



MATHS

BOOKS - PREMIERS PUBLISHERS

DIFFERENTIAL CALCULUS DIFFERENTIABILITY AND METHODS OF DIFFERENTIATION

Worked Examples

1. Find the slope of the tangent line $f(x) = 6x - 1$ at any point $(x_0, f(x_0))$



[Watch Video Solution](#)

2. Find the slope of the tangent line to the graph $f(x) = x^2 - x + 1$ at $(1, 1)$



[View Text Solution](#)

3. Show that the greatest integer function $f(x) = [x]$ is not differentiable at any integer?

 [View Text Solution](#)

4. Differentiate the following with respect to x .

$$y = 3x^5 + 4e^x - 6 \log x + 3 \tan x$$

 [Watch Video Solution](#)

5. Differentiate the following with respect to x .

$$y = \sqrt{1 + \sin 2x}$$

 [Watch Video Solution](#)

6. Differentiate the following with respect to x .

$$y = \sqrt{1 - \cos 2 \frac{x}{1 + \cos 2x}}$$

 [Watch Video Solution](#)



Watch Video Solution

7. Differentiate the following with respect to x .

$$y = 4 \operatorname{cosec} x + 3 \sin^{-1} x$$



View Text Solution

8. Differentiate the following with respect to x .

$$y = \tan^{-1} x + \log(x + 5) + \frac{\cos x}{x}$$



View Text Solution

9. Differentiate the following with respect to x .

$$y = \sin x \cos x$$



View Text Solution

10. Differentiate the following with respect to x .

$$y = (4x^2 + 1)e^{2x}$$



[View Text Solution](#)

11. Differentiate the following with respect to x .

$$y = xe^x \sin x$$



[Watch Video Solution](#)

12. Differentiate the following with respect to x .

$$y = x^2 e^{3x} \tan^{-1} x$$



[Watch Video Solution](#)

13. Differentiate the following with respect to x .

$$y = |x - 2| \text{ find } f(1) \& f(4).$$





Watch Video Solution

14. Find $\frac{dy}{dx}$ if $y = \sqrt[3]{3x^2 - 6x + 1}$.



Watch Video Solution

15. Find $\frac{dy}{dx}$ if

$$y = \tan^{-1}(x^2).$$



Watch Video Solution

16. Find $\frac{dy}{dx}$ if

$$(\tan^{-1} x)^2.$$



Watch Video Solution

17. If $y = (e^x + \log x + 1)^{50}$ find $\frac{dy}{dx}$.





[Watch Video Solution](#)

18. If $y = (3x + 1)^2(x + 4)^3$ find $\frac{dy}{dx}$.



[Watch Video Solution](#)

19. If $y = \left(\frac{x + 3}{2x - 1}\right)^5$ then $f \in ddy/dx$.



[Watch Video Solution](#)

20. $y = \log(\sin x + \cos x)$ show that $\frac{dy}{dx} = \tan\left(\frac{\pi}{4} - x\right)$.



[View Text Solution](#)

21. $y = \log(\tan^{-1}(\sin x))$ find $\frac{dy}{dx}$.



[View Text Solution](#)

22. Differentiate 2^x with respect to x .

 [View Text Solution](#)

23. If $y = \tan^{-1}\left(\frac{1+x^2}{1-x^2}\right)$ Find $\frac{dy}{dx}$.

 [Watch Video Solution](#)

24. Find $\frac{dy}{dx}$ if $y^2 = 4ax$.

 [View Text Solution](#)

25. Find the derivatives of the following :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

 [Watch Video Solution](#)

26. Find $\frac{dy}{dx}$ if $x^3 + y^3 - 3xy + 1 = 0$.

 [View Text Solution](#)

27. If $\sin(y + x) = y^2 + xyf \in ddy/dx$.

 [View Text Solution](#)

28. Find $\frac{dy}{dx}$ at the points corresponding to $x = 1$ to the curve $y^2 + 3xy + 2x^2 = 0$.

 [View Text Solution](#)

29. Differentiate: $y = \sqrt{x^2 + 1} \frac{\sin^3 x}{(x + 1)^5}$.

 [Watch Video Solution](#)

30. If $y = \tan^{-1}\left(\frac{1+x^2}{1-x^2}\right)$ Find $\frac{dy}{dx}$.

 [Watch Video Solution](#)

31. If $f(x) = \sin^{-1}\left(2\frac{x}{1+x^2}\right)$ find $f(x)$.

 [Watch Video Solution](#)

32. If $x = a \sec(\Theta)$ $y = b \tan(\Theta)$ find $\frac{dy}{dx}$.

 [Watch Video Solution](#)

33. If $x = a \sin^3(\Theta)$ $y = a \cos^3(\Theta)$ find $\frac{dy}{dx}$.

 [Watch Video Solution](#)

34. Differentiate $x^{\frac{1}{x}}$ with respect to $x \rightarrow x \log x$.



[View Text Solution](#)

35. Differentiate $\tan^{-1}\left(3x - \frac{x^3}{1 - 3x^2}\right)$ with respect to $\cos^{-1}\left(1 - \frac{x^2}{1 + x^2}\right)$.



[Watch Video Solution](#)

36. Differentiate x^a with respect to a^x .



[View Text Solution](#)

37. If $y = 5x^4 + 3x^3 - 2x^2 + 7x - 1$ find y_3 .



[View Text Solution](#)

38. Find $\frac{d^2y}{dx^2}$ if $x^3 + y^3 = a^3$.



[View Text Solution](#)

39. If $e^{\cot^{-1} x}$ Prove that $(1 + x^2)y_2 + (2x + 1)y_1 = 0$.



Watch Video Solution

40. If $x = a \cos^3(\Theta)$, $y = a \sin^3(\Theta)$ $f \in d \frac{d^2 y}{dx^2}$.



Watch Video Solution

41.

If $x = a(\cos(\Theta) + (\Theta)\sin(\Theta))$, $y = a(\sin(\Theta) - (\Theta)\cos(\Theta))$ show that $\widehat{\frac{d^2 y}{dx^2}}$



Watch Video Solution

42. If $x = \cos(\Theta)$, $y = \sin p(\theta)$. Show that $\widehat{1 - x^2}y_2 - xy_1 + p^2y = 0$.



View Text Solution

[View Text Solution](#)

43. Differentiate $y = \sin x \cdot \sin 2x \cdot \sin 3x \cdot \sin 4x$.

 [Watch Video Solution](#)

44. $y = e^{\sin x^2}$, $f \in ddy/dx$:

 [Watch Video Solution](#)

Solution To Exercise 10 1

1. Find the derivatives of the following functions using first principle.

$$f(x)=6$$

 [Watch Video Solution](#)

2. Find the derivatives of the following functions using first principle.

$$f(x) = -4x + 7$$



Watch Video Solution

3. Find the derivatives of the following functions using first principle.

$$f(x) = -x^2 + 2$$



Watch Video Solution

4. Find the derivatives from the left and from the right at $x=1$ (if they exist) of the following functions. Are the functions differentiable at $x=1$?

$$f(x) = |x - 1|$$



Watch Video Solution

5. Find the derivatives from the left and from the right at $x=1$ (if they exist) of the following functions. Are the functions differentiable at $x=1$?

$$f(x) = \sqrt{1 - x^2}$$



[Watch Video Solution](#)

6. Find the derivatives from the left and from the right at $x=1$ (if they exist) of the following functions. Are the functions differentiable at $x=1$?

$$f(x) = \begin{cases} x, & x \leq 1 \\ x^2, & x > 1 \end{cases}$$



[Watch Video Solution](#)

7. Determine whether the following function is differentiable at the indicated values.

$$f(x) = x|x| \text{ at } x=0$$



[Watch Video Solution](#)

8. Determine whether the following function is differentiable at the indicated values.

$$f(x) = |x^2 - 1| \text{ at } x=1$$

 [Watch Video Solution](#)

9. Determine whether the following function is differentiable at the indicated values.

$$f(x) = |x| + |x - 1| \text{ at } x=0, 1$$

 [Watch Video Solution](#)

10. Determine whether the following function is differentiable at the indicated values.

$$f(x) = \sin|x| \text{ at } x=0$$

 [Watch Video Solution](#)

11. Show that the following functions are not differentiable at the indicated value of x .

$$f(x) = \begin{cases} -x + 2, & x \leq 2 \\ 2x - 4, & x > 2 \end{cases}, x = 2$$



[Watch Video Solution](#)

12. Show that the following functions are not differentiable at the indicated value of x .

$$f(x) = \begin{cases} 3x, & x < 0 \\ -4x, & x \geq 0 \end{cases}, x = 0$$



[Watch Video Solution](#)

13. The graph of f is shown below. State with reasons that x values (the numbers), at which f is not differentiable.



[Watch Video Solution](#)

14. If $f(x) = |x + 100| + x^2$, test whether $f'(-100)$ exist.



[Watch Video Solution](#)

15. Examine the differentiability of functions in \mathbb{R} by drawing the diagrams.

$$|\sin x|$$



[Watch Video Solution](#)

16. Examine the differentiability of functions in \mathbb{R} by drawing the diagrams.

$$|\cos x|$$



[Watch Video Solution](#)

1. Find the derivatives of the following functions with respect to corresponding independent variables.

$$f(x) = x - 3 \sin x$$



[Watch Video Solution](#)

2. Find the derivatives of the following functions with respect to corresponding independent variables.

$$y = \sin x + \cos x$$



[Watch Video Solution](#)

3. Find the derivatives of the following functions with respect to corresponding independent variables.

$$f(x) = x \sin x$$



[Watch Video Solution](#)

4. Find the derivatives of the following functions with respect to corresponding independent variables.

$$y = \cos x - 2 \tan x$$

 [Watch Video Solution](#)

5. Find the derivatives of the following functions with respect to corresponding independent variables.

$$g(t) = t^3 \cos t$$

 [Watch Video Solution](#)

6. Find the derivatives of the following functions with respect to corresponding independent variables.

$$g(t) = 4 \sec t + \tan t$$

 [Watch Video Solution](#)

7. Find the derivatives of the following functions with respect to corresponding independent variables.

$$y = e^x \sin x$$



[Watch Video Solution](#)

8. Find the derivatives of the following functions with respect to corresponding independent variables.

$$y = \frac{\tan x}{x}$$



[Watch Video Solution](#)

9. Find the derivatives of the following functions with respect to corresponding independent variables.

$$y = \frac{\sin x}{1 + \cos x}$$



[Watch Video Solution](#)

10. Find the derivatives of the following functions with respect to corresponding independent variables.

$$y = \frac{x}{\sin x + \cos x}$$



[Watch Video Solution](#)

11. Find the derivatives of the following functions with respect to corresponding independent variables.

$$y = \frac{\tan x - 1}{\sec x}$$



[Watch Video Solution](#)

12. Find the derivatives of the following functions with respect to corresponding independent variables.

$$y = \frac{\sin x}{x^2}$$



[Watch Video Solution](#)

13. Find the derivatives of the following functions with respect to corresponding independent variables:

$$y = \tan(\Theta)(\sin(\Theta) + \cos(\Theta))$$



[Watch Video Solution](#)

14. Find the derivatives of the following functions with respect to corresponding independent variables.

$$y = \cos ecx. \cot x$$



[Watch Video Solution](#)

15. Find the derivatives of the following functions with respect to corresponding independent variables.

$$y = x \sin x \cos x$$



[Watch Video Solution](#)

16. Find the derivatives of the following functions with respect to corresponding independent variables.

$$y = e^{-x} \log x$$



[Watch Video Solution](#)

17. Find the derivatives of the following functions with respect to corresponding independent variables.

$$y = (x^2 + 5) \cdot \log(1 + x)e^{-3x}$$



[Watch Video Solution](#)

18. Find the derivatives of the following functions with respect to corresponding independent variables.

$$y = \sin x^\circ$$



[Watch Video Solution](#)

19. Find the derivatives of the following functions with respect to corresponding independent variables.

$$y = \log_{10} x$$



Watch Video Solution

20. Draw the function $f'(x)$ if $f(x) = 2x^2 - 5x + 3$



Watch Video Solution

Solution To Exercise 10 3

1. Differentiate the following :

$$y = (x^2 + 4x + 6)^5$$



Watch Video Solution

2. Differentiate the following :

$$y = \tan 3x$$



Watch Video Solution

3. Differentiate the following :

$$y = \cos(\tan x)$$



Watch Video Solution

4. Differentiate the following :

$$y = \sqrt[3]{1 + x^3}$$



Watch Video Solution

5. Differentiate the following :

$$y = e^{\sqrt{x}}$$





Watch Video Solution

6. Differentiate the following :

$$y = \sin(e^x)$$



Watch Video Solution

7. Differentiate the following :

$$F(x) = (x^3 + 4x)^7$$



Watch Video Solution

8. Differentiate the following :

$$h(t) = \left(t - \frac{1}{t}\right)^{\frac{3}{2}}$$



Watch Video Solution

9. Differentiate the following :

$$f(t) = \sqrt[3]{1 + \tan t}$$



Watch Video Solution

10. Differentiate the following :

$$y = \cos(a^3 + x^3)$$



Watch Video Solution

11. Differentiate the following :

$$y = e^{-mx}$$



Watch Video Solution

12. Differentiate the following :

$$y = 4 \sec 5x$$





[Watch Video Solution](#)

13. Differentiate the following :

$$y = (2x - 5)^4(8x^2 - 5)^{-3}$$



[Watch Video Solution](#)

14. Differentiate the following :

$$y = (x^2 + 1)\sqrt[3]{x^2 + 2}$$



[Watch Video Solution](#)

15. Differentiate the following :

$$y = xe^{-x^2}$$



[Watch Video Solution](#)

16. Differentiate the following :

$$s(t) = \sqrt[4]{\frac{t^3 + 1}{t^3 - 1}}$$



Watch Video Solution

17. Differentiate the following :

$$f(x) = \frac{x}{\sqrt{7 - 3x}}$$



Watch Video Solution

18. Differentiate the following :

$$y = \tan(\cos x)$$



Watch Video Solution

19. Differentiate the following :

$$y = \frac{\sin^2 x}{\cos x}$$



[Watch Video Solution](#)

20. Differentiate the following :

$$y = 5^{\frac{-1}{x}}$$



[Watch Video Solution](#)

21. Differentiate the following :

$$y = \sqrt{1 + 2 \tan x}$$



[Watch Video Solution](#)

22. Differentiate the following :

$$y = \sin^3 x + \cos^3 x$$



[Watch Video Solution](#)

23. Differentiate the following :

$$y = \sin^2(\cos kx)$$

 [Watch Video Solution](#)

24. Differentiate the following :

$$y = (1 + \cos^2 x)^6$$

 [Watch Video Solution](#)

25. Differentiate the following :

$$y = \frac{e^{3x}}{1 + e^x}$$

 [Watch Video Solution](#)

26. Differentiate the following :

$$y = \sqrt{x + \sqrt{x}}$$





Watch Video Solution

27. Differentiate the following :

$$y = e^{x \cos x}$$



Watch Video Solution

28. Differentiate the following :

$$y = \sqrt{x + \sqrt{x + \sqrt{x}}}$$



Watch Video Solution

29. Differentiate the following :

$$y = \sin(\tan(\sqrt{\sin x}))$$



Watch Video Solution

30. Differentiate the following :

$$y = \sin^{-1} \left(\frac{1 - x^2}{1 + x^2} \right)$$



[Watch Video Solution](#)

Solution To Exercise 10 4

1. Find the derivatives of the following :

$$y = x^{\cos x}$$



[Watch Video Solution](#)

2. Find the derivatives of the following :

$$y = x^{\log x} + (\log x)^x$$



[Watch Video Solution](#)

3. Find the derivatives of the following :

$$\sqrt{xy} = e^{(x-y)}$$



Watch Video Solution

4. Find the derivatives of the following :

$$x^y = y^x$$



Watch Video Solution

5. Find the derivatives of the following :

$$(\cos x)^{\log x}$$



Watch Video Solution

6. Find the derivatives of the following :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



Watch Video Solution

7. Find the derivatives of the following :

$$\sqrt{x^2 + y^2} = \tan^{-1}\left(\frac{y}{x}\right)$$



Watch Video Solution

8. Find the derivatives of the following :

$$\tan(x + y) + \tan(x - y) = x$$



Watch Video Solution

9. if $\cos(xy)=x$, show that $\frac{dy}{dx} = \frac{-(1 + y \sin(xy))}{x \sin xy}$.



Watch Video Solution

10. Find the derivatives of the following :

$$\tan^{-1} \sqrt{\frac{1 - \cos x}{1 + \cos x}}$$

 [Watch Video Solution](#)

11. Find the derivatives of the following :

$$\tan^{-1} \left(\frac{6x}{1 - 9x^2} \right)$$

 [Watch Video Solution](#)

12. Find the derivatives of the following :

$$\cos \left(2 \tan^{-1} \sqrt{\frac{1 - x}{1 + x}} \right)$$

 [Watch Video Solution](#)

13. Find the derivatives of the following :

$$x = a \cos^3 t, y = a \sin^3 t$$



Watch Video Solution

14. Find the derivatives of the following :

$$x = a(\cos t + t \sin t), y = a(\sin t - t \cos t)$$



Watch Video Solution

15. Find the derivatives of the following :

$$x = \frac{1 - t^2}{1 + t^2}, y = \frac{2t}{1 + t^2}$$



Watch Video Solution

16. Find the derivatives of the following :

$$\cos^{-1} \left(\frac{1 - x^2}{1 + x^2} \right)$$



Watch Video Solution

17. Find the derivatives of the following :

$$\sin^{-1}(3x - 4x^3)$$

 [Watch Video Solution](#)

18. Find the derivatives of the following :

$$\tan^{-1}\left(\frac{\cos x + \sin x}{\cos x - \sin x}\right)$$

 [Watch Video Solution](#)

19. Find the derivative of $\sin x^2$ with respect to x^2 .

 [Watch Video Solution](#)

20. Find the derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ with respect to $\tan^{-1} x$.

 [Watch Video Solution](#)

21. If $u = \tan^{-1} \frac{\sqrt{1+x^2} - 1}{x}$ and $v = \tan^{-1} x$, find $\frac{du}{dv}$.

 [Watch Video Solution](#)

22. Find the derivative with $\tan^{-1} \left(\frac{\sin x}{1 + \cos x} \right)$ with respect to $\tan^{-1} \left(\frac{\cos x}{1 + \sin x} \right)$.

 [Watch Video Solution](#)

23. If $y = \sin^{-1} x$ then find y'' .

 [Watch Video Solution](#)

24. If $y = e^{\tan^{-1} x}$, show that $(1 + x^2)y'' + (2x - 1)y' = 0$.

 [Watch Video Solution](#)

25. If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$ show that $(1-x^2)y_2 - 3xy_1 - y = 0$.

 [Watch Video Solution](#)

26. If $x = a(\theta + \sin \theta)$, $y = a(1 - \cos \theta)$ then prove that at $\theta = \frac{\pi}{2}$, $y'' = \frac{1}{a}$.

 [Watch Video Solution](#)

27. If $\sin y = x \sin(a + y)$, then prove that $\frac{dy}{dx} = \frac{\sin^2(a + y)}{\sin a}$, $a \neq n\pi$.

 [Watch Video Solution](#)

28. If $y = (\cos^{-1} x)^2$ prove that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$. Hence find y_2 when $x=0$

 [Watch Video Solution](#)

Solution To Exercise 10 5

1. $\frac{d}{dx} \left(\frac{2}{\pi} \sin x^\circ \right)$ is

A. $\frac{\pi}{180} \cos x^\circ$

B. $\frac{1}{90} \cos x^\circ$

C. $\frac{\pi}{90} \cos x^\circ$

D. $\frac{2}{\pi} \cos x^\circ$

Answer: B



Watch Video Solution

2. If $y = f(x^2 + 2)$ and $f'(3) = 5$, then $\frac{dy}{dx}$ at $x=1$ is

A. 5

B. 25

C. 15

D. 10

Answer: D



Watch Video Solution

3. If $y = \frac{1}{4}u^4$ and $u = \frac{2}{3}x^3 + 5$, then $\frac{dy}{dx}$ is

A. $\frac{1}{27}x^2(2x^3 + 15)^3$

B. $\frac{2}{27}x(2x^3 + 5)^3$

C. $\frac{2}{27}x^2(2x^3 + 5)^3$

D. $-\frac{2}{27}x(2x^3 + 5)^3$

Answer: C



Watch Video Solution

4. If $f(x) = x^2 - 3x$, then the points at which $f(x) = f'(x)$ are

- A. both positive integers
- B. both negative integers
- C. both irrational
- D. one rational and another irrational

Answer: C

 [Watch Video Solution](#)

5. If $y = \frac{1}{a - z}$, then $\frac{dz}{dy}$ is

- A. $(a - z)^2$
- B. $-(z - a)^2$
- C. $(z + a)^2$
- D. $-(z + a)^2$

Answer: A

 [Watch Video Solution](#)

6. If $y = \cos(\sin x^2)$, then $\frac{dy}{dx}$ at $x = \sqrt{\frac{\pi}{2}}$ is

A. -2

B. 2

C. $-2\sqrt{\frac{\pi}{2}}$

D. 0

Answer: D



Watch Video Solution

7. If $y = mx + c$ and $f(0) = f'(0) = 1$, then $f(2)$ is

A. 1

B. 2

C. 3

D. -3

Answer: C



Watch Video Solution

8. If $f(x) = x \tan^{-1} x$, then $f'(1)$ is

A. $1 + \frac{\pi}{4}$

B. $\frac{1}{2} + \frac{\pi}{4}$

C. $\frac{1}{2} - \frac{\pi}{4}$

D. 2

Answer: B



Watch Video Solution

9. $\frac{d}{dx}(e^{(x+5\log x)})$

A. $e^x x^4(x + 5)$

B. $e^x x(x + 5)$

C. $e^x + \frac{x}{5}$

D. $e^x - \frac{x}{5}$

Answer: A

 [Watch Video Solution](#)

10. If the derivative of $(ax - 5)e^{3x}$ at $x=0$ is -13, then the value of a is

A. 8

B. -2

C. 5

D. 2

Answer: D

 [Watch Video Solution](#)

11. $x = \frac{1 - t^2}{1 + t^2}$, $y = \frac{2t}{1 + t^2}$ then $\frac{dy}{dx}$ is

A. $-\frac{y}{x}$

B. $\frac{y}{x}$

C. $-\frac{x}{y}$

D. $\frac{x}{y}$

Answer: C



Watch Video Solution

12. If $x = a \sin \theta$ and $y = b \cos \theta$, then $\frac{d^2y}{dx^2}$ is

A. $\frac{a}{b^2} \sec^3 \theta$

B. $-\frac{b}{a} \sec^3 \theta$

C. $-\frac{b}{a^2} \sec^3 \theta$

D. $-\frac{b^2}{a^2}\sec^3 \theta$

Answer: C

 [Watch Video Solution](#)

13. The differential coefficient of $\log_{10} x$ with respect to $\log_x 10$ is

A. 1

B. $-(\log_{10} x)^2$

C. $(\log_x 10)^2$

D. $\frac{x^2}{100}$

Answer: B

 [Watch Video Solution](#)

14. If $f(x) = x + 2$, then $f'(f(x))$ at $x=4$ is

A. 8

B. 1

C. 4

D. 5

Answer: B

 [Watch Video Solution](#)

15. If $y = \frac{(1-x)^2}{x^2}$, then $\frac{dy}{dx} =$ is

A. $\frac{2}{x^2} + \frac{2}{x^3}$

B. $-\frac{2}{x^2} + \frac{2}{x^3}$

C. $-\frac{2}{x^2} - \frac{2}{x^3}$

D. $-\frac{2}{x^3} + \frac{2}{x^2}$

Answer: D

 [Watch Video Solution](#)

16. If $pv = 81$, then $\frac{dp}{dv}$ at $v=9$ is

A. 1

B. -1

C. 2

D. -2

Answer: B



[Watch Video Solution](#)

17. If $f(x) = \{(x - 5, \text{ if } x \leq 1), (4x^2 - 9, \text{ if } (1 < x < 2), (3x + 4, \text{ if } x \geq 2)\}$ then the right hand derivative of $f(x)$ at $x = 2$ is:

A. 0

B. 2

C. 3

D. 4

Answer: C



Watch Video Solution

18. It is given that $f'(a)$ exists, then $\lim_{x \rightarrow a} \frac{xf(a) - af(x)}{x - a}$ is:

A. $f(a) - af'(a)$

B. $-f(a)$

C. $f(a)$

D. $f(a) + af'(a)$

Answer: A



Watch Video Solution

19. Assertion (A) : $f(x) = \begin{cases} x + 1, & x < 2 \\ 2x - 1, & x \geq 2 \end{cases}$ then $f'(2)$ does not exist.

Reason (R) : $f(x)$ is not continuous at 2.

A. 0

B. 1

C. 2

D. Does not exist

Answer: D



[Watch Video Solution](#)

20.

If $g(x) = (x^2 + 2x + 3)$, $f(x)$ and $f(0) = 5$ and $\lim_{x \rightarrow 0} \frac{f(x) - 5}{x} = 4$ then

A. 22

B. 14

C. 18

D. 12

Answer: A

 [Watch Video Solution](#)

21. If $f(x) = \begin{cases} x + 2 & -1 < x < 3 \\ 5 & x = 3 \\ 8 - x & x > 3 \end{cases}$, then at $x=3$, $f'(x)$ is

A. 1

B. -1

C. 0

D. Does not exist

Answer: D

 [Watch Video Solution](#)

22. The derivative of $f(x) = x|x|$ at $x=-3$ is

A. 6

B. -6

C. Does not exist

D. 0

Answer: A



Watch Video Solution

23. If $f(x) = \begin{cases} 2a - x, & \text{for } -a < x < a \\ 3x - 2a, & \text{for } x \geq a \end{cases}$ then which of the following is true?

A. $f(x)$ is not differentiable at $x=a$

B. $f(x)$ is not discontinuous at $x = a$

C. $f(x)$ is continuous for all x in \mathbb{R}

D. $f(x)$ is differentiable for all $x \geq a$

Answer: A



Watch Video Solution

24. If $f(x) = \{(ax^2) - b, -1$

A. $a = \frac{1}{2}, b = -\frac{3}{2}$

B. $a = -\frac{1}{2}, b = \frac{3}{2}$

C. $a = -\frac{1}{2}, b = -\frac{3}{2}$

D. $a = \frac{1}{2}, b = \frac{3}{2}$

Answer: C



View Text Solution

25.

The number of $p \in \mathbb{R}$ in which the function $f(x) = |x - 1| + |x + 3| + \sin$

A. 3

B. 2

C. 1

D. 4

Answer: B



[View Text Solution](#)

Problems For Practice

1. If $y = \frac{1}{\sqrt[3]{x}} + \log_5 x + 8$, $f \in d \frac{dy}{dx}$.



[Watch Video Solution](#)

2. Differentiate $\log_e x$ from first principles.



[Watch Video Solution](#)

3. $y = a \sin x \log_{10} x + e^{2x}$, $f \in d \frac{dy}{dx}$.



Watch Video Solution

4. $y = \log \sec\left(\frac{\pi}{4} + \frac{x}{2}\right)$, show that $\widehat{\frac{dy}{dx}} = \frac{1}{2} \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$



Watch Video Solution

5. If $y = \cos^{-1}\left(\frac{1-x}{1+x}\right)$, show that $\frac{dy}{dx} = \frac{1}{\sqrt{x}(1+x)}$



Watch Video Solution

6. if $y = x^{x^2}$, show that $\widehat{\frac{dy}{dx}} = x^{x^2} x(1 + 2 \log x)$



Watch Video Solution

7. If $y = x^a + a^x + x^x + a^a$, $d \in d \frac{dy}{dx}$.



Watch Video Solution

8. $f \in d\frac{dy}{dx}$ if $y = (\log x)^{\sin^{-1} x}$

 [Watch Video Solution](#)

9. $f(x) = x^x + x^{\frac{1}{x}}$, $f \in df'(x)$.

 [Watch Video Solution](#)

10. $y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$, $f \in d\frac{dy}{dx}$.

 [Watch Video Solution](#)

11. $y = \sec^{-1}\left(\frac{1 + x^2}{1 - x^2}\right)$, $f \in d\frac{dy}{dx}$.

 [Watch Video Solution](#)

12. Find the derivatives of the following :

$$\tan^{-1} \left(\frac{\cos x + \sin x}{\cos x - \sin x} \right)$$

 [Watch Video Solution](#)

13. If $y = \tan^{-1} \left(\frac{1 + x^2}{1 - x^2} \right)$ Find $\frac{dy}{dx}$.

 [Watch Video Solution](#)

14. $y = \tan^{-1} \left(\frac{\sqrt{x} + \sqrt{a}}{1 - \sqrt{ax}} \right)$, $f \in d \frac{dy}{dx}$.

 [Watch Video Solution](#)

15. $y = \tan^{-1} \left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \right)$, $f \in d \frac{dy}{dx}$.

 [Watch Video Solution](#)

16. $y = \cot^{-1} \left(\frac{\sqrt{1 + \sin x} + \sqrt{1 - \sin x}}{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}} \right)$, $f \in d \frac{dy}{dx}$.

 [Watch Video Solution](#)

17. Differentiate $\cos ec^{-1} \left(\frac{1}{3x - 4x^3} \right)$:

 [Watch Video Solution](#)

18. If $x = a \sec^3 \theta$ and $y = a \tan^3 \theta$ find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{3}$.

 [Watch Video Solution](#)

19. If $x = 2 \cos \theta - \cos 2\theta$, $y = 2 \sin \theta - \sin 2\theta$, $f \in d \frac{dy}{dx}$.

 [Watch Video Solution](#)

20. $x = a \left(\cos \theta + \log \tan \left(\frac{\theta}{2} \right) \right)$, $y = a \sin \theta$, $f \in d \frac{dy}{dx}$.



Watch Video Solution

21. If $x^m y^n = (x + y)^{m+n}$, $f \in d \frac{dy}{dx}$.



Watch Video Solution

22. If $e^x + e^y = e^{x+y}$, $f \in d \frac{dy}{dx}$.



Watch Video Solution

23. If $fax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, $f \in d \frac{dy}{dx}$.



Watch Video Solution

24. If $y = \cos(m \sin^{-1} x)$, prove that

$$(1 - x^2)y_3 - 3xy_2 + (m^2 - 1)y_1 = 0$$



Watch Video Solution

25. Differentiate $\tan^{-1}\left(2\frac{x}{1-x^2}\right)$ with respect to $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

 [Watch Video Solution](#)

26. If $y = \sqrt{(x-1)(x-2)(x-3)}$, $f \in d\frac{dy}{dx}$.

 [Watch Video Solution](#)

27. Differentiate $\cos e^{-1}\left(\frac{1+x^2}{2x}\right)$ with respect to $\cot^{-1}\left(\frac{1-3x^2}{3x-x^3}\right)$.

 [Watch Video Solution](#)

28. $f(x) = \{(x, 0, \leq, x, \leq, 2), (3x, -, 1, 2, <, x, \leq, 3)\}$ then $f'(2^+)$ is:

A. 3

B. 0

C. 2

D. 6

Answer: A



[Watch Video Solution](#)

29. $f(x) = \begin{cases} (x, 0, \leq, x, \leq, 1), & (2x, -, 1, 1, <, x, \leq, 2) \end{cases}$ then $f'(1^-)$ is :

A. 0

B. 1

C. 2

D. none of these

Answer: B



[Watch Video Solution](#)

30. If $f(x) = x^2$ and is differentiable an $[1, 2]f'(c)$ at c where $c \in [1,2]$:

A. $-2c$

B. 0

C. $2c$

D. c^2

Answer: C



[Watch Video Solution](#)

31. $f(x) = \begin{cases} (x, 0, \leq, x, \leq, 1), & (2x, -, 1, 1, <, x, \leq, 2) \end{cases}$ then $f'(1^-)$ is :

A. 1

B. 0

C. $\frac{1}{2}$

D. Does not exist

Answer: D



[Watch Video Solution](#)

32. $f(x) = |x| + |x + 1| + |x - 1| + \cos x$. At how many points on \mathbb{R} , this function is not differentiable:

A. 3

B. 2

C. 1

D. 4

Answer: A



[View Text Solution](#)

33. $f(x) = |x| + |x - 1|$ find $f'(1^+)$:

A. 4

B. 2

C. 1

D. 0

Answer: B



Watch Video Solution

34. $f(x) = \begin{cases} 2x - 3 & 0 \leq x \leq 2 \\ x^2 - 3 & 2 \leq x \leq 4 \end{cases}$ then $f'(2^+)$ and $f'(2^-)$

are:

A. (4,4)

B. (-4,-4)

C. (4,2)

D. (-4,4)

Answer: C

 [Watch Video Solution](#)

35. Compute $f'(0^+)$ if $f(x) = \frac{x(e^{\frac{1}{x}} - 1)}{e^{\frac{1}{x}} + 1}$:

A. Does not exist

B. -1

C. 0

D. 1

Answer: D

 [Watch Video Solution](#)

36. $f(x) = ax^2 + bx + 12$, $f'(2) = 11$, $f'(4) = 15$, Then a and b are:

A. (1,7)

B. (7,1)

C. (2,5)

D. (5,2)

Answer: A

 [View Text Solution](#)

37. $y = \tan^{-1} \sqrt{\frac{1 - \cos x}{1 + \cos x}} \frac{dy}{dx}$ is:

A. 2

B. $\frac{1}{2}$

C. $-\frac{1}{2}$

D. -2

Answer: B

 [Watch Video Solution](#)

38. $y = \log_5 x + 55$ then $\frac{dy}{dx}$ is:

A. $\frac{5}{x}$

B. $\log_5(x^2 - 1)$

C. $\frac{\log_5 e}{x}$

D. $\frac{\log_e 5}{x}$

Answer: C



[View Text Solution](#)

39. Differentiate $\frac{x^2 - 1}{x^2 + 1}$ with respect to x :

A. $\frac{4x}{x^2 + 1}$

B. $\frac{4x}{x^2 - 1}$

C. $\frac{x^2 + 1}{x^2 + 1}$

D. $\frac{4x}{(x^2 + 1)^2}$

Answer: D

 [View Text Solution](#)

40. $y = e^{\sin x^2}$, $f \in ddy/dx$:

A. $y \cos(x^2) 2x$

B. $\sin x^2 e^{\sin x^2 - 1}$

C. $\frac{1}{\sin(x^2)}$

D. $e^{\sin x^2} (2 \sin x \cos x)$

Answer: A

 [Watch Video Solution](#)

41. $y = e^{\sin^{-1} x}$ find y' at $x=0$:

A. e

B. 1

C. 0

D. -1

Answer: B

 [View Text Solution](#)

42. $f(x) = \sqrt{1 + \sin 2x}$ then $f'(x)$ is:

A. $\frac{1}{\sin x + \cos x}$

B. $\cos x + \sin x$

C. $\cos x - \sin x$

D. 1

Answer: C

 [View Text Solution](#)

43. If $y = \tan^{-1}(e^x)$ find $\frac{dy}{dx}$ at $x=0$:

A. 1

B. 0

C. $-\frac{1}{2}$

D. $\frac{1}{2}$

Answer: D



[View Text Solution](#)

44. If $y = \sqrt{x + \sqrt{x + \sqrt{x + \sqrt{x}}}} \rightarrow \infty$, $f \in d\frac{dy}{dx}$:

A. $\frac{1}{2y - 1}$

B. $\frac{1}{1 - 2y}$

C. $yx^{\frac{1}{2}}$

D. $\frac{1}{2xy - 1}$

Answer: A

 [Watch Video Solution](#)

45. Differentiate $\cos ec^{-1}\left(\frac{1}{3x - 4x^3}\right)$:

A. $-\frac{3}{\sqrt{1-x^2}}$

B. $\frac{3}{\sqrt{1-x^2}}$

C. $\frac{3}{1+x^2}$

D. $-\frac{3}{1+x^2}$

Answer: B

 [Watch Video Solution](#)

46. $x = a \sin^2 \theta$, $y = b \cos^2 \theta$ then $\frac{dy}{dx}$:

A. $\frac{a}{b}$

B. $-\frac{a}{b}$

C. $\frac{b}{a}$

D. $-\frac{b}{a}$

Answer: D



[View Text Solution](#)

47. $x = ct, y = \frac{c}{t}$, then $\frac{dy}{dx}$ is:

A. $-\frac{1}{t^2}$

B. $\frac{1}{t^2}$

C. $\frac{1}{t}$

D. $\frac{c}{t^2}$

Answer: A



[View Text Solution](#)

48. Find $\frac{dy}{dx}$ if $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$:

A. $-\frac{b^2x}{a^2y}$

B. $\frac{b^2x}{a^2y}$

C. $\frac{a^2y}{b^2x}$

D. $-\frac{a^2y}{b^2x}$

Answer: B



[View Text Solution](#)

49. $y = e^{ax}$ then $\frac{d^n y}{dx^n} = y_n$ is:

A. e^{anx}

B. ay^n

C. $a^n y$

D. $a^n y^n$

Answer: C



Watch Video Solution

50. If $y = A \cos 4x + b \sin 4x$, $\frac{d^2y}{dx^2} + ky = 0$, find k:



Watch Video Solution

51. $(k + 1)^{15}$ derivative of $(x + 1)^k$ is:

A. 0

B. $|_k$

C. $|_{k+1}$

D. $(x + 1)^{k+1}$

Answer: A



[View Text Solution](#)

52. If $y = e^x \sin ax$ and $\left(d^2 \frac{y}{dx^2} - 2 \frac{dy}{dx} + 5y = 0 \right)$ find a:

A. 4

B. 2

C. -1

D. 0

Answer: B

[View Text Solution](#)

53. If $y = \cos(m \sin^{-1} x)$, prove that

$$(1 - x^2)y_3 - 3xy_2 + (m^2 - 1)y_1 = 0$$

A. $-m^2$

B. $2m$

C. m^2

D. m

Answer: C



Watch Video Solution

54. If $y = 2^{-\frac{1}{x}}$ then $\frac{dy}{dx}$ is:

A. $-\frac{1}{x} 2^{-\frac{1}{x}-1}$

B. $2^{-\frac{1}{x}} (\log 2)$

C. $\frac{2^{-\frac{1}{x}} (\log 2)}{x^2}$

D. $\left(\frac{2^{-\frac{1}{x}}}{x^2} \right)$

Answer: C



View Text Solution

55. If $y = x^x$ then $\frac{dy}{dx}$ is:

A. $x^x(1 + \log x)$

B. $x^x(1 - \log x)$

C. x^x

D. xx^{x-1}

Answer: A



[View Text Solution](#)

56. If $y = f(x^2 + x + 1)$ and $f'(2) = 7$ then $\frac{dy}{dx}$ at $x = 2$ is:

A. 25

B. 35

C. 45

D. 55

Answer: B



Watch Video Solution

57. If $y = ax^2 + b$, $y(0) = 2$ and $y'(1) = 2$ then $f \in dy(2)$:

A. 0

B. 2

C. 4

D. 6

Answer: D



View Text Solution

58. It is given that $f'(a)$ exists, then $\lim_{(x \rightarrow a)} \frac{x^2 f(a) - a^2 f(x)}{x - a}$ is:

A. $a^2 f'(a) - a^2 f(a)$

B. $a^2 f(a) - a^2 f'(a)$

C. $2af(a) - a^2 f'(a)$

D. $2af'(a) - a^2 f'(a)$

Answer: C



Watch Video Solution

59. If $f(x) = 2x+5$ if $x \geq 1$ Then the left hand derivative of $f(x)$ at $x = 1$ is:

A. 2

B. 3

C. 7

D. 1

Answer: A



View Text Solution

60.

If

$$g(x) = (x^3 - 2x + 4)f(x) \text{ and } f(0) = 3 \text{ and } \lim_{(x \rightarrow 0)} \frac{f(x) - 3}{x} = 2$$

A. 1

B. -1

C. 2

D. -2

Answer: C



[Watch Video Solution](#)

61. The number of points in \mathbb{R} in which the function

$$f(x) = |x - 1| + |x - 3| + \sin x \text{ is not differentiable is } \dots\dots\dots .$$

A. 1

B. 2

C. 0

D. many

Answer: A



Watch Video Solution

62. If $f(x) = 3x + 1$ if $x < 3$

A. 1

B. 5

C. 3

D. -1

Answer: C



View Text Solution

63. If $x = a \sin \theta$ and $y = b \cos \theta$, then $\frac{d^2y}{dx^2}$ is

A. $\frac{1}{a} \cos ec^2 \theta$

B. $-\frac{1}{a} \cos ec^2 \theta$

C. $-\frac{1}{a} \cos ec^3 \theta$

D. $\frac{1}{a} \cos ec^3 \theta$

Answer: C

 Watch Video Solution

64. match the following

37.	If $y = \tan^{-1} x$ then $\frac{d^2 y}{dx^2}$ is:	(a) x^{-x}
38.	If $x^2 + y^2 = 1$ then y'' is:	(b) $\frac{2}{1+x^2}$
39.	The derivative of $x \log x$ with respect to x^x :	(c) $\frac{-2x}{(1+x^2)^2}$
40.	$\frac{d}{dx} \left\{ \tan^{-1} \left(\frac{2x}{1+x^2} \right) \right\}$ is:	(d) x
41.	If $f = 2ax$, $g = at^2$ $\frac{dg}{df}$ is:	(e) $-\frac{1}{y^3}$

A. x^{-x}

B. $\frac{2}{1+x^2}$

C. $\frac{-2x}{1+x^2}$

D. x

Answer: C

 [View Text Solution](#)

65. Find the correct statement:

A. If $s = \frac{1}{2}t^2$ is the equation of motion of a particle with usual notations then its velocity at $t = 2$ is 2 units/unit time.

B. If a function is continuous at a point then there is not necessary that the function is differentiable at that point.

C. If $f(x) = |x - 1|$ then $f'(x)$ exists at $x = 1$

D. given $y = 2^x$ $\frac{dy}{dx} = x2^{x-1}$

Answer:



[Watch Video Solution](#)

66. Find the correct statement:

A. If f is differentiable at $x = x_0$ then f is continuous at x_0

B. If $f(x) = u(x)v(x)$ then $f'(x) = \frac{u(x)v' + u'(x)v(x)}{x^2}$

C. $\frac{d}{dx}(\cos^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

D. $f(x) = x|x|$ is not differentiable at $x = 0$

Answer: A



[Watch Video Solution](#)

67. Find the incorrect statement:

A. Discontinuity implies non differentiability

B. $|\sin x|$ is not differentiable at $x = n\pi$

C. If u and v are two differentiable functions with

$$v(x) \neq 0, \text{ then } \frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

D. The slope of the tangent to the curve $y = x^3 + 3x^2 - 1$ at $(1, 3)$ is

4

Answer: D



[View Text Solution](#)

68. Find the incorrect statement:

A. If $y = \tan^{-1}(1 + x^2)$ then $\frac{dy}{dx} = \frac{2x}{x^4 + 2x^2 + 2}$

B. $y = \frac{1}{x}$ then $y' = \frac{2}{x^3}$

C. If $f(x) = x \tan^{-1} x$ then $f'(1)$ is $\frac{1}{2} + \frac{\pi}{4}$

D. $\frac{d}{dx} \left(x^{\frac{p}{q}} \right) = \frac{p}{q} x^{p-q}$

Answer: D



Watch Video Solution

69. Find the odd man out:

A. Derivative of $|x - 2|$ at $x = 2$

B. $f(x) = \lfloor x \rfloor$ at any interger

C. $f(x) = |x^2 - 1|$ at $x = 1$

D. $f(x) = \tan x$ at $x = 0$

Answer: D



Watch Video Solution

70. If $f(x) = |x - 4|$ then find the odd man out:

A. $f'(2)$

B. $f'(3)$

C. $f'(4)$

D. $f'(5)$

Answer: C



View Text Solution

71. Find the odd man from the given answers:

A. a

B. a^a

C. e^a

D. $a \log(ae)$

Answer: D



View Text Solution

72. Assertion : $f(x) = [x]$ is not differentiable.

Reason $f(x) = [x]$ is not continuous at $x = 0$

- A. Reason implies assertion
- B. Assertion is correct reason is correct
- C. Reason is correct Assertion is incorrect
- D. Both Assertion and Reason are incorrect

Answer: A



Watch Video Solution

73. (i) If $y = x^3 + x^2 + x + 1$ then $\frac{dy}{dx}$ at $x = 1$ is 0

(ii) If $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ then $\frac{dy}{dx} = \tan x$

(iii) $y = x^x \frac{dy}{dx} = x^x [1 + \log x]$

(iv) $y = \tan^{-1} x, \frac{dy}{dx} = \frac{1}{1+x^2}$

state which pair of the statement given above are true.

A. (i) and (ii) are true

B. (ii) and (iv) are true

C. (iii) and (iv) are true

D. (i) and (iv) are true

Answer: C



Watch Video Solution