MATHS

BOOKS - PREMIERS PUBLISHERS

DIFFERENTIAL CALCULUS LIMITS AND CONTINUITY

Worked Examples

1. Find $\lim_{x \to 0} f(x), \quad \text{if} \quad f(x) = \sqrt{x}, x \geq 0.$

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2. Find $\lim_{x \to 3} f(x), where f(x) = \frac{|x-3|}{x-3}, x \neq 3.$

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3. Find $\lim_{x \to 0} \left[\frac{2x + |x|}{5x - 3|x|} \right]$ if it exist.

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4. Evaluate $\lim_{x \to 3^+} \lfloor x \rfloor$ and $\lim_{x \to 3^-} (\lfloor x \rfloor)$.

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5. Test the existance of the limit

$$\lim_{x \to 2} \frac{3|x - 2| + (x - 2)}{|x - 2|}, x \neq -2.$$

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6. Evaluate $\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$ if it exist by

(i) tabulation method

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7. Evaluate $\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$ if it exist by

(iii)any other method

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8. Evaluate $\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$ if it exist by

(iii)any other method

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9. Calculate $\lim_{x \to 1} \left(x^3 + 3x^2 - 2x + 5\right)$.

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10. Evaluate $\lim_{x \to a} (6)$for any real number a .

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11. Calculate

(i) $\lim_{x \to 1} (4x + 1)$

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12. Calculate

(ii) $\lim_{x \to -2} \left(- 3x^2 \right).$

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13. Find $\lim_{x \to 0} \left[\frac{x^3 + x^2}{2x^2} + 3x^2 - 4 \right]$

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14. Calculate $\lim_{x \to -1} \left(x^3 + 2 \right)^5$

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15. Calculate $\lim_{x \to 1} \left(3x^2 + 4x - 1\right)(2x + 1)$

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16. Calculate $\lim_{x \to 2} \left[\frac{2x^2 + x + 4}{5x + 2}\right]$

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17. $\lim_{x \to 0} \frac{\sqrt{1 + x} - 1}{x}$

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18. Find $\lim_{x \to 4} \frac{\sqrt{2x + 1} - 3}{\sqrt{x - 2} - \sqrt{2}}$

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19. Evaluate $\lim_{x \to 1} \frac{x^{10} - 1}{x^5 - 1}$

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20. Evaluate $\lim_{x-2} \frac{x^n - 2^n}{x - 2}$

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21. Find $\lim_{x \to 0} \frac{(1+x)^5 - 1}{x}$

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22. Calculate $\lim_{x \to 0} \frac{\sqrt{x+4} - 2}{x}$

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23. Find the realtion between a and b if $\lim_{x \to 2} f(x)$exists where $f(x) = \{[ax + b \text{ if } x > 35ax - 3bx + 2 \text{ if } x < 2$

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24. Calculate $\lim_{x \to 0} \frac{1}{x + x^3}$

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25. Evaluate $\lim_{x \to 3} \frac{1}{(x - 3)^2}$

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26. Calculate $\lim_{x \to \infty} \frac{x^3 + 2x + 5}{3x^2 - 6x + 5}$

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27. Evaluate $\lim_{x \to \infty} \frac{2 + x^3}{-4x + 3}$

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28. Evaluate $\lim_{x \to \infty} \frac{4x^2 - 2x + 3}{2x^2 + 3x - 2}$

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29. Find the left and right limit of

$$f(x) = \frac{x^2 + 1}{(x+1)^2}(x+2) \Big] at x = -1.$$

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30. The concentration of liquid (in grams/litre)is given by $C(t) = \frac{25t}{205 + 5t}$.what happens to the concentrations at $t \to \infty$.

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31. Find $\lim_{n \to \infty} \frac{1^3 + 2^{\#} + \ldots . n^3}{(2n^2 + 1)(2n^4 + 4)}$.

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32. Find $\lim_{n \to \infty} \frac{1}{2.3} + \frac{1}{3.4} + \ldots \ldots$

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33. Show that $\lim_{x \to 0^+} \left[\frac{1}{x} + \frac{2}{x} + \ldots . \left(\frac{10}{x} \right) \right] = 55$

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34. Evaluate $\lim_{x \to 0} (1 + \tan x)^{3 \cot x}$.

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35. Evaluate $\lim_{x \to \infty} \left[\frac{x-a}{x+a}\right]^x$.

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36. Evaluate $\lim_{x \to \infty} \left[\frac{x^2-3x-4}{x^2+3x+2}\right]^x$

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37. Find

(a) $\lim_{x \to 0} \frac{\tan x}{|x|}$

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38. Find

(b) $\lim_{x \to 0} \frac{x|x|}{\tan|x|}$

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39. Evaluate $\lim_{x \to \infty} x\left[4^{\frac{1}{x}} - e^{\frac{1}{x}}\right]$

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40. Discuss the continutity of $f(x) = \sqrt{4 - x^2}$.

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41. Given that`f(x) =[(x^5 -1)/(x-1)] if x ne 1.is continuous at x=1,find k.

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Solution To Exercise 9 1

1. Complete the table using calculator and use the result to estimate the limit.

1 $\lim_{x \to 2} \dfrac{x-2}{x^2-x-2}$

x	1.9	1.99	1.999	2.001	2.01	2.1
$f(x)$						

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2. Complete the table using calculator and use the result to estimate the limit.

$\lim_{x \to 2} \dfrac{x-2}{x^2-4}$

x	1.9	1.99	1.999	2.001	2.01	2.1
$f(x)$						

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3. Complete the table using calculator and use the result to estimate the limit.

$\lim_{x \to 0} \dfrac{\sqrt{x+3}-\sqrt{3}}{x}$

x	−0.1	−0.01	−0.001	0.001	0.01	0.1
$f(x)$						

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4. Complete the table using calculator and use the result to estimate the limit.

$$\lim_{x \to -3} \frac{\sqrt{1-x}-2}{x+3}$$

x	−3.1	−3.01	−3.00	−2.999	−2.99	−2.9
$f(x)$						

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5. Complete the table using calculator and use the result to estimate the limit.

$$\lim_{x \to 0} \frac{\sin x}{x}$$

x	−0.1	−0.01	−0.001	0.001	0.01	0.1
$f(x)$						

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6. Complete the table using calculator and use the result to estimate the limit.

$$\lim_{x \to 0} \frac{\cos x - 1}{x}$$

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$						

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7. Use the graph to find the limits (if it exists).If the limit does not exist ,explain why?

7. $\lim_{x \to 3} (4 - x)$

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8. Use the graph to find the limits (if it exists).If the limit does not exist ,explain why?

$\lim_{x \to 1} (x^2 + 2)$

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9. Use the graph to find the limits (if it exists).If the limit does not exist ,explain why?

$\lim_{x \to 2} f(x)$.

where $f(x) = \begin{cases} 4 - x & x \neq 2 \\ 0 & x = 2 \end{cases}$

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10. Use the graph to find the limits (if it exists).If the limit does not exist ,explain why?

$\lim_{x \to 1} f(x)$

where $f(x) = \begin{cases} x^2 + 2 & x \neq 1 \\ 1 & x = 1 \end{cases}$

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11. Use the graph to find the limits (if it exists).If the limit does not exist ,explain why?

$$\lim_{x \to 3} \frac{1}{x - 3}$$

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12. Use the graph to find the limits (if it exists).If the limit does not exist ,explain why?

$$\lim_{x \to 5} \left[\frac{|x - 5|}{x - 5} \right]$$

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13. Use the graph to find the limits (if it exists).If the limit does not exist ,explain why?

$$\lim_{x \to 1} \sin \pi x$$

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14. Use the graph to find the limits (if it exists).If the limit does not exist ,explain why?

$\lim_{x \to 0} \sec x$

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15. Use the graph to find the limits (if it exists).If the limit does not exist ,explain why?

$\lim_{x \to \left(\frac{\pi}{2}\right)} \tan x$

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16. $f(x) = \begin{cases} x^2, & x \leq 2 \\ 8 - 2x, & 2 < x < 4 \\ 4, & x \geq 4 \end{cases}$

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17. $f(x) = \begin{cases} \sin x, & x < 0 \\ 1 - \cos x, & 0 \leq x \leq \pi \\ \cos x, & x > \pi \end{cases}$

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18. Sketch the graph of a function f that satisfies the given values:

f(0) is underfined $\lim_{x \to 0} f(x) = 4$,

f(2) = 6, $\lim_{x \to 2} f(x) = 3$

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19. Sketch the graph of a function f that satisfies the given values:

f(-2) = 0 , f(2) = 0 , $\lim_{x \to -2} f(x) = 0$

$\lim_{x \to 2} f(x)$ does not exist.

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20. Write a brief description of the meaning of the notation

$\lim_{x \to 8} f(x) = 25$

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21. If f(2) = 4, can you conclude anything about the limit of f(x) as x approaches 2 ?

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22. If the limit of f(x) as x approaches 2 is 4, can you conclude anything about f(2)? Explain reasoning.

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23. Evaluate : $\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$ if it exists by finding $f(3^-)$ and $f(3^+)$.

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24. Verify the existence of $\lim_{x \to 1} f(x)$, where

$$f(x) = \begin{cases} \frac{|x-1|}{x-1}, & \text{for } x \neq 1 \\ 0, & \text{for } x = 1 \end{cases}$$

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Solution To Exercise 9 2

1. Solve : $\lim_{x \to 2} \frac{x^4 - 16}{x - 2}$

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2. $\lim_{x \to 1} \frac{x^m - 1}{x^n - 1}$ is

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3. Evaluate the following limits.

$\lim_{\sqrt{x} \to 3} \frac{x^2 - 81}{\sqrt{x} - 3}$

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4. Evaluate the following limits.

$$\lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}, \; x > 0.$$

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5. $\lim_{x \to 5} \frac{\sqrt{x+4} - 3}{x - 5}$

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6. $\lim_{x \to 2} \frac{\frac{1}{x} - \frac{1}{2}}{x - 2}$

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7. $\lim_{x \to 1} \frac{\sqrt{x} - x^2}{1 - \sqrt{x}}$

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8. $\lim_{x \to 0} \dfrac{\sqrt{x^2+1}-1}{\sqrt{x^2+16}-4}$

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9. $\lim_{x \to 0} \dfrac{\sqrt{1+x}-1}{x}$

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10. Evaluate the following limits.

$\lim_{x \to 1} \dfrac{\sqrt[3]{7+x^3}-\sqrt{3+x^2}}{x-1}$

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11. $\lim_{x \to 2} \dfrac{2-\sqrt{x+2}}{\sqrt[3]{2}-\sqrt[3]{4-x}}$

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12. $\lim_{x \to 0} \frac{\sqrt{1+x^2}-1}{x}$

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13. $\lim_{x \to 0} \frac{\sqrt{1-x}-1}{x^2}$

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14. $\lim_{x \to 5} \frac{\sqrt{x-1}-2}{x-5}$

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15. $\lim_{x \to a} \frac{\sqrt{x-b}-\sqrt{a-b}}{x^2-a^2}(a > b)$

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Solution To Exercise 9 3

1. Find the left and right limits of

$$f(x) = \frac{x^2 - 4}{(x^2 + 4x + 4)(x + 3)}$$ at x = -2

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2. $f(x) = \tan x \quad \text{at} \quad x = \frac{\pi}{2}$

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3. $\lim_{x \to 3} \frac{x^2 - 9}{x^2(x^2 - 6x + 9)}$

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4. $\lim_{x \to \infty} \frac{3}{x - 2} - \frac{2x + 11}{x^2 + x - 6}$

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5. $\lim_{x \to \infty} \frac{x^3 + x}{x^4 - 3x^2 + 1}$

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6. $\lim_{x \to \infty} \frac{x^4 - 5x}{x^2 - 3x + 1}$

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7. $\lim_{x \to \infty} \frac{1 + x - 3x^3}{1 + x^2 + 3x^3}$

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8. $\lim_{x \to \infty} \left(\frac{x^3}{2x^2 - 1} - \frac{x^2}{2x + 1} \right)$

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9. Show that

$$\lim_{x \to \infty} \frac{1 + 2 + 3 + \ldots + n}{3n^2 + 7n + 2} = \frac{1}{6}$$

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10. Show that

$$\lim_{x \to \infty} \frac{1^2 + 2^2 + \ldots + (3n)^2}{(1 + 2 + \ldots + 5n)(2n + 3)} = \frac{9}{25}$$

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11. Show that

$$\lim_{x \to \infty} \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots + \frac{1}{n(n+1)} = 1$$

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12. An important problem in fishery science is to estimate the number of fish presently spawning in streams and use this information to predict

the number of mature fish or "recruits" that will return to the rivers during the reproductive period. If S is the number of spawners and R the number of recruits, "Beverton-Holt spawner recruit function" is R(S) = $\frac{S}{\alpha S + \beta}$ where α and β are positive constants.

Show that this function predicts approximately constant recruitment when the number of spawners is sufficiently large.

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13. A tank contains 5000 litres of pure water. Brine (very salty water) that contains 30 of salt per litre of water is pumped into grams the tank at a rate of 25 litres per minute. The concentration of salt water after t minutes (in grams per litre) is $C(t) = \frac{30t}{200 + t}$.What happens to the concentration as $t \to \infty$?

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Solution To Exercise 9 4

1. $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^{7x}$

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2. $\lim_{x \to 0} (1 + x)^{\frac{1}{3x}}$

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3. $\lim_{x \to \infty} \left(1 + \frac{k}{x}\right)^{\frac{m}{x}}$

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4. $\lim_{x \to \infty} \left(\frac{2x^2 + 3}{2x^2 + 5}\right)^{8x^2 + 3}$

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5. $\lim_{x \to \infty} \left(1 + \frac{3}{x}\right)^{x+2}$

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6. $\lim_{x \to 0} \frac{\sin^3\left(\frac{x}{2}\right)}{x^3}$

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7. $\lim_{x \to 0} \frac{\sin \alpha x}{\sin \beta x}$

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8. $\lim_{x \to 0} \frac{\tan 2x}{\sin 5x}$

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9. $\lim_{\alpha \to 0} \frac{\sin(\alpha^n)}{(\sin \alpha)^m}$

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10. $\lim_{x \to 0} \frac{\sin(a+x) - \sin(a-x)}{x}$

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11. $\lim_{x \to 0} \frac{\sqrt{x^2 + a^2} - a}{\sqrt{x^2 + b^2} - b}$

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12. $\lim_{x \to 0} \frac{2 \arcsin x}{3x}$

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13. $\lim_{x \to 0} \frac{1 - \cos x}{x^2}$

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14. $\lim_{x \to 0} \frac{\tan 2x}{x}$

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15. $\lim_{x \to 0} \frac{2^x - 3^x}{x}$

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16. $\lim_{x \to 0} \frac{3^x - 1}{\sqrt{x+1} - 1}$

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17. $\lim_{x \to 0} \frac{1 - \cos^2 x}{x \sin 2x}$

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18. $\lim_{x \to \infty} x\left[3^{\frac{1}{x}} + 1 - \cos\left(\frac{1}{x}\right) - e^{\frac{1}{x}}\right]$

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19. $\lim_{x \to \infty} \{x[\log(x+a) - \log(x)]\}$

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20. $\lim_{x \to \pi} \frac{\sin 3x}{\sin 2x}$

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21. $\lim_{x \to \frac{\pi}{2}} (1 + \sin x)^{2\operatorname{cosec} x}$

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22. $\lim_{x \to 0} \frac{\sqrt{2} - \sqrt{1 + \cos x}}{\sin^2 x}$

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23. $\lim_{x \to 0} \frac{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}}{\tan x}$

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24. Evaluate the following limits:

$\lim_{x \to \infty} \left[\frac{x^2 - 2x + 1}{x^2 - 4x + 3} \right]^x$

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25. $\lim_{x \to 0} \frac{e^x - e^{-x}}{\sin x}$

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26. $\lim_{x \to 0} \frac{e^{ax} - e^{bx}}{x}$

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27. $\lim_{x \to 0} \frac{\sin x(1 - \cos x)}{x^3}$

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28. $\lim_{x \to 0} \frac{\tan x - \sin x}{x^3}$

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Solution To Exercise 9 5

1. Prove that $f(x) = 2x^2 + 3x - 5$ is continuous at all points in $\mathbb{R}$.

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2. Examine the continuity of the following :

x + sinx

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3. Examine the continuity of the following :

$x^2 \cos x$

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4. Examine the continuity of the following :

$e^x \tan x$

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5. Examine the continuity of the following :

$e^{2x} + x^2$

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6. Examine the continuity of the following :

x.ln x

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7. Examine the continuity of the following :

$$\frac{\sin x}{x^2}$$

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8. Examine the continuity of the following :

$$\frac{x^2 - 16}{x + 4}$$

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9. Examine the continuity of the following :

$|x+2|+|x-1|$

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10. Examine the continuity of the following :

$$\frac{|x-2|}{|x+1|}$$

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11. Examine the continuity of the following :

$\cot x + \tan x$

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12. Find the points of discontinuity of the function f, where

$$f(x) = \begin{cases} 4x+5, & \text{if } x \leq 3 \\ 4x-5, & \text{if } x > 3 \end{cases}$$

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13. Find the points of discontinuity of the function f, where

$$f(x) = \begin{cases} x+2, & \text{if } x \geq 2 \\ x^2, & \text{if } x < 2 \end{cases}$$

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14. Find the points of discontinuity of the function f, where

$$f(x) = \begin{cases} x^3 - 3, & \text{if } x \leq 2 \\ x^2 + 1, & \text{if } x > 2 \end{cases}$$

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15. Find the points of discontinuity of the function f, where

$$f(x) = \begin{cases} \sin x, & 0 \leq x \leq \frac{\pi}{4} \\ \cos x, & \frac{\pi}{4} < x < \frac{\pi}{2} \end{cases}$$

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16. At the given point x_0 discover whether the given function is continuous or discontinuous citing the reasons for your answer:

$$x_0 = 1, f(x) = \begin{cases} \frac{x^2-1}{x-1}, & x \neq 1 \\ 2, & x = 1 \end{cases}$$

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17. At the given point x_0 discover whether the given function is continuous or discontinuous citing the reasons for your answer:

$$x_0 = 3, f(x) = \begin{cases} \frac{x^2-9}{x-3}, & \text{if } x \neq 3 \\ 5, & \text{if } x = 3 \end{cases}$$

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18. Show that the function $\begin{cases} \frac{x^3-1}{x-1}, & \text{if } x \neq 1 \\ 3, & \text{if } x = 1 \end{cases}$ is continuous on $(-\infty, \infty)$

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19. For what value of α is this function $f(x) = \begin{cases} \frac{x^4-1}{x-1}, & \text{if } x \neq 1 \\ \alpha, & \text{if } x = 1 \end{cases}$ continuous at x = 1?

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20. Let $f(x) = \begin{cases} 0, & \text{if } x < 0 \\ x^2, & \text{if } 0 \leq x < 2 \\ 4, & \text{if } x \geq 2 \end{cases}$. Graph the function. Show that f(x) continuous on $(-\infty, \infty)$.

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21. If f and g are continuous functions with f(3) = 5 and $\lim_{x \to 3} [2f(x) - g(x)] = 4$, find g(3).

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22. Find the points at which f is discontinuous. At which of these points f is continuous from the right, from the left, or neither? Sketch the graph of f.

$$f(x) = \begin{cases} 2x + 1, & \text{if } x \leq -1 \\ 3x, & \text{if } -1 < x < 1 \\ 2x - 1, & \text{if } x \geq 1 \end{cases}$$

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23. Find the points at which f is discontinuous. At which of these points f is continuous from the right, from the left, or neither? Sketch the graph of f.

$$f(x) = \begin{cases} (x - 1)^3, & \text{if } x < 0 \\ (x + 1)^3, & \text{if } x \geq 0 \end{cases}$$

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24. A function f is defined as follows :

$$f(x) = \begin{cases} 0 & \text{for} \quad x < 0 \\ x & \text{for} \quad 0 \leq x < 1 \\ -x^2 + 4x - 2 & \text{for} \quad 1 \leq x < 3 \\ 4 - x & \text{for} \quad x \geq 3 \end{cases}$$

Is the function continuous ?

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25. Which of the following functions f has a removable discontinuity at $x = x_0$? If the discontinuity is removable, find a function g that agrees with f for $x \neq x_0$ and is continuous on $\mathbb{R}$.

$$f(x) = \frac{x^2 - 2x - 8}{x + 2}, x_0 = -2$$

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26. Which of the following functions f has a removable discontinuity at $x = x_0$? If the discontinuity is removable, find a function g that agrees

with f for $x \neq x_0$ and is continuous on $\mathbb{R}$.

$$f(x) = \frac{x^3 + 64}{x + 4}, x_0 = -4$$

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27. Which of the following functions f has a removable discontinuity at $x = x_0$? If the discontinuity is removable, find a function g that agrees with f for $x \neq x_0$ and is continuous on $\mathbb{R}$.

$$f(x) = \frac{3 - \sqrt{x}}{9 - x}, x_0 = 9$$

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28. Find the constant b that makes g continuous on $(-\infty, \infty)$ $g(x) = \begin{cases} x^2 - b^2 & \text{if } x < 4 \\ bx + 20 & \text{if } x \geq 4 \end{cases}$

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29. Consider the function $f(x) = x\sin\frac{\pi}{x}$. What value must we give f(0) in order to make the function continuous everywhere?

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30. The function $f(x) = \frac{x^2 - 1}{x^3 - 1}$ is not defined at x=1.What value must we give f(1) in oreder to make f(x)continuous at x=1?

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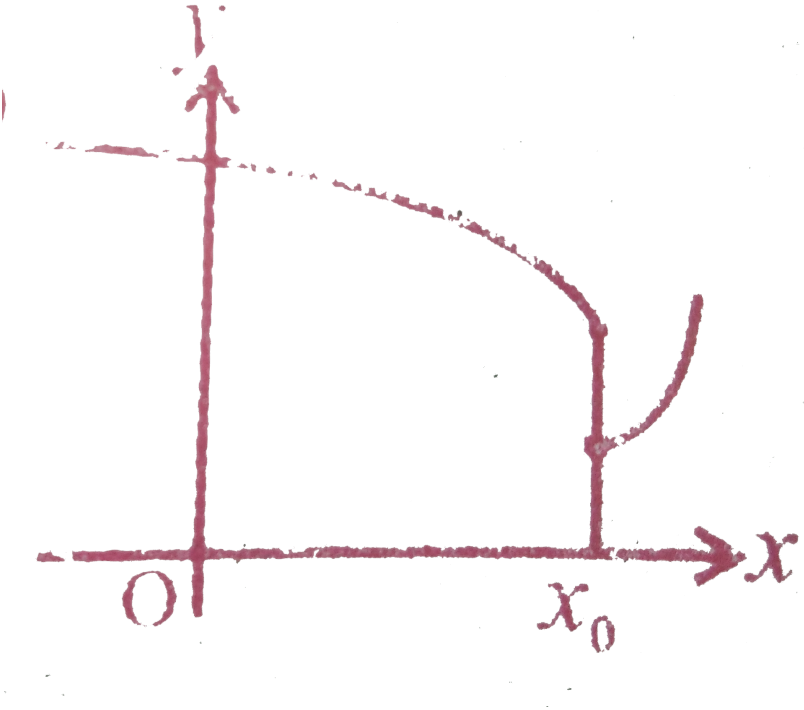
31. State how continuity is destroyed at $x = x_0$ for each of the following graphs.



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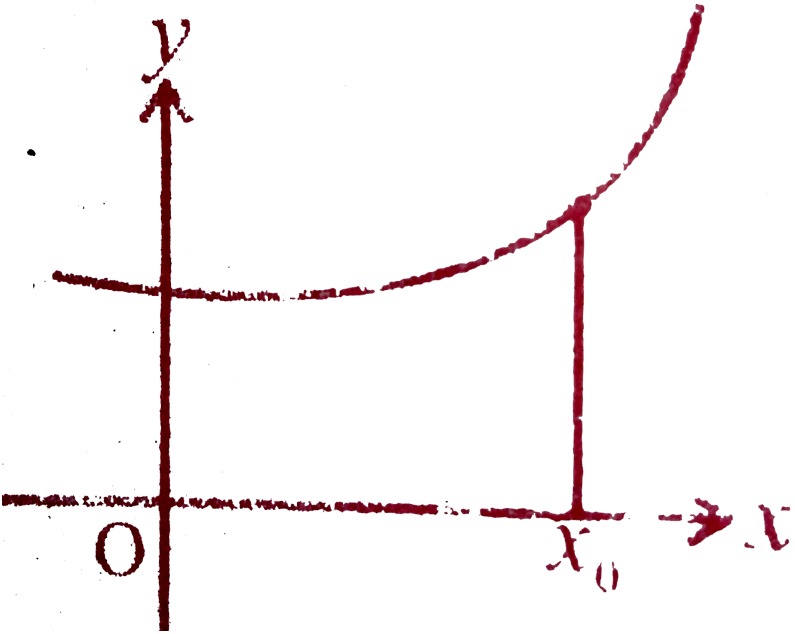
32. State how continuity is destroyed at $x = x_0$ for each of the following graphs.



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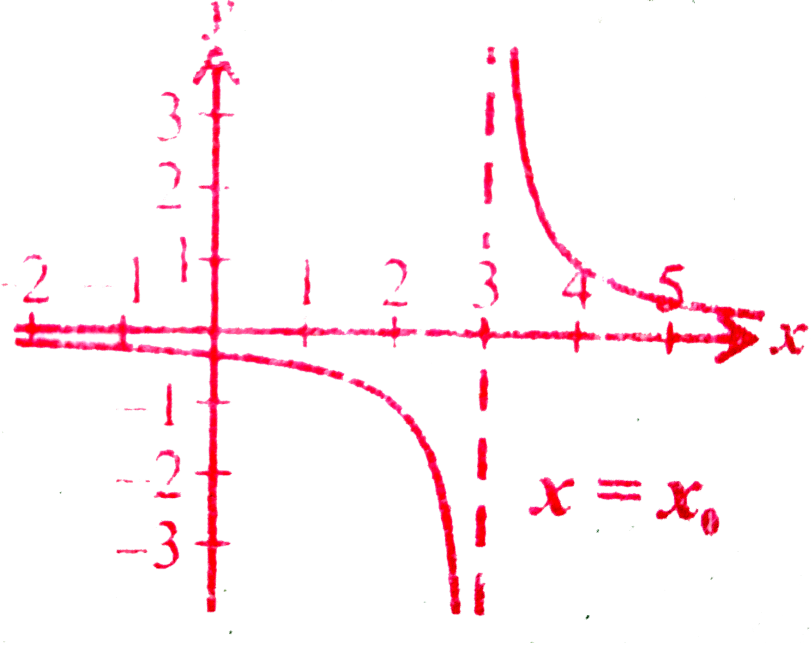
33. State how continuity is destroyed at $x = x_0$ for each of the following graphs.



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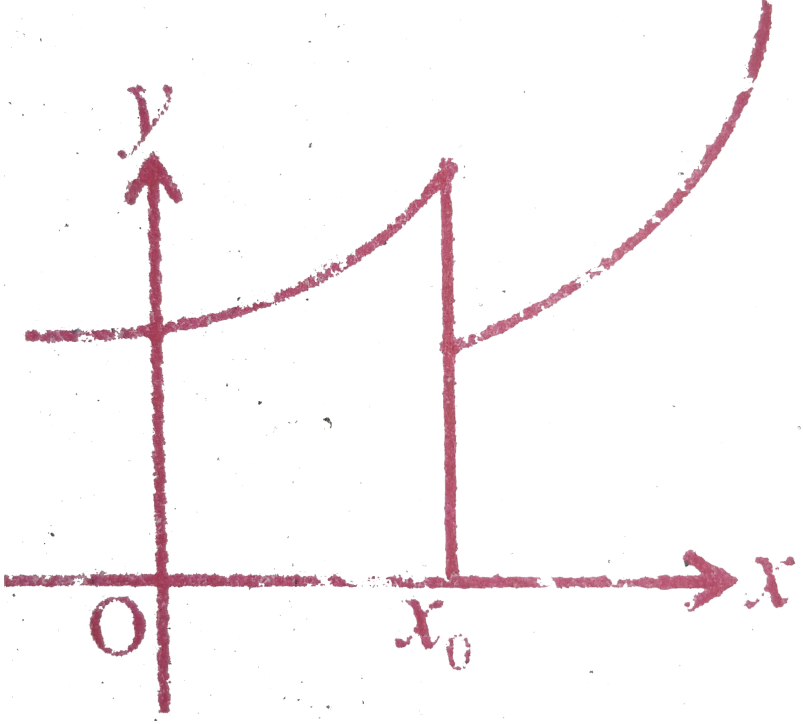
34. State how continuity is destroyed at $x = x_0$ for each of the following graphs.



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Solution To Exercise 9 6

1. Choose the correct or the most suitable answer from the given four alternatives.

$$\lim_{x \to 0} \frac{\sin x}{x}$$

A. 1

B. 0

C. ∞

D. $-\infty$

Answer: A

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2. Choose the correct or the most suitable answer from the given four alternatives.

$$\lim_{x \to \left(\frac{\pi}{2}\right)} \frac{2x - \pi}{\cos x}$$

A. 2

B. 1

C. -2

D. 0

Answer: C

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3. Choose the correct or the most suitable answer from the given four alternatives.

$$\lim_{x \to 0} \frac{\sqrt{1 - \cos 2x}}{x}$$

A. 0

B. 1

C. $\sqrt{2}$

D. does not exist

Answer: D

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4. Choose the correct or the most suitable answer from the given four alternatives.

$$\lim_{\theta \to 0} \frac{\sin \sqrt{\theta}}{\sqrt{\sin \theta}}$$

A. 1

B. -1

C. 0

D. 2

Answer: A

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5. Choose the correct or the most suitable answer from the given four alternatives.

$$\lim_{x \to \infty} \left[\frac{x^2 + 5x + 6}{x^2 + x - 6} \right]^x \text{ is:}$$

A. e^4

B. e^2

C. e^3

D. 1

Answer: A

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6. Choose the correct or the most suitable answer from the given four alternatives.

$$\lim_{x \to \infty} \frac{\sqrt{x^2 - 1}}{2x + 1}$$

A. 1

B. 0

C. -1

D. $\frac{1}{2}$

Answer: D

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7. Choose the correct or the most suitable answer from the given four alternatives.

$\lim_{x \to 0} \frac{a^x - b^x}{x} =:$

A. logab

B. $\log\left(\frac{a}{b}\right)$

C. $\log\left(\frac{b}{a}\right)$

D. $\frac{a}{b}$

Answer: B

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8. Choose the correct or the most suitable answer from the given four alternatives.

$\lim_{x \to 0} \frac{8^x - 4^x - 2^x + 1^x}{x^2} =$

A. $2\log 2$

B. $2(\log 2)^2$

C. log2

D. 3log2

Answer: B

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9. If $f(x) = x(-1)\left[\frac{1}{x}\right] x \leq 0$, then the value of $\lim_{x \to 0} f(x)$ is equal to

A. -1

B. 0

C. $2^{x \to 0}$

D. 4

Answer: B

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10. Choose the correct or the most suitable answer from the given four alternatives.

$\lim_{x \to 3} \lfloor x \rfloor =$

A. 2

B. 3

C. does not exist

D. 0

Answer: C

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11. Let the function f be defined by $f(x) = \begin{cases} 3x0 \leq x \leq 1 \\ -3x + 51 < x \leq 2 \end{cases}$, then:

A. $\lim_{x \to 1} f(x) = 1$

B. $\lim_{x \to 1} f(x) = 3$

C. $\lim_{x \to 1} f(x) = 2$

D. $\lim_{x \to 1} f(x)$ does not exist

Answer: D

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12. If $f: R \to R$ is defined by $f(x) = \lfloor x - 3 \rfloor + |x + 4| f$ or $x \varepsilon R, then \lim_{x \to 3^-} f(x)$ is equal to:

A. -2

B. -1

C. 0

D. 1

Answer: C

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13. $\lim_{x \to 0} \frac{xe^x - \sin 2x}{x}$ is :

A. 1

B. 2

C. 3

D. 0

Answer: D

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14. If $\lim_{x \to 0} \frac{\sin px}{\tan 3x} = 4$, then the value of p is

A. 6

B. 9

C. 12

D. 4

Answer: C

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15. $\lim_{\alpha \to \frac{\pi}{4}} \frac{\sin \alpha - \cos \alpha}{\alpha - \frac{\pi}{4}}$ is

A. $\sqrt{2}$

B. $\frac{1}{\sqrt{2}}$

C. 1

D. 2

Answer: A

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16. Choose the correct or the most suitable answer from the given four alternatives.

$\lim_{n \to \infty} \left[\frac{1}{n^2} + \frac{2}{n^2} + \frac{3}{n^2} + \ldots + \frac{n}{n^2} \right]$ is :

A. $\frac{1}{2}$

B. 0

C. 1

D. ∞

Answer: A

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17. Choose the correct or the most suitable answer from the given four alternatives.

$\lim_{x \to 0}$ [e^(sinx)-1]/x $=$:

A. 1

B. e

C. $\frac{1}{e}$

D. 0

Answer: A

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18. $\lim_{x \to 0} \frac{e^{\tan x} - e^{x}}{\tan x - x} =$

A. 1

B. e

C. $\frac{1}{2}$

D. 0

Answer: A

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19. The value of $\lim_{x \to 0} \frac{\sin x}{\sqrt{x}}$ is

A. 1

B. -1

C. 0

D. ∞

Answer: D

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20. The value of $\lim_{x \to k^-} x - [x]$, where k is an integer

A. -1

B. 1

C. 0

D. 2

Answer: B

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21. At $x = \frac{3}{2}$ the function $f(x) = \frac{|2x-3|}{2x-3}$ is

A. continuous

B. discontinuous

C. differentiable

D. non -zero

Answer: B

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22. Let f : $\mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \begin{cases} x & x \text{ is irrational} \\ 1-x & x \text{ is rational} \end{cases}$ then f is

A. discontinuous at $x = \frac{1}{2}$

B. continuous at $x = \frac{1}{2}$

C. continuous everywhere

D. discontinuous everywhere

Answer: B

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23. The function $f(x) = \begin{cases} \frac{x^2-1}{x^3+1} & x \neq 1 \\ P & x = -1 \end{cases}$ is not defined for x = -1.

The value of f(-1) so that the function extended by this value is continuous is

A. $\frac{2}{3}$

B. $-\frac{2}{3}$

C. 1

D. 0

Answer: B

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24. Let f be a continuous function on [2, 5]. If f takes only rational values for all x and f(3) = 12, then f(4.5) is equal to

A. $\frac{f(3)+f(4.5)}{7.5}$

B. 12

C. 17.5

D. $\frac{f(4.5)-f(3)}{1.5}$

Answer: B

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25. Let a function f be defined by $f(x)=\frac{x-|x|}{x}f \text{ or } x \neq 0 \text{ and } f(0)=2$.Then f is :

A. continuous no where

B. continuous everywhere

C. continuous for all x except x=1

D. continuous for all x except x=0

Answer: D

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Problems For Practice

1. Evaluate : $\lim_{x \to 0} \frac{\sqrt{x+4}-2}{x}$ by filling the table.

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$						

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2. Test the existance of the limit

$$\lim_{x \to 2} \frac{3|x-2|+(x-2)}{|x-2|}, x \neq -2.$$

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3. Find

$f\left[(-3)^{-}\right]$ and $f\left[(-3)^{+}\right]$ if $\left\{f(x)=\frac{|x+3|}{x+3}, x \neq 0 f(0)=0, x-0\right.$

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4. Choose the correct or the most suitable answer from the given four alternatives.

$$\lim_{x \to 1} \frac{x^2-5x+6}{x^2-3x+4}$$

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5. Evaluate : $\lim_{x \to \infty} \frac{(3+x)^5-3^5}{x}$

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6. Evaluate : $\lim_{n \to \infty} \frac{1^3+2^3+\ldots+n^3}{\left(1^2+2^2+\ldots n^2\right)n}$

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7. Evaluate : $\lim_{\theta \to 0} \left[\frac{1 - \cos \theta}{\theta^2} \right]$.

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8. Evaluate : $\lim_{x \to 1} \frac{\sqrt[3]{26 + x^3} - \sqrt{15 + x^2}}{x - 1}$

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9. Evaluate : $\lim_{x \to 0} \frac{\sqrt{x^2 + 16} - 4}{\sqrt{x^2 + 9} - 3}$

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10. Find $\lim_{x \to \infty} \frac{3x^2 + 2x - 1}{(x + 5)(x - 7)}$

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11. Find $\lim_{x \to \infty} \frac{1+2+3+\ldots+n}{4n^2-3n+1}$

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12. Evaluate $\lim_{x \to \infty} x \sin\left(\frac{1}{x}\right)$

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13. Find $\lim_{x \to \left(\frac{\pi}{2}\right)} (1+\cos x)^{3 \sec x}$

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14. Find $\lim_{x \to 0} \frac{\log(1+x)-x}{x}$

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15. Evaluate $\lim_{x \to 0} \frac{4^x - 1}{\sqrt{1 + 2x} - 1}$

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16. Find $\lim_{x \to 0} \frac{\sqrt{1 + \sin^2 2x} - \sqrt{1 - \sin^2 2x}}{x \tan 2x}$

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17. Find $\lim_{x \to \left(\frac{\pi}{2}\right)} (1 + \cos x)^{3 \sec x}$

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18. Find the point of discontinuity

$f(x) = \{3x + 5, \quad \text{if} \quad x \leq 23x - 5, \quad \text{if} \quad x > 2$

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19. For what value of a is the function is continues at x=2

$$f(x) = \begin{cases} \dfrac{x^4 - 16}{x - 2}, & \text{if } x \neq 2a, \quad \text{if } x = 2 \end{cases}$$

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20. A function f is defined as follows,is it continuous?

$$f(x) = \{0, x < 0x, 0 \leq x < 1x^2 - 2, 1 \leq x < 2x + 35x - x, x \geq 2$$

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21. What is $\lim_{x \to 0} \dfrac{e^{x^2} - 1}{x^2}$

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22. Show that $f(x) = \dfrac{|3x - 2|}{3x - 2}$ is continuous at $x = \dfrac{2}{3}$

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23. Evaluate : $\lim_{x \to \infty} \frac{27^x - 9^x - 3^x + 1}{x}$

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24. Choose the correct option for the following

$\lim_{\theta \to 0} \sqrt{\frac{1 - \cos 9\theta}{1 - \cos 16\theta}}$ is :

A. $\frac{3}{4}$

B. $\frac{9}{16}$

C. $\frac{16}{9}$

D. $\frac{4}{3}$

Answer: A

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25. Evaluate $\lim_{x \to 0} (1 + \tan x)^{3 \cot x}$.

A. e^3

B. e^{-3}

C. 1

D. 0

Answer: A

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26. Choose the correct option for the following

$\lim_{x \to \infty} \left[\frac{x+3}{x-3}\right]^{x-3}$ is :

A. e^3

B. e^6

C. e^{-3}

D. e^9

Answer: B

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27. Choose the correct option for the following

$$\lim_{x \to \infty} \left[\tan^{-1} x . (1+x)^{\left(\frac{1}{x}\right) . x^{-1}} \right] :$$

A. 1

B. e

C. $\frac{1}{e}$

D. none of these

Answer: B

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28. Choose the correct option for the following

$$\lim_{x \to 0} \frac{2^x - 1}{\sqrt{1+3x} - 1} \text{ is ,}$$

A. $\left(\frac{1}{2}\right) \log 2$

B. $\left(\frac{3}{2}\right)\log 2$

C. $\left(\frac{2}{3}\right)\log 2$

D. $\log 2$

Answer: C

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29. Choose the correct option for the following

$\lim_{x \to 0} \frac{\tan(x - |x|)}{x - |x|}$ is :

A. -1

B. 0

C. does not exist

D. 1

Answer: C

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30. Choose the correct option for the following

$$\lim_{x \to \infty} \left[\frac{2x^2+1}{2x^2+3}\right]^{4x^2+6} \text{ is :}$$

A. e^{-2}

B. e^{2}

C. e^{4}

D. e^{-4}

Answer: D

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31. Choose the correct option for the following

$$\lim_{x \to 0} \frac{\tan\left(x^5\right)}{(\tan x)^5} \text{ is ,}$$

A. $\left(\frac{1}{5}\right)$

B. 5

C. 0

D. 1

Answer: D

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32. Choose the correct option for the following

$\lim_{x \to 0} \frac{\sin^n\left(\frac{x}{2}\right)}{x^n}$ is

A. $\frac{1}{2^n}$

B. $\frac{1}{2}$

C. 2^n

D. 0

Answer: A

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33. $\lim_{x \to 0} \dfrac{1 - \cos^2 x}{x \sin 2x}$

A. 2

B. 0

C. 4

D. $\dfrac{1}{2}$

Answer: A

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34. Choose the correct option for the following

$\lim_{x \to \infty} \dfrac{\sqrt{x^2 + 4} + 5}{\sqrt{x^2 + 4} + 3}$

A. $\dfrac{9}{4}$

B. $\dfrac{3}{2}$

C. $\frac{2}{3}$

D. $\frac{4}{9}$

Answer: B

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35. Choose the correct option for the following

$$\lim_{x \to 0} \frac{\sqrt{x^2 + 16} - 4}{x^2}$$

A. $\frac{1}{2}$

B. $\frac{1}{8}$

C. $\frac{1}{16}$

D. $\frac{1}{4}$

Answer: B

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36. Choose the correct option for the following

$$\lim_{x \to 9} \frac{\sqrt{x-4}-\sqrt{5}}{x^2-81} \text{ is :}$$

A. 1

B. $\frac{1}{45}$

C. $\frac{\sqrt{5}}{180}$

D. $\frac{1}{90}$

Answer: C

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37. Choose the correct option for the following

$$\lim_{x \to \infty} \left[\frac{x^2+5x+6}{x^2+2x+1}\right]^x \text{ is :}$$

A. e^4

B. e^6

C. e^2

D. $\frac{1}{e^2}$

Answer: C

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38. For what value of k is this function $f(x) = \begin{cases} \frac{x^3 - 8}{x - 2} & \text{if } x \neq 2k \\ \text{if } x = 2 \end{cases}$ is continuous on $(-\infty, \infty)$:

A. 2

B. 8

C. 4

D. 12

Answer: D

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39. The point of discontiuity for

$f(x) = \begin{cases} (x-2)^3, & \text{if } x \leq 0 \\ (x+2)^3, & \text{if } x > 0 \end{cases}$:

A. ± 8

B. -2

C. 2

D. 0

Answer: D

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40. Choose the correct option for the following

$\lim_{x \to \infty} \left(\frac{1}{n^3} + \frac{4}{n^3} + \frac{9}{n^3} + \frac{16}{n^3} + \ldots + \frac{1}{n} \right)$ is :

A. $\frac{1}{3}$

B. $\frac{1}{2}$

C. 0

D. $\frac{1}{6}$

Answer: A

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41. Let a function f be defined by $f(x) = \frac{x - |x|}{x} f \text{ or } x \neq 0 \text{ and } f(0) = 2$.Then f is :

A. comtinuous for all x except x=0

B. continuous everywhere

C. continuos nowhere

D. continuous for all x except x=1

Answer: A

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42. Let the function be defined as

$f(x) = \{4x0 < x \leq 1(- 4x + 5)1 < x < 2$ then $\lim_{x \to 1} f(x)$ is :

A. 4

B. does not exist

C. -4

D. 1

Answer: B

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43. $\lim_{x \to 0} \frac{xe^{2x} + \tan x}{x}$ is :

A. 1

B. 2

C. 0

D. does not exist

Answer: B

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44. Find $\lim_{x \to 0} \frac{xe^x - \sin 2x}{2x}$

A. 1

B. 0

C. $\frac{1}{2}$

D. $-\frac{1}{2}$

Answer: D

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45. The value of $\lim_{x \to k^-} x - [x]$, where k is an integer

A. -1

B. 1

C. 0

D. 2

Answer: C

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46. $\lim_{n \to \infty} \left(\frac{1}{n^4} + \frac{8}{n^4} + \frac{27}{n^4} + \frac{64}{n^4} + \ldots \right)$ is:

A. $\frac{1}{4}$

B. $\frac{1}{2}$

C. 1

D. 4

Answer: A

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47. $\lim_{x \to 0} \frac{\sin mx}{\sin nx}$ is :

A. m-n

B. mn

C. $\frac{n}{m}$

D. $\frac{m}{n}$

Answer: D

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48. $\lim_{x \to 0} \frac{xe^x - \sin 2x}{x}$ is :

A. 2

B. 0

C. -1

D. 1

Answer: C

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49. If $f: R \to R$ is defined by $f(x) = |x-2| + |x+2| f \text{ or } x \varepsilon R, then \lim_{x \to 2^-} f(x)$ is :

A. -2

B. -1

C. 0

D. 1

Answer: C

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50. $\lim_{x \to 2^-} 2\lfloor x \rfloor$ is :

A. 2

B. -4

C. 0

D. 4

Answer: A

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51. $\lim_{\theta \to 0} \frac{\sin\sqrt{2\theta}}{\sqrt{\sin 2\theta}}$ is :

A. 0

B. 2

C. -1

D. 1

Answer: D

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52. $\lim_{x \to 0} (1 + \sin x)^{3 \cos ecx} is :$

A. 3

B. e^3

C. e

D. not defined

Answer: B

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53. $\lim_{x \to 0} \frac{4^x - 1}{\sqrt{x+1} - 1}$ is :

A. 1

B. log16

C. log9

D. log4

Answer: B

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54.

31.	$\lim_{x \to 0} \frac{\tan x}{x}$	(a) $\log a$
32.	$\lim_{x \to 0} \frac{e^x - 1}{x}$	(b) e^a
33.	$\lim_{x \to 0} \frac{a^x - 1}{x}$	(c) a
34.	$\lim_{x \to 0} \frac{\sin^{-1} ax}{x}$	(d) 1
35.	$\lim_{x \to \infty} \left(1 + \frac{a}{x}\right)^x$	(e) 2

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55. Find the odd one out :

If a and g are continuous at $x = x_o$,then

A. f+g is continuous at $x = x_o$

B. f.g is continuous at $x = x_o$

C. $\frac{f}{g}$ is continuous at $x = x_o$ if $g(x) \neq 0$

D. f-g is not continuous at $x = x_o$

Answer: D

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56. Find the odd one out :f(x)=[x-|x|]/x x ne 0

A. continuous no where

B. continuous everywhere

C. continuous except x=5

D. continuous for all x except x=0

Answer: D

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57. Assertion : $\lim_{x \to 0} \sin\left(\frac{1}{x}\right) = 0$

Reason: If f,g,h :$I \leq R$such that $g(x) \leq f(x) \leq h(x)$for all x in a deleted neighbouhood of x_o contained in I and if $\lim_{x \to 0} g(x) = \lim_{x \to x_o} h(x) = l$ .Then $\lim_{x \to x_o} f(x) = l$.[sandwich theorem]

A. R is the best tool to prove A

B. R is not enough to prove A

C. A is not true

D. both A and R are incorrect

Answer: A

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58. (i) $\lim_{x \to 0} \frac{2xe^x - \sin 2x}{x}$ is zero.

(ii) $\lim_{n \to \infty} \frac{1^2 + 2^2 + \ldots n^2}{n^3} is \frac{2}{6}$

(iii) $\lim_{x \to 1} \frac{x^7 - 1}{x - 1} = 6$

(iv) $\lim_{\theta \to 0} \frac{\sin 2\theta}{3\theta} = \frac{3}{2}$

State which pair of statements given above are correct.

A. (iii) and (iv)

B. (i) and (ii)

C. (ii) and (iv)

D. (i) and (iii)

Answer: B

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