MATHS

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APPLICATIONS OF DIFFERENTIAL CALCULUS

Worked Example

1. For the functions $f(x) = x^2, x \in [0, 2]$ compute the average rate of changes in the

subintervals $[0, 0.5], [0.5, 1], [1, 1.5], [1.5, 2]$ and the instantaneous rate of changes at the points x = 0.5 , 1, 1.5, 2

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2. A rod is of length 30m, insulated at both ends. The temperature of the rod is given by T = x (30-x) where x is length. Prove that the rate of change of temperature at the mid point of the rod vanishes.

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3. The length l meters of a metal rod at temperature $\theta^\circ C$ is given by $l = 2 + 0.5\theta + 0.4\theta^2$. Determine the rate of change of length with respect to temperature (i) where $\theta = 100^\circ C$, (ii) when $\theta = 20^\circ C$.

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4. The equation of motion is given by $s(t) = 2t^3 - 6t^2 + 6$. At what time the velocity and accelerations are zero?

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5. The distance in meters described by a car in time t seconds is given by $s = 4t^3 - 3t^2 + 6t - 1.$ Determine the velocity and acceleration when (i) t = 0, (ii) t =2 seconds.

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6. A particle moves along a horizontal line such that its equation of motion is $s(t) = 2t^3 - 15t^2 + 24t - 2$, s in meters and t in second.

At what time the particle is at rest

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7. A particle moves along a horizontal line such that its equation of motion is $s(t) = 2t^3 - 15t^2 + 24t - 2$, s in meters and t in second.

At what time the particle changes its direction

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8. A particle moves along a horizontal line such that its equation of motion is $s(t) = 2t^3 - 15t^2 + 24t - 2$, s in meters and t in second.

Find the total distance travelled by the particle in the first 2 seconds.

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9. The rate of increase of the volume of air in a baloon is $400cm^3/s$. Find the rate of increase of its radius, when its diameter is 20 cm

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10. The supply equation is given $p(x+2)=4x+90$, where p is the price/unit in rupees and x is the no. of units find the rate at which the price is changing with respect to time when 39 units are available and the supply is increasing at the rate of 5 units/week.

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11. Gravel is being clumped form a conveyor belt at a rate of $60 ft^3 / \min$ and is coasrsened such that it forms a pie in the shape of a cone where base diameter and height and always equal. How fast is the height of the pile increasing when the pile is 15 ft high ?

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12. A ship A which is at distance of 30km to the north of P moves at 15 km/hr and another ship B which is at a distance of 40 km to the east of P moves at 20 km/hr. How fast the distance between the cars changing?

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13. Find the equations of tangent and normal to the curve $y = x^2 - 2x - 3$ at the point (2, 3)

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14. For what value of x the tangent to the curve $y = 3x^2 + 4x + 1$ is parallel to the line $y + 2x - 3 = 0$

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15. Show that the equation of the normal to the curve $x = a\cos^3\theta, y = a\sin^3\theta$ at $'\theta'$ is $x\cos\theta - y\sin\theta = a\cos 2\theta.$

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16. If the curve $y^2 = x$ and $xy = k$ are orthogonal then prove that $8k^2 = 1$

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17. Find the equation of the normal to $y = x^2 - 2x$ that is parallel to $x + 4y - 1 = 0$

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18. If the curves $ax^2 + by^2 = 1$ and $a_1x^2 + b_1y^2 = 1$ intersect each other orthogonally then show that $\frac{1}{a} - \frac{1}{b} = \frac{1}{a_1} - \frac{1}{b_1}$

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19. Show that $x^2 - y^2 = a^2 and xy = c^2$ cut orthogonally

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20. Verify Rolle's theorem for the funnction $f(x) = 4x^3 - 9x$ in $-\frac{3}{2} \leq x \leq \frac{3}{2}$

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21. Apply Rolle's theorem to find points on the curve $y = -1 + \cos 2x$ where the tangent is parallel to x axis $(0, \pi)$

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22. Prove, Using the Rolle's theorem that between any two distinct real zeros of the polynomial $a_n x^n + a_{n-1} x^{n-1} + \dots . a_1 x + a_0$ there is a zero of the polynomial .

$n a_n x^{n-1} + (n-1) a_{n-1} x^{n-2} + \dots + a_1$

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23. Verify Rolle's theorem for the following

$f(x) = |x| - 1 \leq x \leq 1$

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24. Verify Rolle's theorem for the following

$f(x) = x^3 - 3x + 3 \text{ in } 0 \leq x \leq 1$

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25. Verify Lagrange 's law of the mean for f(x) = x^3 on [-2, 2].

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26. Verify Lagrange's mean value theorem for

$f(x) = \frac{1}{x}$ in $[1, 2]$

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27. At 2.00 pm a car's speedometer reads 30 miles/hr. At 2.10 it reads 50 miles/hr . Show that sometime between 2.00 and 2.10 the acceleration is exactly 120 miles/hr^2.

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28. Verify Lagrange's theorem for $f(x) = x^{\frac{2}{3}}$ in $[-2, 2]$

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29. Suppose that $f(0) = -3$, and $f'(x) \leq 5$ for all values of x how large can f(2) possibly be?

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30. Expand $f(x) = e^{-2x}$ as a Maclaurin's series

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31. Expand $\tan x$ in ascending power of x upto 5^{th} power for $\frac{-\pi}{2} < x < \frac{\pi}{2}$

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32. Expand f(x) = $\cos x$ in Taylors series about $x = \frac{\pi}{4}$

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33. Evaluate $\lim_{x \to 0} \frac{\sin mx}{\sin nx}$

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34. Find $\lim_{x \to \infty} \frac{\sin\left(\frac{1}{x}\right)}{\tan\left(\frac{1}{x}\right)}$

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35. Evaluate $\lim_{\theta \to 0} \frac{1 - \cos m\theta}{1 - \cos n\theta}$

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36. Evaluate $\lim_{x \to \frac{\pi}{2}} \frac{\log \sin x}{(\pi - 2x)^2}$

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37. Evaluate $\lim_{x \to 0} \frac{\tan^{-1} x}{x}$

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38. Evaluate $\lim_{x-2} \frac{x^n - 2^n}{x - 2}$

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39. Evaluate $\lim_{x \to 0} x^3 \log x$

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40. Evaluate $\lim_{x \to 1} x^{\frac{1}{x-1}}$

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41. Evaluate $\lim_{x \to \infty} (1+x)^{\frac{1}{\log x}}$

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42. The current at time t in a coil with resistance R inductance L and subjected to a constant electromotive force E is given By $I = \frac{E}{R}\left(1 - e^{\frac{-Rt}{L}}\right)$ obtain a suitable formula to be used where R is small.

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43. Prove that $f(x) = x^2 - x + 1$ is neither increasing nor decreasing in [0,1]

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44. Prove that $f(x) = x^2 - 6x + 3$ is strictly increasing in $(3, \infty)$

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45. Find the absolute maximum and absolute minimum values of the function

$f(x) = 2x^3 - 3x^2 + 2\text{in} \quad -\frac{1}{2} \leq x \leq 4$

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46. Find the intervals of monotonicity and hence find the local maximum for the function

$f(x) = x^2 - 6x + 2$

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47. Find the extrema of $f(x) = \log(1 + x) - \frac{x}{1 + x}, x > -1$

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48. Find the intervals of monotonicity and local extrema of the function $f(x) = x \log x + x$

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49. Find the intervals of monotonicity and local extreme of the function $f(x) = \frac{1}{1+x^2}$

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50. Determine the intervals of concavity of the curve $f(x) = (x-2)^3(x-4), x \in R$ and the points of inflection if any.

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51. Determine the intervals of concavity of the curve $y = 2 + \sin x \quad \text{in} \quad (-\pi, \pi)$

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52. Find the local extremum of the function $f(x) = x^3 - 12x$

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53. Find the local extrema of the function $f(x) = 3x^4 - 4x^3$

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54. Find the local maximum and local minimum of the function $x^4 - 3x^3 + 3x^2 - x$

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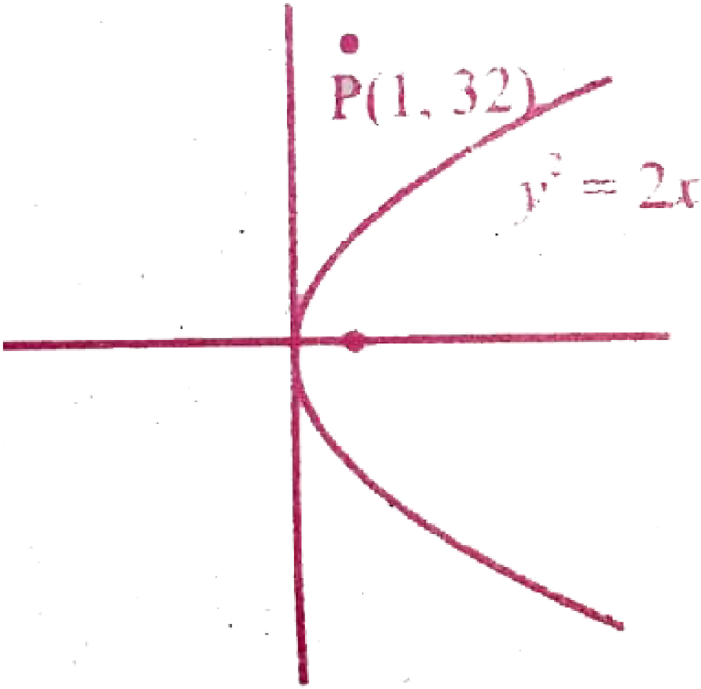
55. Find a point on the parabola $y^2 = 2$ that is closest to the point (1, 32)



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56. A closed box (cuboid) with a square base is to have a volume 2000c.c, The material for the top and bottom of the box is to cost Rs 3 per square cm and the material for the sides is to cost Rs 1.50 per square cm. If the cost of the material is to be least find the dimension of the box.

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57. A garden is to be formed in a rectangular shape and projected by wire fence. What is the largest possible area of the fenced garden with 60 metres of wire.

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58. Find the asymptotes of the curve $xy = c^2$

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59. Find the slant (oblique) asymptote of the function $f(x) = \frac{x^2 + 3x + 5}{x + 2}$

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60. Sketch the curve $y = f(x) = x^2 - 5x + 6$

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61. Find the asymptotes of the curve $f(x) = \frac{3x^2 - 27}{x^2 - 25}$

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62. Sketch the curve $y = f(x) = x^2 - 5x + 4$

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63. Sketch the graph of $y = \frac{3x}{x^2 - 4}$

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Solution To Exercise 7 1

1. A particle moves along a straight line in suc a way that after t second its distance from the origin is $s = 2t^2 + 3t$ metres.

Find the instantaneous velocities at t =3 and t =6 seconds.

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2. A particle moves along a straight line in suc a way that after t second its distance from the origin is $s = 2t^2 + 3t$ metres.

Find the instantaneous velocities at t =3 and t =6 seconds.

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3. A camera is accidentally knocked off an edge of a cliff 400 ft high. The camera falls a distance of $s = 16t^2$ in t seconds.

How long does the camera fall before it hits the ground?

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4. A camera is accidentally knocked off an edge of a cliff 400 ft high. The camera falls a distance of $s = 16t^2$ in t seconds.

What is the average velocity with which the camera falls during the last 2 seconds?

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5. A camera is accidentally knocked off an edge of a cliff 400 ft high. The camera falls a distance of $s = 16t^2$ in t seconds.

What is the instantaneous velocity of the camera when it hits the ground?

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6. A particle moves along a horizontal line such that its equation of motion is $s(t) = 2t^3 - 15t^2 + 24t - 2$, s in meters and t in second.

At what time the particle changes its direction

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7. A particle moves along a line according to the law $s(t) = 2t^3 - 9t^2 + 12t - 4$, where $t \geq 0$.

Find the total distance travelled by the particle in the first 4 seconds.

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8. A particle moves along a line according to the law $s(t) = 2t^3 - 9t^2 + 12t - 4$, where $t \geq 0$.

Find the particle's acceleration each time the velocity is zero.

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9. If the volume of a cube of side length x is $V = x^3$. Find the rate of change of the volume with respect to x when x = 5 units.

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10. If the mass m(x) (in kilograms) of a thin rod of length x(in metres) is given by, $m(x) = \sqrt{3x}$ then what is the rate of change of mass with respect to the length when it is x = 27 meters.

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11. A stone is dropped into a pond causing ripples in the form of concentric circles. The radius r of the outer ripple is increasing at a constant rate at 2 cm per second. When the radius is 5 cm find the rate of changing of the total area of the disturbed water?

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12. A conical water tank with vertex down of 12 meters height has a radius of 5 meters at the top. If water flows into the tank at a rate 10 cubic m/min, how fast is the depth of the water increases when the water is 8 metres deep?

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13. A ladder 17 metre long is leaning against the wall. The base of the ladder is pulled away from the wall at a rate of 5m/s. When the base of the ladder is 8 metres from the wall.

How fast is the top of the ladder moving down the wall?

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14. A ladder 17 metre long is leaning against the wall. The base of the ladder is pulled away from the wall at a rate of 5m/s. When the base of the ladder is 8 metres from the wall.

At what rate, the area of the triangle formed by the ladder, wall, and the floor, is changing?

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15. A police jeep, approaching an orthogonal intersection from the northern direction, is chasing a speeding car that has turned and moving straight east. When the jeep is 0.6 km north of the intersection and the car is 0.8 km to the east. The police deteremine with a radar that the distance between them and the cae is increasing at 20 km/hr. If the jeep is moving at 60 km/hr at the instant of measurement, what is the speed of the car?

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Solution To Exercise 7 2

1. Find the slope of the tangent to the following curves at the respective given points.

$y = x^4 + 2x^2 - x$ at $x = 1$

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2. Find the slope of the tangent to the following curves at the respective given points.

$x = a\cos^3 t, y = b\sin^3 t$ at $t = \frac{\pi}{2}$

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3. Find the point on the curve $y = x^2 - 5x + 4$ at which the tangent is parallel to the line 3x + y = 7.

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4. Find the points on the curve $y = x^3 - 6x^2 + x + 3$ where the normal is parallel to the line $x + y = 1729$

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5. Find the points on the curve $y^2 - 4xy = x^2 + 5$ for which the tangent is horizontal.

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6. Find the tangent and normal to the following curves at the given points on the curve.

$y = x^2 - x^4$ at $(1, 0)$

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7. Find the tangent and normal to the following curves at the given points on the curve.

$y = x^4 + 2e^x \text{ at } (0, 2)$

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8. Find the tangent and normal to the following curves at the given points on the

curve.

$y = x \sin x \quad \text{at} \quad \left(\frac{\pi}{2}, \frac{\pi}{2}\right)$

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9. Find the tangent and normal to the following curves at the given points on the curve.

$x = \cos t, y = 2 \sin^2 t \quad \text{at} t = \frac{\pi}{3}$

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10. Find the equation of the tangents to the curve $y = 1 + x^3$ for which the tangent is orthogonal with the line $x + 12y = 12$.

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11. Find the equations of the tangents to the curve $y = \frac{x+1}{x-1}$ which are parallel to the line $x + 2y = 6$

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12. Find the equation of tangent and normal to the curve given by x = 7 $\cos t$ and $y = 2\sin t, t \in R$ at any point on the curve.

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13. Show that $x^2 - y^2 = a^2 and xy = c^2$ cut orthogonally

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Solution To Exercise 7 3

1. Explain why Rolle's theorem is not applicable to the following functions in the respective intervals.

$$f(x) = \left|\frac{1}{x}\right|, x \in [-1, 1]$$

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2. Explain why Rolle's theorem is not applicable to the following functions in the respective intervals.

$f(x) = \tan x, x \in [0, \pi]$

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3. Explain why Rolle's theorem is not applicable to the following functions in the respective intervals.

$f(x) = x - 2\log x, x \in [2, 7]$

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4. Using the Rolle's theorem, determine the values of x at which the tangent is parallel to the x-axis for the following functions :

$f(x) = x^2 - x, x \in [0, 1]$

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5. Using the Rolle's theorem, determine the values of x at which the tangent is parallel to the x-axis for the following functions :

$f(x) = \frac{x^2 - 2x}{x + 2}, x \in [-1, 6]$

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6. Using the Rolle's theorem, determine the value of x at which the tangent is parallel to the x-axis for the following functions:

$$f(x) = \sqrt{x} - \frac{x}{3}, \in [0, 9]$$

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7. Explain why Lagrange's mean value theorem is not applicable to the following functions in the respective intervals:

$$f(x) = \frac{x+1}{x}, x \in [-1, 2]$$

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8. Explain why Lagrange's mean value theorem is not applicable to the following functions in the respective intervals :

$$f(x) = |3x+1|, x \in [-1, 3]$$

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9. Show that the value in the conclusion of the mean value theorem for

$f(x) = \frac{1}{x}$ on a closed interval of positive number [a,b] in $\sqrt{a}b$

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10. A race car driver is racing at 20^{th} km. If his speed never exceeds 150 km/hr, what is the maximum distance he can cover in the next two hours.

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11. Does there exist a differentiable function f(x) such that $f(0) = -1, f(2) = 4$ and $f'(x) \leq 2$ for all x. Justify your answer.

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12. Show that there lies a point on the curve $f(x) = x(x+3)e^{\frac{\pi}{2}}, \ -3 \leq x \leq 0$ where tangent drawn is parallel to the x-axis.

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13. Using mean value theorem prove that for, $a > 0, b > 0, \left|e^{-a} - e^{-b}\right| < |a - b|$.

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Solution To Exercise 7 4

1. Write the Maclaurin series expansion of the following functions :

e^x

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2. Write the Maclaurin series expansion of the following functions :

$\sin x$

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3. Write the Maclaurin series expansion of the following functions :

$\cos x$

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4. Write the Maclaurin series expansion of the following functions :

$\log(1-x), \ -1 \leq x \leq 1$

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5. Write the Maclaurin series expansion of the following functions :

$\tan^{-1}(x), \ -1 \le x \le 1$

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6. Write the Maclaurin series expansion of the following functions :

$\cos^2 x$

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7. Write down the Taylor series expansion, of the function $\log x$ about x = 1 upto three non

zero terms for $x > 0$.

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8. Expand $\sin x$ in ascending powers $x - \frac{\pi}{4}$ upto three non-zero terms.

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9. Expand the polynomial $f(x) = x^2 - 3x + 2$ in powers of x - 1.

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Solution To Exercise 7 5

1. Evaluate the following limits, if necessary use 1'Hopital Rule:

$$\lim_{x \to 0} \frac{1 - \cos x}{x^2}$$

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2. Evaluate the following limits, if necessary use l' Hopital Rule :

$$\lim_{x \to \infty} \frac{2x^2 - 3}{x^2 - 5x + 3}$$

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3. Evaluate the following limits, if necessary use 1'Hopital Rule:

$$\lim_{x \to \infty} \frac{x}{\log x}$$

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4. Evaluate the following limits, if necessary use l' Hopital Rule :

$$\lim_{x \to \frac{\pi}{2}^-} \frac{\sec x}{\tan x}$$

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5. Evaluate the following limits, if necessary use l' Hopital Rule :

$$\lim_{x \to \infty} e^{-x}\sqrt{x}$$

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6. Evaluate the following limits, if necessary use l' Hopital Rule :

$$\lim_{x \to 0} \left(\frac{1}{\sin x} - \frac{1}{x}\right)$$

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7. $\lim_{x \to 1} \left(\frac{2}{x^2 - 1} - \frac{x}{x - 1}\right)$

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8. Evaluate the following limits, if necessary use l' Hopital Rule :

$$\lim_{x \to o^+} x^x$$

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9. Evaluate the following limits, if necessary use l' Hopital Rule :

$$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^{x}$$

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10. $\lim_{x \to \frac{x}{2}} (\sin x)^{\tan x}$

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11. Evaluate the following limits, if necessary use l' Hopital Rule :

$$\lim_{x \to 0^+} (\cos x)^{\frac{1}{x^2}}$$

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12. If an initial amount A_0 of money is invested at an interest rate r compounded n times a year, the value of the investment after t years is $A = A_0\left(1 + \frac{1}{n}\right)^{nt}$. If the interest is compounded continuously, (that is as $n \to \infty$

), show that the amount after t years is $A = A_0e^{rt}$.

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Solution To Exercise 7 6

1. Find the absolute extrema of the following functions on the given closed interval.

$f(x) = x^3 - 12x + 10, [1, 2]$

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2. Find the absolute extrema of the following functions on the given closed interval.

$f(x) = 3x^4 - 4x^3, [-1, 2]$

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3. Find the absolute extrema of the following functions on the given closed interval.

$f(x) = 6x^{\frac{4}{3}} - 3x^{\frac{1}{3}}, [-1, 1]$

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4. Find the absolute extrema of the following functions on the given closed interval.

$$f(x) = 2\cos x + \sin 2x, \left[0, \frac{\pi}{2}\right]$$

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5. Find the intervals of monotonicities and hence find the local extremum for the following functions:

$$f(x) = 2x^3 + 3x^2 - 12x$$

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6. Find the intervals of monotonicities and hence find the local extremum for the following functions:

$$f(x) = \frac{x}{x-5}$$

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7. Find the intervals of monotonicities and hence find the local extremum for the following functions:

$$f(x) = \frac{e^x}{1-e^x}$$

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8. Find the intervals of monotonicities and hence find the local extremum for the following functions:

$$f(x) = \frac{x^3}{3} - \log x$$

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9. Find the intervals of monotonicities and hence find the local extremum for the following functions:

$f(x) = \sin x \cos x + 5, x \in (0, 2\pi)$

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Solution To Exercise 7 7

1. Find intervals of concavity and points of inflexion for the following functions:

$f(x) = x(x - 4)^3$

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2. Find intervals of concavity and points of inflexion for the following functions:

$f(x) = \sin x + \cos x, 0 < x < 2\pi$

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3. Find intervals of concavity and points of inflexion for the following functions:

$f(x) = \frac{1}{2}\left(e^{x} - e^{-x}\right)$

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4. Find the local extrema for the following functions using second derivative test :

$f(x) = -3x^5 + 5x^3$

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5. Find the local extrema for the following functions using second derivative test :

$f(x) = x \log x$

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6. Find the local extrema for the following functions using second derivative test :

$f(x) = x^2 e^{-2x}$

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7. For the function $f(x) = 4x^3 + 3x^2 - 6x + 1$ find the intervals of monotonicity, local extrema, intervals of concavity and points of inflection.

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Solution To Exercise 7 8

1. Find two positive numbers whose sum is 12 and their product is maximum.

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2. Find two positive numbers whose product is 20 and their sum is minimum.

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3. Find the smallest possible value of $x^2 + y^2$ given that x + y = 10.

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4. A garden is to be laid out in a rectangular area and protected by wire fence. What is the largest possible area of the fenced garden with 40 metres of wire.

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5. A rectangular page is to contain $24cm^2$ of print. The margins at the top and bottom of the page are 1.5 cm and the margins at other sides of the page is 1 cm. What should be the dimensions of the page so that the area of the paper used is minimum.

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6. A farmer plans to fence a rectangular pasture adjacent to a river. The pasture must contain 1,80,000 sq. mtrs in order to provide enough grass for herds. No fencing is needed along the river. What is the length of the minimum needed fencing material?

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7. Find the dimensions of the rectangle with maximum area that can be inscribed in a circle of radius 10 cm.

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8. Prove that among all the rectangles of the given perimeter, the square has the maximum area.

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9. Find the dimensions of the largest rectangle that can be inscribed in a semi circle of radius r cm.

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10. A manufacturer wants to design an open box having a square base and a surface area of 108 sq. cm. Determine the dimensions of the box for the maximum volume.

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11. The volume of a cylinder is given by the formula $V = \pi r^2 h$. Find the greatest and least values of V if r + h = 6.

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12. A hollow cone with base radius a cm and height b cm is placed on a table . Show that the volume of the largest cylinder that can be hidden underneath is $\frac{4}{9}$ times volume of the cone .

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Solution To Exercise 7 9

1. Find the asymptotes of the following curves:

$$f(x) = \frac{x^2}{x^2 - 1}$$

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2. Find the asymptotes of the following curves:

$$f(x) = \frac{x^2}{x + 1}$$

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3. Find the asymptotes of the following curves:

$$f(x) = \frac{x^2 - 6x - 1}{x + 3}$$

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4. Find the asymptotes of the following curves:

$$f(x) = \frac{x^2 + 6x - 4}{3x - 6}$$

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5. Sketch the graphs of the following functions:

$$y = -\frac{1}{3}\left(x^3 - 3x + 2\right)$$

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6. Sketch the graphs of the following functions:

$$y = x\sqrt{4 - x}$$

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7. Sketch the graphs of the following functions:

$$y = \frac{x^2 + 1}{x^2 - 4}$$

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8. Sketch the graphs of the following functions:

$$y = \frac{x^3}{24} - \log x$$

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Solution To Exercise 7 10

1. The volume of a sphere is increasing in volume at the rate of $3\pi cm^3/\sec$. The rate of change of its radius when radius is $\frac{1}{2}$ cm

A. 3 cm/s

B. 2 cm/s

C. 1 cm/s

D. $\frac{1}{2}cm/s$

Answer: A

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2. A balloon rises straight up at 10m/s. An observer is 40 m away from the spot where the balloon left the ground. Find the rate of change of the balloon's angle of elevation in radian per second when the balloon is 30 metres above the ground.

A. $\frac{3}{25}$ radians/sec

B. $\frac{4}{25}$ radians/sec

C. $\frac{1}{5}$ radians/sec

D. $\frac{1}{3}$ radians/sec

Answer: B

Watch Video Solution

3. The position of a particle moving along a horizontal line of any time t is given by $s(t) = 3t^2 - 2t - 8$. The time at which the particle is at rest is

A. t=0

B. $t = \frac{1}{3}$

C. t=1

D. t=3

Answer: B

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4. A stone is thrown up vertically. The height it reaches at time t seconds is given by $x = 80t - 16t^2$. The stone reaches the maximum height in time t seconds is given by

A. 2

B. 2.5

C. 3

D. 3.5

Answer: B

Watch Video Solution

5. Find the point on the curve $6y = x^3 + 2$ at which y-coordinate changes 8 times as fast as x-coordinate is:

A. (4,11)

B. $(4, \ -11)$

C. $(-4, 11)$

D. $(-4, \ -11)$

Answer: A

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6. The abscissa of the point on the curve $f(x) = \sqrt{8 - 2x}$ at which the slope of the tangent is -0.25 ?

A. -8

B. -4

C. -2

D. 0

Answer: B

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7. The slope of the line normal to the curve $f(x) = 2\cos 4x$ at $x = \frac{\pi}{12}$ is

A. $-4\sqrt{3}$

B. -4

C. $\frac{\sqrt{3}}{12}$

D. $4\sqrt{3}$

Answer: C

Watch Video Solution

8. The tangent to the curve $y^2 - xy + 9 = 0$ is vertical when ....

A. y = 0

B. $y = \pm\sqrt{3}$

C. $y = \frac{1}{2}$

D. $y = \pm 3$

Answer: D

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9. Angle between $y^2 = x$ and $x^2 = y$ at the origin is

A. $\tan^{-1}\left(\frac{3}{4}\right)$

B. $\tan^{-1}\left(\frac{4}{3}\right)$

C. $\frac{\pi}{2}$

D. $\frac{\pi}{4}$

Answer: C

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10. What is the value of the limit $\lim_{x \to 0}\left(\cot x - \frac{1}{x}\right)$?

A. 0

B. 1

C. 2

D. $\leq$

Answer: A::B

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11. The function $\sin^4 x + \cos^4 x$ is increasing in the interval

A. $\left[\frac{5\pi}{8}, \frac{3\pi}{4}\right]$

B. $\left[\frac{\pi}{2}, \frac{5\pi}{8}\right]$

C. $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

D. $\left[0, \frac{\pi}{4}\right]$

Answer: C

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12. The number given by the Rolle's theorem for the function $x^3 - 3x^2, x \in [0, 3]$ is

A. 1

B. $\sqrt{2}$

C. $\frac{3}{2}$

D. 2

Answer: D

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13. The number given by the Mean value theorem for the function $\frac{1}{x}, x \in [1, 9]$ is

A. 2

B. 2.53

C. 3

D. 3.5

Answer: C

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14. The minimum value of the function $|3-x|+9$ is

A. 0

B. 3

C. 6

D. 9

Answer: D

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15. The maximum slope of the tangent to the curve $y = e^x \sin x, x \in [0, 2\pi]$ is at:

A. $x = \frac{\pi}{4}$

B. $x = \frac{\pi}{2}$

C. $x = \pi$

D. $x = \frac{3\pi}{2}$

Answer: B

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16. The maximum value of the functions $x^2 e^{-2x}, x > 0$ is ....

A. $\frac{1}{e}$

B. $\frac{1}{2e}$

C. $\frac{1}{e^2}$

D. $\frac{4}{e^4}$

Answer: C

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17. One of the closest points on the curve $x^2 - y^2 = 4$ to the point (6,0) is

A. (2,0)

B. $(\sqrt{5}, 1)$

C. $(3, \sqrt{5})$

D. $(\sqrt{13}, -\sqrt{3})$

Answer: C

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18. The maximum value of the product of two positive numbers, when their sum of the squares is 200, is

A. 100

B. $25\sqrt{7}$

C. 28

D. $24\sqrt{14}$

Answer: A

Watch Video Solution

19. The curve $y = ax^4 + bx^2$ with $ab > 0$

A. has no horizontal tangent

B. is concave up

C. is concave down

D. has no points of inflection

Answer: D

Watch Video Solution

20. The point of inflection of the curve $y=(x-1)^3$ is

A. (0,0)

B. (0, 1)

C. (1, 0)

D. (1, 1)

Answer: C

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Problems For Practice Choose The Correct Answer

1. A particle moves so that the distance moved is according to the law $s(t) = \frac{t^3}{3} - t^2 + 3$. At what time the velocity and acceleration are zero respectively ?

A. t =1

B. t =0

C. t =2

D. $t = \frac{1}{2}$

Answer: A

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2. The slope of the normal to the curve $y = 3x^2$ at the point whose abscissa is 2 is:

A. $\frac{1}{12}$

B. $-\frac{1}{12}$

C. $\frac{1}{13}$

D. $\frac{1}{14}$

Answer: B

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3. The value of c is Rolle's theoram for the function f(x) = $\cos\left(\frac{x}{2}\right)$ on $[\pi, 3\pi]$ is:

A. $\frac{\pi}{2}$

B. 2π

C. $\frac{3\pi}{2}$

D. 0

Answer: B

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4. The curves $2x^2 + 3y^2 = 1$ and $cx^2 + 4y^2 = 1$ cut each other orthogonally then the value of c is:

A. $\frac{1}{3}$

B. 3

C. $\frac{5}{12}$

D. $\frac{12}{5}$

Answer: D

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5. The angle between $y^2 = x$ and $x^2 = y$ at the origin is

A. $2\tan^{-1}\left(\frac{3}{4}\right)$

B. $\tan^{-1}\frac{4}{3}$

C. $\frac{\pi}{2}$

D. $\frac{\pi}{4}$

Answer: C

View Text Solution

6. The curve $y = ax^3 + bx^2 + cx + d$ has a point of inflexion at x=1 , then :

A. $a + b0$

B. $a + 3b = 0$

C. $3a + b = 0$

D. $3a - b = 0$

Answer: C

Watch Video Solution

7. In a given semi circle of diameter 4 cm a rectangle is to be inscribed . The maximum area of the rectangle is ....

A. 2

B. 4

C. 8

D. 16

Answer: B

Watch Video Solution

8. $\lim_{x \to 0} \frac{2x}{\tan 2x}$ is

A. 1

B. -1

C. 0

D. ∞

Answer: A

Watch Video Solution

9. $\lim\limits_{x \to 0} \dfrac{a^x - b^x}{c^x - d^x}$ is

A. ∞

B. 0

C. $\log\left(\dfrac{ab}{cd}\right)$

D. $\dfrac{\log\left(\frac{a}{b}\right)}{\log\left(\frac{c}{d}\right)}$

Answer: D

View Text Solution

10. If $s = t^3 - 4t^2 + 7$ the velocity when the acceleration is zero is

A. $\frac{32}{3}$ m/sec

B. $\frac{-16}{3}$ m/sec

C. $\frac{16}{3}$ m/sec

D. $\frac{-32}{3}$ m/sec

Answer: B

Watch Video Solution

11. Which of the following functions is increasing in $(0, \infty)$?

A. e^x

B. $\frac{1}{x}$

C. $-x^2$

D. x^{-2}

Answer: A

Watch Video Solution

12. The slope of the tangent to the curve $y = 3x^2 + 4\cos x$ at $x = 0$ is

A. 4

B. 2

C. 3

D. -1

Answer: A

Watch Video Solution

13. The stationary point of $f(x) = x^{\frac{3}{5}}(4 - x)$

occurs at x=

A. a) $\frac{3}{2}$

B. b) $\frac{2}{3}$

C. c) 0

D. d) 4

Answer: A

View Text Solution

14. The angle between the curve $x^2 - y^2 = 8$ and $\frac{x^2}{25} + \frac{y^2}{9} = 1$ is

A. $\frac{\pi}{3}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{2}$

Answer: D

Watch Video Solution

15. The radius of a cylinder is increasing at the rate of 2 cm/sec and the height is decreasing at the rate of 3 cm/sec. The rate of change of volume when the radius is 3 cm and height is 5 cm is:

A. 23π

B. 33π

C. 43π

D. 53π

Answer: B

Watch Video Solution

16. $\lim_{x \to \infty} \frac{x^2}{e^x}$ is

A. 2

B. 0

C. ∞

D. 1

Answer: B

Watch Video Solution

17. f is a differentiable function defined on an interval I with positive derivative . Then f is ....

A. increasing on I

B. decreasing on I

C. strictly increasing on I

D. strictly decreasing on I

Answer: C

Watch Video Solution

18. For what values of x the rate of increase of $x^3 - 2x^2 + 3x + 8$ is twice the rate of increase of x.

A. $\left(-\frac{1}{3}, -3\right)$

B. $\left(\frac{1}{3}, 3\right)$

C. $\left(-\frac{1}{3}, 3\right)$

D. $\left(\frac{1}{3}, 1\right)$

Answer: D

Watch Video Solution

19. The least possible perimeter of a rectangule of area $400m^2$ is

A. 100m

B. 40m

C. 80m

D. 60m

Answer: C

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20. The equation of the normal to the curve $y = x^3$ at $(1, 1)$ is

A. $3x + y - 4 = 0$

B. $3x + y = 4$

C. $x - 3y - 4 = 0$

D. $x + 3y - 4 = 0$

Answer: D

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21. The equation of the tangent to the curve $y=\frac{x^3}{5}$ at $\left(-1,-\frac{1}{5}\right)$ is

A. $5y+3x-2=0$

B. $5y-3x=2$

C. $3x+5y=2$

D. $3x+3y-2=-6$

Answer: B

Watch Video Solution

22. Which of the following is concave down:

A. $y = -x^2$

B. $y = x^2$

C. $y = e^x$

D. $y = x^2 + 2x - 3$

Answer: A

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23. If

$f(a) = 2, f'(a) = 1, g(a) = -1, g'(a) = 2.$

Then $\lim_{x \to a} \frac{g(x)f(a) - g(a)f(x)}{x - a}$ is

A. 5

B. -5

C. 3

D. -3

Answer: A

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24. The velocity v of a particle moving along a straight line when at a distance x from the origin is given by $a + bv^2 = x^2$ where a and b are constants then the acceleration is:

A. $\frac{b}{x}$

B. $\frac{a}{x}$

C. $\frac{x}{b}$

D. $\frac{x}{a}$

Answer: C

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25. The value of c in Lagrange's mean value theorem for the function $f(x) = x^2 + 2x - 1$ in (0, 1) is

A. -1

B. 1

C. 0

D. $\frac{1}{2}$

Answer: D

26. The function $f(x) = x^2$ is decreasing in

A. $(-\infty, \infty)$

B. $(-\infty, 0)$

C. $(0, \infty)$

D. $(-2, \infty)$

Answer: B

27. $y = -e^x$ is

A. concave up for $x > 0$

B. concave down for $x > 0$

C. concave up everywhere

D. concave down everywhere

Answer: D

Watch Video Solution

28. If f has a local extremum at a and if f'(a) exists then

A. $f'(a) < 0$

B. $f'(a) > 0$

C. $f'(a) = 0$

D. $f''(a) = 0$

Answer: C

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29. A spherical snowball is melting in such a way that its volume is decreasing at a rate of $1cm^3 / \min$. The rate at which the diameter is decreaseing when the diameter is 10 cms is ..

Watch Video Solution

30. $f(x) = \sqrt{x}, a = 1, b = 4$ find c in Lagrange's mean value theorm:

A. $\frac{9}{4}$

B. $\frac{3}{2}$

C. $\frac{1}{2}$

D. $\frac{1}{4}$

Answer: A

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31. The point of curve $y = 2x^2 - 6x - 4$ at which the tangent is parallel to x-axis is

A. $\left(\frac{5}{2}, -\frac{17}{2}\right)$

B. $\left(\frac{-5}{2}, \frac{-17}{2}\right)$

C. $\left(\frac{-5}{2}, \frac{17}{2}\right)$

D. $\left(\frac{3}{2}, -\frac{17}{2}\right)$

Answer: D

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32. The curve $y = 3e^{x}$ and $y = \frac{a}{3}e^{-x}$ intersect orthogonally if a =

A. -1

B. 1

C. $\frac{1}{3}$

D. 3

Answer: B

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33. $f'(x) = x^2 - 5x + 4$ then f(x) is decreasing in

A. $(-\infty, 0)$

B. $(1, 4)$

C. $(4, \infty)$

D. everywhere

Answer: B

Watch Video Solution

34. If the normal to the curve $x^{2/3} + y^{2/3} = a^{2/3}$ makes an angle θ with the x -axis then the slope of the normal is ....

A. $-\cot\theta$

B. $\tan\theta$

C. $-\tan\theta$

D. $\cot\theta$

Answer: D

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35. For the curve $x = e^t \cos t, y = e^t \sin t$ the tangent line is parallel to x-axis when t is equal to

A. $-\frac{\pi}{4}$

B. $\frac{\pi}{4}$

C. 0

D. $\frac{\pi}{3}$

Answer: A

Watch Video Solution

36. If $y = 6x - x^3$ and x increases at the rate of 5 units/sec the rate of change of slope when x =3 is

A. -90 units/sec

B. 90 units/sec

C. 180 units/sec

D. -180 units/sec

Answer: A

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37. If the length of the diagonal of a square is increasing at the rate of 0.1 cm/sec what is the rate of increase of its area when the side is $\frac{15}{\sqrt{2}}$ cm?

A. $1.5cm^2/\sec$

B. $3cm^2/\sec$

C. $3\sqrt{2}cm^2/\sec$

D. $0.15cm^2/\sec$

Answer: A

Watch Video Solution

38. The equation of the normal to the curve $\theta = \frac{1}{t}$ at $\left(-3, \frac{-1}{3}\right)$ is:

A. $3\theta = 27t - 80$

B. $5\theta = 27t - 80$

C. $3\theta = 27t + 80$

D. $\theta = \frac{1}{t}$

Answer: C

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39. The point of inflection of the curve $y = x^4$

is at:

A. `

B.

C.

D.

Answer:

Watch Video Solution

40. The angle between the curve $y = e^{mx}$ and $y = e^{-mx} m > 1$ is

A. $\tan^{-1}\left(\frac{2m}{m^2 - 1}\right)$

B. $\tan^{-1}\left(\frac{2m}{1 - m^2}\right)$

C. $\tan^{-1}\left(-\frac{2m}{1 + m^2}\right)$

D. $\tan^{-1}\left(\frac{2m}{m^2 + 1}\right)$

Answer: B

Problems For Practice Answer The Following Question

1. The angular displacement θ radians of a fly wheel varies with time t seconds and follow the equation $\theta = 9t^2 - 2t^3$. Determine the angular velocity and acceleration of the fly wheel when time t =1 seconds and (ii) The time when the angular acceleration is zero.

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2. The distance s meters moved by a particle travelling in a straight line t seconds is given by $s = 45t + 11t^2 - t^3$. Find the acceleration when the particle comes to rest.

Watch Video Solution

3. A particle moves along a horizontal line such that its position at any time t is given by $s(t) = t^3 - 6t^2 + 9t + 1$, s in meters and t in seconds.

At what time the particle is at rest?

Watch Video Solution

4. A particle moves along a horizontal line such that its position at any time t is given by $s(t) = t^3 - 6t^2 + 9t + 1$, s in meters and t in seconds.

At what time the particle is at rest?

Watch Video Solution

5. A particle moves along a horizontal line such that its equation of motion is $s(t) = 2t^3 - 15t^2 + 24t - 2$, s in meters and t in second.

Find the total distance travelled by the particle in the first 2 seconds.

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6. A missile fixed from ground level raises x metres vertically upwards in t seconds and

$x = 100t - \frac{25}{2}t^2$. Find

Initial velocity

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7. A missile fixed from ground level raises x metres vertically upwards in t seconds and $x = 100t - \frac{25}{2}t^2$. Find

The time when height of the missile is max

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8. A missile fixed from ground level raises x metres vertically upwards in t seconds and $x = 100t - \frac{25}{2}t^2$. Find

The maximum height reached

Watch Video Solution

9. A missile fixed from ground level raises x metres vertically upwards in t seconds and $x = 100t - \frac{25}{2}t^2$. Find

Velocity with which the missile strikes the ground.

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10. A water tank has the shape of an invertd circular cone with base radius 2 metres and height 4 metres. If water is being pumped into the tank at the rate of $2m^3/mm$. Find the rate at which the water level is rising when the water is 3m deep

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11. At a particular instant ship A is 100 km west of ship B, ship A is sailing at a speed of 35 km/hr and ship B is sailing north at 25 km/hr. How fast is the distance between the ships changes after 4 hours

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12. A particle is fired straight up from the ground to each a height of x feet in t seconds, where x $(t) = 128t - 16t^2$.

(1) Compute the maximum height of the particle reached.

(2) What is the velocity when the particle hits the ground ?

Watch Video Solution

13. Find the equation of tangent and normal to the curve $y = x^2 + 3x + 2$ at (0, 2)

Watch Video Solution

14. For what value of x the tangent of the curve $y = x^3 - 3x^2 + 2x - 5$ is parallel to $x + y + 3 = 0$

Watch Video Solution

15. Find the angle between the curves, $\frac{x^2}{8} + \frac{y^2}{2} = 1$ and $\frac{x^2}{4} - \frac{y^2}{2} = 1$

View Text Solution

16. Find the equation of the tangent to the curve $x^2 + y^2 = 5^2$ which are parallel to $2x + 3y = 6$

Watch Video Solution

17. Find the equation of normal to $y = x^3 - 3x$ that is parallel to $2x + 18y - 9 = 0$

Watch Video Solution

18. Verify Rolle's theorem for $f(x)=\sqrt{1-x^2},\ -1\le x\le 1$

Watch Video Solution

19. Using Rolle's theorem find the point on the curve $y=x^2+1,\ -2\le x\le 2$ where the tangent is parallel to x-axis.

Watch Video Solution

20. Verify mean value theorem for the function

$f(x) = x^3 - 5x^2 \_ 2x$ in [1, 3]

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21. Expand $f(x) = e^{3x}$ on a maclaurin's series

Watch Video Solution

22. Evaluate: $\lim_{x \to 2} \frac{\sin \pi x}{2 - x}$

Watch Video Solution

23. Evaluate: $\lim_{x \to \infty} \frac{x^{\frac{1}{2}} - 2\tan^{-1}\left(\frac{1}{x}\right)}{\left(\frac{1}{x}\right)}$

Watch Video Solution

24. Evaluate: $\lim_{x \to 0} x^{\sin x}$

Watch Video Solution

25. Prove , using mean value theorem, that

$|\sin\alpha - \sin\beta| \leq |\alpha - \beta|, \alpha, \beta \in \mathbb{R}.$

Watch Video Solution

26. Expand $f(x) = \frac{1}{x}$ about x =2 upto four terms

Watch Video Solution

27. Evaluate $\lim_{\theta \to 0} \frac{1 - \cos m\theta}{1 - \cos n\theta}$

Watch Video Solution

28. Evaluate: $\lim_{x \to 0} x \log x$

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29. Determine for which values of x the function $y = \frac{x-2}{x+1}, x \neq -1$ is strictly increasing or decreasing.

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30. Discuss the monotonicity of $f(x) = 2x^3 - 15x^2 + 36x + 1$

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31. Find the critical number and stationary point $f(x) = 2x - 3x^2$

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32. Find the absolute extreme for $f(x) = 1 - 2x - x^2$ in (-4,1)`

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33. Find the local maximum and local minimum of $f(x) = 2x^3 + 5x^2 - 4x$

Watch Video Solution

34. Find the absolute extreme of the function

$f(x) = 2\cos x$ in $[0, 2\pi]$

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35. Find the points of inflection of

$y = x^4 - 6x^2 + 8x - 1$

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36. Obtain Maclaurin's series for f(x) = log sec x

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37. Resistance to motion F of a moving vehicle is given by $F = \frac{5}{x} + 100x$. Determined the minimum value of the resistance.

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38. Find the point on the parabola $y^2 = 2x$ that is closest to the point (1, 4)

Watch Video Solution

39. Find the asymptotes of the function

$$f(x) = \frac{1}{x+1}$$

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