MATHS

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THEORY OF EQUATIONS

Example

1. If α and β are the roots of $3x^2 + 7x - 5 = 0$ form the equations whose roots are $\alpha - 1$ and $\beta - 1$.

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2. If α and β are the roots of $2x^2 - 3x + 7 = 0$, form the equations where roots are $\alpha^2 + 2$ and $\beta^2 + 2$.

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3. If α, β, γ are the roots of the equations $x^3 + px^2 + qx + r = 0$ find the value of $\sum \frac{1}{\alpha}$

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4. If α, β, γ are the roots of the equations $x^3 + px^2 + qx + r = 0$ find the value of $\sum \frac{1}{\alpha\beta}$

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5. Find the sum of the squares of the roots of $3x^3 - 2x^2 + 4x + 1 = 0$

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6. Solve $x^3 + 6x^2 + 11x + 6 = 0$ given that the roots are in the ratio 1:2:3.

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7. Form the equations whose roots are reciprocals of the roots of cubic equation.

$x^3 + ax^2 + bx + c = 0$

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8. Find the value of a if the equation $2x^2 - (a+1)x + (a-1) = 0$ has equal roots.

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9. Find the monic polynomial equation of minimum degree with real coefficients given that $\sqrt{3} + i$ is a root.

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10. Find a polynomial equation of minimum degree with rational coefficients , having $3-\sqrt{5}$ as a root.

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11. Form a polynomial equation with integer coefficients with $\sqrt{\frac{\sqrt{3}}{\sqrt{5}}}$ is a root.

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12. Show that the equation $3x^2-x+7=0$ can not be satisfied by any real values of x.

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13. If $x^2 + (k+2)x + (k+26) = 0$ has equal roots , find k.

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14. If a and b are rational show that the equation $x^2 - 2ax + \left(a^2 - b^2 - 6ab - 9\right) = 0$ are rational.

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15. Solve : $x^4 - 3x^2 - 4 = 0$

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16. Solve : $x^3 - 8x^2 + 19x - 12 = 0$

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17. Solve : $3x^3 + 2x^2 - 4x - 3.$

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18. Solve : $4x^3 + 3x^2 - 4x - 3 = 0$

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19. Find the condition that the roots of the equation $ax^3 + bx^2 + cx + d = 0$ are in A.P.

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20. Find the condition that the roots of $x^3 + px^2 + qx + r = 0$ are in G.P.

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21. Find the condition that the roots of $ax^3 + bx^2 + cx + d = 0$ are in H.P.

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22. Solve if the roots of $x^3 - 12x^2 + 39x - 28 = 0$ are in AP.

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23. Solve : $(x-5)(x-3)(x+2)(x+4) + 45 = 0$

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24. Solve : (5x+2)(5x-3)(2x-1)(2x+1)=2.

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25. Solve : $x^3 - 5x^2 - 9x + 45 = 0$

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26. Find the sum of the squares of the roots of $3x^3 - 2x^2 + 4x + 1 = 0$

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27. Solve: $x^3 - 5x^2 + 5x - 1 = 0$

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28. Solve : $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$

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29. $2\sin^2 x - 7\sin x + 3 = 0$

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30. Show that $x^5 - 2x^4 - x + 2$ has atleast 2 imaginary roots.

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31. Discuss the nature of the roots of the following roots of the polynomials.

$x^{18} + 3x^{14} + 70x^6 + 25x^2 + 70$

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32. Discuss the nature of the roots of the following roots of the polynomials.

$x^5 - 19x^4 + 2x^3 + 5x^2 + 11$

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Exercise 3 1

1. If the sides of a cunbic box are increased by 1, 2, 3 units respectively to form a cuboid, then the volume is increased by 52 cubic units. Find the volume of the cuboid.

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2. Equation with roots 1, 2 and 3

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3. Equation with roots 1, 1 and -2

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4. Construct a cubic equation with roots 2,-2 and 4

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5. If α, β and γ are the roots of the cubic equation $x^3 + 2x^2 + 3x + 4 = 0,$ for a cubic equation roots are

$2\alpha, 2\beta, 2\gamma$

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6. If α, β and γ are the roots of the cubic equation $x^3 + 2x^2 + 3x + 4 = 0$, for a cubic equation roots are

$$\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$$

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7. If α, β and γ are the roots of the cubic equation $x^3 + 2x^2 + 3x + 4 = 0$, for a cubic equation roots are

$$-\alpha, -\beta, -\gamma$$

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8. Solve the equation $3x^3 - 16x^2 + 23x - 6 = 0$ if the product of two roots is 1.

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9. Find the sum of squares of roots of the equation $2x^4 - 8x^3 + 6x^2 - 3 = 0$

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10. Solve the equation $x^3 - 9x^2 + 14x + 24 = 0$ if it is given that two of its roots are in the ratio 3:2.

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11. If α, β and γ are the roots of the polynomlal equation $ax^3 + bx^2 + cx + d = 0$, find the value of $\Sigma \frac{a}{\beta\gamma}$ in terms of the coefficients.

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12. If α, β, γ and δ are the roots of the polynomial eqauation $2x^4 + 5x^3 - 7x^2 - 8 = 0$, find a quadratic equation with integer corddicients whose roots are $\alpha + \beta + \gamma + \delta$ and $\alpha\beta\gamma\delta$.

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13. If p and q are the roots of the equation $lx^2 + nx + n = 0$, show that $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0.$

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14. If the equation $x^2 + px + q = 0$ and $x^2 + p'x + q' = 0$ have common roots, show that it must be equal to $\frac{pq' - p'q}{q - q'}$ or $\frac{q - q'}{p' - p}$.

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15. Formulate into a mathematical problem to find a number such that when its cube root is added to it, the result is 6.

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16. A 12 metre tell tree was broken into Two it was found that the height of the part which was left standing was the cube root of the length of the part that was cut away. Formulate this into a mathematical problem to find the height of the part which was cut away.

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Exercise 3 2

1. If k is real , discuss the nature of the roots of the polynomial equation $2x^2 + kx + k = 0$, in terms of k.

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2. Find a polynomial equation of minimum degree with rational coefficients , having $2 + \sqrt{3}$ as a root.

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3. Find a polynomial equation of minimum degree with rational coefficients, having 2i+3 as a root.

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4. Find a polynomial equation of minimum degree with rational coefficients, having $\sqrt{5}-\sqrt{3}$ as a root.

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5. Provet that a stralght line and parabola connot intersect at more than two points.

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Exercise 3 3

1. Solve the cubic equation : $2x^3 - x^2 - 18x + 9 = 0$ if sum of two of its roots vanishes.

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2. Solve the equation $9x^3 - 36x^2 + 44x - 16 = 0$ if the roots form an arithmetic progression.

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3. Solve the equation $3x^3 - 26x^2 + 52x - 24 = 0$ if its roots form a geometric progression.

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4. Determine k and solve the equation $2x^3 - 6x^2 + 3x + k = 0$ if one of its roots is twice the sum of the other two roots.

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5. Find all zeros of the polynomial $x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135$, if it is

known that 1 + 2i and $\sqrt{3}$ are two of its zeros.

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6. Solve the cubic equation : $2x^3 - 9x^2 + 10x = 3$

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7. Solve the cubic equation : $8x^3 - 2x^2 - 7x + 3 = 0$

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8. Solve the equation : $x^4 - 14x^2 + 45 = 0$

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Exercise 3 4

1. Solve : $(x-5)(x-7)(x+6)(x+4)=504$

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2. Solve : $(x-4)(x-7)(x-2)(x+1)=16$

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3. Solve : $(2x-1)(x+3)(x-2)(2x+3)+20=0$

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Exercise 3 5

1. Solve the equation

$$\sin^2 x - 5\sin x + 4 = 0$$

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2. Solve the equation

$$12x^3 + 8x = 29x^2 - 4$$

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3. Examine for the rational roots of

$2x^3 - x^2 - 1 = 0$

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4. Examine for the rational roots of

$x^8 - 3x + 1 = 0$

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5. Solve : $8x^{\frac{3}{2n}} - 8x^{\frac{-3}{2n}} = 63$

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6. Sove : $2\sqrt{\frac{x}{a}}+3\sqrt{\frac{a}{x}}=\frac{b}{a}+\frac{6a}{b}$

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7. Solve : $6x^4-35x^3+62x^2-35x+6=0$

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8. Solve the equations : $x^4+3x^3-3x-1=0$

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9. Find all real numbers satisfying $4^x - 3\left(2^{x+2}\right) + 2^5 = 0$

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10. Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ if it is known that $\frac{1}{3}$ is a solution.

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Exercise 3 6

1. Discuss the maximum possible number of positive the negative roots of the polynomial equation $9x^9 - 4x^8 + 4x^7 - 3x^6 + 2x^5 + x^3 + 7x^2 + 7x + 2 = 0$

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2. Discuss the maximum possibel number of positive the negative zeros of the polynomials $x^2 - 5x + 6$ and $x^2 - 5x + 16$. Also draw rough sketch of the graphs.

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3. Show that the equation $x^9 - 5x^5 + 4x^4 + 2x^2 + 1 = 0$ has at least 6 imaginary solutions.

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4. Determine the number of positive and negative roots of the equation $x^9 - 5x^8 - 14x^7 = 0$.

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5. Find the exact number of real zeros and imaginary of the polynomial $x^9 + 9x^7 + 7x^5 + 5x^3 + 3x$.

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Exercise 3 7

1. A zero of $x^3 + 64$is

A. 0

B. 4

C. 4i

D. -4

Answer: D

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2. If find g are polynomials of derrees m and n respectively, and if h(x) = $(f \circ g)(x)$, then the degree of h is

A. mn

B. m+n

C. m^n

D. n^m

Answer: A

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3. A polynomial equation in x of degree n always has :

A. n distinct roots

B. n real roots

C. n imaginary roots

D. at most one root.

Answer: A

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4. If α, β, γ are the roots of the equations $x^3 + px^2 + qx + r = 0$ find the value of $\sum \frac{1}{\alpha}$

A. $-\frac{q}{r}$

B. $-\frac{p}{r}$

C. $\frac{q}{r}$

D. $-\frac{q}{p}$

Answer: A

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5. According to the rational root equation which number is not possible rational root of $4x^7 + 2x^3 - 10x^2 - 3$:

A. -1

B. $\frac{5}{4}$

C. $\frac{4}{5}$

D. 5

Answer: C

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6. The polynomial $x^3 - kx^2 + 9x$ has three real zeros if and only if, k satifies

A. $|k| \leq 6$

B. $k = 0$

C. $|k| > 6$

D. $|k| \geq 6$

Answer: D

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7. The number of real numbers in $[0, 2\pi]$ satisfying $\sin^4 x - 2\sin^2 x + 1$ is .....

A. 2

B. 4

C. 1

D. ∞

Answer: A

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8. If $x^3 + 12x^2 + 10ax + 1999$ definitely has positive zero , if and only if .....

A. $a \geq 0$

B. $a > 0$

C. $a < 0$

D. $a \leq 0$

Answer: C

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9. The polynomial $x^3 + 2x + 3$ has :

A. one negative and two real roots

B. one positive and two imaginary roots

C. three real roots

D. no solution

Answer: A

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10. The number of positive zeros of polynomial $\sum_{r=0}^{n} .^{n} C_r(-1)^r x^r$ is .....

A. 0

B. n

C. $< n$

D. r

Answer: B

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Problems For Practice Choose The Correct Answer

1. The equation whose roots are $1+\sqrt{2}i$ and $1-\sqrt{2}i$ is :

A. $x^2-2x-3=0$

B. $x^2-2x+3=0$

C. $x^2+2x+3=0$

D. $x^2+2x-3=0$

Answer: B

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2. Find the maximum possible number of real roots of the equation. $x^5 - 6x^2 - 4x + 5 = 0$.

A. 5

B. 4

C. 3

D. 2

Answer: B

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3. If the equation $x^2 - ax + a + 2 = 0$ has equal roots then 'a' will be :

A. $2 + \sqrt{12}$

B. $2, \ -2$

C. $2 - \sqrt{12}$

D. $2 + \sqrt{12}, 2 - \sqrt{12}$

Answer: D

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4. The number of solution of $x^2 + |x + 1| = 1$ is :

A. 1

B. No solution

C. 2

D. 3

Answer: C

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5. If α, β, γ are the roots of the equation $x^3 - 3x + 2 = 0$ then the equation whose roots are $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$ is :

A. $2x^3 - 3x^2 + 1 = 0$

B. $x^3 - \frac{1}{3}x + \frac{1}{2} = 0$

C. $2x^3 + 3x^2 - 1 = 0$

D. $x^3 - 2x^3 + 3 = 0$

Answer: A

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6. The sum of square of roots of the equation $2x^4 + x^2 + x + 2 = 0$ is :

A. -1

B. 1

C. $-\frac{1}{2}$

D. -2

Answer: A

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7. If α, β, γ are the roots of the equation $x^3 - 4x^2 + 3x - 1 = 0$, then the equation whose roots are $(\alpha + \beta + \gamma), \alpha\beta\gamma$ is :

A. $x^2 - 4x + 5 = 0$

B. $x^2 - 5x - 4 = 0$

C. $x^2 - 5x + 4 = 0$

D. $x^3 + 5x + 4 = 0$

Answer: C

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8. Form a polynomial equation with integer coefficients with $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as a root.

A. $2x^4 + 3 = 0$

B. $3x^4 + 2 = 0$

C. $2x^4 - 3 = 0$

D. $3x^4 - 2 = 0$

Answer: D

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9. A straight line and parabola cannot intersect at more than ___ points.

A. 4

B. 3

C. 2

D. 1

Answer: C

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10. If a,b,c are rational , the roots of equation $x^2 - 2ax + \left(a^2 - b^2 + 2bc - c^2\right) = 0$ are :

A. rational

B. irrational

C. equal

D. imaginary

Answer: A

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11. If $x^2 + ax + 1$ is a factor of $ax^3 + bx + c$, them find the conditions.

A. $c^3 = ab^3$

B. $a^3 = bc^3$

C. $a^3 = b^3c$

D. $b^3 = ca^3$

Answer: D

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12. According to the rational root equation which number is not possible rational root of $4x^7 + 2x^3 - 10x^2 - 3$:

A. 1

B. $\frac{1}{4}$

C. $\frac{3}{4}$

D. $\frac{4}{3}$

Answer: D

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13. $P(x) = x^5 - 4x^3 + 2x^2 + x + 7$ cannot have more than positive real root.

A. -1

B. -3

C. 2

D. ± 10

Answer: C

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14. The value of k for which the equation $x^2 + kx + 25 = 0$ has equal roots is

A. -1

B. -3

C. 2

D. ± 10

Answer: D

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15. If $\alpha\beta\gamma$ are the roots of $x^3 - 3x^2 + 4x + 1 = 0$ then $\sum \frac{1}{\alpha\beta}$ is

A. -1

B. -3

C. 2

D. ± 10

Answer: B

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16. The only real root of $x^3 + 2x^2 + 2x + 1 = 0$ is

A. -1

B. -3

C. 2

D. ± 10

Answer: A

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17. Find the correct statement for the following :

A. If α and β are the roots of $x^2 + 3x + 7 = 0$ then this equation whose roots are $\alpha + 1$ and $\beta + 1$ is $x^2 + x + 5 = 0$

B. The quadratic equation $ax^2 + bx + c = 0$ will have equal roots if its discriminant $b^2 - 4ac$ is negative

C. A zero of $x^3 + 8 = 0$ is -8

D. The number of real roots is $(0, 2\pi)$ satisfying $\sin^2 x - 3\sin x + 2 = 0$ is 4.

Answer: A

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18. Find the incorrect statement for the following :

A. A straight line will cut a circle at the most 2 points in the xy plane .

B. For an odd degree reciprocal equation of Type I, x=-1 must be solution

C. $3x^2 - 10x + 3 = 0$ is a reciprocal equation.

D. 3 is a root of the equation $x^3 + 2x^2 - x + 11 = 0$

Answer: D

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19. Identify the correct pair from the following statements:

(i)For an odd degree reciprocal equation of Type II x=1 is a solution .

(ii)For an even degree reciprocal equation of Type II, the middle term must be zero.

(iii) The no. of positive roots of a polynomial p(x) cannot be less than the no. of the sign changes in coefficients of P(x).

(iv)Polynomial of degree 4 is called cubic equation.

A. (i)&(ii)

B. (i)&(iii)

C. (ii)&(iv)

D. (iii)&(iv)

Answer: A

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20. Find the odd one out :

If $\sqrt{p}+\sqrt{q}$ is a root of a polynomial equation with rational coefficient then

A. $\sqrt{p}-\sqrt{q}$

B. $-\sqrt{p}+\sqrt{q}$

C. $-\sqrt{p}-\sqrt{q}$

D. $\sqrt{p+q}$ are also roots of the same equation.

Answer: D

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21. Assertion: The polynomial $5x^9 + 3x^5 - x^4 - 2x^2 + 5$ has atleast six imaginary roots.

Reason: Descretes rule of sign.

A. (R) is one of the reason of prove (A)

B. Find maximum no. of positive roots , then find maximum no. of negative roots. Using (R) and hence (A) can be proved .

C. (A) is wrong

D. (R) cannot give current discussion to solve (A)

Answer: B

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22. Find the odd one out : Given $x^3 - 5x^2 + 6x = 0$

A. 0

B. 1

C. 2

D. 3

Answer: B

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23. If $x^2 + 2(k+2)x + 9k = 0$ has equal roots then k is :

A. (4) or (1)

B. (3) or (2)

C. (-4) or (1)

D. (-1) or (-3)

Answer: A

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Problems For Practice Answer The Following

1. If α and β are the roots of $2x^2 - 5x + 3 = 0$ form the equation whose roots are α^2 and β^2

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2. If α and β are the roots of $2x^2 - 5x + 3 = 0$ form the equation whose roots are $\alpha + \beta$ and $\frac{1}{\alpha} + \frac{1}{\beta}$

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3. If α, β, γ are the roots of $x^3 - 2x^2 + 3x - 4 = 0$ find the value of $\sum \frac{1}{\beta\gamma}$

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4. If α, β, γ are the roots of $x^3 - 2x^2 + 3x - 4 = 0$ find the value of $\sum \alpha^2$

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5. Solve the equation $3x^3 - 7x^2 - 7x + 3 = 0$ given that product of two of its roots is equal to 1.

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6. Solve : $x^3 - 15x^2 + 66x + 80 = 0$ given that the roots as in AP.

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7. Solve $x^3 + 7x^2 + 14x + 8 = 0$ given that the roots are in GP.

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8. Solve : $3x^3 - x^2 + x + 1 = 0$ given the roots are in HP.

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9. Find the polynomial equation with integer coefficients with $\sqrt{\frac{\sqrt{3}}{\sqrt{4}}}$ as a root.

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10. Solve : $2x^3 - 3x^2 - 14x + 5 = 0$ given that 2-3i is a root

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11. Solve : $x^3 - x^2 - 4x - 2 = 0$ given that $1 + \sqrt{3}$ is a root .

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12. Solve : $x^6 - 6x^5 + 10x^4 - 9x^2 + 6x - 2 = 0$ given that $\left(2 + \sqrt{3}\right)$ and (1+i) are roots.

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13. Solve : $x^6 - 6x^5 + 13x^4 - 12x^3 + 7x^2 - 6x - 5 = 0$ if $\left(1 + \sqrt{2}\right)$ and (2+i) are roots.

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14. Solve $2x^3 - 11x^2 + 10x - 1 = 0$

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15. Solve $x^3 - 3x^2 - 9x - 5$

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16. Solve $x^4 - 4x^2 - 5 = 0$

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17. Solve $6x^6 - 35x^5 + 56x^4 - 56x^2 + 35x - 6 = 0$

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18. Solve $3x^4 - 10x^3 + 10x - 3 = 0$

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19. Show that $3x^9 - 10x^5 + 3x^4 + 2x^2 + 7 = 0$ has at least 6 imaginary roots.

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20. Find the least no. of imaginary roots of $2x^9 + 3x^5 - 5x^4 - 9x^2 + 2$

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21. (x+1)(x-3)(x+2)(x-4)+6=0

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