty)$  (d) none of these

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**18.** If 
$$f(x) = \left\{\frac{1-\cos x}{x\sin x}, x \neq 0 \text{ and } \frac{1}{2}, x = 0 \text{ then at } x = 0, f(x) \text{ is} \right.$$
  
(a)continuous and differentiable (b)differentiable but not continuous (c)continuous but not differentiable (d)neither continuous nor differentiable

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19. If  $f(x)=iggl\{rac{1}{1+e^{rac{1}{x}}},x
eq 00,x=0,thenf(x) ext{ is continuous as}$ 

well as differentiable at x=0 continuous but not differentiable at

x=0 differentiable but not continuous at x=0 none of these

**20.** If f(x) = |3 - x| + (3 + x), where (x) denotes the least integer greater than or equal to x, then f(x) is continuous and differentiable at x = 3 continuous but not differentiable at x = 3 differentiable but not continuous at x = 3 neither differentiable nor continuous at x = 3



21. Let  $f(x) = a + b|x| + c|x|^4$ , where a, bandc are real constants. Then, f(x) is differentiable at x = 0, if a = 0 (b) b = 0 (c) c = 0 (d) none of these

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22. The function  $f(x) = rac{\sin(\pi [x-\pi])}{4+{[x]}^2}$  , where [] denotes the greatest

integer function, (a) is continuous as well as differentiable for all

 $x\in R$  (b) continuous for all x but not differentiable at some x (c) differentiable for all x but not continuous at some x .(d) none of these



23. Let 
$$f(x) = \left\{ax^2 + 1, x > 1; x + \frac{1}{2}, x \le 1. then, f(x) \text{ is derivable at } x = 1, \text{ if } a = 2 \text{ (b) } a = 1 \text{ (c) } a = 0 \text{ (d) } a = \frac{1}{2} \right\}$$

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24. Show that the function defined by g(x) = x - [x] is discontinuous at all integral points. Here [x] denotes the greatest integer less than or equal to x.



**25.** It A is a symmetric matrix, write whether  $A^T$  is symmetric or skew -

symmetric matrix.

**26.** If  $f(x)=ig\{x^2+3x+a,x\leq 1,bx+2,f ext{ or }x>1 ext{ is everywhere}$ 

differentiable, find the values of *aandb*.

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27. Let f(x) = |x| and  $g(x) = |x^3|$ , then (a)f(x) and g(x) both are continuous at x = 0 (b)f(x) and g(x) both are differentiable at x = 0(c)f(x) is differentiable but g(x) is not differentiable at x = 0 (d)f(x)and g(x) both are not differentiable at x = 0

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**28.** Is  $|\sin x|$  differentiable? What about  $\cos |x|$ ?

29. Find the values of aandb so that the function  $f(x)=ig\{x^2+3x+a, ext{ if } x\leq 1bx+2, ext{ if } x>1 ext{ is differentiable}$  at each  $x\in R$ .

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**30.** The function  $f(x) = e^{-|x|}$  is continuous everywhere but not differentiable at x = 0 continuous and differentiable everywhere not continuous at x = 0 none of these

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**31.** Discuss the continuity and differentiability of 
$$f(x) = \{1-x, x < 1(1-x)(2-x), 1 \le x \le 2(3-x), x > 2$$

32. If 
$$y = (x-1)\log(x-1) - (x+1)\log(x+1)$$
, prove that : $\frac{dy}{dx} = \log\left(\frac{x-1}{1+x}\right)$ 

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**33.** The set of points where the function f(x) = x|x| is differentiable is

 $(\,-\infty,\infty)$  (b)  $(\,-\infty,0)\cup(0,\infty)$   $(0,\infty)$  (d)  $[0,\infty)$ 

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**34.** The function  $f(x) = \sin^{-1}(\cos x)$  is (a) . discontinuous at x = 0

(b). continuous at x=0 (c) . differentiable at x=0 (d) . non of these

**35.** Let 
$$f(x) = \left\{ \frac{1}{|x|} f \text{ or } |x| \ge 1ax^2 + bf \text{ or } |x| < 1 \Leftrightarrow (x) \text{ is continuous and differentiable at any point, then  $a = \frac{1}{2}, b = -\frac{3}{2}$  (b)$$

$$a=\ -rac{1}{2}, b=rac{3}{2}\,a=1, b=\ -1$$
 (d) none of these

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**36.** If  $f(x) = \sqrt{1 - (\sqrt{1} - x^2)}$ , then f(x) is (a)continuous on [-1, 1] and differentiable on (-1, 1) (b) continuous on [-1,1] and differentiable on  $(-1,0) \cup (0,1)$  (C) continuous and differentiable on [-1, 1](d) none of these

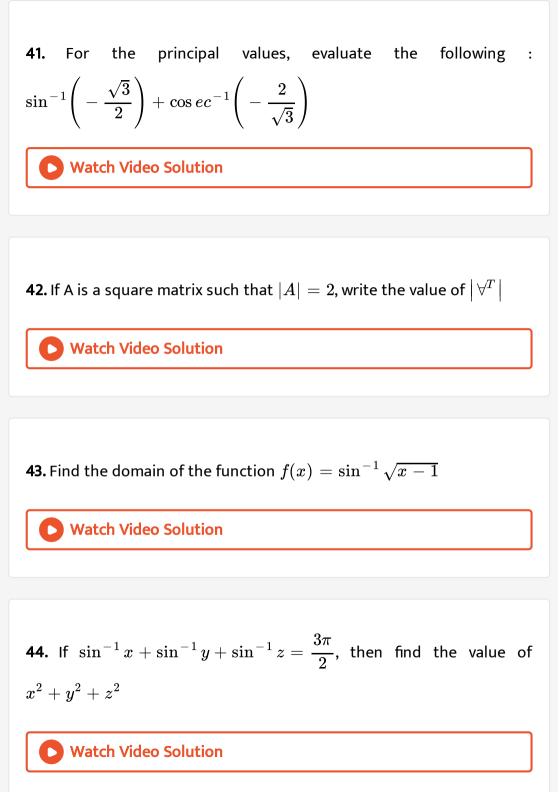


37. If  $f(x) = a |\sin x| + b e^{|x|} + c |x|^3$  and if f(x) is differentiable at

 $x\,=\,0$  then

38. Solve the following equation for x ::  

$$\cos(\tan^{-1} x) = \sin\left(\cot^{-1}\left(\frac{3}{4}\right)\right)$$
  
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39. Find the principal values the following ::  
 $\sin^{-1}\left\{\cos\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right\}$   
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40. Solve the following equation for x ::  
 $\cos(\tan^{-1} x) = \sin\left(\cot^{-1}\left(\frac{3}{4}\right)\right)$   
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**45.** If 
$$f(x) = \left(\frac{x^l}{x^m}\right)^{l+m} \left(\frac{x^m}{x^n}\right)^{m+n} \left(\frac{x^n}{x^l}\right)^{n+l}$$
, then find f'(x).

**46.** Show that f(x) = |x| is not differentiable at x = 0.

**47.** Evaluate : 
$$\tan\left(\cos^{-1}\left(-\frac{7}{25}\right)\right)$$

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**48.** For what choice of a and b is the function  $f(x) = \left\{x^2, x \leq c \text{ and } ax + b, x > c \text{ is differentiable at } x = c 
ight.$ 

**49.** Discuss the differentiability of f(x)=xert xert at x=0



**50.** Show that the function f(x)={  $x^2 \sin\left(rac{1}{x}
ight)$ , when x 
eq 0 and 0 when

x=0 } is differentiable at x=0.

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**51.** If 
$$f(2) = 2andf'(2) = 1$$
, then find  $(\lim_{x \to 2} \frac{xf(2) - 2f(x)}{x - 2})$ 

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52. If  $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$ , then find the value of  $x^2 + y^2 + z^2$ 

53. Show that f(x) = |x| is not differentiable at x = 0 .



**54.** Show that the function  $f(x) = \{x-1 \ , \ ext{ if } \ x < 2$ 

2x-3 , if  $x\geq 2$  is not

differentiable at x=2 .

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**55.** Show that the function 
$$f(x) = \left\{ \begin{pmatrix} x^2 \sin\left(rac{1}{x}\right) & ext{if} & x 
eq 0 \\ 0 & ext{if} & x = 0 \end{pmatrix} 
ight\}$$
 is

differentiable at x=0 and f'(0) = 0

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**56.** Show that  $f(x) = x^2$  is differentiable at x = 1 and find f'(1) .

57. Show that the function f(x) = |x-1| for all  $x \in R$  , is not

differentiable at x=1 .

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**58.** Discuss the differentiability of f(x) = x |x| at x = 0 .

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**59.** Show that the function 
$$f(x) = \begin{cases} x \sin \frac{1}{x} & , when \ x \neq 0 & 0, when \ x = 0 \ ext{is continuous} \end{cases}$$

but not differentiable at x=0 .

60. Discuss the differentiability of 
$$f(x) = \left\{xe - \left(\frac{1}{|x|} + \frac{1}{x}\right), x \neq 0x, x = 0atx = 0$$
  
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61. If  $f(x)$  is differentiable at  $x = a$ , find  $(\lim_{x \to a} \frac{x^2 f(a) - a^2 f(x)}{x - a}$ .  
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62. For what choice of  $a$  and  $b$  is the function  $f(x) = \left\{x^2, x \leq c \ ax + b, x > c$  is differentiable at  $x = c$ .  
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63. If  $f(2) = 4$  and  $f'(2) = 1$ , then find  $(\lim_{x \to 2} \frac{x f(2) - 2f(x)}{x - 2}$ .

**64.** A function  $f\colon R o R$  satisfies that equation f(x+y)=f(x)f(y)for all  $x,\ y\in R$ , f(x)
eq 0. Suppose that the function f(x) is differentiable at x=0 and f'(0)=2. Prove that  $f'(x)=2\ f(x)$ .

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**65.** Show that f(x) = |x-3| is continuous but not differentiable at

x = 3.

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**66.** Show that  $f(x) = x^{1/3}$  is not differentiable at x = 0 .

$$f(x) = ig\{ 12x - 13, \quad ext{if} \quad x \leq 3 \quad 2x^2 + 5, \quad ext{if} \quad x > 3 \quad ext{is}$$

differentiable at x=3 . Also, find  $f^{\,\prime}(3)$  .



**68.** Show that the function f defined as follows `f(x)={3x-2, 0 < x <=1,

 $2x^2 - x$ , 1 < x <= 2, 5x-4, x > 2 is continuous at x=2, but not differentiable thereat.

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69. Discuss the continuity and differentiability of the function f(x) = |x| + |x - 1| in the interval (-1, 2).

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that

70. Find whether the following function is differentiable at x = 1 and x = 2 or not :  $f(x) = ig\{x, x < 12 - x, 1 \leq x \leq 2 - 2 + 3x - x^2, x > 2ig\}$ 

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71. Show that the function 
$$f(x)=igg\{x^m\sinigg(rac{1}{x}igg),x=0,0,\ x
eq 0,\ x=0$$
 is differentiable at  $x=0$  , if  $m>1$ 

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72. Show that the function  $f(x)=iggl\{x^m\siniggl(rac{1}{x}iggr),\ x
eq 0$  ,  $0\ x=0$  is

differentiable at x = 0 , if (m>1)

73. Show that the function 
$$f(x) = \left\{ x^m \sin\left(rac{1}{x}
ight), \ x 
eq 00, \qquad x=0 \quad ext{ is neither }$$

continuous nor differentiable, if  $m \leq 0$ 

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74. Find the values of 
$$a$$
 and  $b$  so that the function  
 $f(x) = \{x^2 + 3x + a, \text{ if } x \leq 1$   
 $bx + 2, \text{ if } x > 1$  is differentiable at each x in R.  
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75. Show that the function 
$$f(x)=\{|2x-3|~[x],~x\geq 1;\ \sin\Bigl(rac{\pi x}{2}\Bigr),~x<1$$
 is continuous at

x = 1 .

76. If 
$$f(x)=igg\{ax^2-b,$$
 if  $|x|<1rac{1}{|x|},$  if  $|x|\ge 1$  is

differentiable at x=1 , find  $a,\ b$ 

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77. Find the values of a and b , if the function f(x) defined by  $f(x)=ig\{x^2+3x+a,\ x\leq 1bx+2,\ x>1\ ext{is differentiable at}\ x=1\,.$ 

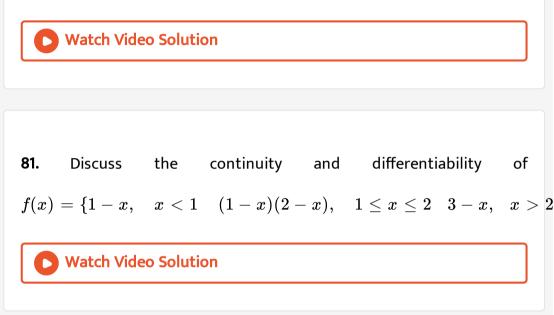
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78. If 
$$f(x)=x^2+2x+7$$
 , find  $f$  '  $(3)$  .

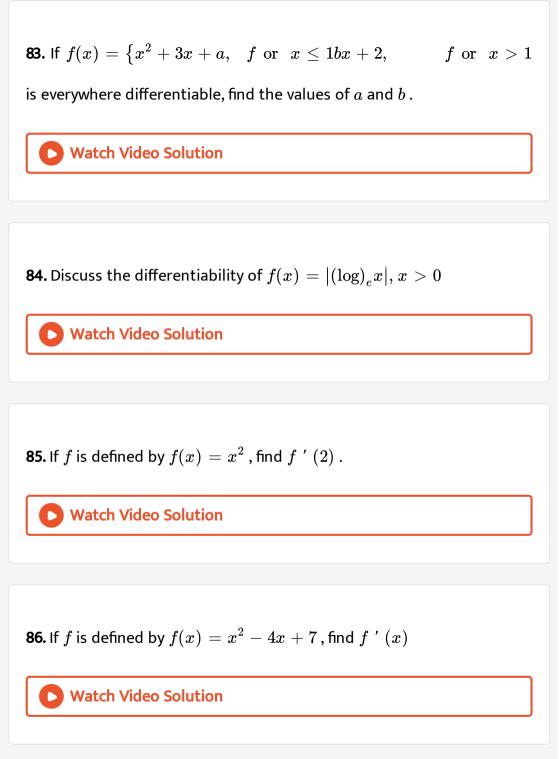


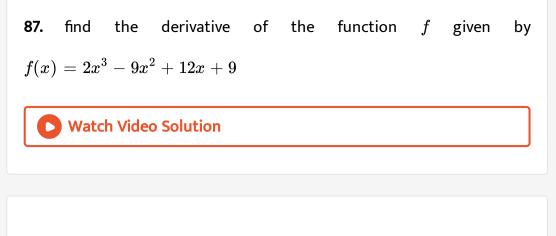
**79.** Find f ' (2) and f ' (5) when  $f(x) = x^2 + 7x + 4$  .

80. For the function f given by  $f(x) = x^2 - 6x + 8$  , prove that f ' (5) = 4 .



**82.** Discuss the differentiability of f(x) = |x-1| + |x-2|





**88.** If for the function  $\operatorname{Phi}(x) = \lambda x^2 + 7x - 4, \ \operatorname{Phiprime}^{\hat{}}(5) = 97$  , find  $\lambda$  .

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**89.** If 
$$f(x) = x^3 + 7x^2 + 8x - 9$$
, find  $f'(4)$ .

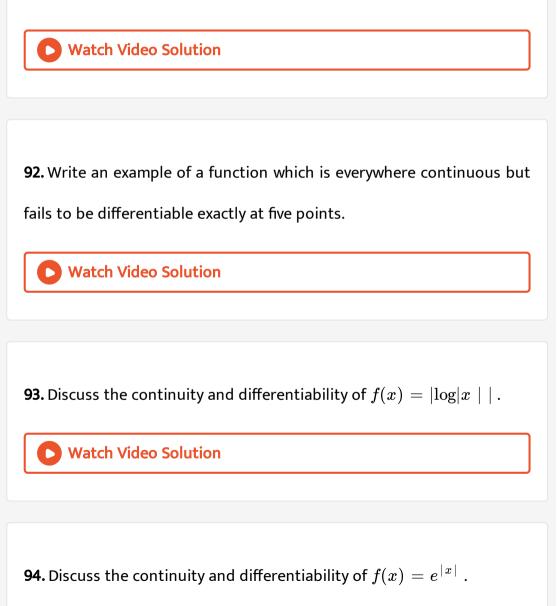
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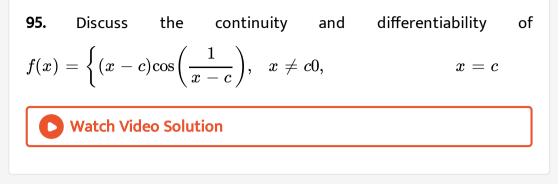
**90.** Find the derivative of the function f defined by f(x) = mx + c at

x = 0.

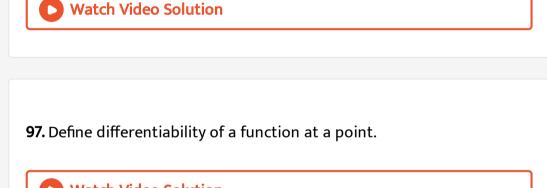
**91.** Examine the differentiability of the function f defined by  $f(x)=\{2x+3,$ 

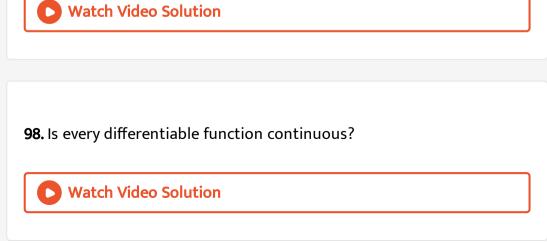
if -3 <= x< -2, x+1, if -2<=x<0, x+2, if 0<=x<=1

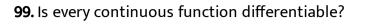


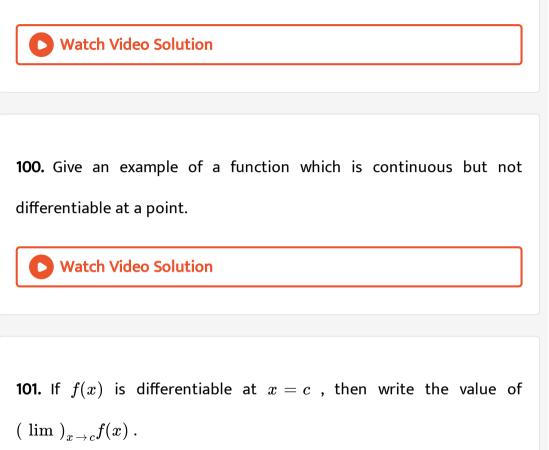


**96.** Is  $|\sin x|$  differentiable? What about  $\cos |x|$ ?



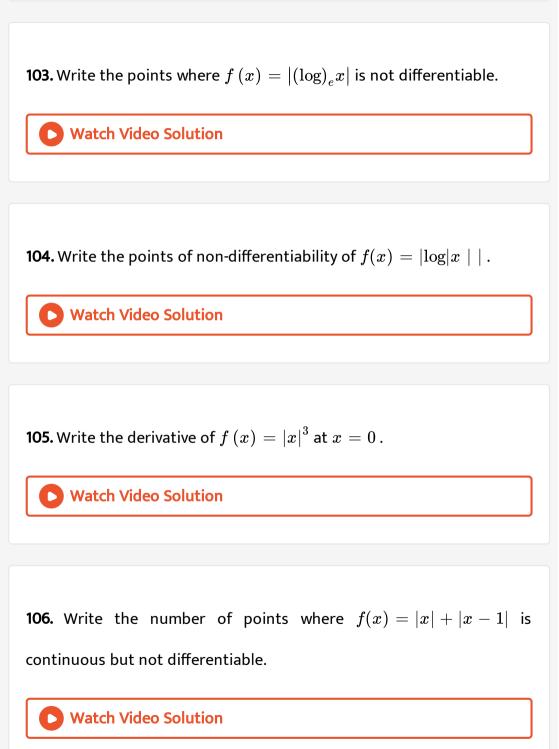






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102. If f(x) = |x - 2| write whether f ' (2) exists or not.



107. If  $(\lim_{x \to c} \frac{f(x) - f(c)}{x - c}$  exists finitely, write the value of  $(\lim_{x \to c} f(x)$ .

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108. Write the value of the derivative of f(x) = |x-1| + |x-3| at

x=2 .

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109. If 
$$f(x)=\sqrt{x^2+9}$$
 , write the value of  $(\ \lim\ )_{x
ightarrow 4}rac{f(x)-f(4)}{x-4}$ 

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110. Let f(x)=|x| and  $g(x)=\left|x^3\right|$  , then (a).f(x) and g(x) both are continuous at x=0 (b) f(x) and g(x) both are differentiable at x=0

(c) f(x) is differentiable but g(x) is not differentiable at x=0 (d)

f(x) and g(x) both are not differentiable at x=0



111. The function  $f(x) = \sin^{-1}(\cos x)$  is discontinuous at x = 0 (b)

continuous at x=0 (c) differentiable at x=0 (d) none of these

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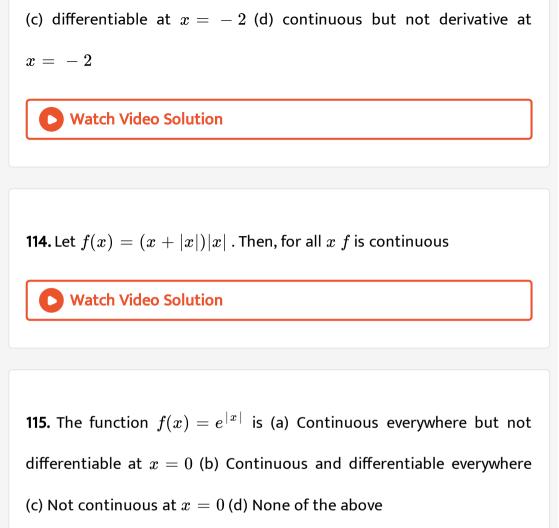
**112.** The set of points where the function f(x) = x|x| is differentiable

is (a) 
$$(\,-\infty,\,\infty)$$
 (b)  $(\,-\infty,\,0)\cup(0,\,\infty)$  (c)  $(0,\,\infty)$  (d)  $[0,\,\infty]$ 

## Watch Video Solution

113. If 
$$f(x) = iggl\{ rac{|x+2|}{ anu^{-1}(x+2)}, \ x 
eq -2, \qquad x=-2$$
 ,

then (a).f(x) is continuous at  $x=\ -2$  (b) not continuous at  $x=\ -2$ 



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116. The function  $f(x) = |\cos x|$  is (a) everywhere continuous and differentiable (b) everywhere continuous but not differentiable at

 $(2n+1)\pi/2$  ,  $n\in Z$  (c) neither continuous nor differentiable at  $(2n+1)\pi/2$  ,  $n\in Z$  (d) none of these

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**117.** If  $f(x) = \sqrt{1 - \sqrt{1 - x^2}}$ , then f(x) is (a) continuous on [-1, 1] and differentiable on (-1, 1) (b) continuous on [-1, 1] and differentiable on ( - 1, 0 ) U ( 0, 1 )` (c) continuous and differentiable on [-1, 1] (d) none of these

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118. If  $f(x) = a | \sin x | + b \, e^{\, |\, x\,|} + c \, |x|^3$  and if f(x) is differentiable at

x=0 , then a=b=c=0 (b)  $a=0,\;b=0;\;c\in R$  (c)

$$b=c=0, \;\; a\in R$$
 (d)  $c=0, \;\; a=0, \;\; b\in R$ 

119. If 
$$f(x) = x^2 + rac{x^2}{1+x^2} + rac{x^2}{\left(1+x^2
ight)^2} + \ldots + rac{x^2}{\left(1+x^2
ight)^n} + ,$$

then at  $x=0,\,f(x)$  (a)has no limit (b) is discontinuous (c)is continuous but not differentiable (d) is differentiable

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120. If 
$$f(x)=ig|(\log)_e xig|$$
 , then (a)  $f'ig(1^+ig)=1$  (b)  $f'ig(1^-ig)=-1$  (c)  $f'(1)=1$  (d)  $f'(1)=-1$ 

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121. If  $f(x) = |(\log)_e |x||$ , then (a)f(x) is continuous and differentiable for all x in its domain] (b) f(x) is continuous for all x in its domain but not differentiable at  $x = \pm 1$  (c) f(x) is neither continuous nor differentiable at  $x = \pm 1$  (d) none of these

122. Let 
$$f(x)=egin{cases} rac{1}{|x|}&f ext{ or }|x|\geq 1\ ax^2+b&f ext{ or }|x|<1 \end{pmatrix}$$
 . If  $f(x)$  is continuous and

differentiable at any point, then

A. 
$$a = rac{1}{2}, b = -rac{3}{2}$$
  
B.  $a = -rac{1}{2}, b = rac{3}{2}$ 

$$C. a = 1, b = -1$$

D. none of these

#### Answer: B

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123. The function f(x) = x - [x], where [·] denotes the greatest integer function is (a) continuous everywhere (b) continuous at integer points only (c) continuous at non-integer points only (d) differentiable everywhere 124. Let  $f(x)=ig\{ax^2+1,\quad x>1,\quad x+1/2,\quad x\leq 1$  Then, f(x) is derivable at x=1 , if a=2 (b) b=1 (c) a=0 (d) a=1/2

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125. Let  $f(x)=|\sin x|$  . Then, (a) f(x) is everywhere differentiable. (b) f(x) is everywhere continuous but not differentiable at  $x=n\,\pi,\ n\in Z$  (c) f(x) is everywhere continuous but not differentiable at  $x=(2n+1)rac{\pi}{2}$ ,  $n\in Z$ .(d) none of these

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126. Let  $f(x) = |\cos x|$  (a) Then, f(x) is everywhere differentiable (b) f(x) is everywhere continuous but not differentiable at  $x = n\pi$ ,  $n \in Z$  (c) f(x) is everywhere continuous but not differentiable at  $x = (2n + 1) \frac{\pi}{2}$ ,  $n \in Z$  (d) none of these 127. The function  $f(x)=1+|{\cos x}|$  is (a) continuous no where (b) continuous everywhere (c) not differentiable at x=0 (d) not differentiable at  $x=n\pi,\ n\in Z$ 

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128. The  $f(x)=|\cos x|$  (a)Then, f (x) is everywhere differentiable at  $x=(2n+1) \pi/2, \ n\in Z$  (b) continuous but not differentiable at  $x=(2n+1) \pi/2, \ n\in Z$  (c) neither differentiable nor continuous at  $x=n\pi, \ n\in Z$  (d) none of these

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129. The function  $f(x)=rac{\sin(\pi[x-\pi])}{4+\left[x
ight]^2}$  , where [] denotes the

greatest integer function,(a) is continuous as well as differentiable for

all  $x \in R$  (b) continuous for all x but not differentiable at some x (c) differentiable for all x but not continuous at some x .(d) none of these

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130. Let  $f(x) = a + b |x| + c |x|^4$  , where  $a, \ b, \ {
m and} \ c$  are real constants.

Then, f(x) is differentiable at x = 0 , if

A. a = 0

- B. b = 0
- C. c = 0

D. none of these

#### Answer: B

131. If f(x) = |3 - x| + [3 + x] , where [x] denotes the least integer greater than or equal to x , then f(x) is

A. (a) continuous and differentiable at  $x\,=\,3$ 

B. (b) continuous but not differentiable at x=3

C. (c) differentiable but not continuous at x=3

D. (d) neither differentiable nor continuous at x=3

#### Answer: null

132. If 
$$f(x) = \begin{cases} \frac{1}{1+e^{1/x}}, & x \neq 00, \\ x = 0 \end{cases}$$
, then  $f(x)$  is continuous as well as differentiable at  $x = 0$  (b) continuous but not differentiable at  $x = 0$  (c) differentiable but not continuous at  $x = 0$  (d) none of these

**133.** If  $f(x) = \left\{ \frac{1 - \cos x}{x \sin x}, x \neq 0 \text{ and } \frac{1}{2}, x = 0 \text{ then at } x = 0, f(x) \right\}$ is (a)continuous and differentiable (b)differentiable but not continuous (c)continuous but not differentiable (d)neither continuous nor differentiable

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134. The set of points where the function f(x) given by  $f(x) = |x - 3| \cos x$  is differentiable, is R (b)  $R - \{3\}$  (c)  $(0, \infty)$  (d) none of these

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**135.** Let  $f(x)=\{1, x \le -1, |x|, -1 \le x \le 1, 0, x \ge 1\}$ . Then, f is (a) continuous at

x=-1 (b) differentiable at x=-1 (c) everywhere continuous (d) everywhere

#### differentiable



