MATHS

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COMPLEX NUMBERS

Worked Example

1. Simplify the following :

i^9

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2. Simplify the following :

i^{2748}

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3. Simplify the following :

i^{1603}

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4. Simplify the following :

i^{-1920}

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5. Simplify the following :

$$\frac{1}{i}$$

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6. Simplify the following :

$$i^{-74} + i^{88}$$

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7. Simplify the following :

$$\sum_{n=1}^{100} i^n$$

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8. Simplify the following :

$ii^2i^3 \ldots\ldots\ldots\ldots\ldots i^{80}$

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9. Find x and y for which of the following is satisfied.

$$\frac{(1+i)x-2i}{3+i}+\frac{(2-3i)y+i}{3-i}=i$$

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10. Reduce $\frac{4+3i}{5-4i}$ in $x+iy$ form, find its real and imaginary parts.

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11. If $z = \left(\frac{1+i}{1-i}\right)^9$ find $z + \frac{1}{z}$

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12. If $\frac{z-1}{z+3} = \frac{3+4i}{2}$ find $\bar{z}$

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13. If $z_1 = 3 - 4i, z_2 = 2 + i$ find

$\overline{z_1 z_2}$

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14. If $z_1 = 3 - 4i$, $z_2 = 2 + i$ find

$$\overline{\left(\frac{z_1}{z_2}\right)}$$

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15. If $z_1 = 3 - 4i$, $z_2 = 2 + i$ find

$$\overline{\left(\frac{z_2}{z_1}\right)}$$

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16. If $z_1 = 3 - 4i$, $z_2 = 2 + i$ find

$$z_1\bar{z}_1 + z_2\bar{z}_2$$

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17. If $z = \dfrac{(1+i)(2+i)}{(3+i)}$ find z^{-1} and $\bar{z}$.

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18. Show that $(i)\left(3+i\sqrt{5}\right)^{12} + \left(3-i\sqrt{5}\right)^{12}$ is real and $(ii)\left(\dfrac{13-i}{5-3i}\right)^{15} - \left(\dfrac{7+4i}{2+3i}\right)^{15}$ ia purely imaginary.

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19. If $z_1 = 4+3i,\quad z_2 = 12-5i,\quad z_3 = 8+6i$ find $|z_1|, |z_2|, |z_3|, |z_1+z_2|$ and $|z_3-z_2|$

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20. Find the following

$$|3 - 4i|$$

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21. Find the following

$$|(2 + i)(3 - i)|$$

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22. Find the following

$$\left|\frac{-4 + 2i}{2 + 3i}\right|$$

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23. Find the following

$$\left|\frac{(1+i)(1+2i)}{(1+3i)}\right|$$

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24. Find the following

$$\left|\frac{i(1+2i)^2}{(2+i)^4}\right|$$

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25. Which one of the points $1+2i,\ 2+3i,\ 1-i$ is nearest to the origin.

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26. For what values of x and y the numbers $-3 + ix^2y$ and $x^2 + y + 4i$ and complex conjugate to each other.

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27. If $|z| = 1$ show that $4 \leq |z + 4 - 3i| \leq 6$.

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28. Show that the complex numbers $3 + 3i$, $-3 - 3i\left(-3\sqrt{3} + i3\sqrt{3}\right)$ are the vertices of an equilateral triangle in the complex plane.

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29. Let z_1, z_2 and z_3 be complex numbers such that $|z_1| = |z_2| = |z_3| = 1$ then prove that $|z_1 + z_2 + z_3| = |z_1 z_2 + z_2 z_3 + z_3 z_1|$

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30. Show that the points represented by the complex numbers $7 + 9i$, $-3 + 7i$, $3 + 3i$ from a right angled triangle on the Argand diagram.

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31. Find the square root of $(-5 + 12i)$

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32. If $z = 1 + 3i$ then show that z, iz and $z + zi$ form a right angled isosceles triangle.

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33. Show that $|2z + 5 - i| = 4$ represents a circle find its centre and radius.

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34. Show that $|z + 2 - i| < 2$ represents interior points of a circle find its centre and radius.

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35. Obtain the cartesian form of the locus of z in each of the following.

$|z+2|=|z-3i|$

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36. Obtain the cartesian form of the locus of z in each of the following.

$|3(x+iy)+2-3i|=2$

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37. Find the modulus and principal argument of the following complex numbers .

$1 + i\sqrt{3}$

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38. Find the modulus and principal argument of the following complex numbers .

$-1 + i\sqrt{3}$

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39. Find the modulus and principal argument of the following complex numbers .

$-1 - i\sqrt{3}$

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40. Find the modulus and principal argument of the following complex numbers .

$1 - i\sqrt{3}$

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41. Represent the complex numbers

$-1 + i$ and $\left(-\sqrt{3} - i\right)$ in polar form.

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42. Find the principal argument Arg z.

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43. Find the product of z_1 and z_2

If $z_1 = 2\left(\cos.\ \frac{\pi}{3} + i\sin.\ \frac{\pi}{3}\right)$ and $z_2 = 3\left(\cos.\ \frac{2\pi}{3} + i\sin.\ \frac{2\pi}{3}\right)$

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44. Simplify : $\frac{(\cos 6\theta + i\sin 6\theta)(\cos 2\theta + i\sin 2\theta)}{(\cos 4\theta - i\sin 4\theta)(\cos 3\theta + i\sin 3\theta)}$

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45. Represents the variable complex number z. Find the locus of P if $arg\left(\frac{z-1}{z+3}\right)=\frac{\pi}{2}$.

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46. If $z=(\cos\theta+i\sin\theta)$, show that $z^n+(1)/(z^n)=2\cos n\theta$ and z^(n)-(1)(z^(n))=2isinntheta

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47. Simplify $\left(\sin.\frac{\pi}{4}-i\cos.\frac{\pi}{4}\right)^{12}$

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48. If n is positive integer prove that

$$\left(\frac{1+\sin\theta+i\cos\theta}{1+\sin\theta-i\cos\theta}\right)^n=\cos n\left(\frac{\pi}{2}-\theta\right)+i\sin n\left(\frac{\pi}{2}-\theta\right)$$

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49. Simplify

$(\sqrt{3}+i)^8$

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50. Simplify

$(3-i\sqrt{3})^6$

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51. Find the cube roots of unity.

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52. Find the fourth roots of unity.

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53. Solve $z^4 - 16i = 0$ where z to c complex no.

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54. Find all the cube roots of $(1+i\sqrt{3})^{\frac{1}{3}}$

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55. Suppose z_1, z_2 and z_3 are the vertices of an equilateral triangle inscribed in the circle |z| = 2. If $z_1 = 1 + i\sqrt{3}$ then find z_2 and z_3.

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Solution To Exercise 2 1

1. Simplify the following:

$i^{1947} + i^{1950}$

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2. $i^{1948} - i^{-1869}$

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3. Simplify the following:

$$\sum_{n-1}^{12} i^n$$

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4. Simplify : $i^{59} + \frac{1}{i^{59}}$.

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5. Simplify the following:

$i\, i^2 i^3 \ldots\ldots i^{2000}$

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6. Simplify the following:

$$\sum_{n=1}^{10} i^{n+50}$$

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Solution To Exercise 2 2

1. Evaluate the following if $z = 5 - 2i$ and $w = -1 + 3i$

$z + w$

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2. Evaluate the following if $z = 5 - 2i$ and $w = -1 + 3i$

$z - iw$

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3. Evaluate the following if $z = 5 - 2i$ and $w = -1 + 3i$

$2z + 3w$

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4. Evaluate the following if $z = 5 - 2i$ and $w = -1 + 3i$

zw

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5. Evaluate the following if $z = 5 - 2i$ and $w = -1 + 3i$

$z^2 + 2zw + w^2$

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6. Evaluate the following if $z = 5 - 2i$ and $w = -1 + 3i$

$(z + w)^2$

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7. Given the complex number z = 2 +3i, represent the complex numbers in Argand diagram.

z, iz, and z + iz

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8. Given the complex number z = 2 +3i, represent the complex numbers in Argand diagram.

z, -iz, and z - iz.

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9. Find the values of the real numbers x and y, if the complex numbers

(3 - i) x - (2-i) y + 2i + 5 and 2x + (-1 + 2i)y + 3 + 2i are equal

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Solution To Exercise 2 3

1. If $z_1 =$ 1 - 3i, z_2 = -4i and z_3 = 5, show that

$(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3)$

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2. If $z_1 = 1 - 3i$, $z_2 = -4i$ and $z_3 = 5$, show that

$(z_1 z_2) z_3 = z_1 (z_2 z_3)$

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3. If $z_1 = 3$, $z_2 = 7i$, and $z_3 = 5 + 4i$, show that

$z_1 (z_2 + z_3) = z_1 z_2 + z_1 z_3$

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4. If $z_1 = 3$, $z_2 =$ - 7i, and $z_3 = 5 + 4i$, show that

$(z_1 + z_2) z_3 = z_1 z_3 + z_2 z_3$

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5. If $z_1 = 2 + 5i, z_2 = -3 - 4i,$ and $z_3 = 1+$ I, find the additive and multiplicative inverse of z_1, z_2 and z_3.

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Solution To Exercise 2 4

1. Write in the rectangular form

$$\overline{(5 + 9i) + (2 - 4i)}$$

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2. Write in the rectangular form

$$\frac{10 - 5i}{6 + 2i}$$

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3. Write in the rectangular form

$$\overline{3i} + \frac{1}{2 - i}$$

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4. If z = x + iy, find in rectangular form.

$$\text{Re}\left(\frac{1}{z}\right)$$

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5. If z = x + iy, find in rectangular form.

Re$\left(\overline{iz}\right)$

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6. If z = x + iy, find in rectangular form.

Im (3z + $4\bar{z}$ - 4i)

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7. If $z_1 = 2 - i$ and $z_2 = -4$ + 3i, find the inverse of $z_1 z_2$ and $\frac{z_1}{z_2}$.

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8. The complex numbers u, v and w are related by $\frac{1}{u} = \frac{1}{v} + \frac{1}{w}$. If v = 3 -4i and w = 4 + 3i , find u in rectangular form.

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9. Prove the following properties:

z is real if and only if z = $\bar{z}$

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10. Prove the following properties:

Re(z) = $\frac{z + \bar{z}}{2}$ and Im(z) = $\frac{z - \bar{z}}{2i}$

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11. Find the least value of the positve interger n for which

$$\left(\sqrt{3}+i\right)^{n}$$

real

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12. Show that

$\left(2+i\sqrt{3}\right)^{10}-\left(2-i\sqrt{3}\right)^{10}$ is purely imaginary.

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13. Show that

$$\left(\frac{19-7i}{9+i}\right)^{12}+\left(\frac{20-5i}{7-6i}\right)^{12}$$ is real.

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Solution To Exercise 2 5

1. Find the modulus of the complex number

$$\frac{2i}{3+4i}$$

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2. Find the modulus of the complex number

$$\frac{2-i}{1+i}+\frac{1-2i}{1-i}$$

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3. Find the modulus of the complex number

$$(1-i)^{10}$$

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4. Find the modulus of the complex number

2i(3 - 4i)(4 - 3i)

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5. For any two complex number z_1 and z_2, such that $|z_1| = |z_2| = 1$ and $z_1 z_2 \neq -1$, then show that $\frac{z_1 + z_2}{1 + z_1 z_2}$ is real number.

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6. Which one of the points 10 - 8i, 11 + 6i is closest to 1 + i.

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7. If |z| = 3, show that 7 $\leq |z + 6 - 8i| \leq$ 13.

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8. If |z| = 1, show that 2 $\leq \left|z^3 - 3\right| \leq 4.$

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9. If $\left|z - \frac{2}{z}\right|$ = 2. show that the greatest and least value of |z| are $\sqrt{3} + 1$ and $\sqrt{3}$ - 1 respectively.

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10. If $z_1, z_2,$ and z_3 are three complex numbers such that

$|z_1| = 1, \quad |z_2| = 2, \quad |z_3| = 3$ and $|z_1 + z_2 + z_3| = 1,$

show that $|9z_1z_2 + 4z_1z_3 + z_2z_3| = 6.$

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11. If the area of the triangle formed by the vertices z, iz, and z + iz is 50 square units, find the value of |z|.

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12. Show that the equation $z^3 + 2\bar{z} = 0$ has five solutions.

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13. Find the square roots of

(i) 4 + 3i

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14. Find the square roots of

- 6 + 8i

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15. Find the square roots of

- 5 - 12i.

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Solution To Exercise 2 6

1. If z = x + iy is a complex number such that $\left|\frac{z-4i}{z+4i}\right|$ = 1

show that the locus of z is real axis.

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2. If z =x + iy is complex number such that Im$\left(\frac{2z+1}{iz+1}\right)$ = 0, show that the locus of z is

$2x^2 + 2y^2 + x - 2y$ = 0.

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3. Obtain the Cartesian form of the locus of z = x + iy in each of cases:

$[\text{Re}(iz)]^2$ = 3

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4. Obtain the Cartesian form of the locus of z = x + iy in each of cases:

Im[1 - i) z + 1] = 0

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5. Obtain the Cartesian form of the locus of z = x + iy in each of cases:

|z + i| = |z - 1|

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6. Obtain the Cartesian form of the locus of z = x + iy in each of cases:

$\bar{z} = z^{-1}$

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7. Show that the equations represent a circle, and , find its centre and radius.

$|z - 2 - i| = 3$

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8. Show that the equations represent a circle, and , find its centre and radius.

$|2z + 2 - 4i| = 2$

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9. Show that the equations represent a circle, and , find its centre and radius.

$|3z - 6 + 12i| = 8$

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10. Obtain the Cartesian equation for the locus of z = x + iy in each of the cases:

$|z - 4| = 16$

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11. Obtain the Cartesian equation for the locus of z = x + iy in each of the cases:

$|z-4|^2 - |z-1|^2$ = 16

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Solution To Exercise 2 7

1. Write in polar form of the complex numbers.

2 + $i2\sqrt{3}$

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2. Write in polar form of the complex numbers.

3 - i $\sqrt{3}$

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3. Write in polar form of the complex numbers.

$-2-2i$

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4. Write in polar form of the complex numbers.

$$\frac{i-1}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}$$

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5. Find the rectangular form of the complex numbers.

$$\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)$$

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6. Find the rectangular form of the complex numbers.

$$\frac{\cos\frac{\pi}{6}-i\sin\frac{\pi}{6}}{2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)}$$

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7. Given $(x_1+iy_1)(x_2+iy_2)\ldots(x_n+iy_n)$ = a + ib, show that

$$\left(x_1^2+y_1^2\right)\left(x_2^2+y_2^2\right)\left(x_3^2+y_3^2\right)\ldots\left(x_n^2+y_n^2\right)=a^2+b^2$$

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8. Given $(x_1 + iy_1)(x_2 + iy_2)\ldots(x_n + iy_n)$ = a + ib, show that

$$\sum_{r=1}^{n} \tan^{-1} \frac{y_r}{x_r} = \tan^{-1}\left(\frac{b}{a}\right) + 2k\pi, \quad \text{k} \quad \in \mathbb{Z}$$

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9. If $\dfrac{1+z}{1-z}$ = cos $2\theta + i \sin 2\theta$, show that z = i tanθ.

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10. If cos $\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0$,

show that

$\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha + \beta + \gamma)$ and

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11. If $\cos \alpha + \cos \beta + \cos \gamma = \sin \alpha + \sin \beta + \sin \gamma = 0,$

show that

$\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha + \beta + \gamma)$

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12. If z = x + iy and $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$. Show that

$x^2 + y^2 + 3x - 3y + 2 = 0.$

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Solution To Exercise 2 8

1. If $\omega \neq 1$ is a cube root of unity, then show that $\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2}+\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}=1$

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2. Show that $\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5+\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5=-\sqrt{3}$

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3. Find the value of $\left(\frac{1+\sin\frac{\pi}{10}+i\cos\frac{\pi}{10}}{1+\sin\frac{\pi}{10}-i\cos\frac{\pi}{10}}\right)^{10}$.

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4. If 2 cos α = x + $\frac{1}{x}$ and 2 cos β = y + $\frac{1}{y}$, show that

$$\frac{x}{y} + \frac{y}{x} = 2\cos(\alpha - \beta)$$

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5. If 2 cos α = x + $\frac{1}{x}$ and 2 cos β = y + $\frac{1}{y}$, show that

$$xy - \frac{1}{xy} = 2i\sin(\alpha - \beta)$$

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6. If 2 cos α = x + $\frac{1}{x}$ and 2 cos β = y + $\frac{1}{y}$, show that

$$\frac{x^m}{y^n} - \frac{y^n}{x^m} = 2i\sin(m\alpha - n\beta)$$

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7. If 2 cos α = x + $\frac{1}{x}$ and 2 cos β = y + $\frac{1}{y}$, show that

$$x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\alpha + n\beta)$$

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8. Solve the equation $z^3 + 27$ = 0

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9. If $\omega \neq 1$ is a cube root of unity, show that the roots of the equation $(z-1)^3 + 8 = 0$ are $-1, 1-2\omega, 1-2\omega^2$.

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10. Find the value of $\sum_{k=1}^{8}\left(\cos\frac{2k\pi}{9}+i\frac{\sin 2k\pi}{9}\right)$

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11. If $\omega \pm$ 1 is a cube root of unity, show that

$$\left(1-\omega+\omega^2\right)^6+\left(1+\omega-\omega^2\right)^6=128$$

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12. If $\omega \pm$ 1 is a cube root of unity, show that

$(1+\omega)\left(1+\omega^2\right)\left(1+\omega^4\right)\left(1+\omega^8\right)\ldots\left(1+\omega^{2^{11}}\right)$ = 1

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13. If z = 2 - 2i, find the rotation of z by θ radians in the counter clockwise direction about the origin when

$$\theta = \frac{\pi}{3}$$

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14. If z = 2 - 2i, find the rotation of z by θ radians in the counter clockwise direction about the origin when

$$\theta = \frac{2\pi}{3}$$

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15. If z = 2 - 2i, find the rotation of z by θ radians in the counter clockwise direction about the origin when

$\theta = \frac{3\pi}{2}$

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16. Prove that the values of $4\sqrt{-1}$ are $\pm\frac{1}{\sqrt{2}}(1 \pm i)$.

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Solution To Exercise 2 9

1. $i^n + i^{n+1} + i^{n+2} + i^{n+3}$

A. 0

B. 1

C. -1

D. i`

Answer: A

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2. The value of $\sum_{i=1}^{13} \left(n^n + i^{n-1}\right)$ is

A. $1 + i$

B. i

C. 1

D. 0

Answer: A

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3. The area of the triangle formed by the complex numbers z, iz, and z + iz in the Argand's diagram is

A. $\frac{1}{2}|z|^2$

B. $|z|^2$

C. $\frac{3}{2}|z|^2$

D. $2|z|^2$

Answer: A

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4. The conjugate of a complex number is $\frac{1}{i-2}$. Then, the complex number is

A. $\frac{1}{i+2}$

B. $\frac{-1}{i+2}$

C. $\frac{-1}{i-2}$

D. $\frac{1}{i-2}$

Answer: B

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5. If z $=\frac{\left(\sqrt{3}+i\right)^{3}(3i+4)^{2}}{(8+6i)^{2}}$, then |z| is equal to

A. 0

B. 1

C. 2

D. 3

Answer: C

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6. If z is non zero complex number, such that 2i $z^2 = \bar{z}$, then |z| is

A. $\frac{1}{2}$

B. 1

C. 2

D. 3

Answer: A

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7. If |z-2 + i| $\leq$ 2, then the greatest value of |z| is

A. $\sqrt{3}-2$

B. $\sqrt{3}+2$

C. $\sqrt{5}-2$

D. $\sqrt{5}+2$

Answer: D

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8. If $|z - \frac{3}{z}|$ = 2, then the least value of |z| is

A. 1

B. 2

C. 3

D. 5

Answer: A

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9. If |z| = 1, then the value of $\frac{1+z}{1+\bar{z}}$.

A. z

B. $\bar{z}$

C. $\frac{1}{z}$

D. 1

Answer: A

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10. The solution of the equation |z| -z = 1 + 2i is

A. $\frac{3}{2} - 2i$

B. $-\frac{3}{2} + 2i$

C. $2 - \frac{3}{2}i$

D. $2+\frac{3}{2}i$

Answer: A

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11. If $|z_1|$ = 1, $|z_2| = 2$, $|z_3| = 3$ and $|9z_1z_2+4z_1z_3+z_2z_3=12|$, then the value of $|z_1+z_2+z_3|$ is

A. 1

B. 2

C. 3

D. 4

Answer: B

12. If z is a complex number such that z $\in \mathbb{C} \backslash \mathbb{R}$, and z + $\frac{1}{z} \in \mathbb{R}$, then |z| is

A. 0

B. 1

C. 2

D. 3

Answer: B

13. $z_1, z_3,$ and z_3 are complex numbers such that $z_1 + z_2 + z_3 = 0$ and $|z_1| = |z_2| = |z_3|$ = 1 then $z_1^2 + z_2^2 + z_3^3$

A. 3

B. 2

C. 1

D. 0

Answer: D

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14. If $\dfrac{z-1}{z+1}$ is purely imaginary, then |z| is

A. $\frac{1}{2}$

B. 1

C. 2

D. 3

Answer: B

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15. If z = x + iy is a complex number such that |z + 2| = |z - 2|, then the locus of z is

A. real axis

B. imaginary axis

C. ellipse

D. circle

Answer: B

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16. The principal argument of $\frac{3}{-1+i}$ is

A. $\frac{-5\pi}{6}$

B. $\frac{-2\pi}{3}$

C. $\frac{-3\pi}{4}$

D. $\frac{-\pi}{2}$

Answer: C

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17. The principal argument of $(\sin 40^\circ + i\cos 40^\circ)^5$ is

A. -110°

B. -70°

C. 70°

D. 110°

Answer: A

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18. If (1 + i)(1 + 2i)(1 + 3i)....(1 + ni) = x + iy, then 2.5.10... $(1+n^2)$ is

A. 1

B. i

C. $x^2 + y^2$

D. $1 + n^2$

Answer: C

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19. If $\omega \neq$ 1 is a cubic root of unit and $(1+\omega)^7$ = A + Bω, then (A, B) equals

A. $(1, 0)$

B. $(-1, 1)$

C. $(0, 1)$

D. $(1, 1)$

Answer: D

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20. The principal argument of the complex number $\frac{\left(1+i\sqrt{3}\right)^2}{4i\left(1-i\sqrt{3}\right)}$ is

A. $\frac{2\pi}{3}$

B. $(\pi)(6)$

C. $\frac{5\pi}{6}$

D. $\frac{\pi}{2}$

Answer: D

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21. If α and β are the roots of x^2 + x + 1 = 0, then $\alpha^{2020} + \beta^{2020}$ is

A. -2

B. -1

C. 1

D. 2

Answer: B

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22. The product of all four values of $\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)^{\frac{3}{4}}$ is

A. -2

B. -1

C. 1

D. 2

Answer: C

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23. If $\omega \neq 1$ is a cubit root unity and $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} =$ 3 k, then k is equal to

A. 1

B. -1

C. $\sqrt{3}i$

D. $-\sqrt{3}i$

Answer: D

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24. The value of $\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{10}$ is

A. $cis\frac{2\pi}{3}$

B. $cis\frac{4\pi}{3}$

C. $-cis\frac{2\pi}{3}$

D. $-cis\frac{4\pi}{3}$

Answer: A

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25. If $\omega = cis\frac{2\pi}{3}$, then number of distinct roots of $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0.$

A. 1

B. 2

C. 3

D. 4

Answer: A

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Problem For Practice

1. If $\frac{1-i}{1+i}$ is a root of the equation

$ax^2+bx+1=0.$

Where a, b are real than (a, b) is :

A. $(1, 1)$

B. $(1, \ -1)$

C. $(0, 1)$

D. $(1, 0)$

Answer: D

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2. If is a cubeth root of unity root of :

$\left(1-\omega+\omega^2\right)^4+\left(1+\omega-\omega^2\right)^4$ is :

A. 0

B. 32

C. -16

D. -32

Answer: C

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3. The arguments of the roots of a complex number differ by :

A. $\frac{2\pi}{n}$

B. $\frac{\pi}{n}$

C. $\frac{3\pi}{n}$

D. $\frac{4\pi}{n}$

Answer: A

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4. If $z_1 = 4 + 5i$ and $z_2 = -3 + 2i$ that $\frac{z_1}{z_2}$ is :

A. $\frac{2}{13} - \frac{22}{13}i$

B. $\frac{-2}{13} + \frac{22}{13}i$

C. $\frac{-2}{13} - \frac{23}{13}i$

D. $\frac{2}{13} + \frac{22}{13}i$

Answer: C

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5. The number of values of $(\cos\theta + i\sin\theta)^{p/q}$ where p and q are non zero integers prime to each other is :

A. p

B. q

C. $p+q$

D. $p-q$

Answer: B

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6. If $-i+3$ is a root of $x^2-6x+k=0$. Then the value of k is :

A. 5

B. $\sqrt{5}$

C. $\sqrt{10}$

D. 10

Answer: D

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7. If $x^2 + y^2 = 1$, then the value of $\frac{1 + x + iy}{1 + x - iy}$.

A. $2x$

B. $x - iy$

C. $-2iy$

D. $x + iy$

Answer: D

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8. The value of $\left(\frac{-1+i\sqrt{3}}{2}\right)^{100} + \left(\frac{-1-i\sqrt{3}}{2}\right)^{100}$ is :

A. 2

B. 0

C. -1

D. 1

Answer: C

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9. The principal value of $arg(z)$ lies in the interval :

A. $\left[0, \frac{\pi}{2}\right]$

B. $[-\pi, \pi]$

C. $[0, \pi]$

D. $[-\pi, 0]$

Answer: B

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10. If $-\bar{z}$ lies in the third quadrant then z lies in the :

A. first quadrant

B. second quadrant

C. third quadrant

D. fourth quadrant

Answer: D

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11. If P is $\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)$ and OP is rotated through an angle $\frac{\pi}{2}$ in the clockwise direction then the new position of P is :

A. $\frac{1}{2}i - i\frac{\sqrt{3}}{2}$

B. $\frac{-\sqrt{3}}{2} - \frac{1}{2}$

C. $-\frac{1}{2}i - i\frac{\sqrt{3}}{2}$

D. $\frac{\sqrt{3}}{2} - \frac{i}{2}$

Answer: D

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12. If the amplitude of a complex number is $\frac{\pi}{2}$ then the number is :

A. purely imaginery

B. purely real

C. 0

D. neither real nor imaginary

Answer: A

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13. If $A+iB=(a_1+ib_1)(a_2+ib_2)(a_3+ib_3)$ then A^2+B^2 is :

A. $a_1^2+a_2^2+a_3^2+b_1^2+b_2^2+b_3^2$

B. $(a_1+a_2+a_3)^2+(b_1+b_2+b_3)^2$

C. $(a_1^2+b_1^2)(a_2^2+b_2^2)(a_3^2+b_3^2)$

D. $(a_1^2+a_2^2+a_3^2)+(b_1^2+b_2^2+b_3^2)$

Answer: C

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14. The points $z_1 z_2 z_3 z_4$ in the complex plane are the vertices of a parallelgram taken in order if and only if :

A. $z_1 + z_4 = z_2 + z_3$

B. $z_1 + z_3 = z_2 + z_4$

C. $z_1 + z_2 = z_3 + z_4$

D. $z_1 - z_2 = z_3 - z_4$

Answer: B

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15. If ω is the cube root of unity, then then value of $(1-\omega)\left(1-\omega^2\right)\left(1-\omega^4\right)\left(1-\omega^8\right)$ is

A. 9

B. -9

C. 16

D. 32

Answer: A

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16. Find the correct statement : The cube roots of unity are :

A. in G.P. with common ratio ω

B. in G.P with common ratio ω^2

C. in A.P with common differnce ω

D. in A.P with common difference ω^2

Answer: A

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17. Which of the following is incorrect.?

A. $Re(z) \leq |z|$

B. $Im(z) \leq |z|$

C. $z\bar{z} = |z|^2$

D. $Re(z) \geq z$

Answer: A

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18. Which of the following is incorrect ?

A. $|z_1 + z_2| \leq |z_1| + |z_2|$

B. $|z_1 - z_2| \leq |z_1| + |z_2|$

C. $|z_1 - z_2| \geq ||z_1| + |z_2 ||$

D. $|z_1 + z_2| \geq |z_1| + |z_2|$

Answer: D

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19. Which of the following is incorrect ?

A. $\bar{z}$ is the mirror image of z on the real axis

B. polar form of $\bar{z}$ is (r, θ)

C. $-z$ is a point, symmetrical to z about the origin

D. the polar form of $-z$ is $(-r, -\theta)$

Answer: D

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20. Which of the following is correct statement ?

A. $\overline{z_1 + z_2} = \bar{z}_1 - \bar{z}_2$

B. $|z_1 + z_2| = |z_1| + |z_2|$

C. $|z_1 z_2| = |z_1||z_2|$

D. Multiplying by i means rotation through angle $\frac{\pi}{2}$ in the clockwise direction

Answer: C

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21. If $z^n = \cos\frac{n\pi}{3} + i\sin\frac{n\pi}{3}$, then $z_1, z_2 \dots.. Z_6$ is

A. 1

B. -1

C. i

D. $-i$

Answer: B

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22. Which of the following is incorrect regarding n^{th} roots of unity ?

A. There are n distinct roots

B. The roots are in GP with common ratio $\frac{2\pi}{n}$

C. The argument are in AP with common difference $\frac{2\pi}{n}$

D. Product of the roots $= 0$

Answer: D

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23. Find the correct staements in the following? If P represents the complex number z and if $|2z-1| = 2|z|$ then the locus of z.

A. the straight line $4x - 1 = 0$

B. the straight line $4y - 1 = 0$

C. the straight line $x - y = 0$

D. the circle $x^2 + y^2 - 4x - 1 = 0$

Answer: A

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24. If $a = 3 + i$ and $z = 2 - 3i$ then the points az, $3az$ and $-az$ are:

A. vertices of a right angled triangle

B. vertices of an equilateral triangle

C. vertices of an isosceles triangle

D. collinear

Answer: D

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25. If $|z - z_1| = |z - z_2|$ then the locus of z is :

A. a circle with centre at origin

B. a circle with centre at z_1

C. a straight line passing through origin

D. the perpendicular bisector of the line joining z_1 and z_2

Answer: D

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26. Find the odd one out .

A. $\frac{1}{2}+i\frac{\sqrt{3}}{2}$

B. $\frac{-1}{2}-i\frac{\sqrt{3}}{2}$

C. $\frac{\sqrt{3}}{2}+\frac{i}{2}$

D. $\frac{1}{2}+\frac{i}{2}$

Answer: D

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27. $\frac{1+e^{-i\theta}}{1+e^{i\theta}}$ =

A. $\cos\theta + i\sin\theta$

B. $\cos\theta - i\sin\theta$

C. $\sin\theta - i\cos\theta$

D. $\sin\theta + i\cos\theta$

Answer: B

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28. Match the following Column I to Column II

28.	z is a complex no. the arg z + arg $\bar{z}$ is:	(a) 2
29.	$\frac{i^{22} - i^{24}}{i^{23} - i^{25}}$	(b) 1
30.	If $x = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4}$ then $x^8 + \frac{1}{x^8}$ is	(c) $-i$
31.	$\arg(z-1) = \frac{\pi}{6}$ $\arg(z+1) = 2\frac{\pi}{3}$ then $(z) =$	(d) $\frac{1}{2}$
32.	$\left\lvert \frac{z}{2\bar{z}} \right\rvert$ is	(e) 0

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29. choose the correct pair of statement :

$(i)(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta, x$ is an integer

$(ii)(\sin\theta + i\cos\theta)^n = \sin n\theta + i\cos\theta n\theta,$ n is an integer

(iii) If $z = \cos\theta + i\sin\theta$, then $z^{-1} = \cos\theta - i\sin\theta$

$(iv)\dfrac{z+\bar{z}}{2} = Re(z)$

A. (i) and (iv)

B. (i) and (iii)

C. (i) and (ii)

D. (i) and (iii)

Answer: B

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30. Find the incorrect pair of statements:

$(i) Re(z) \geq |z|$

$(ii) |z^n|$ is not equal to $|z|^n$

$(iii) |z_1 + z_2| \leq |z_1| + |z_2|$

$(iv) z.\bar{z} = |z|^2$

A. (i) and (iii)

B. (i) and (ii)

C. (iii) and (iv)

D. (i) and (iv)

Answer: B

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31. Find the odd one out .

A. $|1+i|+|1+2i|$

B. $(i)^{24}$

C. $(1+i)^2-(1-i)^2$

D. $\left|\frac{2+i}{3+i}\right|$

Answer: C

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32. If ω is the cube root of unity then

Assertion : $\left(1-\omega+\omega^2\right)^6+\left(1+\omega-\omega^2\right)^6=128$

Reason : $1+\omega+\omega^2=0$

A. R is the only clue to prove (A)

B. We can prove by expansion of each term in the LHS

C. (A) is wrong (R) is correct.

D. $\omega^3 = 1$ is also needed to prove (A)

Answer: D

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33. Simplify $\left[\dfrac{i^4 + i^9 + i^{16}}{3 - 2i^5 - i^{10} - i^{15}}\right]^{10}$

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34. Find x and y if $\sqrt{x^2 + 3x + 8} + (h + 4)i = y(2 + i)$

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35. P represents the variable complex number z find the locus of z if :

$$IM\left(\frac{2z+1}{iz+1}\right) = -2$$

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36. Represents the variable complex number z. Find the locus of P if $arg\left(\frac{z-1}{z+3}\right) = \frac{\pi}{2}$.

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37. P represents the variable complex number z find the locus of z if :

$$Re\left(\frac{z+1}{z+i}\right)=1$$

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38. Simplify : $\frac{(\cos 3\theta+i\sin 3\theta)^2(\cos 3\theta-i\sin 3\theta)^{-3}}{(\cos 4\theta+i\sin 4\theta)^{-4}(\cos\theta+i\sin\theta)^2}$

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39. If α and β are the roots of $x^2-2x+2=0$ and $\cot\theta=y+1, show\frac{(y+\alpha)^n-(y+\beta)^n}{\alpha-\beta}=\frac{\sin n\theta}{\sin^n\theta}$

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40. If α and β are the roots of $x^2 - 2x + 2 = 0$ the prove that $\alpha^n - \beta^n = i2^{n+1}\sin\left(\frac{n\pi}{3}\right)$ .Here find $\alpha^9 - \beta^9$.

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41. Solve $x^9 - x^5 + x^4 - 1 = 0$

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42. show that

$$\left(\frac{-1 + i\sqrt{3}}{2}\right)^5 + \left(\frac{-1 - i\sqrt{3}}{2}\right)^5 = -1$$

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43. Find all values of $\left(\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)^{\frac{3}{4}}$ and hence find the product of all the values .

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44. If $x=\cos\theta+i\sin\theta$ Prove that $\frac{x^{2n}-1}{x^{2n}+1}=i\tan\theta$

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45. Prove that $\left(\frac{1+\sin.\frac{\pi}{8}+i\cos.\frac{\pi}{8}}{1+\sin.\frac{\pi}{8}-i\cos.\frac{\pi}{8}}\right)^{8}=-1$

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