MATHS

BOOKS - CENGAGE MATHS (ENGLISH)

COMPLEX NUMBERS AND QUADRATIC EQUATIONS

Others

1. Show that the equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has no real solution.

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2. Solve for $x : 4^x - 3^{x-1/2} = 3^{x+1/2} - 2^{2x-1}$.

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3. Solve for x: $\sqrt{x+1} - \sqrt{x-1} = 1$.

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4. If $x, y \in R and 2x^2 + 6xy + 5y^2 = 1, then$ a. $|x| \leq \sqrt{5}$ b. $|x| \geq \sqrt{5}$ c. $y^2 \leq 2$ d. $y^2 \leq 4$

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5. If the roots $x^5 - 40x^4 + Px^3 + Qx^2 + Rx + S = 0$ are n G.P. and the sum of their reciprocals is 10, then $|S|$ is 4 b. 6 c. 8 d. none of these

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6. Show that for any triangle with sides $a, b, and c 3(ab + bc + ca) < (a + b + c)^2 < 4(bc + ca + ab)$. When are the first two expressions equal ?

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7. For what values of m, does the system of equations 3x+my=m and 2x-5y=20 has a solution satisfying the conditions x > 0, y > 0 ?

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8. Show that the square to $\left(\sqrt{26-15\sqrt{3}}\right)/\left(5\sqrt{2}-\sqrt{38+5\sqrt{3}}\right)$ is a rational number.

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9. If α, β are the roots of $x^2+px+q=0 and y, \delta$ are the roots of $x^2+rx+s=0$, evaluate $(\alpha-\gamma)(\alpha-\delta)(\beta-\gamma)(\beta-\delta)$ in terms of $p, q, r, ands$. Deduce the condition that the equation has a common root.

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10. Let $f(x) = x^2 + bx + c, where b, c \in R$ If $f(x)$ is a factor of both $x^4 + 6x^2 + 25$ and $3x^4 + 4x^2 + 28x + 5$, then the least value of $f(x)$ is: (a.) 2 (b.) 3 (c.) 5/2 (d.) 4

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11. If the equation $ax^2 + bx + c = x$ has no real roots, then the equation $a\left(ax^2 + bx + c\right)^2 + b\left(ax^2 + bx + c\right) + c = x$ will have a. four real roots b. no real root c. at least two least roots d. none of these

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12. The value of expression $x^4 - 8x^3 + 18x^2 - 8x + 2$ when $x = 2 + \sqrt{3}$ a. 2 b. 1 c. 0 d. 3

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13. The exhaustive set of values of a for which inequation $(a-1)x^2-(a+1)x+a-1\geq 0$ is true $\forall x>2$ $(a)(-\infty,1)$ $(b)\left[\frac{7}{3},\infty\right)$ $(c)\left[\frac{3}{7},\infty\right)$ (d) none of these

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14. If p, q, r, s are rational numbers and the roots of $f(x)=0$ are eccentricities of a parabola and a rectangular hyperbola, where $f(x)=px^3+qx^2+rx+s$,then $p+q+r+s=$ a. p b. $-p$ c. $2p$ d. 0

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15. If $\left|z-\left(\frac{1}{z}\right)\right|=1$, then a. $(|z|)_{max}=\frac{1+\sqrt{5}}{2}$ b. $(|z|)_{m\in}=\frac{\sqrt{5}-1}{2}$ c. $(|z|)_{max}=\frac{\sqrt{5}-2}{2}$ d. $(|z|)_{m\in}=\frac{\sqrt{5}-1}{\sqrt{2}}$

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16. zo is one of the roots of the equation $z^n\cos\theta_0 + z^{n-1}\cos\theta_2 + \ldots\ldots + z\cos\theta_{n-1} + \cos\theta_n = 2$, where $\theta \in R$, then

(A) $|z_0| < \frac{1}{2}$

(B) $|z_0| > \frac{1}{2}$

(C) $|z_0| = \frac{1}{2}$

(D)None of these

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17. If a_0, a_1, a_2, a_3 are all the positive, then $4a_0x^3 + 3a_1x^2 + 2a_2x + a_3 = 0$ has least one root in $(-1, 0)$ if (a) $a_0 + a_2 = a_1 + a_3$ and $4a_0 + 2a_2 > 3a_1 + a_3$ (b) $4a_0 + 2a_2 < 3a_1 + a_3$ (c) $4a_0 + 2a_2 = 3a_1 + a_0$and $4a_0 + a_2 < a_1 + a_3$ (d) none of these

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18. If $1, z_1, z_2, z_3, \ldots\ldots, z_{n-1}$ be the n, nth roots of unity and ω be a non-real complex cube root of unity, then $\prod_{r=1}^{n-1}\left(\omega - z_r\right)$ can be equal to

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19. If $ax^2 + bx + c = 0$ has imaginary roots and $a - b + c > 0$ then the set of points (x, y) satisfying the equation $\left|a\left(x^2 + \frac{y}{a}\right) + (b+1)x + c\right| = \left|ax^2 + bx + c\right| + |x + y|$ consists of the region in the xy - $plane$ which is on or above the bisector of I and III quadrant on or above the bisector of II and IV quadrant on or below the bisector of I and III quadrant on or below the bisector of II and IV quadrant

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20. All the values of $'a'$ for which the quadratic expression $ax^2 + (a-2)x - 2$ is negative for exactly two integral values of x may lie in

(a) $\left[1, \frac{3}{2}\right]$ (b) $\left[\frac{3}{2}, 2\right)$ (c) $[1, 2)$ (d) $[-1, 2)$

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21. If the equation $z^3 + (3 + i)\left(z^2\right) - 3z - (m + i) = 0,\ \ m \in R$, has at least one real root, then sum of possible values of m, is

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22. If $a + b + c = 0, a^2 + b^2 + c^2 = 4, then a^4 + b^4 + c^4$ is_____.

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23. Let P(x) and Q(x) be two polynomials. If $f(x) = P\left(x^4\right) + xQ\left(x^4\right)$ is divisible by $x^2 + 1$, then

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24. Find the solution set of the system $x + 2y + z = 1 \quad 2x - 3y - w = 2$

$x \geq 0, y \geq 0, z \geq 0, w \geq 0$

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25. If $amp\left(z_1 z_2\right) = 0 and \left|z_1\right| = \left|z_2\right| = 1, then \quad z_1 + z_2 = 0$ b. $z_1 z_2 = 1$ c. $z_1 = z_2$ d. none of these

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26. mn squares of equal size are arranged to form a rectangle of dimension m by n, where m and n are natural numbers. Two square will be called neighbors if they have exactly one common side. A number is written in each square such that the number written in any square is the arithmetic mean of the numbers written in its neighboring squares. Show that this is possible only if all the numbers used are equal.

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27. If the points $A(z), B(-z), andC(1-z)$ are the vertices of an equilateral triangle $ABC,$ then (a)sum of possible z is $\frac{1}{2}$ (b)sum of possible z is 1 (c)product of possible z is $\frac{1}{4}$ (d)product of possible z is $\frac{1}{2}$

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28. Form a quadratic equation whose roots are $-4and6$.

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29. If $\left|\frac{z-z_1}{z-z_2}\right| = 3, wherez_1andz_2$ are fixed complex numbers and z is a variable complex number, then z lies on a (a).circle with z_1 as its interior point (b).circle with z_2 as its interior point (c).circle with z_1 as its exterior point (d).circle with z_2 as its exterior point

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30. If a, b, c are odd integere then about that $ax^2 + bx + c = 0$, does not have rational roots

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31. if $arg(z + a) = \frac{\pi}{6}$ and $arg(z - a) = \frac{2\pi}{3}$ then

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32. Values $(s)(-i)^{\frac{1}{3}}$ is/are $\frac{\sqrt{3} - i}{2}$ b. $\frac{\sqrt{3} + i}{2}$ c. $\frac{-\sqrt{3} - i}{2}$ d. $\frac{-\sqrt{3} + i}{2}$

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33. if $\cos\theta$, $\sin\phi$, $\sin\theta$ are in g.p then check the nature of roots of $x^2 + 2\cot\phi . x + 1 = 0$

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34. Given $z = \left(1 + i\sqrt{3}\right)^{100}$, then $[Re(z)/Im(z)]$ equals (a)2^{100} b. 2^{50} c. $\frac{1}{\sqrt{3}}$

d. $\sqrt{3}$

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35. If a ,b ,c are non zero rational no then prove roots of equation $\left(abc^2\right)x^2 + 3a^2cx + b^2cx - 6a^2 - ab + 2b^2 = 0$ are rational.

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36. If $ab + bc + ca = 0$, then solve $a(b - 2c)x^2 + b(c - 2a)x + c(a - 2b) = 0$.

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37. $If(\cos\theta + i\sin\theta)(\cos2\theta + i\sin2\theta)\ldots.(\cos n\theta + i\sin n\theta) = 1$ then the value of θ is:

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38. The polynomial $x^6 + 4x^5 + 3x^4 + 2x^3 + x + 1$ is divisible by______ where ω is one of the imaginary cube roots of unity. (a) $x + \omega$ (b) $x + \omega^2$ (c) $(x + \omega)\left(x + \omega^2\right)$ (d) $(x - \omega)\left(x - \omega^2\right)$

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39. If roots of equation $3x^2 + 5x + 1 = 0$ are $\left(\sec\theta_1 - tan\theta_1\right)$ and $\left(cosec\theta_2 - \cot\theta_2\right)$. Then find the equation whose roots are $\left(\sec\theta_1 + tan\theta_1\right)$ and $\left(cosec\theta_2 + \cot\theta_2\right)$.

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40. If roots of the equation $ax^2 + bx + c = 0$ be a quadratic equation and α,β are its roots then f(−x)=0 is an equation whose roots

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41. Find the principal argument of the complex number

$$\frac{(1+i)^5\left(1+\sqrt{3i}\right)^2}{-2i\left(-\sqrt{3}+i\right)}$$

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42. Form a quadratic equation with real coefficients whose one root is

$3-2i^{\cdot}$

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43. Number of solutions of the equation $z^3+\frac{3(\bar{z})^2}{|z|}=0$ where z is a complex number is

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44. If the roots of the quadratic equation $x^2 + px + q = 0$ are $\tan 30^0 and \tan 15^0$, respectively, then find the value of $2 + q - p$.

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45. If $x and y$ are complex numbers, then the system of equations $(1 + i)x + (1 - i)y = 1, 2ix + 2y = 1 + i$ has Unique solution No solution Infinite number of solutions None of these

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46. If $a, b, and c$ are in A.P. and one root of the equation $ax^2 + bc + c = 0 is 2$, the find the other root.

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47. If $z = x + iy\left(x, y \in R, x \neq -\frac{1}{2}\right)$, the number of values of z satisfying $|z|^n = z^2|z|^{n-2} + z|z|^{n-2} + 1.\ (n \in N, n > 1)$ is

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48. If $K + \left|K + z^2\right| = z\ |\ ^2\left(K \in R^-\right)$, then possible argument of z is

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49. If α is the root (having the least absolute value) of the equation $x^2 - bx - 1 = 0\left(b \in R^+\right)$, then prove that $-1 < \alpha < 0$.

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50. If α, β are roots of $x^2 - 3x + a = 0$, $a \in R$ and $\alpha < 1 < \beta$ then find the value of a.

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51. If z=x+iy and $x^2 + y^2 = 16$, then the range of $||x| - |y||$ is

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52. If $a < b < c < d$, then for any real non-zero λ, the quadratic equation $(x - a)(x - c) + \lambda(x - b)(x - d) = 0$,has real roots for

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53. If $k > 0$, $|z| = |w| = k$, and $\alpha = \frac{z - \bar{w}}{k^2 + z\bar{w}}$, then $Re(\alpha)$ (A) 0 (B) $\frac{k}{2}$ (C) k (D) None of these

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54. The quadratic $x^2 + ax + b + 1 = 0$ has roots which are positive integers, then $\left(a^2 + b^2\right)$ can be equal to a.50 b. 37 c. 61 d. 19

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55. The sum of values of x satisfying the equation $\left(31+8\sqrt{15}\right)^{x^2-3}+1=\left(32+8\sqrt{15}\right)^{x^2-3}$ is (a) 3 (b) 0 (c) 2 (d) none of these

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56. Let a complex number $\alpha, \alpha \neq 1$, be a rootof hte euation $z^{p+q}-z^p-z^q+1=0, wherep, q$ are distinct primes. Show that either $1+\alpha+\alpha^2+\ +\alpha^{p-1}=0$ or $1+\alpha+\alpha^2+\ +\alpha^{q-1}=0$, but not both together.

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57. If α, β are real and distinct roots of $ax^2+bx-c=0andp, q$ are real and distinct roots of $ax^2+bx-|c|=0, where(a>0)$, then (a)$\alpha, \beta \in (p, q)$ (b). $\alpha, \beta \in [p, q]$ (c). $p, q \in (\alpha, \beta)$ (d). none of these

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58. Let $a \neq 0$ and $p(x)$ be a polynomial of degree greater than 2. If $p(x)$ leaves remainders a and -a when divided respectively, by $x + a$ and $x - a$, the remainder when $p(x)$ is divided by $x^2 - a^2$ is (a) $2x$ (b) $-2x$ (c) x (d) $-x$

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59. Prove that there exists no complex number z such that $|z| < \frac{1}{3}$ and $\sum_{n=1}^{n} a_r z^r = 1$, where $|a_r| < 2$.

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60. A quadratic equation with integral coefficients has two different prime numbers as its roots. If the sum of the coefficients of the equation is prime, then the sum of the roots is a. 2 b. 5 c. 7 d. 11

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61. Find the centre and radius of the circle formed by all the points represented by $z = x + iy$ satisfying the relation $\left|\frac{z - \alpha}{z - \beta}\right| = k(k \neq 1)$, where α and β are the constant complex numbers given by $\alpha = \alpha_1 + i\alpha_2, \beta = \beta_1 + i\beta_2$.

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62. If a, b, c are three distinct positive real numbers, the number of real and distinct roots of $ax^2 + 2b|x| - c = 0$ is 0 b. 4 c. 2 d. none of these

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63. Find the non-zero complex number z satisfying $z = iz^2$.

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64. Let a, b and c be real numbers such that $4a + 2b + c = 0$ and $ab > 0$ .Then the equation $ax^2 + bx + c = 0$ has (A) real roots (B) Imaginary roots (C) exactly one root (D) roots of same sign

A. only one root

B. null

C. null

D. null

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65. If $|z| \leq 1, |w| \leq 1$, then show that $|z - w|^2 \leq (|z| - |w|)^2 + (argz - argw)^2$

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66. If α, β are the roots of the equation $x^2 - 2x + 3 = 0$ obtain the equation whose roots are $\alpha^3 - 3\alpha^2 + 5\alpha - 2$ and $\beta^3 - \beta^2 + \beta = 5$

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67. For complex numbers z and w, prove that $|z|^2 w - |w|^2 z = z - w$, if and only if $z = w$ or $z\bar{w} = 1$.

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68. If α, β are the roots of the equation $ax^2 + bx + c = 0$, then the value of $\frac{a\alpha^2 + c}{a\alpha + b} + \frac{a\beta^2 + c}{a\beta + b}$ is a. $\frac{b(b^2 - 2ac)}{4a}$ b. $\frac{b^2 - 4ac}{2a}$ c. $\frac{b(b^2 - 2ac)}{a^2 c}$ d. none of these

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69. Let z_1 and z_2 be the roots of the equation $z^2 + pz + q = 0$, where the coefficients p and q may be complex numbers. Let A and B represent z_1 and z_2 in the complex plane, respectively. If $\angle AOB = \theta \neq 0$ and $OA = OB$, where O is the origin, prove that $p^2 = 4q\cos^2(\theta/2)$.

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70. If $a \in (-1, 1)$, then roots of the quadratic equation $(a-1)x^2 + ax + \sqrt{1-a^2} = 0$ are

A. a. Real

B. b. Imaginary

C. c. both equal

D. d. none of these

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71. The maximum value of $\left|arg\left(\frac{1}{1-z}\right)\right|$ for |z|=1,$z \neq 1$ is given by.

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72. If one root is square of the other root of the equation $x^2 + px + q = 0$, then the relation between $p and q$ is $p^3 - q(3p - 1) + q^2 = 0$ $p^3 - q(3p + 1) + q^2 = 0$ $p^3 + q(3p - 1) + q^2 = 0$ $p^3 + q(3p + 1) + q^2 = 0$

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73. If $z^4 + 1 = \sqrt{3}i$ (A) z^3 is purely real (B) z represents the vertices of a square of side $2^{\frac{1}{4}}$ (C) z^9 is purely imaginary (D) z represents the vertices of a square of side $2^{\frac{3}{4}}$

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74. Let α, β be the roots of the quadratic equation $ax^2 + bx + c = 0$ and $\delta = b^2 - 4a \cdot If\alpha + \beta, \alpha^2 + \beta^2\alpha^3 + \beta^3$ are in G.P. Then a. $= 0$ b. $\neq 0$ c. $b = 0$ d. $c = 0$

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75. If $x = a + bi$ is a complex number such that $x^2 = 3 + 4i$ and $x^3 = 2 + 11i$, where $i = \sqrt{-1}$, then $(a + b)$ equal to

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76. Let α, β be the roots of $x^2 - x + p = 0 and \gamma, \delta$ are roots of $x^2 - 4x + q = 0.$ If $\alpha, \beta, \gamma, \delta$ are in G.P., then the integral value of $pandq$, respectively, are $-2, \ -32$ b. $-2, 3$ c. $-6, 3$ d. $-6, \ -32$

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77. If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then the set of possible values of a is (1) $(-3, 3)$ (2) $(-3, \infty)$ (3) $(3, \infty)$ (4) $(-\infty, -3)$

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78. If $f(x) = x^2 + 2bx + 2c^2$ and $g(x) = -x^2 - 2cx + b^2$ are such that min $f(x) > max g(x)$, then the relation between b and c is

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79. Let z be a complex number such that the imaginary part of z is nonzero and $a = z^2 + z + 1$ is real. Then a cannot take the value

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80. For the equation $3x^2 + px + 3 = 0, p > 0,$ if one of the root is square of the other, then p is equal to $1/3$ b. 1 c. 3 d. $2/3$

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81. Let z, w be complex numbers such that $\bar{z} + i\bar{w} = 0$ and $argzw = \pi$ Then $argz$ equals

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82. Let $f(x) = \left(1 + b^2\right)x^2 + 2bx + 1$ and let $m(b)$ be the minimum value of $f(x)^{\cdot}$ As b varies, the range of $m(b)$ is (a) $[0, \}$ b. $\left(0, \frac{1}{2}\right)$ c. $\frac{1}{2}, 1$ d. $(0, 1]$

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83. For any two complex numbers z_1 and z_2, , prove that Re (z_1 z_2) = Re z_1 Re z_2 – Imz_1 Imz_2

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84. If $\alpha and \beta$ are the roots of the equation $x^2 + bx + c = 0$,where $c < 0 < b$ then (a) 0<α<β (b) $\alpha < 0 < \beta^2 < \alpha^2$ (c) α<β<0 (d) $\alpha < 0 < \alpha^2 < \beta^2$

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85. If $\omega(\neq 1)$ be an imaginary cube root of unity and $\left(1 + \omega^2\right)^n = \left(1 + \omega^4\right)^n$, then the least positive value of n is (a) 2 (b) 3 (c) 5 (d) 6

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86. If $b > a$, then the equation $(x - a)(x - b) - 1 = 0$ has (a) both roots in (a, b) (b) both roots in $(- \infty, a)$ (c) both roots in $(b, + \infty)$ (d)one root in $(- \infty, a)$ and the other in $(b, + \infty)$

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87. Let $z_1 and z_2$ be complex numbers such that $z_1 \neq z_2$ and $|z_1| = |z_2|$. If z_1 has positive real part and z_2 has negative imaginary part, then $\frac{z_1 + z_2}{z_1 - z_2}$ may be (a)zero (b) real and positive (c)real and negative (d) purely imaginary

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88. The equation $\sqrt{x+1} - \sqrt{x-1} = \sqrt{4x-1}$ has a. no solution b. one solution c. two solution d. more than two solutions

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89. If z_1, z_2 are complex number such that $\frac{2z_1}{3z_2}$ is purely imaginary number, then find $\left|\frac{z_1 - z_2}{z_1 + z_2}\right|$.

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90. If the roots of the equation $x^2 - 2ax + a^2 - a - 3 = 0$ are real and less than 3, then (a)$a < 2$ b. $2 < \ - a \leq 3$ c. `34

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91. If $z(1 + a) = b + icanda^2 + b^2 + c^2 = 1,$ then $[(1 + iz)/(1 - iz) =$

A. $\dfrac{a + ib}{1 + c}$

B. $\dfrac{b - ic}{1 + a}$

C. $\dfrac{a + ic}{1 + b}$

D. none of these

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92. A value of b for which the equation $x^2 + bx - 1 = 0, x^2 + x + b = 0$ have one root in common is $-\sqrt{2}$ b. $-i\sqrt{3}$ c. $\sqrt{2}$ d. $\sqrt{3}$

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93. If z_1, z_2, z_3 are three complex, numbers and

$A = \begin{bmatrix} argz_1 & argz_3 & argz_3 \\ argz_2 & argz_2 & argz_1 \\ argz_3 & argz_1 & argz_2 \end{bmatrix}$ Then A divisible by $arg(z_1 + z_2 + z_3)$ b. $arg(z_1, z_2, z_3)$ c. all numbers d. cannot say

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94. Let p and q be real numbers such that $p \neq 0, p^3 \neq q,$ and $p^3 \neq -q$. If α and β are nonzero complex numbers satisfying $\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$, then a quadratic equation having α/β and β/α as its roots is A. $(p^3 + q)x^2 - (p^3 + 2q)x + (p^3 + q) = 0$ B. $(p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$ C. $(p^3 + q)x^2 - (5p^3 - 2q)x + (p^3 - q) = 0$ D. $(p^3 + q)x^2 - (5p^3 + 2q)x + (p^3 + q) = 0$

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95. If $\cos\alpha + 2\cos\beta + 3\cos\gamma = \sin\alpha + 2\sin\beta + 3\sin\gamma = 0$, then the value of $\sin3\alpha + 8\sin3\beta + 27\sin3\gamma$ is sin(a+b+gamma) b. $3\sin(\alpha + \beta + \gamma)$ c. $18\sin(\alpha + \beta + \gamma)$ d. $\sin(\alpha + 2\beta + 3)$

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96. Let α, β be the roots of the equation $x^2 - px + r = 0$ and $\alpha/2, 2\beta$ be the roots of the equation $x^2 - qx + r = 0$, then the value of r is (1) $\frac{2}{9}(p - q)(2q - p)$ (2) $\frac{2}{9}(q - p)(2p - q)$ (3) $\frac{2}{9}(q - 2p)(2q - p)$ (4) $\frac{2}{9}(2p - q)(2q - p)$

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97. If center of a regular hexagon is at the origin and one of the vertices on the Argand diagram is $1 + 2i$, then its perimeter is $2\sqrt{5}$ b. $6\sqrt{2}$ c. $4\sqrt{5}$ d. $6\sqrt{5}$

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98. Let a, b, c be the sides of a triangle, where $a \neq b \neq c$ and $\lambda \in R$. If the roots of the equation $x^2 + 2(a + b + c)x + 3\lambda(ab + bc + ca) = 0$ are real. Then a.$\lambda < \frac{4}{3}$ b.$\lambda > \frac{5}{3}$ c.$\lambda \in \left(\frac{1}{3}, \frac{5}{3}\right)$ d.$\lambda \in \left(\frac{4}{3}, \frac{5}{3}\right)$

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99. Let z be a complex number satisfying equation $z^p - z^{-q} = 0$, *where* $p, q \in N$, *then* (A) if $p = q$, then number of solutions of equation will be infinite. (B) if $p = q$, then number of solutions of equation will be finite. (C) if $p \neq q$, then number of solutions of equation will be $p + q + 1$. (D) if $p \neq q$, then number of solutions of equation will be $p + q^{\cdot}$

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100. Let S be the set of all non-zero real numbers such that the quadratic equation $\alpha x^2 - x + \alpha = 0$ has two distinct real roots $x_1 and x_2$ satisfying the inequality $\left|x_1 - x_2\right| < 1$. Which of the following intervals is (are) a subset (s) of S? $\left(\frac{1}{2}, \frac{1}{\sqrt{5}}\right)$ b. $\left(\frac{1}{\sqrt{5}}, 0\right)$ c. $\left(0, \frac{1}{\sqrt{5}}\right)$ d. $\left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$

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101. A complex number z is rotated in anticlockwise direction by an angle α and we get z' and if the same complex number z is rotated by an angle α in clockwise direction and we get z'' then

A. z',z,z" are in G.P

B. z'2+z"2=2z2cos2alpha

C. z'+z"=2zcosalpha

D. z',z,z" are in H.P

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102. For real x, the function $\frac{(x - a)(x - b)}{x - c}$ will assume all real values provided a)$a > b > c$ b)$a < b < c$ c) $a > c < b$ d) $a \le c \le b$

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103. If z_1, z_2 are two complex numbers $(z_1 \ne z_2)$ satisfying $\left|z_1^2 - z_2^2\right| = \left|z_1^2 + z_2^2 - 2(z_1)(z_2)\right|$, then a.$\frac{z_1}{z_2}$ is purely imaginary b. $\frac{z_1}{z_2}$ is purely real c. $\left|argz_1 - argz_2\right| = \pi$ d. $\left|argz_1 - argz_2\right| = \frac{\pi}{2}$

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104. The quadratic equation $p(x) = 0$ with real coefficients has purely imaginary roots. Then the equation $p(p(x)) = 0$ has A. only purely imaginary roots B. all real roots C. two real and purely imaginary roots D. neither real nor purely imaginary roots

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105. If from a point P representing the complex number z_1 on the curve $|z| = 2$, two tangents are drawn from P to the curve $|z| = 1$, meeting at points $Q(z_2)$ and $R(z_3)$, then :

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106. Let α and β be the roots $x^2 - 6x - 2 = 0$, with $\alpha > \beta$ If $a_n = \alpha^n - \beta^n$ for or $n \geq 1$ then the value of $\dfrac{a_{10} - 2a_8}{2a_9}$ is (a) 1 (b) 2 (c) 3 (d) 4

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107. If $|z - 3| = \min\{|z - 1|, |z - 5|\}$, then $Re(z)$ equals to

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108. For the following question, choose the correct answer from the codes (a), (b), (c) and (d) defined as follows: Statement I is true, Statement

II is also true; Statement II is the correct explanation of Statement I. Statement I is true, Statement II is also true; Statement II is not the correct explanation of Statement I. Statement I is true; Statement II is false Statement I is false; Statement II is true. Let a, b, c, p, q be the real numbers. Suppose α, β are the roots of the equation $x^2 + 2px + q = 0$ and $\alpha, \frac{1}{\beta}$ are the roots of the equation $ax^2 + 2bx + c = 0$, where $\beta^2 \notin \{-1, 0, 1\}$. Statement I $(p^2 - q)(b^2 - ac) \geq 0$ and Statement II $b \notin pa$ or $c \notin qa$.

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109. Minimum value of| z1- z2| as z1& z2 over the curves √3

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110. All the values of m for whilch both the roots of the equation $x^2 - 2mx + m^2 - 1 = 0$ are greater than -2 but less than 4 lie in the interval

A `-2 B. $m > 3$

C. `-1 D. $1 < m < 4$

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111. If $p = a + b\omega + c\omega^2$, $q = b + c\omega + a\omega^2$, and $r = c + a\omega + b\omega^2$, where $a, b, c \neq 0$ and ω is the complex cube root of unity, then (a) $p + q + r = a + b + c$ (b) $p^2 + z^2 + r^2 = a^2 + b^2 + c^2$ (c) $p^2 + z^2 + r^2 = -2(pq + qr + rp)$ (d) none of these

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112. If the roots of the quadratic equation $\left(4p - p^2 - 5\right)x^2 - (2p - 1)x + 3p = 0$ lie on either side of unit, then the number of integer values of p is a.1 b. 2 c. 3 d. 4

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113. If $z_1 = 5 + 12i$ and $\left|z_2\right| = 4$, *then*

A. (a) maximum $\left(\left|z_1 + iz_2\right|\right) = 17$

B. (b) minimum $\left(\left|z_1 + (1 + i)z_2\right|\right) = 13 + 4\sqrt{2}$

C. (c) minimum $\left|\dfrac{z_1}{z_2 + \dfrac{4}{z_2}}\right| = \dfrac{13}{4}$

D. (d) maximum $\left|\dfrac{z_1}{z_2 + \dfrac{4}{z_2}}\right| = \dfrac{13}{3}$

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114. If roots of $x^2 - (a - 3)x + a = 0$ are such that at least one of them is greater than 2, then a. $a \in [7, 9]$ b. $a \in [7, \infty]$ c. $a \in [9, \infty)$ d. $a \in [7, 9]$

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115. If $|z-1|-1, then\ arg((z-1-i)/z)$ can be equal to $\pi/4$ $(z-2)/z$ is purely imaginary number $(z-2)/z$ is purely real number If $arg(z)=\theta, wherez \neq 0 andth\eta$ is acute, then $1-2/z=itan\theta$

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116. Let $f(x)=ax^2+bx+c$a ,b ,c $\in R$. If $f(x)$ takes real values for real values of x and non-real values for non-real values of x , then (a)$a=0$ (b) $b=0$ (c) $c=0$ (d) nothing can be said about a, b, c.

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117. Write a linear equation representing a line which is parallel to y-axis and is at a distance of 2 units on the positive side of x-axis

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118. If both roots of the equation $ax^2 + x + c - a = 0$ are imaginary and $c > -1$, then

A. a) 3a >2+4c

B. b) 3a<2+4c

C. c) c < a

D. d) none of these

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119. If $|z| = 1$ and $w = \frac{z - 1}{z + 1}$ (where $z \neq -1$), then $Re(w)$ is 0 (b) $\frac{1}{|z + 1|^2}$ $\left|\frac{1}{z + 1}\right|, \frac{1}{|z + 1|^2}$ (d) $\frac{\sqrt{2}}{|z|1|^2}$

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120. The set of all possible real values of a such that the inequality $(x - (a - 1))\left(x - \left(a^2 - 1\right)\right) < 0$ holds for all $x \in (- 1, 3)$ is $(0, 1)$ b. $(\infty, - 1]$ c. $(- \infty, - 1)$ d. $(1, \infty)$

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121. Column I, Column II (Locus) parallelogram, p. $z_1 - z_4 = z_2 - z_3$ rectangle, q. $|z_1 - z_3| = |z_2 - z_4|$ rhombus, r. $\frac{z_1 - z_2}{z_3 - z_4}$ is purely real square, s. $\frac{z_1 - z_3}{z_2 - z_4}$ is purely imaginary , t. $\frac{z_1 - z_2}{z_3 - z_2}$ is purely imaginary

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122. The interval of a for which the equation $tan^2 x - (a - 4)\tan x + 4 - 2a = 0$ has at least one solution $\forall x \in [0, \pi/4]$ $a \in (2, 3)$ b. $a \in [2, 3]$ c. $a \in (1, 4)$ d. $a \in [1, 4]$

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123. The range of a for which the equation $x^2 + ax - 4 = 0$ has its smaller root in the interval $(-1, 2)is$ a. $(-\infty, -3)$ b. $(0, 3)$ c. $(0, \infty)$ d. $(-\infty, -3) \cup (0, \infty)$

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124. Let z and ω be two complex numbers such that $|z| \leq 1, |\omega| \leq 1$ and $|z - i\omega| = \left|z - i\bar{\omega}\right| = 2,$ then z equals (a)$1 \text{ or } i$ (b). $i \text{ or } -i$ (c). $1 \text{ or } -1$ (d). $i \text{ or } -1$

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125. Consider the equation $x^2 + 2x - n = 0$ where $n \in N$ and $n \in [5, 100]$ The total number of different values of n so that the given equation has integral roots is a.8 b. 3 c. 6 d. 4

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126. If n_1, n_2 are positive integers, then $(1+i)^{n_1} + \left(1+i^3\right)^{n_1} + \left(1+i^5\right)^{n_2} + \left(1+i^7\right)^{n_2}$ is real if and only if :

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127. The total number of values a so that $x^2 - x - a = 0$ has integral roots, where $a \in Nand6 \leq a \leq 100$, is equal to a.2 b. 4 c. 6 d. 8

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128. If $i = \sqrt{-1}$, then $4 + 5\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{334} + 3\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{365}$ is equal to

(1) $1 - i\sqrt{3}$ (2) $-1 + i\sqrt{3}$ (3) $i\sqrt{3}$ (4) $-i\sqrt{3}$

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129. Let $P(x) = x^3 - 8x^2 + cx - d$ be a polynomial with real coefficients and with all it roots being distinct positive integers. Then number of possible value of c is__________.

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130. If $arg(z) < 0,$ then $arg(-z) - arg(z)$ equals π (b) $-\pi$ (d) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$

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131. Let $P(x) = \frac{5}{3} - 6x - 9x^2 and Q(y) = -4y^2 + 4y + \frac{13}{2}\cdot$ if there exists unique pair of real numbers (x, y) such that $P(x)Q(y) = 20$, then the value of $(6x + 10y)$ is _____.

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132. If z_1, z_2, z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1$ then $|z_1 + z_2 + z_3|$ is equal to

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133. if a $< c < b,$ then check the nature of roots of the equation

$(a - b)^2 x^2 + 2(a + b - 2c)x + 1 = 0$

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134. Q. Let z_1 and z_2 be nth roots of unity which subtend a right angle at the origin, then n must be the form $4k$.

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135. If $a + b + c = 0$ then check the nature of roots of the equation

$4ax^2 + 3bx + 2c = 0 where a, b, c \in R$

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136. The complex numbers z_1, z_2 and z_3 satisfying $\frac{z_1 - z_3}{z_2 - z_3} = \frac{1 - i\sqrt{3}}{2}$ are the vertices of triangle which is (1) of area zero (2) right angled isosceles(3) equilateral (4) obtuse angled isosceles

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137. Find the value of a for which the sum of the squares of the roots of the equation $x^2 - (a - 2)x - a - 1 = 0$ assumes the least value.

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138. For all complex numbers z_1, z_2 satisfying $|z_1| = 12$ and $|z_2 - 3 - 4i| = 5$, find the minimum value of $|z_1 - z_2|$

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139. If $x_1, and x_2$ are the roots of $x^2 + (\sin\theta - 1)x - \frac{1}{2}\left(\cos^2\theta\right) = 0$, then find the maximum value of $x_1^2 + x_2^2$

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140. If $y = \sec\left(\tan^{-1}x\right), then \frac{dy}{dx} at x = 1$ is (a) $\frac{\cos\pi}{4}$ (b) $\frac{\sin\pi}{2}$ (c) $\frac{\sin\pi}{6}$ (d) $\frac{\cos\pi}{3}$

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141. If $p, q \in \{1, 2, 3, 4, 5\}$, then find the number of equations of form $p^2x^2 + q^2x + 1 = 0$ having real roots.

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142. If $a^2 + b^2 = 1$ t h e n $\frac{1 + b + ia}{1 + b - ia} = 1$ b. 2 c. $b + ia$ d. $a + ib$

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143. Find the domain and the range of $f(x) = \sqrt{x^2 - 3x}$.

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144. Show that the equation $Z^4 + 2Z^3 + 3Z^2 + 4Z + 5 = 0$ has no root which is either purely real or purely imaginary.

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145. Find the domain and range of $f(x) = \sqrt{3 - 2x - x^2}$

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146. If $x \in (0, \pi/2) and \cos x = 1/3$, then prove that

$$\sum_{n=0}^{\infty} \frac{\cos nx}{3^n} = \frac{3(3 - \cos x)}{10 - 6\cos x + \cos^2 x}$$

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147. Prove that if the equation $x^2 + 9y^2 - 4x + 3 = 0$ is satisfied for real values of $xandy, thenx$ must lie between 1 and 3 and$y$ must lie between-1/3 and 1/3.

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148. Let $Z_p = r_p\left(\cos\theta_p + i\sin\theta_p\right), p = 1, 2, 3and\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} = 0.$ Consider the ABC formed formed by $\frac{\cos 2\theta_1 + i\sin 2\theta_1}{Z_1}, \frac{\cos 2\theta_2 + i\sin 2\theta_2}{Z_2}, \frac{\cos 2\theta_3 + i\sin 2\theta_3}{Z_3}$ Prove that origin lies inside triangle ABC.

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149. Find the least value of $\frac{\left(6x^2 - 22x + 21\right)}{\left(5x^2 - 18x + 17\right)}$ for real x.

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150. Let a, b and c be any three nonzero complex number. If $|z| = 1$ and $'z'$ satisfies the equation $az^2 + bz + c = 0$, prove that $a.\bar{a} = c.\bar{c}$ and $|a||b| = \sqrt{ac(\bar{b})^2}$

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151. Find the range of the function $f(x) = x^2 - 2x - 4$.

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152. If $x = 9^{\frac{1}{3}} 9^{\frac{1}{9}} 9^{\frac{1}{27}} \ldots \to \infty$, $y = 4^{\frac{1}{3}} 4^{-\frac{1}{9}} 4^{\frac{1}{27}} \ldots . \to \infty$ and $z = \sum_{r=1}^{\infty} (1 + i)^{-r}$ then , the argument of the complex number $w = x + yz$ is

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153. Find the linear factors of $2x^2 - y^2 - x + xy + 2y - 1$.

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154. If $a < 0, b > 0,$ then $\sqrt{a}\sqrt{b}$ is equal to (a)-$\sqrt{|a|b}$ (b) $\sqrt{|a|bi}$ (c) $\sqrt{|a|b}$ (d) none of these

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155. The value(s) of m for which the expression $2x^2 + mxy + 3y^2 - 5y - 2$ can be factorized in to two linear factors are:

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156. The number of solutions of $z^2 + \bar{z} = 0$ is

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157. If $a_1x^3 + b_1x^2 + c_1x + d_1 = 0$ and $a_2x^3 + b_2x^2 + c_2x + d_2 = 0$ have a pair of repeated common roots, then prove that

$$\begin{vmatrix} 3a_1 & 2b_1 & c_1 \\ 3a_2 & 2b_2 & c_2 \\ a_2b_1 - a_1b_2 & c_1a_2 - c_2a_1 & d_1a_2 - d_2a_1 \end{vmatrix} = 0$$

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158. Consider the equation $10z^2 - 3iz - k = 0, wherez$ is a following complex variable and $i^2 = -1.$ Which of the following statements ils true? (a)For real complex numbers k , both roots are purely imaginary. (b)For all complex numbers k , neither both roots is real. (c)For all purely imaginary numbers k , both roots are real and irrational. (d)For real negative numbers k , both roots are purely imaginary.

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159. If $x - c$ is a factor of order m of the polynomial $f(x)$ of degree n (1 < m < n) , then find the polynomials for which $x = c$ is a root.

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160. If $z_1 and z_2$ are two complex numbers such that $|z_1| = |z_2| and arg(z_1) + arg(z_2) = \pi$, then show that $z_1, = -\bar{z}_2$.

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161. Solve the equation $x^3 - 13x^2 + 15x + 189 = 0$ if one root exceeds the other by 2.

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162. Let z_1, z_2, z_3 be the three nonzero complex numbers such that $z_2 \neq 1, a = |z_1|, b = |z_2| and c = |z_3|$. Let $|abcbcacab| = 0$ $ar\frac{g(z_3)}{z_2} = arg\left(\frac{z_3 - z_1}{z_2 - z_1}\right)^2$ orthocentre of triangle formed by $z_1, z_2, z_3, is z_1 + z_2 + z_3$ if triangle formed by z_1, z_2, z_3 is equilateral, then its area is $\frac{3\sqrt{3}}{2}|z_1|^2$ if triangle formed by z_1, z_2, z_3 is equilateral, then $z_1 + z_2 + z_3 = 0$

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163. If $\tan\theta$ and $\sec\theta$ are the roots of $ax^2 + bx + c = 0$, then prove that $a^4 = b^2(b^2 - 4ac)$.

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164. Given that the complex numbers which satisfy the equation $|z\bar{z}^3| + |\bar{z}z^3| = 350$ form a rectangle in the Argand plane with the length of its diagonal having an integral number of units, then area of rectangle is 48 sq. units if z_1, z_2, z_3, z_4 are vertices of rectangle, then $z_1 + z_2 + z_3 + z_4 = 0$ rectangle is symmetrical about the real axis $arg(z_1 - z_3) = \frac{\pi}{4}$ or $\frac{3\pi}{4}$

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165. If the roots of the equation $x^2 - bx + c = 0$ are two consecutive integers, then find the value of $b^2 - 4c$

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166. If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then the set of possible values of a is (1) $(-3, 3)$ (2) $(-3, \infty)$ (3) $(3, \infty)$ (4) $(-\infty, -3)$

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167. For what real values of a do the roots of the equation $x^2 - 2x - \left(a^2 - 1\right) = 0$ lie between the roots of the equation $x^2 - 2(a+1)x + a(a-1) = 0$.

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168. If P and Q are represented by the complex numbers z_1 and z_2 such that $\left|\frac{1}{z_2}+\frac{1}{z_1}\right|=\left|\frac{1}{z_2}-\frac{1}{z_1}\right|$, then a) $OPQ(where O)$ is the origin of equilateral OPQ is right angled. b) the circumcenter of $OPQ is \frac{1}{2}\left(z_1+z_2\right)$ c) the circumcenter of $OPQ is \frac{1}{3}\left(z_1+z_2\right)$

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169. Find the value of a for which the equation a $\sin\left(x+\frac{\pi}{4}\right)=\sin 2x+9$ will have real solution.

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170. Given $z=f(x)+ig(x)$ where $f, g:(0,1)\to(0,1)$ are real valued functions. Then which of the following does not hold good?

a.$z=\frac{1}{1-ix}+i\frac{1}{1+ix}$

b. $z=\frac{1}{1+ix}+i\frac{1}{1-ix}$

c. $z = \frac{1}{1 + ix} + i\frac{1}{1 + ix}$

d. $z = \frac{1}{1 - ix} + i\frac{1}{1 - ix}$

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171. Let a, b and c be real numbers such that $a + 2b + c = 4$. Find the maximum value of $(ab + bc + ca)$

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172. If $z = x + iy$, then the equation $\left|\frac{2z - i}{z + 1}\right| = m$ does not represents a circle, when m is (a) $\frac{1}{2}$ (b). 1 (c). 2 (d). `3

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173. Prove that for real values of x, $\left(ax^2 + 3x - 4\right)/\left(3x - 4x^2 + a\right)$ may have any value provided a lies between 1 and 7.

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174. Given that the two curves $arg(z) = \frac{\pi}{6}$ and $\left|z - 2\sqrt{3}i\right| = r$ intersect in two distinct points, then a. $[r] \neq 2$ b. $0 < r < 3$ c. $r = 6$ d. $3 < r < 2\sqrt{3}$

(Note : [r] represents integral part of r)

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175. Let $x^2 - (m - 3)x + m = 0(m\varepsilon R)$ be a quadratic equation . Find the values of m for which the roots are (ix)one root is smaller than 2 & other root is greater than 2 (x) both the roots are greater than 2 (xi) both the roots are smaller than 2 (xii)exactly one root lies in the interval (1;2) (xiii) both the roots lies in the interval (1;2) (xiv) atleast one root lies in the interval (1;2) (xv) one root is greater than 2 and the other root is smaller than 1

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176. A particle P starts from the point $z_0 = 1 + 2i$, where $i = \sqrt{-1}$. It moves first horizontally away from origin by 5 units and then vertically away from origin by 3 units to reach a point z_1. From z_1 the particle moves $\sqrt{2}$ units in the direction of the vector $\hat{i} + \hat{j}$ and then it moves through an angle $\frac{\pi}{2}$ in anticlockwise direction on a circle with centre at origin, to reach a point z_2. The point z_2 is given by (a)$6 + 7i$ (b) $-7 + 6i$ (c) $7 + 6i$ (d) $-6 + 7i$

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177. Prove that for all real values of x and $y, x^2 + 2xy + 3y^2 - 6x - 2y \geq -11.$

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178. Let $z = x + iy$ be a complex number where $x and y$ are integers. Then, the area of the rectangle whose vertices are the roots of the equation $z\bar{z}$

^3 + zbar z^3 = 350 is (a)48 (b) 32 (c) 40 (d) 80

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179. The values of 'a' for which the equation $\left(x^2+x+2\right)^2-(a-3)\left(x^2+x+2\right)\left(x^2+x+1\right)+(a-4)\left(x^2+x+1\right)^2=0$ has atlesast one real root is:

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180. A man walks a distance of 3 units from the origin towards the North-East $\left(N45^0E\right)$ direction.From there, he walks a distance of 4 units towards the North-West $\left(N45^0W\right)$ direction to reach a point P Then, the position of P in the Argand plane is (a) $3e^{\frac{i\pi}{4}}+4i$ (b) $(3-4i)e^{\frac{i\pi}{4}}$ $(4+3i)e^{\frac{i\pi}{4}}$ (d) $(3+4i)e^{\frac{i\pi}{4}}$

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181. Find the values of a for which the equation $\sin^4 x + a\sin^2 x + 1 = 0$ will have a solution.

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182. If $|z| = 1 and z \neq \pm 1$, then all the values of $\frac{z}{1 - z^2}$ lie on (a)a line not passing through the origin (b)$|z| = \sqrt{2}$ (c)the x-axis (d) the y-axis

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183. Find all the value of m for which the equation $\sin^2 x - (m - 3)\sin x + m = 0$ has real roots.

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184. Let $A\left(z_1\right)$ and $\left(z_2\right)$ represent two complex numbers on the complex plane. Suppose the complex slope of the line joining A and B is defined as

$\frac{z_1 - z_2}{\bar{z}_1 - \bar{z}_2}$.If the line l_1, with complex slope ω_1, and l_2, with complex slope $omeg_2$, on the complex plane are perpendicular then prove that $\omega_1 + \omega_2 = 0$.

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185. If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then the set of possible values of a is (1) $(-3, 3)$ (2) $(-3, \infty)$ (3) $(3, \infty)$ (4) $(-\infty, -3)$

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186. Let $z_1 and z_2$ be complex numbers such that $z_1 \neq z_2$ and $|z_1| = |z_2|$. If z_1 has positive real part and z_2 has negative imaginary part, then $\frac{z_1 + z_2}{z_1 - z_2}$ may be (a) zero (b) real and positive (c) real and negative (d) purely imaginary

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187. Find the condition if the roots of $ax^2 + 2bx + c = 0$ and $bx^2 - 2\sqrt{acx} + b = 0$ are simultaneously real.

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188. Locus of complex number satisfying $\arg\left[\frac{z - 5 + 4i}{z + 3 - 2i}\right] = \frac{\pi}{4}$ is the arc of a circle whose radius is $5\sqrt{2}$ whose radius is 5 whose length (of arc) is $\frac{15\pi}{\sqrt{2}}$ whose centre is $-2 - 5i$

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189. Solve $\left(x^2 - 5x + 7\right)^2 - (x - 2)(x - 3) = 1.$

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190. If α is a complex constant such that $\alpha z^2 + z + \bar{\alpha} = 0$ has a real root, then (a) $\alpha + \bar{\alpha} = 1$ (b) $\alpha + \bar{\alpha} = 0$ (c)$\alpha + \bar{\alpha} = -1$ (d)the absolute value of the real root is 1

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191. Solve the equation $x^4 - 5x^2 + 4 = 0$.

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192. The complex number z satisfies $z + |z| = 2 + 8i$. find the value of $|z| - 8$

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193. Solve $\dfrac{x^2 - 2x - 3}{x + 1} = 0$.

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194. If ω is a cube root of unity and $(1+\omega)^7 = A + B\omega$ then find the values of A and B`

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195. Solve $\left(x^3 - 4x\right)\sqrt{x^2 - 1} = 0.$

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196. The complex number $\sin x + i\cos 2x$ and $\cos x - i\sin 2x$ are conjugate to each other when

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197. Solve $\dfrac{2x-3}{x-1} + 1 = \dfrac{9x - x^2 - 6\cdot}{x-1}$

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198. The points, $z_1, z_2, z_3, z_4,$ in the complex plane are the vertices of a parallelogram taken in order, if and only if (a)$z_1 + z_4 = z_2 + z_3$ (b) $z_1 + z_3 = z_2 + z_4$ (c)$z_1 + z_2 = z_3 + z_4$ (d) None of these

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199. Using differentiation method check how many roots of the equation $x^3 - x^2 + x - 2 = 0$ are real?

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200. If $z = x + iy$ and $w = \frac{1 - iz}{z - i}$, then $|w| = 1$ implies that in the complex plane (A)z lies on imaginary axis (B) z lies on real axis (C)z lies on unit circle (D) None of these

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201. If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then the set of possible values of a is (1) $(-3, 3)$ (2) $(-3, \infty)$ (3) $(3, \infty)$ (4) $(-\infty, -3)$

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202. $|z - 4| < |z - 2|$ represents the region given by: (a) $Re(z) > 0$ (b) $Re(z) < 0$ (c) $Re(z) > 3$ (d) None of these

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203. Find how many roots of the equations $x^4 + 2x^2 - 8x + 3 = 0$.

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204. If $z = \left[\left(\frac{\sqrt{3}}{2}\right) + \frac{i}{2}\right]^5 + \left[\left(\frac{\sqrt{3}}{2}\right) - \frac{i}{2}\right]^5$, then a. $Re(z) = 0$ b. $Im(z) = 0$ c. $Re(z) > 0$ d. $Re(z) > 0, Im(z) < 0$

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205. How many real solutions does the equation $x^7 + 14x^5 + 16x^3 + 30x - 560 = 0$ have?

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206. The complex numbers $z = x + iy$ which satisfy the equation $\left|\frac{z - 5i}{z + 5i}\right| = 1$ lie on (a) The x-axis (b) The straight line $y = 5$ (c) A circle passing through the origin (d) Non of these

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207. Solve $\sqrt{5x^2 - 6x + 8} - \sqrt{5x^2 - 6x - 7} = 1.$

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208. The smallest positive integer n for which $\left(\frac{1+i}{1-i}\right)^n = 1$ is (a)8(b) 16 (c) 12(d) None of these

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209. Solve $\sqrt{3x^2 - 7x - 30} + \sqrt{2x^2 - 7x - 5} = x + 5.$

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210. If the cube roots of unity are $1, \omega, \omega^2,$ then the roots of the equation $(x-1)^3 + 8 = 0$ are a. $-1, 1+2\omega, 1+2\omega^2$ b. $-1, 1-2\omega, 1-2\omega^2$ c. $-1, -1, -1$ d. none of these

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211. If $x = \left(7 + 4\sqrt{3}\right),$ prove that $x + 1/x = 14$

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212. Prove that the locus of midpoint of line segment intercepted between real and imaginary axes by the line $az + az + b = 0$, $whereb$ is a real parameterand a is a fixed complex number with nondzero real and imaginary parts, is $az + az = 0$.

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213. Solve $\sqrt{5x^2 - 6x + 8} - \sqrt{5x^2 - 6x - 7} = 1$.

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214. Show that: $\sum_{r=0}^{n-1} \left|z_1 + \alpha^r z_2\right|^2 = n\left(\left|z_1\right|^2 + \left|z_2\right|^2\right)$, $where, \alpha; r = 0, 1, 2, \ldots, (n - 1)$, are the nth roots of unity and z_1, z_2 are any two complex numbers.

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215. Solve $\sqrt{x^2 + 4x - 21} + \sqrt{x^2 - x - 6} = \sqrt{6x^2 - 5x - 39.}$

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216. If $\alpha = (z - i)(z + i)$, show that, when z lies above the real axis, α will lie within the unit circle which has center at the origin. Find the locus of αasz travels on the real axis from $- \infty \rightarrow + \infty^{\cdot}$

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217. Solve $4^x + 6^x = 9^{x^{\cdot}}$

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218. Let x_1, x_2 are the roots of the quadratic equation $x^2 + ax + b = 0, where a, b$ are complex numbers and y_1, y_2 are the roots of the quadratic equation $y^2 + |a|y + |b| = 0$. If $\left|x_1\right| = \left|x_2\right| = 1$, then prove that $\left|y_1\right| = \left|y_2\right| = 1$

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219. Solve $3^{2x^2 - 7x + 7} = 9$.

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220. Plot the region represented by $\frac{\pi}{3} \leq arg\left(\frac{z + 1}{z - 1}\right) \leq \frac{2\pi}{3}$ in the Argand plane.

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221. How many solutions does the equation $\frac{8^x + 27^x}{12^x + 18^x} = \frac{7}{6}$ have? (A) Exactly one (B) Exactly two (C) Finitely many (D) Infinitely many

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222. Is the following computation correct? If not give the correct computation : $\sqrt{(-2)}\sqrt{(-3)} = \sqrt{(-2)(-3)} = \sqrt{(-6)}$

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223. Consider an equilateral triangle having verticals at point $A\left(\frac{2}{\sqrt{3}}e^{\frac{l\pi}{2}}\right), B\left(\frac{2}{\sqrt{3}}e^{\frac{-i\pi}{6}}\right)$ and $C\left(\frac{2}{\sqrt{3}}e^{\frac{-5\pi}{6}}\right)$. If $P(z)$ is any point an its incircle, then $AP^2 + BP^2 + CP^2$

A. 4

B. 4

C. 3

D. -3

Answer: A

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224. Find the number of real roots of the equation $(x-1)^2+(x-2)^2+(x-3)^2=0.$

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225. Find the value of (i) $\frac{i^{592}+i^{590}+i^{588}+i^{586}+i^{584}}{i^{582}+i^{580}+i^{578}+i^{576}+i^{574}}(i)-1$ (ii) $(1+i)^6+(1-i)^6$

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226. Let z, z_0 be two complex numbers. It is given that $|z|=1$ and the numbers $z, z_0, z\bar{\_}(0), 1$ and 0 are represented in an Argand diagram by the points P,P_0,Q,A and the origin, respectively. Show that $\triangle POP_0$ and $\triangle AOQ$ are congruent. Hence, or otherwise, prove that

$$\left|z-z_0\right| = \left|\bar{z}z_0-1\right| = \left|z\bar{z}_0-1\right|.$$

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227. Show that the equation $az^3 + bz^2 + \bar{b}z + \bar{a} = 0$ has a root α such that $|\alpha| = 1, a, b, z$ and α belong to the set of complex numbers.

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228. If $n \geq 3$ and $1, \alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_{n-1}$ are the n,nth roots of unity, then find value of $\sum \sum{}_{1 \le i < j \le n-1} \alpha_i \alpha_j$

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229. How many roots of the equation $3x^4 + 6x^3 + x^2 + 6x + 3 = 0$ are real ?

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230. If the roots of $(z - 1)^n = i(z + 1)^n$ are plotted in ten Arg and plane, then prove that they are collinear.

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231. Find the value of k if $x^3 - 3x + k = 0$ has three real distinct roots.

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232. Let $z = t^2 - 1 + \sqrt{t^4 - t^2}, where t \in R$ is a parameter. Find the locus of z depending upon t, and draw the locus of z in the Argand plane.

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233. If $|z| = 1$, then prove that points represented by $\sqrt{(1 + z)/(1 - z)}$ lie on one or other of two fixed perpendicular straight lines.

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234. If ω is an imaginary fifth root of unity, then find the value of $\log_2\left|1 + \omega + \omega^2 + \omega^3 - 1/\omega\right|$.

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235. a, b, and c are all different and non-zero real numbers on arithmetic progression. If the roots of quadratic equation $ax^2 + bx + c = 0$ $are\ \alpha\ and\ \beta$ such that $\frac{1}{\alpha} + \frac{1}{\beta}, \alpha + \beta, and \alpha^2 + \beta^2$ are in geometric progression the value of a/c will be____.

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236. Let $x^2 + y^2 + xy + 1 \geq a(x + y) \forall x, y \in R,$ then the number of possible integer (s) in the range of a is_________.

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237. If $\alpha = e^{i2\pi/7} and f(x) = a_0 + \sum_{k=0}^{20} a_k x^k,$ then prove that the value of $f(x) + f(\alpha x) + ... + f\left(\alpha^6 x\right)$ is independent of α

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238. If $w = \alpha + i\beta$, where $\beta \neq 0$ and $z \neq 1$, satisfies the condition that $\left(\frac{w - \bar{w}z}{1 - z}\right)$ is a purely real, then the set of values of z is $|z| = 1, z \neq 2$ (b) $|z| = 1 and z \neq 1$ (c)$z = \bar{z}$ (d) None of these

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239. If z is a non real root of $\sqrt[7]{-1}$, then find the value of $z^{86} + z^{175} + z^{289}$.

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240. The quadratic equation $x^2 + mx + n = 0$ has roots which are twice those of $x^2 + px + m = 0 adm, n and p \neq 0$. The n the value of n/p is ____.

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241. Let $\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$, then value of the determinant

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1-\omega^2 & -\omega^2 \\ 1 & \omega^2 & \omega^4 \end{bmatrix}$$ is

(a) 3ω

(b) $3\omega(\omega - 1)$

(c) $3\omega^2$

(d) $3\omega(1 - \omega)$

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242. All the value of k for which the quadratic polynomial $f(x) = 2x^2 + kx + 2 = 0$ has equal roots is ______. (a) 4 (B) +4,-4 (c) +3,-3 (d) 2

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243. If $a = \cos(2\pi/7) + i\sin(2\pi/7)$, then find the quadratic equation whose roots are $\alpha = a + a^2 + a^4$and $\beta = a^3 + a^5 + a^6$.

A. $x^2 - x + 2 = 0$

B. $x^2 + x - 2 = 0$

C. $x^2 - x - 2 = 0$

D. $x^2 + x + 2 = 0$

Answer: D

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244. Let complex numbers α and $\frac{1}{\alpha}$ lies on circle $(x - x_0)^2 + (y - y_0)^2 = r^2$ and $(x - x_0)^2 + (y - y_0)^2 = 4r^2$ respectively. If $z_0 = x_0 + iy_0$ satisfies the equation $2|z_0|^2 = r^2 + 2$ then $|\alpha|$ is equal to

A. (a) $\frac{1}{\sqrt{2}}$

B. (b) $\frac{1}{2}$

C. (c) $\frac{1}{\sqrt{7}}$

D. (d) $\frac{1}{3}$

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245. If $\left|\frac{z}{|\bar{z}|} - \bar{z}\right| = 1 + |z|$, then prove that z is a purely imaginary number.

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246. If $x = -5 + 2\sqrt{-4}$, find the value of $x^4 + 9x^3 + 35x^2 - x + 4$.

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247. Let $a, b, andc$ be rel numbers which satisfy the equation $a + \frac{1}{bc} = \frac{1}{5}, b + \frac{1}{ac} = \frac{-1}{15}, andc + \frac{1}{ab} = \frac{1}{3}$. The value of $\frac{c-b}{c-a}$ is equal to

_______.

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248. The value of $i^{1+3+5+\ldots..+(2n+1)}$ is_______.

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249. a, b, c are integers, not all simultaneously equal, and ω is cube root of unity $(\omega \neq 1)$, then minimum value of $\left|a+b\omega+c\omega^2\right|$ is 0 b. 1 c. $\frac{\sqrt{3}}{2}$ d. $\frac{1}{2}$

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250. a, b, c are reals such that a+b+c=3 and $\frac{1}{a+b}+\frac{1}{b+c}+\frac{1}{c+a}=\frac{10}{3}$. The value of $\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}$ is_______.

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251. If $z + 1/z = 2\cos\theta$, prove that $\left|\left(z^{2n} - 1\right)/\left(z^{2n} + 1\right)\right| = |\tan n\theta|$

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252. If α, β are the roots of the quadratic equation $ax^2 + bx + c = 0$, then which of the following expression will be the symmetric function of roots

a. $\left|\log\left(\frac{\alpha}{\beta}\right)\right|$ b. $\alpha^2\beta^5 + \beta^2\alpha^5$ c. $tan(\alpha - \beta)$ d. $\left(\log\left(\frac{1}{\alpha}\right)\right)^2 + (\log\beta)^2$

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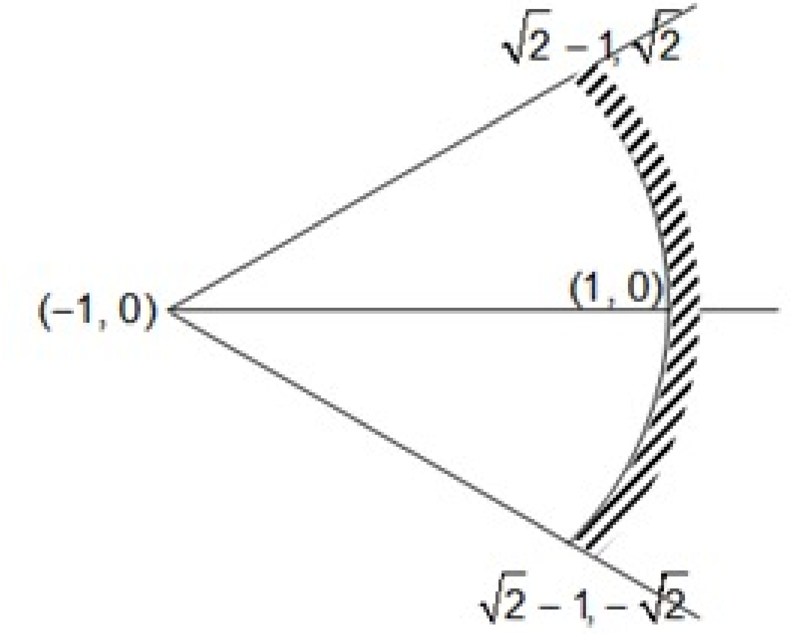
253. The locus of z which lies in shaded region (excluding the boundaries) is best represented by Fig



A. $z : |z + 1| > 2, |arg(z + 1)| < \frac{\pi}{4}$

B. $z : |z - 1| > 2, |arg(z - 1)| < \frac{\pi}{4}$

C. $z : |z + 1| < 2, |arg(z + 1)| < \frac{\pi}{2}$

D. $z : |z - 1| < 2, |arg(z - 1)| < \frac{\pi}{2}$

Answer: A

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254. Prove that the roots of the equation $x^4 - 2x^2 + 4 = 0$ forms a rectangle.

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255. If a, b, c are non-zero real numbers, then find the minimum value of the expression $\left(\frac{\left(a^4 + 3a^2 + 1\right)\left(b^4 + 5b^2 + 1\right)\left(c^4 + 7c^2 + 1\right)}{a^2b^2c^2}\right)$ which is not divisible by prime number.

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256. If $z_1 and z_2$ are two nonzero complex numbers such that = $\left|z_1 + z_2\right| = \left|z_1\right| + \left|z_2\right|$, then $arg z_1 - arg z_2$ is equal to $-\pi$ b. $\frac{\pi}{2}$ c. 0 d. $\frac{\pi}{2}$ e. π

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257. if diagonals of a parallelogram bisect each other,prove that its a rhombus

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258. If a, b, c and u, v, w are the complex numbers representing the vertices of two triangles such that $(c = (1 - r)a + rb$ and $w = (1 - r)u + rv,$ where r is a complex number, then the two triangles (a)have the same area (b) are similar (c)are congruent (d) None of these

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259. If $z = \cos\theta + i\sin\theta$ is a root of the equation $a_0 z^n + a_2 z^{n-2} + \ + a_{n-1} z^{+} a_n = 0,$ then prove that $a_0 + a_1 \cos\theta + a_2^{\cos 2}\theta + \ + a_n \cos n\theta = 0 \ a_1 \sin\theta + a_2^{\sin 2}\theta + \ + a_n \sin n\theta = 0$

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260. If ω is an imaginary cube root of unity, then $\left(1 + \omega - \omega^2\right)^7$ is equal to

A. 128ω

B. -128ω

C. $128\omega^2$

D. $-128\omega^2$

Answer: D

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261. If $z = x + iy$ is a complex number with $x, y \in Q$ and $|z| = 1$, then show that $\left|z^{2n} - 1\right|$ is a rational numberfor every $n \in N$.

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262. Referred to the principal axes as the axes of co ordinates find the equation of hyperbola whose focii are at $\left(0, \pm\sqrt{10}\right)$ and which passes through the point $(2, 3)$

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263. Find the area bounded by $|argz| \leq \pi/4$ and $|z - 1| < |z - 3|$

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264. $\sum_{k=1}^{6}\left(\sin, \frac{2\pi k}{7} - i\cos, \frac{2\pi k}{7}\right) = ?$

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265. If the equation $ax^2 + bx + c = 0(a > 0)$ has two real roots $\alpha and \beta$ such that $\alpha < -2$ and $\beta > 2$, then which of the following statements is/are true? (a)$a - |b| + c < 0$ (b)$c < 0, b^2 - 4ac > 0$ (c) $4a - 2|b| + c < 0$ (d) $9a - 3|b| + c < 0$

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266. If fig shows the graph of $f(x) = ax^2 + bx + c, then$ Fig a. $c < 0$ b. $bc > 0$ c. $ab > 0$ d. $abc < 0$

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267. If det $|\ 6i\ \text{-}\ 3i1 \quad 43i\ \text{-}\ 1 \quad 203i\ |\ = x + iy,$ then a.$x = 3, y = 1$ b. $x = 1, y = 3$ c. $x = 0, y = 3$ d. $x = 0, y = 0$

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268. Let $z = x + iy$ be a complex number, where $xandy$ are real numbers. Let $AandB$ be the sets defined by $A = \{z : |z| \leq 2\} andB = \left\{z : (1\ \text{-}\ i)z + (1 + i)\bar{z} \geq 4\right\}$. Find the area of region $A \cap B$

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269. If $z=\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5+\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5$, then prove that $Im(z)=0$.

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270. The value of $\sum_{n=1}^{13}\left(i^n+i^{n+1}\right)$, where $i=\sqrt{-1}$ equals (A) i (B) $i-1$ (C) $-i$ (D) 0

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271. If $c\neq 0$ and the equation $p/(2x)=a/(x+c)+b/(x-c)$ has two equal roots, then p can be $\left(\sqrt{a}-\sqrt{b}\right)^2$ b. $\left(\sqrt{a}+\sqrt{b}\right)^2$ c. $a+b$ d. $a-b$

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272. If the equation $4x^2-x-1=0$ and $3x^2+(\lambda+\mu)x+\lambda-\mu=0$ have a root common, then the irrational values of λ and μ are (a) $\lambda=\frac{-3}{4}$ b. $\lambda=0$

c. $\mu = \frac{3}{4}$ b. $\mu = 0$

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273. Express the following in $a + ib$ form: $\frac{(\cos 2\theta - i\sin 2\theta)^4 (\cos 4\theta + i\sin 4\theta)^{-5}}{(\cos 3\theta + i\sin 3\theta)^{-2} (\cos 3\theta - i\sin 3\theta)^{-9}}$

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274. The roots of the equation $t^3 + 3at^2 + 3bt + c = 0 are z_1, z_2, z_3$ which represent the vertices of an equilateral triangle. Then $a^2 = 3b$ b. $b^2 = a$ c. $a^2 = b$ d. $b^2 = 3a$

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275. Solve the equation $(x - 1)^3 + 8 = 0$ in the set C of all complex numbers.

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276. If 'z, lies on the circle $|z-2i|=2\sqrt{2}$, then the value of $arg\left(\frac{z-2}{z+2}\right)$ is the equal to

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277. If the equation whose roots are the squares of the roots of the cubic $x^3-ax^2+bx-1=0$ is identical with the given cubic equation, then (A) $a=0, b=3$ (B) $a=b=0$ (C) $a=b=3$ (D) $a, b,$ are roots of $x^2+x+2=0$

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278. If $\sqrt{3}+i=(a+ib)(c+id)$, then find the value of $\tan^{-1}(b/a)+\tan^{-1}(d/c)$

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279. P(z) be a variable point in the Argand plane such that |z|=minimum $\{|z-1, \; |z+1 \; | \; \}$, then $z+\bar{z}$ will be equal to a. -1 or 1

b. 1 but not equal to-1 c. -1 but not equal to 1 d. none of these

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280. If the equation $ax^2+bx+c=0, a, b, c, \in R$ have none-real roots, then $c(a-b+c)>0$ b. $c(a+b+c)>0$ c. $c(4a-2b+c)>0$ d. none of these

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281. Prove that the equation $Z^3+iZ-1=0$ has no real roots.

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282. The locus of point z satisfying $Re\left(\frac{1}{z}\right) = k, wherek$ is a non zero real number, is a. a straight line b. a circle c. an ellipse d. a hyperbola

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283. If $p(q - r)x^2 + q(r - p)x + r(p - q) = 0$ has equal roots, then prove that $\frac{2}{q} = \frac{1}{p} + \frac{1}{r}\cdot$

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284. Find the square root $9 + 40i^{\cdot}$

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285. Let $\alpha, \beta \in R^{\cdot}$ If α, β^2 are the roots of quadratic equation $x^2 - px + 1 = 0. and\alpha^2, \beta$ are the roots of quadratic equation

$x^2 - qx + 8 = 0$, then find p, q, α, β

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286. Let a be a complex number such that $|a| < 1 and z_1, z_2, z_3, \ldots$ be the vertices of a polygon such that $z_k = 1 + a + a^2 + \ldots + a^{k-1}$ for all $k = 1, 2, 3, Then z_1, z_2$ lie within the circle (a) $\left|z - \frac{1}{1 - a}\right| = \frac{1}{|a - 1|}$ (b) $\left|z + \frac{1}{a + 1}\right| = \frac{1}{|a + 1|}$ (c) $\left|z - \frac{1}{1 - a}\right| = |a - 1|$ (d) $\left|z + \frac{1}{a + 1}\right| = |a + 1|$

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287. Let $\lambda \in R$. If the origin and the non-real roots of $2z^2 + 2z + \lambda = 0$ form the three vertices of an equilateral triangle in the Argand plane, then λ is (a.)1 (b) $\frac{2}{3}$ (c.) 2 (d.) -1

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288. If the ratio of the roots of the equation $x^2 + px + q = 0$ are equal to ratio of the roots of the equation $x^2 + bx + c = 0$, then prove that $p^2c = b^2q$.

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289. Let $z = 1 - t + i\sqrt{t^2 + t + 2}$, where t is a real parameter.the locus of the z in argand plane is

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290. If $\sin\theta, \cos\theta$ be the roots of $ax^2 + bx + c = 0$, then prove that $b^2 = a^2 + 2a \cdot$

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291. Express the following complex numbers in $a + ib$ form: $\frac{(3 - 2i)(2 + 3i)}{(1 + 2i)(2 - i)}$

(ii) $\frac{2 - \sqrt{-25}}{1 - \sqrt{-16}}$

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292. If a, b, c are nonzero real numbers and $az^2 + bz + c + i = 0$ has purely imaginary roots, then prove that $a = b^2c^{\cdot}$

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293. If $z^2 + z|z| + \left|z^2\right| = 0,$ then the locus z is a. a circle b. a straight line c. a pair of straight line d. none of these

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294. If the sum of the roots of the equation $\frac{1}{x+a}+\frac{1}{x+b}=1/c$ is zero, the prove that the product of the root is $\left(-\frac{1}{2}\right)\left(a^2+b^2\right)$.

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295. Solve the equation $x^2+px+45=0$. it is given that the squared difference of its roots is equal to 144

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296. Find the least positive integer n such that $\left(\frac{2i}{1+i}\right)^n$ is a positive integer.

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297. $z_1 and z_2$ lie on a circle with center at the origin. The point of intersection z_3 of he tangents at $z_1 and z_2$ is given by $\frac{1}{2}\left(z_1 + (z)_2\right)$ b. $\frac{2z_1z_2}{z_1 + z_2}$ c. $\frac{1}{2}\left(\frac{1}{z_1} + \frac{1}{z_2}\right)$ d. $\frac{z_1 + z_2}{(z)_1(z)_2}$

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298. If α, β are the roots of the equation $2x^2 - 35x + 2 = 0$, the find the value of $(2\alpha - 35)^3(2\beta - 35)^{3^{\cdot}}$

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299. If one root of the equation $z^2 - az + a - 1 = 0$ is (1+i), where a is a complex number then find the root.

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300. If $|z_1| = |z_2| = |z_3| = 1$ and $z_1 + z_2 + z_3 = 0$ then the area of the triangle whose vertices are z_1, z_2, z_3 is $3\sqrt{3}/4$ b. $\sqrt{3}/4$ c. 1 d. 2

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301. Simplify: $\dfrac{\sqrt{5 + 12i} + \sqrt{5 - 12i}}{\sqrt{5 + 12i} - \sqrt{5 - 12i}}$

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302. Find a quadratic equation whose product of roots x_1 and x_2 is equal to 4 and satisfying the relation $\dfrac{x_1}{x_1 - 1} + \dfrac{x_2}{x_2 - 1} = 2.$

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303. If $\sqrt{5 - 12i} + \sqrt{-5 - 12i} = z$, then principal value of $argz$ can be

A. a. $\dfrac{\pi}{4}$

B. b. - $\frac{\pi}{4}$

C. c. $\frac{3\pi}{4}$

D. d. - $\frac{3\pi}{4}$

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304. If $(x + iy)(p + iq) = \left(x^2 + y^2\right)i$, prove that $x = q, y = p$.

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305. If a and $b(\neq 0)$ are the roots of the equation $x^2 + ax + b = 0$, then find the least value of $x^2 + ax + b(x \in R)$.

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306. Let A, B, C, D be four concyclic points in order in which $AD: AB = CD: CB$. If A, B, C are repreented by complex numbers a, b, c representively, find the complex number associated with point D.

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307. Convert $\frac{1 + 3i}{1 - 2i}$ into the polar form.

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308. If the sum of the roots of the equation $(a + 1)x^2 + (2a + 3)x + (3a + 4) = 0$ is -1, then find the product of the roots.

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309. Let the altitudes from the vertices A, B and Cof the triangle e ABCmeet its circumcircle at D, E and F respectively and z_1, z_2 and z_3

represent the points D, E and F respectively. If $\frac{z_3 - z_1}{z_2 - z_1}$ is purely real then the triangle ABC is

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310. For $|z - 1| = 1$, show that $\tan\left\{\frac{arg(z - 1)}{2}\right\} - \left(\frac{2i}{z}\right) = - i$

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311. The quadratic polynomial $p(x)$ has the following properties:$p(x) \geq 0$ for all real numbers, $p(1) = 0$ and $p(2) = 2$. Find the value of $p(3)$ is________.

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312. If $z_1 = 9y^2 - 4 - 10ix,\ z_2 = 8y^2 - 20i,$ where $z_1 = \bar{z}_2,$ then find $z = x + iy$

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313. If $arg(z_1) = 170^0$ and $arg(z_2) = 70^0$, then find the principal argument of z_1z_2.

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314. z_1, z_2 and z_3 are the vertices of an isosceles triangle in anticlockwise direction with origin as in center , then prove that z_2, z_1 and kz_3 are in G.P. where $k \in R^+$.

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315. function f , R $\rightarrow$ R , $f(x) = \frac{3x^2 + mx + n}{x^2 + 1}$, if the range of function is [-4,3), find the value of |m+n| is ....

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316. If z_1 and z_2 are conjugate to each other , find the principal argument of $\left(-z_1z_2\right)$.

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317. If a is a complex number such that $|a| = 1$, then find the value of a, so that equation $az^2 + z + 1 = 0$ has one purely imaginary root.

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318. If $x^2 + px + 1$ is a factor of the expression $ax^3 + bx + c$, then $a^2 - c^2 = ab$ b. $a^2 + c^2 = -ab$ c. $a^2 - c^2 = -ab$ d. none of these

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319. Find the value of expression

$$\left(\frac{\cos\pi}{2} + i\sin\left(\frac{\pi}{2}\right)\right)\left(\cos\left(\frac{\pi}{2^2}\right) + i\sin\left(\frac{\pi}{2^2}\right)\right)......\infty$$

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320. Solve for z, i.e. find all complex numbers z which satisfy $|z|^2 - 2iz + 2c(1 + i) = 0$ where c is real.

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321. If α, β are the roots of $x^2 - px + q = 0$ and α', β' are the roots of $x^2 - p'x + q' = 0$, then the value of $(\alpha - \alpha')^2 + (\beta + \alpha')^2 + (\alpha - \beta')^2 + (\beta - \beta')^2$ is

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322. If a, b are complex numbers and one of the roots of the equation $x^2 + ax + b = 0$ is purely real, whereas the other is purely imaginary, prove that $a^2 - (\bar{a})^2 = 4b$.

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323. If $|z_1| = |z_2| = 1$, then prove that $|z_1 + z_2| = \left|\frac{1}{z_1} + \frac{1}{z_2}\right|$

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324. If $(ax^2 + c)y + (a'x^2 + c') = 0$ and x is a rational function of y and ac is negative, then

a. $ac' + c'c = 0$

b. $a/a' = c/c'$

c. $a^2 + c^2 = a'^2 + c'^2$

d. $aa' + cc' = 1$

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325. Find the principal argument of the complex number

$\frac{\sin(6\pi)}{5} + i\left(1 + \frac{\cos(6\pi)}{5}\right)$.

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326. For $x \in (0, 1)$, prove that $i^{2i+3}\ln\left(\frac{i^3x^2 + 2x + i}{ix^2 + 2x + i^3}\right) = \frac{1}{e^\pi}\left(\pi - 4\tan^{-1}x\right)$

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327. The sum of the non-real root of $\left(x^2 + x - 2\right)\left(x^2 + x - 3\right) = 12$ is -1 b. 1

c. -6 d. 6

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328. If n is a positive integer, prove that $\left|Im\left(z^n\right)\right| \leq n|Im(z)||z|^{n-1}$.

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329. The number of roots of the equation $\sqrt{x-2}\left(x^2-4x+3\right)=0$ is (A) Three (B) Four (C) One (D) Two

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330. If $z_1, z_2 and z_3, z_4$ are two pairs of conjugate complex numbers, then find the value of $\arg\left(z_1/z_4\right)+arg\left(z_2/z_3\right)^{.}$

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331. Prove that following inequalities: $\left|\frac{z}{|z|}-1\right| \le |argz|$ (ii) $|z-1| \le |z| + ||z|-1|$

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332. If $x = 1 + i$ is a root of the equation $= x^3 - ix + 1 - i = 0$, then the other real root is 0 b. 1 c. -1 d. none of these

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333. Find the modulus, argument, and the principal argument of the complex numbers. $\dfrac{i - 1}{i\left(1 - \cos\left(\frac{2\pi}{5}\right)\right) + \sin\left(\frac{2\pi}{5}\right)}$

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334. Find the principal argument of the complex number $\dfrac{(1 + i)^5\left(1 + \sqrt{3i}\right)^2}{-2i\left(-\sqrt{3} + i\right)}$

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335. Column I, Column II: possible argument of $z = a + ib$ $ab > 0$, p. $\tan^{-1}\left|\frac{b}{a}\right|$ $ab < 0$, q. $\pi\tan^{-1}\left|\frac{b}{a}\right|$ $a^2 + b^2 = 0$, r. $\frac{\tan^{-1}b}{a}$ $ab = 0$, s. $\pi + \frac{\tan^{-1}b}{a}$, t. not defined , u. 0 or $\frac{\pi}{2}$

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336. If the expression $x^2 + 2(a + b + c) + 3(bc + c + ab)$ is a perfect square, then $a = b = c$ b. $a = \pm b = \pm c$ c. $a = b \neq c$ d. $noneofthese$

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337. Find the point of intersection of the curves $arg(z - 3i) = \frac{3\pi}{4} andarg(2z + 1 - 2i) = \pi/4.$

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338. The curve $y = (\lambda + 1)x^2 + 2$ intersects the curve $y = \lambda x + 3$ in exactly one point, if λ equals $\{-2, 2\}$ b. $\{1\}$ c. $\{-2\}$ d. $\{2\}$

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339. Column I, Column II (one of the values of z) $z^4 - 1 = 0$, p. $z = \frac{\cos\pi}{8} + i\frac{\sin\pi}{8}$ $z^4 + 1 = 0$, q. $z = \frac{\cos\pi}{8} - i\frac{\sin\pi}{8}$ $iz^4 + 1 = 0$, r. $z = \frac{\cos\pi}{4}i\frac{\sin\pi}{4}$ $iz^4 - 1 = 0$, s. $z = \cos 0 + i\sin 0$

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340. Let $z and w$ be two nonzero complex numbers such that $|z| = |w| and arg(z) + arg(w) = \pi$ Then prove that $z = -\bar{w}$

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341. The number of irrational roots of the equation $\frac{4x}{x^2 + x + 3} + \frac{5x}{x^2 - 5x + 3} = -\frac{3}{2}$ is (a)4 b. 0 c. 1 d. 2

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342. If $\left|z + \bar{z}\right| + \left|z - \bar{z}\right| = 2$ then z lies on (a) a straight line (b) a set of four lines (c) a circle (d) None of these

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343. If one vertex of the triangle having maximum area that can be inscribed in the circle $|z - i| = 5is3 - 3i$, then find the other vertices of the triangle.

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344. The number of complex numbers z satisfying $|z - 3 - i| = |z - 9 - i| and |z - 3 + 3i| = 3$ are a. one b. two c. four d. none of these

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345. If the equation $x^2 - 3px + 2q = 0 and x^2 - 3ax + 2b = 0$ have a common roots and the other roots of the second equation is the reciprocal of the other roots of the first, then $(2 - 2b)^2$. a.$36pa(q - b)^2$ b. $18pa(q - b)^2$ c. $36bq(p - a)^2$ d. $18bq(p - a)^2$

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346. Solve the equation $3^{x^2 - x} + 4^{x^2 - x} = 25$.

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347. If t and c are two complex numbers such that $|t| \neq |c|, |t| = 1 and z = \frac{at + b}{t - c}, z = x + iy$. Locus of z is (where a, b are complex numbers) a. line segment b. straight line c. circle d. none of these

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348. Consider the circle $|z| = r$ in the Argand plane, which is in fact the incircle of triangle ABC. If contact points opposite to the vertices A, B, C are $A_1(z_1), B(z_2) and C_1(z_3)$, obtain the complex numbers associated with the vertices A, B, C in terms of $z_1, z_2 and z_3$.

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349. Solve the equation $12x^4 - 56x^3 + 89x^2 - 56x + 12 = 0$.

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350. P is a point on the argand diagram on the circle with OP as diameter two points taken such that $\angle POQ = \angle QOR = \theta$. If O is the origin and P, Q, R are are represented by complex z_1, z_2, z_3 respectively then show that $z_2^2\cos 2\theta = z_1 z_3 \cos^2\theta$

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351. Locus of z if $arg[z - (1 + i)] = \{(3\pi/4 when|z| < = |z - 2|), (-\pi/4 when|z| > |z - 4|)\}$ is straight lines passing through (2, 0) straight lines passing through (2, 0) (1, 1) a line segment a set of two rays

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352. Solve the equation $(x + 2)(x + 3)(x + 8) \times (x + 12) = 4x^2$.

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353. Given $\alpha, \beta,$ respectively, the fifth and the fourth non-real roots of units, then find the value of $(1+\alpha)(1+\beta)\left(1+\alpha^2\right)\left(1+\beta^2\right)\left(1+\alpha^4\right)\left(1+\beta^4\right)$

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354. Solve the equation $(x-1)^4+(x-5)^4=82.$

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355. If the six roots of $x^6=-64$ are written in the form $a+ib$, where a and b are real, then the product ofthose roots for which $a>0$ is

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356. The maximum area of the triangle formed by the complex coordinates z, z_1, z_2 which satisfy the relations $\left|z-z_1\right|=\left|z-z_2\right|$ and

$\left|z - \frac{z_1 + z_2}{2}\right| \le r$,where $r > \left|z_1 - z_2\right|$ is

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357. Solve $\sqrt{x + 5} + \sqrt{x + 21} = \sqrt{6x + 40.}$

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358. If $z_r : r = 1, 2, 3, , 50$ are the roots of the equation $\sum_{r=0}^{50} z^r = 0$, then find the value of $\sum_{r=0}^{50} \frac{1}{z_r - 1}$

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359. The complex number associated with the vertices A, B, C of ΔABC are $e^{i\theta}, \omega, \bar{\omega}$, respectively [where $\omega, \bar{\omega}$ are the com plex cube roots of unity and $\cos\theta > Re(\omega)$], then the complex number of the point where angle bisector of A meets cumcircle of the triangle, is

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360. Evaluate $x = \sqrt{6 + \sqrt{6 + \sqrt{6 + \infty}}}$.

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361. If a complex number z satisfies $|2z + 10 + 10i| \leq 5\sqrt{3} - 5$, then the least principal argument of z is

A. $-\frac{5\pi}{6}$

B. $-\frac{11\pi}{12}$

C. $-\frac{3\pi}{4}$

D. $-\frac{2\pi}{3}$

Answer: A

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362. If $1, \alpha_1, \alpha_2, , \alpha_{n-1}$ are the nth roots of unity, prove that $\left(1 - \alpha_1\right)\left(1 - \alpha_2\right)\left(1 - \alpha_{n-1}\right) = n^{.}$ Deduce that $\frac{\sin\pi}{n}\frac{\sin(2\pi)}{n}\frac{\sin((n-1)\pi)}{n} = \frac{n}{2^{n-1}}$

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363. If $n > 1$, show that the roots of the equation $z^n = (z+1)^n$ are collinear.

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364. If the expression $ax^4 + bx^3 - x^2 + 2x + 3$ has remainder $4x + 3$ when divided by $x^2 + x - 2,$ find the value of $aandb^{.}$

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365. If $\left|z_2 + iz_1\right| = \left|z_1\right| + \left|z_2\right|$ and $\left|z_1\right| = 3$ and $\left|z_2\right| = 4$, then the area of $\triangle ABC$, if affixes of $A, B,$ and C are $z_1, z_2,$ and $\left[\frac{z_2 - iz_1}{1 - i}\right]$ respectively, is

A. $\frac{5}{2}$

B. 0

C. $\frac{25}{2}$

D. $\frac{25}{4}$

Answer: D

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366. What is the locus of w if $w = \frac{3}{z} and |z - 1| = 1$?

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367. If z is complex number, then the locus of z satisfying the condition $|2z - 1| = |z - 1|$ is (a)perpendicular bisector of line segment joining 1/2 and 1 (b)circle (c)parabola (d)none of the above curves

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368. Find the remainder when $x^3 + 4x^2 - 7x + 6$ is divided by $x - 1$.

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369. What is the locus of z if $\left|\left|z - \cos^{-1}\cos 12\right| - \left|z - \sin^{-1}s \in 12\right|\right| = 8(\pi - 3)$?

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370. Use the factor theorem to find the value of k for which $(a + 2b), wherea, b \neq 0$ is a factor of $a^4 + 32b^4 + a^3b(k + 3)$

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371. If z is a complex number lying in the fourth quadrant of Argand plane and $\left|\left[\frac{kz}{k+1}\right]+2i\right|>\sqrt{2}$ for all real value of$k(k\neq -1)$, then range of $\arg(z)$ is $\left(\frac{\pi}{8},0\right)$ b. $\left(\frac{\pi}{6},0\right)$ c. $\left(\frac{\pi}{4},0\right)$ d. none of these

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372. If $z=(\lambda+3)+i\sqrt{5-\lambda^2}$ then the locus of Z is

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373. Let z be a complex number having the argument θ ,$0<\theta<\frac{\pi}{2}$, and satisfying the equation$|z-3i|=3$. Then find the value of $\cot\theta-\frac{6}{z}$

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374. Given that $x^2 + x - 6$ is a factor of $2x^4 + x^3 - ax^2 + bx + a + b - 1$, find the value of a and b.

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375. If z is any complex number such that $|3z - 2| + |3z + 2| = 4$, then identify the locus of z.

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376. If p, q, r are positive and are in A.P., the roots of quadratic equation $px^2 + qx + r = 0$ are all real for a. $\left|\frac{r}{p} - 7\right| \geq 4\sqrt{3}$ b. $\left|\frac{p}{r} - 7\right| \geq 4\sqrt{3}$ c. all p and r d. no p and r

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377. $A(z_1), B(z_2), C(z_3)$ are the vertices of the triangle ABC (in anticlockwise). If $\angle ABC = \pi/4$ and $AB = \sqrt{2}(BC)$, then prove that $z_2 = z_3 + i(z_1 - z_3)$.

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378. If $\left|z^2 - 1\right| = |z|^2 + 1$, then z lies on (a) The Real axis (b)The imaginary axis (c)A circle (d)An ellipse

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379. $A(z_1), B(z_2), C(z_3)$ are the vertices of he triangle ABC (in anticlockwise). If $\angle ABC = \pi/4$ and $AB = \sqrt{2}(BC)$, then prove that $z_2 = z_3 + i(z_1 - z_3)$.

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380. The number of points of intersection of two curves $y = 2\sin x and y = 5x^2 + 2x + 3 is$ 0 b. 1 c. 2 d. ∞

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381. If $|z| = 1,$ then the point representing the complex number $-1 + 3z$ will lie on a. a circle b. a parabola c. a straight line d. a hyperbola

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382. If one vertex of a square whose diagonals intersect at the origin is $3(\cos\theta + i\sin\theta)$, then find the two adjacent vertices.

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383. If $\alpha and \beta$ are the roots of $x^2 + px + q = 0 and \alpha^4, \beta^4$ are the roots of $x^2 - rx + s = 0$, then the equation $x^2 - 4qx + 2q^2 - r = 0$ has always. A. one positive and one negative root B . two positive roots C . two negative roots D . cannot say anything

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384. Find the center of the are represented by $arg[(z - 3i)/(z - 2i + 4)] = \pi/4$.

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385. Let $\left|z_r - r\right| \leq r, \forall r = 1, 2, 3, \ldots, n$ Then $\left|\sum_{r=1}^{n} Z_r\right|$ is less than n b. $2n$ c. $n(n + 1)$ d. $\frac{n(n + 1)}{2}$

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386. If $a^2 + b^2 + c^2 = 1, thenab + bc + ca$ lie in the interval $\left[\frac{1}{3}, 2\right]$ b. $[-1, 2]$

c. $\left[-\frac{1}{2}, 1\right]$ d. $\left[-1, \frac{1}{2}, \right]$

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387. $z_1 andz_2$ are the roots of $3z^2 + 3z + b = 0$. if $O(0), \left(z_1\right), \left(z_2\right)$ form an equilateral triangle, then find the value of b

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388. Consider the given equation $11z^{10} + 10iz^9 + 10iz - 11 = 0$, then |z| is

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389. Let α, β be the roots of the equation $(x - a)(x - b) = c, c \neq 0$. Then the roots of the equation $(x - \alpha)(x - \beta) + c = 0$ are a, c b. b, c c. a, b d.

$a + c, b + c$

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390. If $8iz^3 + 12z^2 - 18z + 27i = 0,$ then (a). $|z| = \frac{3}{2}$ (b). $|z| = \frac{2}{3}$ (c). $|z| = 1$ (d). $|z| = \frac{3}{4}$

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391. Let $z_1, z_2 and z_3$ represent the vertices $A, B, and C$ of the triangle ABC, respectively, in the Argand plane, such that $\left|z_1\right| = \left|z_2\right| = \left|z_3\right| = 5.$ Prove that $z_1 \sin 2A + z_2 \sin 2B + z_3 \sin 2C = 0.$

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392. Let a, b, c be real numbers, $a \neq 0.$ If α is a zero of $a^2x^2 + bx + c = 0, \beta$ is the zero of $a^2x^2 - bx - c = 0$ and $0 < \alpha < \beta$ then prove that the equation $a^2x^2 + 2bx + 2c = 0$ has a root γ that always satisfies $\alpha < \gamma < \beta.$

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393. If $\left(x^2 + px + 1\right)$ is a factor of $\left(ax^3 + bx + c\right), then\ a^2 + c^2 = -ab$ b. $a^2 - c^2 = -ab$ c. $a^2 - c^2 = ab$ d. none of these

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394. If $|z| < \sqrt{2} - 1, then\left|z^2 + 2z\cos\alpha\right| is$ a. less than 1 b. $\sqrt{2} + 1$ c. $\sqrt{2} - 1$ d. none of these

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395. On the Argand plane $z_1, z_2 and z_3$ are respectively, the vertices of an isosceles triangle ABC with $AC = BC$ and equal angles are θ. If z_4 is the incenter of the triangle, then prove that $\left(z_2 - z_1\right)\left(z_3 - z_1\right) = (1 + \sec\theta)\left(z_4 - z_1\right)^2$.

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396. If complex number $z(z \neq 2)$ satisfies the equation $z^2 = 4z + |z|^2 + \frac{16}{|z|^3}$,then the value of $|z|^4$ is_____.

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397. Both the roots of the equation $(x - b)(x - c) + (x - a)(x - c) + (x - a)(x - b) = 0$ are always a. positive b. real c. negative d. none of these

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398. Find the locus of the points representing the complex number z for which$|z + 5|^2 - |z - 5|^2 = 10$.

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399. The equation $x - \frac{2}{x - 1} = 1 - \frac{2}{x - 1}$ has a. no root b. one root c. two equals roots d. Infinitely many roots

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400. Identify the locus of z if $\bar{z} = \bar{a} + \frac{r^2}{z - a}$.

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401. If the expression $(1 + ir)^3$ is of the form of $s(1 + i)$ for some real 's' where 'r' is also real and $i = \sqrt{-1}$

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402. Two towns A and B are 60 km a part. A school is to be built to serve 150 students in town A and 50 students in town B. If the total distance to be travelled by all 200 students is to be as small as possible, then the school be built be (a) town B (b) 45 k from town A (c) town A (d) 45 km from town B

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403. Find the amplitude of $\sin\alpha + i(1 - \cos\alpha)$

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404. Modulus of non zero complex number z satisfying $\bar{z} + z = 0$ and $|z|^2 - 4zi = z^2$ is ________.

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405. Find the condition on a, b, c, d such that equations $2ax^3 + bx^2 + cx + d = 0$ and $2ax^2 + 3bx + 4c = 0$ have a common root.

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406. Let $z = 9 + bi, whereb$ is nonzero real and $i^2 = -1$. If the imaginary part of $z^2 and z^3$ are equal, then $\frac{b}{3}$ is ______.

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407. If $z_1 and z_2$ are two complex numbers and $c > 0$, then prove that $|z_1 + z_2|^2 \leq (1 + c)|z_1|^2 + (1 + c^{-1})|z_2|^2$.

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408. Let $f(x), g(x)$, and $h(x)$ be the quadratic polynomials having positive leading coefficients and real and distinct roots. If each pair of them has a common root, then find the roots of $f(x) + g(x) + h(x) = 0$.

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409. Find the minimum value of $|z - 1$ if $||z - 3| - |z + 1|\ |\ = 2$.

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410. If $x = \omega - \omega^2 - 2$ then , the value of $x^4 + 3x^3 + 2x^2 - 11x - 6$ is (where ω is a imaginary cube root of unity)

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411. If a, b, c be the sides of ABC and equations $ax^2 + bx + c = 0$ and $5x^2 + 12x + 13 = 0$ have a common root, then find $\angle C^{\cdot}$

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412. Find the greatest and the least value of $\left|z_1 + z_2\right|$ if $z_1 = 24 + 7iand\left|z_2\right| = 6.$

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413. If the complex numbers x and y satisfy $x^3 - y^3 = 98i and x - y = 7i, then xy = a + ib, where a, b, \in R$. The value of $(a + b)/3$ equals _____.

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414. If $b^2 < 2ac$, then prove that $ax^3 + bx^2 + cx + d = 0$ has exactly one real root.

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415. If z is any complex number such that $|z + 4| \leq 3$, then find the greatest value of $|z + 1|$.

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416. If z_1, z_2 and z_3, are the vertices of an equilateral triangle ABC such that $|z_1 - i| = |z_2 - i| = |z_3 - i|$.then $|z_1 + z_2 + z_3|$equals:

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417. If two roots of $x^3 - ax^2 + bx - c = 0$ are equal in magnitude but opposite in signs, then prove that $ab = c$

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418. For any complex number z find the minimum value of $|z| + |z - 2i|$

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419. The greatest positive argument of complex number satisfying $|z - 4| = Re(z)$ is

A. $\frac{\pi}{3}$

B. $\frac{2\pi}{3}$

C. $\frac{\pi}{2}$

D. $\frac{\pi}{4}$

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420. If $\alpha, \beta and \gamma$ are the roots of $x^3 + 8 = 0$ then find the equation whose roots are $\alpha^2, \beta^2 and \gamma^2$.

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421. Prove that the distance of the roots of the equation $\left|\sin\theta_1\right|z^3 + \left|\sin\theta_2\right|z^2 + \left|\sin\theta_3|z + |\sin\theta_4| = |3\right|$ from z=0 is greater than $2/3$.

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422. Let $z_1 and z_2$ be two distinct complex numbers and let $z = (1 - t)z_1 + tz_2$ for some real number t with $0 < t < 1$. If $a r g(w)$ denotes the principal argument of a nonzero complex number w, then

$|z - z_1| + |z - z_2| = |z_1 - z_2|$ $(z - z_1) = (z - z_2)$

$|z - z_1 z - (z)_1 z_2 - z_1 (z)_2 - (z)_1| = 0$

$arg(z - z_1) = arg(z_2 - z_1)$

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423. If α, β, γ are the roots of the equation $x^3 - px + q = 0,$ then find the cubic equation whose roots are $\frac{\alpha}{1 + \alpha}, \frac{\beta}{1 + \beta}, \frac{\gamma}{1 + \gamma}$.

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424. If $|z_1 - 1| \leq 1, |z_2 - 2| \leq 2, |z_3 - 3| \leq 3,$ then find the greatest value of $|z_1 + z_2 + z_3|$.

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425. Let w = $\left(\sqrt{3} + \frac{\iota}{2}\right)$ and $P = \left\{w^n : n = 1, 2, 3, \ldots .\right\}$, Further $H_1 = \left\{z \in C : Re(z) > \frac{1}{2}\right\}$ and $H_2 = \left\{z \in c : Re(z) < -\frac{1}{2}\right\}$ Where C is set of all complex numbers. If $z_1 \in P \cap H_1, z_2 \in P \cap H_2$ and O represent the origin, then $\angle Z_1OZ_2$ =

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426. If the roots of equation $x^3 + ax^2 + b = 0 are \alpha_1, \alpha_2$ and $\alpha_3 (a, b \neq 0)$, then find the equation whose roots are $\frac{\alpha_1\alpha_2 + \alpha_2\alpha_3}{\alpha_1\alpha_2\alpha_3}, \frac{\alpha_2\alpha_3 + \alpha_3\alpha_1}{\alpha_1\alpha_2\alpha_3}, \frac{\alpha_1\alpha_3 + \alpha_1\alpha_2}{\alpha_1\alpha_2\alpha_3}$

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427. If z is a complex number, then find the minimum value of $|z| + |z - 1| + |2z - 3|$.

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428. Let $|z| = 2$ and $w = \frac{z + 1}{z - 1}$, $where z, w, \in C$ (where C is the set of complex numbers). Then product of least and greatest value of modulus of w is________.

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429. If $\alpha, \beta and \gamma$ are roots of $2x^3 + x^2 - 7 = 0$, then find the value of $\sum \left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)$.

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430. if z is complex no satisfies the condition | Z |>3. Then find the least value of | Z+ 1/ Z |

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431. If α is the nth root of unity, then $1 + 2\alpha + 3\alpha^2 + \rightarrow n$ terms equal to

a. $\frac{-n}{(1 - \alpha)^2}$ b. $\frac{-n}{1 - \alpha}$ c. $\frac{-2n}{1 - \alpha}$ d. $\frac{-2n}{(1 - \alpha)^2}$

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432. Let $r, s, andt$ be the roots of equation $8x^3 + 1001x + 2008 = 0$. Then find the value of $(r + s)^3 + (s + t)^3 + (t + r)^3$.

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433. Locate the region in the Argand plane determined by $z^2 + z^2 + 2\left|z^2\right| < (8i(z - z))$.

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434. Given z is a complex number with modulus 1. Then the equation $\left[\frac{1 + ia}{1 - ia}\right]^4 = z$ has all roots real and distinct two real and two imaginary three roots two imaginary one root real and three imaginary

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435. The number of value of k for which $\left[x^2 - (k - 2)x + k^2\right] \times \left[x^2 + kx + (2k - 1)\right]$ is a perfect square is a.2 b. 1 c. 0 d. none of these

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436. For any complex number z prove that $|Re(z)| + |Im(z)| \leq \sqrt{2}|z|$

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437. The point $z_1 = 3 + \sqrt{3}i$ and $z_2 = 2\sqrt{3} + 6i$ are given on a complex plane. The complex number lying on the bisector of the angel formed by the vectors $z_1 and z_2$ is $z = \frac{\left(3+2\sqrt{3}\right)}{2} + \frac{\sqrt{3}+2}{2} i$ $z = 5 + 5i$ $z = -1 - i$ none of these

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438. The total number of integral values of a so that $x^2 - (a+1)x + a - 1 = 0$ has integral roots is equal to a. 1 b. 2 c. 4 d. none of these

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439. If $w = \frac{z}{z - \left(\frac{1}{3}\right)i}$ and $|w| = 1$, then find the locus of z

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440. Let C_1 and C_2 be two circles with C_2 lying inside C_1 A circle C lying inside C_1 touches C_1 internally and C_2 externally. Identify the locus of the centre of C

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441. The number of positive integral solutions of $x^4 - y^4 = 3789108$ is a.0

b. 1 c. 2 d. 4

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442. The region of argand diagram defined by $|z - 1| + |z + 1| \leq 4$ (1) interior of an ellipse (2) exterior of a circle (3) interior and boundary of an ellipse (4) none of these

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443. z_1, z_2, z_3, z_4 are distinct complex numbers representing the vertices of a quadrilateral $ABCD$ taken in order. If

$z_1 - z_4 = z_2 - z_3 and a r g\left[\left(z_4 - z_1\right)/\left(z_2 - z_1\right)\right] = \pi/2$, the quadrilateral is a. rectangle b. rhombus c. square d. trapezium

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444. If α, β are the roots of $x^2 + px + q = 0 and x^{2n} + p^n x^n + q^n = 0 and if (\alpha/\beta), (\beta/\alpha)$ are the roots of $x^n + 1 + (x + 1)^n = 0, the \cap (\in N)$ a. must be an odd integer b. may be any integer c. must be an even integer d. cannot say anything

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445. If $(\log)_{\sqrt{3}}\left(\frac{|z|^2 - |z| + 1}{2 + |z|}\right) > 2,$ then locate the region in the Argand plane which represents $\dot{z}$

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446. If $z = \frac{\left(1 + i\sqrt{3}\right)^2}{4i\left(1 - i\sqrt{3}\right)}$ is a complex number then a. $arg(z) = \frac{\pi}{4}$ b. $arg(z) = \frac{\pi}{2}$ c. $|z| = \frac{1}{2}$ d. $|z| = 2$

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447. If α, β, γ are such that $\alpha + \beta + \gamma = 2, \alpha^2 + \beta^2 + \gamma^2 = 6, \alpha^3 + \beta^3 + \gamma^3 = 8, then \alpha^4 + \beta^4 + \gamma^4$ is a. 18 b. 10 c. 15 d. 36

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448. If $z = \frac{3}{2 + \cos\theta + \text{i}\sin\theta}$ then locus of z is straight line a circle having center on the y-axis a parabola a circle having center on the x-axis

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449. If $z = x + iy$ such that $|z + 1| = |z - 1|$ and $arg\left(\frac{z - 1}{z + 1}\right) = \frac{\pi}{4}$, then find z.

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450. If $xy = 2(x + y), x \le y$ and $x, y \in N,$ then the number of solutions of the equation are a. two b. three c. no solution d. infinitely many solutions

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451. If $Im\left(\frac{z - 1}{e^{\theta i}} + \frac{e^{\theta i}}{z - 1}\right) = 0$, then find the locus of z

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452. If $pandq$ are distinct prime numbers, then the number of distinct imaginary numbers which are pth as well as qth roots of unity are. a. $\min(p, q)$ b. $\min(p, q)$ c. 1 d. $zero$

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453. The number of real solutions of the equation $(9/10)^x = -3 + x - x^2$ is

a. 2 b. 0 c. 1 d. none of these

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454. What is locus of z if $\left|z - 1 - \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)\right| + \left|z + \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) - \frac{\pi}{2}\right| = 1$?

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455. If $|z - 2 - i| = |z|\sin\left(\frac{\pi}{4} - argz\right)|$, where $i = \sqrt{-1}$,then locus of z, is

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456. The number of integral values of a for which the quadratic equation $(x + a)(x + 1991) + 1 = 0$ has integral roots are a. 3 b.0 c.1 d. 2

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457. ω is an imaginary root of unity.

Prove that

(i) $\left(a + b\omega + c\omega^2\right)^3 + \left(a + b\omega^2 + c\omega\right)^3 = (2a - b - c)(2b - a - c)(2c - a - b)$

(ii) If $a + b + c = 0$ then prove that $\left(a + b\omega + c\omega^2\right)^3 + \left(a + b\omega^2 + c\omega\right)^3 = 27abc.$

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458. If z is a complex number having least absolute value and $|z - 2 + 2i| = 1$,then $z =$ $\left(2 - 1/\sqrt{2}\right)(1 - i)$ b. $\left(2 - 1/\sqrt{2}\right)(1 + i)$ c. $\left(2 + 1/\sqrt{2}\right)(1 - i)$ d. $\left(2 + 1/\sqrt{2}\right)(1 + i)$

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459. If the equation $\cot^4 x - 2\operatorname{cosec}^2 x + a^2 = 0$ has at least one solution, then the sum of all possible integral values of a is equal to a. 4 b. 3 c. 2 d. 0

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460. Which of the following is equal to $\sqrt[3]{-1}$ a. $\dfrac{\sqrt{3}+\sqrt{-1}}{2}$ b. $\dfrac{-\sqrt{3}+\sqrt{-1}}{\sqrt{-4}}$ c. $\dfrac{\sqrt{3}-\sqrt{-1}}{\sqrt{-4}}$ d. $-\sqrt{-1}$

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461. ω is an imaginary root of unity. Prove that $\left(a+b\omega+c\omega^2\right)^3+\left(a+b\omega^2+c\omega^{\square}\right)^3=(2a-b-c)(2b-a-c)(2c-a-b)$.

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462. The number of real solutions of $|x| + 2\sqrt{5 - 4x - x^2} = 16$ is/are a. 6 b. 1 c. 0 d. 4

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463. If $|z - 1| + |z + 3| \leq 8$, then prove that z lies on the circle.

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464. If $z_1 and z_2$ are the complex roots of the equation $(x - 3)^3 + 1 = 0, then z_1 + z_2$ equal to 1 b. 3 c. 5 d. 7

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465. If the quadratic equation $ax^2 + bx + 6 = 0$ does not have real roots and $b \in R^+$, then prove that $a > max\left\{\frac{b^2}{24}, b - 6\right\}$

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466. If the equation $|z - a| + |z - b| = 3$ represents an ellipse and $a, b \in C, where a$ is fixed, then find the locus of b

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467. If $\left|z^2 - 3\right| = 3|z|$, then the maximum value of $|z|$ is a.1 b. $\frac{3 + \sqrt{21}}{2}$ c. $\frac{\sqrt{21} - 3}{2}$ d. none of these

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468. What is the minimum height of any point on the curve $y = x^2 - 4x + 6$ above the x-axis?

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469. Find the locus of point z if z , i ,and iz , are collinear.

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470. If $|z - 1| \leq 2 and \left|\omega z - 1 - \omega^2\right| = a$ where ω is cube root of unity , then complete set of values of a is $a. 0 \leq a \leq 2$ b. $\frac{1}{2} \leq a \leq \frac{\sqrt{3}}{2}$ c. $\frac{\sqrt{3}}{2} - \frac{1}{2} \leq a \leq \frac{1}{2} + \frac{\sqrt{3}}{2}$ d. $0 \leq a \leq 4$

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471. What is the maximum height of any point on the curve $y = - x^2 + 6x - 5$ above the x-axis?

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472. Consider an ellipse having its foci at $A\left(z_1\right) and B\left(z_2\right)$ in the Argand plane. If the eccentricity of the ellipse be e and it is known that origin is an interior point of the ellipse, then prove that $e \in \left(0, \frac{|z_1 - z_2|}{|z_1| + |z_2|}\right)$

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473. The roots of the cubic equation $(z + ab)^3 = a^3$, such that $a \neq 0$, respresent the vertices of a trinagle of sides of length

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474. Find the largest natural number a for which the maximum value of $f(x) = a - 1 + 2x - x^2$ is smaller than the minimum value of $g(x) = x^2 - 2ax + 10 - 2a$.

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475. In the Argands plane what is the locus of $z(\neq 1)$ such that

$$arg\left\{\frac{3}{2}\left(\frac{2z^2 - 5z + 3}{3z^2 - z - 2}\right)\right\} = \frac{2\pi}{3}\cdot$$

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476. If ω is a complex nth root of unity, then $\sum_{r=1}^{n}(ar + b)\omega^{r-1}$ is equal to

A.. $\frac{n(n+1)a}{2}$

B. $\frac{nb}{1+n}$

C. $\frac{na}{\omega - 1}$

D. none of these

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477. Let $f(x) = ax^2 + bx + c$ be a quadratic expression having its vertex at (3, -2) and value of $f(0) = 10$. Find $f(x)$.

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478. If $|z| = 2 and \dfrac{z_1 - z_3}{z_2 - z_3} = \dfrac{z - 2}{z + 2}$, then prove that z_1, z_2, z_3 are vertices of a right angled triangle.

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479. If $\left|\dfrac{z_1}{z_2}\right| = 1$ and $arg\left(z_1 z_2\right) = 0$, then a. $z_1 = z_2$ b. $\left|z_2\right|^2 = z_1 \cdot z_2$ $c.\ z_1 \cdot z_2 = 1$ d. none of these

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480. Find the least value of n such that $(n - 2)x^2 + 8x + n + 4 > 0,\ \forall x \in R, where n \in N$

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481. The common roots of the equation $Z^3 + 2Z^2 + 2Z + 1 = 0$ and $Z^{1985} + Z^{100} + 1 = 0$ are

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482. If $z_1 + z_2 + z_3 + z_4 = 0$ where $b_i \in R$ such that the sum of no two values being zero and $b_1z_1 + b_2z_2 + b_3z_3 + b_4z_4 = 0$ where z_1, z_2, z_3, z_4 are arbitrary complex numbers such that no three of them are collinear, prove that the four complex numbers would be concyclic if $|b_1b_2||z_1 - z_2|^2 = |b_3b_4||z_3 - z_4|^2$.

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483. If the inequality $(mx^2 + 3x + 4)/(x^2 + 2x + 2) < 5$ is satisfied for all $x \in R$, then find the value of m

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484. If $|(z - 2)/(z - 3)| = 2$ represents a circle, then find its radius.

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485. If z_1 is a root of the equation $a_0 z^n + a_1 z^{n-1} + \ldots\ldots . + (a_{n-1})z + a_n = 3$, where $|a_i| < 2$ for $i = 0, 1, \ldots . n$, then (a). $|z| = \frac{3}{2}$ (b). $|z| < \frac{1}{4}$ (c). $|z| > \frac{1}{4}$ (d). $|z| > \frac{1}{3}$

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486. If $f(x) = (a_1 x + b_1)^2 + (a_2 x + b_2)^2 + \ldots + (a_n x + b_n)^2$, then prove that $(a_1 b_1 + a_2 b_2 + \ldots + a_n b_n)^2 \leq (a_1^2 + a_2^2 + \ldots + a_n^2)(b_1^2 + b_2^2 + . . + b_n^2)$.

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487. If the imaginary part of $(2z + 1)/(iz + 1)$ is -2, then find the locus of the point representing in the complex plane.

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488. If $|2z - 1| = |z - 2| and z_1, z_2, z_3$ are complex numbers such that $\left|z_1 - \alpha\right| < \alpha, \left|z_2 - \beta\right| < \beta$, then= $\left|\frac{z_1 + z_2}{\alpha + \beta}\right|$a) $< |z|$b. $< 2|z|c.$ $> |z|$ d. $> 2|z|$

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489. If c is positive and $2ax^2 + 3bx + 5c = 0$ does not have any real roots, then prove that $2a - 3b + 5b > 0$.

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490. Find the number of complex numbers which satisfies both the equations $|z - 1 - i| = \sqrt{2} and |z + 1 + i| = 2$.

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491. Let ω be the complex number $\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)$. Then the number of distinct complex cos numbers z satisfying

$$\Delta = \begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0 \text{ is}$$

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492. If $ax^2 + bx + 6 = 0$ does not have distinct real roots, then find the least value of $3a + b^{\cdot}$

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493. $|z - 2 - 3i|^2 + |z - 4 - 3i|^2 = \lambda$ represents the equation of the circle with least radius. find the value of λ

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494. Match the statements/expressions given in column I with the values given in Column II. Column I, Column II In R^2, if the magnitude of the projection vector of the vector $\alpha\hat{i} + \beta\hat{j}$ on $\sqrt{3}\hat{i} + \hat{j} is \sqrt{3}$ and if $|\alpha|$ is /are, (p) 1 Let $a and b$ be real numbers such that the function $f(x) = \{ -3ax^2 - 2, x < 1bx + a^2, x \geq 1$ Differentiable for all $x \in R$ Then possible value (s) of a is/are, (q) 2 Let $\omega \neq 1$ be a complex cube root of unity. If $\left(3 - 3\omega + 2\omega^2\right)^{4n+3} + \left(2 + 3\omega - 3\omega^2\right)^{4n+3} + \left(-3 - 2\omega + 3\omega^2\right)^{4n+3} = 0$, then possible values (s) of n is /are, (r) 3 Let the harmonic mean of two positive real numbers $a and b$ be 4. If q is a positive real number such that $a, 5, q, b$ is an arithmetic progressin, then the values $(s) of |q - a|$ is /are, (s) 4 , (t) 5

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495. A quadratic trinomial $P(x) = ax^2 + bx + c$ is such that the equation $P(x) = x$ has no real roots. Prove that in this case equation $P(P(x)) = x$ has no real roots either.

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496. If $\left(\sqrt{8}+i\right)^{50}=3^{49}(a+ib)$, then find the value of a^2+b^2.

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497. Let $a, b, c \in Q^+$ satisfying $a > b > c$. Which of the following statement(s) hold true of the quadratic polynomial $f(x)=(a+b-2c)x^2+(b+c-2a)x+(c+a-2b)$? a. The mouth of the parabola $y=f(x)$ opens upwards b. Both roots of the equation $f(x)=0$ are rational c. The x-coordinate of vertex of the graph is positive d. The product of the roots is always negative

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498. Find number of values of complex numbers ω satisfying the system of equation $z^3=-\left(\bar{\omega}\right)^7$ and $z^5 . \omega^{11}=1$

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499. Match the statements in column-I with those I column-II [Note: Here z takes the values in the complex plane and $Im(z) and Re(z)$ denote, respectively, the imaginary part and the real part of z] Column I, Column II: The set of points z satisfying $|z - i|z|| - |z + i|z| \,|\, | = 0$ is contained in or equal to, p. an ellipse with eccentricity 4/5 The set of points z satisfying $|z + 4| + |z - 4| = 10$ is contained in or equal to, q. the set of point z satisfying $Imz = 0$ If $|\omega| = 1$, then the set of points $z = \omega + 1/\omega$ is contained in or equal to, r. the set of points z satisfying $|Imz| \leq 1$, s. the set of points z satisfying $|Rez| \leq 1$, t. the set of points z satisfying $|z| \leq 3$

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500. If $x, y \in R$ satify the equation $x^2 + y^2 - 4x - 2y + 5 = 0$, then the value of the expression $\left[\left(\sqrt{x} - \sqrt{y}\right)^2 + 4\sqrt{xy}\right] / \left(x + \sqrt{xy}\right)$ is $\sqrt{2} + 1$ b. $\frac{\sqrt{2} + 1}{2}$ c. $\frac{\sqrt{2} - 1}{2}$ d. $\frac{\sqrt{2} + 1}{\sqrt{2}}$

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501. If $|z - iRe(z)| = |z - Im(z)|$, then prove that z , lies on the bisectors of the quadrants.

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502. For any integer k, let $\alpha_k = \cos\left(\frac{k\pi}{7}\right) + i\sin\left(\frac{k\pi}{7}\right), where i = \sqrt{-1}.$ Value of the expression $\frac{\sum_{k=1}^{12}\left|\alpha_{k+1} - \alpha_k\right|}{\sum_{k=1}^{3}\left|\alpha_{4k-1} - \alpha_{4k-2}\right|}$ is

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503. If $x = 1 + \frac{1}{3 + \frac{1}{2 + \frac{1}{3 + \frac{1}{2}}}}$ a $\frac{52}{2}$ b. $\frac{55}{71}$ c. $\frac{60}{52}$ d. $\frac{71}{55}$

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504. Show that $\left(x^2+y^2\right)^4 = \left(x^4-6x^2y^2+y^4\right)^2 + \left(4x^3y-4xy^3\right)^2.$

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505. Let $\omega = e^{\frac{i\pi}{3}}$ and a, b, c, x, y, z be non-zero complex numbers such that $a+b+c=x,\ a+b\omega+c\omega^2=y,\ a+b\omega^2+c\omega=z$.Then, the value of $\dfrac{|x|^2+|y|^2|+|y|^2}{|a|^2+|b|^2+|c|^2}$

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506. Find the values of a for which all the roots of the euation $x^4-4x^3-8x^2+a=0$ are real.

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507. If z is any complex number satisfying $|z-3-2i| \le 2$ then the maximum value of $|2z-6+5i|$ is

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508. Let $\left|\left(\left(\bar{z}_1\right) - 2\left(\bar{z}_2\right)\right)/\left(2 - z_1\left(\bar{z}_2\right)\right)\right| = 1$ and $\left|z_2\right| \neq 1$,where z_1 and z_2 are complex numbers. Show that $\left|z_1\right| = 2$.

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509. If $x = 2 + 2^{2/3} + 2^{1/3}$, then the value of $x^3 - 6x^2 + 6x$ is (a)3 b. 2 c. 1 d. -2

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510. Let $1, w, w^2$ be the cube root of unity. The least possible degree of a polynomial with real coefficients having roots $2w, (2 + 3w), \left(2 + 3w^2\right), \left(2 - w - w^2\right)$ is ____________.

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511. If $z_1 and z_2$ are complex numbers and $u = \sqrt{z_1 z_2}$, then prove that

$$|z_1| + |z_2| = \left|\frac{z_1 + z_2}{2} + u\right| + \left|\frac{z_1 + z_2}{2} - u\right|$$

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512. The least value of the expression $x^2 + 4y^2 + 3z^2 - 2x - 12y - 6z + 14$ is

a.1 b. no least value c. 0 d. none of these

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513. If ω is an imaginary cube root of unity, then $\left(1 + \omega - \omega^2\right)^7$ is equal to

128ω (b) -128ω $128\omega^2$ (d) $-128\omega^2$

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514. If $|z| = 1$ and let $\omega = \frac{(1 - z)^2}{1 - z^2}$, then prove that the locus of ω is equivalent to $|z - 2| = |z + 2|$

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515. If $x = 2 + 2^{2/3} + 2^{1/3}$, then the value of $x^3 - 6x^2 + 6x$ is

A. a. 3

B. b. 2

C. c. 1

D. d. -2

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516. Let $z = x + iy$. Then find the locus of $P(z)$ such that $\frac{1 + \bar{z}}{z} \in R$.

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517. $\frac{(\cos\theta + i\sin\theta)^4}{(\sin\theta + i\cos\theta)^5}$ is equal to.

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518. Find the values of k for which $\left|\frac{x^2 + kx + 1}{x^2 + x + 1}\right| < 2, \forall x \in R$

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519. Identify locus z if $Re(z + 1) = |z - 1|$

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520. If z is a complex number satisfying $z^4 + z^3 + 2z^2 + z + 1 = 0$ then the set of possible values of z is

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521. Solve the equation $\sqrt{a\left(2^x - 2\right) + 1} = 1 - 2^x, x \in R$.

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522. If $\left|z_1\right| = 1, \left|z_2\right| = 2, \left|z_3\right| = 3, and \left|9z_1z_2 + 4z_1z_3 + z_2z_3\right| = 12,$ then find the value of $\left|z_1 + z_2 + z_3\right|$.

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523. Let $Z_1 = (8 + i)\sin\theta + (7 + 4i)\cos\theta$ and $Z_2 = (1 + 8i)\sin\theta + (4 + 7i)\cos\theta$ are two complex numbers. If $Z_1 \cdot Z_2 = a + ib$ where $a, b \in R$ then the largest value of $(a + b)\forall\theta \in R$, is

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524. For $a < 0$, determine all real roots of the equation $x^2 - 2a|x - a| - 3a^2 = 0$.

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525. Let $A = \{a \in R\}$ the equation $(1 + 2i)x^3 - 2(3 + i)x^2 + (5 - 4i)x + a^2 = 0$ has at least one real root. Then the value of $\frac{\sum a^2}{2}$ is______.

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526. Express the following in $a + ib$ form: $\frac{(\cos\alpha + i\sin\alpha)^4}{(\sin\beta + i\cos\beta)^5}$

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527. Find the root of equation $2x^2 + 10x + 20 = 0$.

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528. Suppose that z is a complex number the satisfies $|z - 2 - 2i| \leq 1$. The maximum value of $|2z - 4i|$ is equal to ______.

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529. If $1/x + x = 2\cos\theta$, then prove that $x^n + 1/x^n = 2\cos n\theta$

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530. If $ax^2 + bx + c = 0$ and $bx^2 + cx + a = 0$ have a common root and a, b, and c are nonzero real numbers, then find the value of $\left(a^3 + b^3 + c^3\right)/abc$

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531. Find the roots of the equation $2x^2 - x + \frac{1}{8} = 0$

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532. If $|z+2-i|=5$ then the maximum value of $|3z+9-7i|$ is K, then find k

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533. If $x^2+3x+5=0 and ax^2+bx+c=0$ have common root/roots and $a, b, c \in N$, then find the minimum value of $a+b+c$

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534. Find the minimum value of the expression $E=|z|^2+|z-3|^2+|z-6i|^2$ (where $z=x+iy, x, y \in R$)

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535. The area bounded by the curves $argz=\frac{\pi}{3}$ and $argz=2\frac{\pi}{3}$ and $arg\left(z-2-2i\sqrt{3}\right)=\pi$ in the argand plane is

(in sq. units)

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536. If $\alpha \neq \beta and \alpha^2 = 5\alpha - 3 and \beta^2 = 5\beta - 3.$ find the equation whose roots are $\alpha / \beta and \beta / \alpha$.

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537. Express $\dfrac{1}{1 - \cos\theta + 2i\sin\theta}$ in the form $x + iy$.

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538. a, b, c are three complex numbers on the unit circle $|z| = 1,$ such that $abc = a + b + c$. Then find the value of $|ab + bc + ca|$.

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539. If α, β are the roots of lthe equation $2x^2 - 3x - 6 = 0$, find the equation whose roots are $\alpha^2 + 2$ and $\beta^2 + 2$.

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540. If z_1, z_2, z_3 are distinct nonzero complex numbers and $a, b, c \in R^+$ such that $\frac{a}{|z_1 - z_2|} = \frac{b}{|z_2 - z_3|} = \frac{c}{|z_3 - z_1|}$ Then find the value of $\frac{a^2}{z_1 - z_2} + \frac{b^2}{z_2 - z_3} + \frac{c^2}{z_3 - z_1}$

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541. If $|z_1| = 15$ and $|z_2 - 3 - 4i| = 5$, *then*

A. a.$(|z_1 - z_2|)_{\min} = 5$

B. b. $(|z_1 - z_2|)_{\min} = 10$

C. c. $(|z_1 - z_2|)_{max} = 20$

D. d. $(|z_1 - z_2|)_{max} = 25$

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542. Determine the values o m for which equations $3x^2 + 4mx + 2 = 0 and 2x^2 + 3x - 2 = 0$ may have a common root.

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543. If $z = \frac{\left(\sqrt{3} + i\right)^{17}}{(1 - i)^{50}}$, then find $amp(z)$.

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544. A rectangle of maximum area is inscribed in the circle $|z - 3 - 4i| = 1$. If one vertex of the rectangle is $4 + 4i$, then another adjacent vertex of this rectangle can be a. $2 + 4i$ b. $3 + 5i$ c. $3 + 3i$ d. $3 - 3i$

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545. If α, β are the roots of the equation $ax^2 + bx + c = 0$, then find the roots of the equation $ax^2 - bx(x - 1) + c(x - 1)^2 = 0$ in term of α and $\dot{\beta}$

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546. If $\frac{3\pi}{2} < \alpha < 2\pi$ then the modulus argument of $(1 + \cos 2\alpha) + i\sin 2\alpha$

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547. The value of z satisfying the equation $\log z + \log z^2 + \dot{} + \log z^n = 0 is$

(a) $\frac{\cos(4m\pi)}{n(n+1)} + i\frac{\sin(4m\pi)}{n(n+1)}, m = 0, 1, 2...$

(b) $\frac{\cos(4m\pi)}{n(n+1)} - i\frac{\sin(4m\pi)}{n(n+1)}, m = 0, 1, 2...$

(c) $\frac{\sin(4m\pi)}{n(n+1)} + i\frac{\sin(4m\pi)}{n(n+1)}, m = 0, 1, 2, ...$ (d) 0

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548. If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less then $\sqrt{5}$, then find the set of possible value of a

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549. find the differtiation of (i) $\tan(\sec x)$ (ii) $\sin(\tan x)$

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550. Roots of the equation are $(z + 1)^5 = (z - 1)^5$ are

(a) $\pm i\tan\left(\frac{\pi}{5}\right), \pm i\tan\left(\frac{2\pi}{5}\right)$

(b) $\pm i\cot\left(\frac{\pi}{5}\right), \pm i\cot\left(\frac{2\pi}{5}\right)$

(c) $\pm i\cot\left(\frac{\pi}{5}\right), \pm i\tan\left(\frac{2\pi}{5}\right)$

(d)none of these

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551. Find the value of a for which one root of the quadratic equation $\left(a^2 - 5a + 3\right)x^2 + (3a - 1)x + 2 = 0$ is twice as large as the other.

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552. If $\left|z_1 - z_0\right| = \left|z_2 - z_0\right| = a$ and $amp\left(\frac{z_2 - z_0}{z_0 - z_1}\right) = \frac{\pi}{2}$, then find z_0

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553. Which of the following represents a points in an Argand pane, equidistant from the roots of the equation $(z + 1)^4 = 16z^4$? a. $(0, 0)$ b. $\left(-\frac{1}{3}, 0\right)$ c. $\left(\frac{1}{3}, 0\right)$ d. $\left(0, \frac{2}{\sqrt{5}}\right)$

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554. If the harmonic mean between roots of $\left(5+\sqrt{2}\right)x^2 - bx + 8 + 2\sqrt{5} = 0 is 4$, then find the value of b

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555. If $n \in N > 1$, then the sum of real part of roots of $z^n = (z+1)^n$ is equal to

A. a. $\frac{n}{2}$

B. b. $\frac{(n-1)}{2}$

C. c. $\frac{n}{2}$

D. d. $\frac{(1-n)}{2}$

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556. If z_1, z_2, z_3, z_4 are the affixes of four point in the Argand plane, z is the affix of a point such that $|z - z_1| = |z - z_2| = |z - z_3| = |z - z_4|$, then z_1, z_2, z_3, z_4 are

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557. Find the values of the parameter a such that the rots α, β of the equation $2x^2 + 6x + a = 0$ satisfy the inequality $\alpha/\beta + \beta/\alpha < 2$.

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558. Solve the equation $z^3 = z(z \neq 0)$

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559. If $z = \omega, \omega^2$ where ω is a non-real complex cube root of unity, are two vertices of an equilateral triangle in the Argand plane, then the third vertex may be represented by a,$z = 1$ b. $z = 0$ c. $z = -2$ d. $z = -1$

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560. Let $\alpha and \beta$ be the solutions of the quadratic equation $x^2 - 1154x + 1 = 0$, then the value of $\alpha^{\frac{1}{4}} + \beta^{\frac{1}{4}}$ is equal to ______.

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561. If $\left(\frac{1+i}{1-i}\right)^m = 1,$ then find the least positive integral value of $m^{\cdot}$

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562. If $, Z_1, Z_2, Z_3, \ldots\ldots. Z_{n-1}$ are n^{th} roots of unity then the value of $\frac{1}{3 - Z_1} + \frac{1}{3 - Z_2} + \ldots\ldots\ldots + \frac{1}{3 - Z_{n-1}}$ is equal to

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563. If $a, b, c \in R^{+} and 2b = a + c,$ then check the nature of roots of equation $ax^2 + 2bx + c = 0.$

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564. If z is a complex number such taht $z^2 = (\bar{z})^2,$ then find the location of z on the Argand plane.

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565. If $z^3 + (3 + 2i)z + (-1 + ia) = 0$ has one real roots, then the value of a lies in the interval $(a \in R)$ $(-2, 1)$ b. $(-1, 0)$ c. $(0, 1)$ d. $(-2, 3)$

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566. Determine the value of k for which $x + 2$ is a factor of $(x + 1)^7 + (2x + k)^3$.

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567. Find the complex number z satisfying $Re\left(z^2 = 0\right), |z| = \sqrt{3.}$

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568. Given that the expression $2x^3 + 3px^2 - 4x + p$ hs a remainder of 5 when divided by $x + 2$, find the value of $p^{\cdot}$

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569. z_1andz_2 are two distinct points in an Argand plane. If $a\left|z_1\right| = b\left|z_2\right|(wherea, b \in R)$, then the point $\left(az_1/bz_2\right) + \left(bz_2/az_1\right)$ is a point on the line segment [-2, 2] of the real axis line segment [-2, 2] of the imaginary axis unit circle $|z| = 1$ the line with $argz = \tan^{-1}2$

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570. Consider two complex numbers $\alpha and \beta$ as $\alpha = [(a + bi)/(a - bi)]^2 + [(a - bi)/(a + bi)]^2$, where a ,b , in R and $\beta = (z - 1)/(z + 1), where|z| = 1$, then find the correct statement: both $\alpha and \beta$ are purely real both $\alpha and \beta$ are purely imaginary α is purely real and β is purely imaginary β is purely real and α is purely imaginary

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571. In how many points the graph of $f(x) = x^3 + 2x^2 + 3x + 4$ meets the x-axis ?

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572. If $x^2 + x + 1 = 0$ then the value of $\left(x + \frac{1}{x}\right)^2 + \left(x^2 + \frac{1}{x^2}\right)^2 + \dots + \left(x^{27} + \frac{1}{x^{27}}\right)^2$ is

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573. If $(a+ib)(c+id)(e+if)(g+ih)=A+iB$, then show that $\left(a^2+b^2\right)\left(c^2+d^2\right)\left(e^2+f^2\right)\left(g^2+h^2\right)=A^2+B^2$

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574. Analyze the roots of the equation $(x-1)^3+(x-2)^3+(x-4)^3+(x-5)^3=0$ by differentiation method.

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575. Solve the equation $|z|=z+1+2i$

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576. If $z=i^{i^i}$ where $i=\sqrt{-1}$ then $|z|$ is equal to

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577. Find the values of a for which the roots of the equation $x^2 + a^2 = 8x + 6a$ are real.

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578. If α and β are different complex numbers with $|\beta| = 1$, then find $\left|\frac{\beta - \alpha}{1 - \bar{\alpha}\beta}\right|$.

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579. If $z = i\log\left(2 - \sqrt{3}\right)$, then $\cos z =$ a. -1 b. $\frac{-1}{2}$ c. 1 d. 2

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580. If $f(x) = x^3 - x^2 + ax + b$ is divisible by $x^2 - x$, then find the value of $f(2)$.

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581. If $z = x + iy$ and $w = (1 - iz)/(z - i)$ and $|w| = 1$, then show that z is purely real.

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582. If the equation $z^4 + a_1z^3 + a_2z^2 + a_3z + a_4 = 0$ where a_1, a_2, a_3, a_4 are real coefficients different from zero has a pure imaginary root then the expression $\frac{a_3}{a_1a_2} + \frac{a_1a_4}{a_2a_3}$ has the value equal to

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583. If $f(x) = x^3 - 3x^2 + 2x + a$ is divisible by $x - 1$, then find the remainder when $f(x)$ is divided by $x - 2$.

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584. If $z_1 and z_2$ are two complex numbers and $c > 0$, then prove that $|z_1 + z_2|^2 \leq (1 + c)|z_1|^2 + (1 + c^{-1})|z_2|^2$.

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585. Suppose A is a complex number and $n \in N$, such that $A^n = (A + 1)^n = 1$, then the least value of n is 3 b. 6 c. 9 d. 12

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586. Find the value of p for which $x + 1$ is a factor of $x^4 + (p - 3)x^3 - (3p - 5)x^2 + (2p - 9)x + 6$. Find the remaining factor for this value of p.

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587. If z_1, z_2, z_3 be the affixes of the vertices A, B and C of a triangle having centroid at G such ;that $z = 0$ is the mid point of AG then $4z_1 + z_2 + z_3 =$

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588. The number of complex numbers z such that $|z| = 1$ and $\left|\frac{z}{\bar{z}} + \frac{\bar{z}}{z}\right| = 1$ is $arg(z) \in [0, 2\pi)$) then a. 4 b. 6 c. 8 d. more than 8

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589. Given that $x^2 - 3x + 1 = 0,$ then the value of the expression $y = x^9 + x^7 + x^{-9} + x^{-7}$ is divisible by prime number?

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590. If $iz^4 + 1 = 0,$ then prove that z can take the value $\cos\pi/8 + i\sin\pi/8.$

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591. Find the value of x such that $\dfrac{(x+\alpha)^n-(x+\beta)^n}{\alpha-\beta}=\dfrac{\sin(n\theta)}{\sin^n\theta}$, where α and β are the roots of the equation $t^2-2t+2=0$.

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592. Suppose $a, b, c \in I$ such that the greatest common divisor for x^2+ax+b and x^2+bx+c is $(x+1)$ and the least common multiple of x^2+ax+b and x^2+bx+c is $\left(x^3-4x^2+x+6\right)$. Then the value of $|a+b+c|$ is equal to ________.

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593. Find the value of following expression: $\left[\dfrac{1-\dfrac{\cos\pi}{10}+i\dfrac{\sin\pi}{10}}{1-\dfrac{\cos\pi}{10}-i\dfrac{\sin\pi}{10}}\right]^{10}$

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594. Dividing $f(z)$ by $z - i$, we obtain the remainder i and dividing it by $z + i$, we get the remainder 1 + i, then remainder upon the division of $f(z)$ by $z^2 + 1$ is

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595. If the roots of the cubic equation, $x^3 + ax^2 + bx + c = 0$ are three consecutive positive integers, then the value of $\left(a^2/b + 1\right)$ is equal to?

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596. If $z_1, z_2 \in C, z_1^2 + z_2^2 \in R, z_1\left(z_1^2 - 3z_2^2\right) = 2$ and $z_2\left(3z1^2 - z2^2\right) = 11$, then the value of $z12 + z22$ is 10 b. 12 c. 5 d. 8

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597. If $\cos\alpha + \cos\beta + \cos\gamma = 0 and also \sin\alpha + \sin\beta + \sin\gamma = 0$, then prove that

$\cos 2\alpha + \cos 2\beta + \cos 2\gamma$ $= \sin 2\alpha + \sin 2\beta + \sin 2\gamma = 0$

$\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha + \beta + \gamma)$

$\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha + \beta + \gamma)$

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598. If $x + y + z = 12$ and $x^2 + y^2 + z^2 = 96$ and $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 36$, then the value $x^3 + y^3 + z^3$ divisible by prime number is______.

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599. Prove that $(1 + i)^n + (1 - i)^n = 2^{\frac{n+2}{2}}.\cos\left(\frac{n\pi}{4}\right)$, where n is a positive integer.

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600. The set $\left\{Re\left(\frac{2iz}{1-z^2}\right): zisacomplexvmber, |z| = 1, z = \pm 1\right\}$ is_______.

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601. If the equation $x^2 + ax + bc = 0$ and $x^2 - bx + ca = 0$ have a common root, then $a + b + c$ =

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602. If $arg\left[z_1\left(z_3 - z_2\right)\right] = arg\left[z_3\left(z_2 - z_1\right)\right]$, then find prove that O, z_1, z_2, z_3 are concyclic, where O is the origin.

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603. If $x - iy = \sqrt{\frac{a - ib}{c - id}}$ prove that $\left(x^2 + y^2\right)^2 = \frac{a^2 + b^2}{c^2 + d^2}$

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604. If $x^3 + 3x^2 - 9x + c$ is of the form $(x - \alpha)^2(x - \beta)$, then c is equal to a.27 b. -27 c. 5 d. -5

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605. If $x = a + b, y = a\alpha + b\beta$ and $z = a\beta + b\alpha$, where α and β are the imaginary cube roots ofunity, then $xyz =$

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606. If $z = (a + ib)^5 + (b + ia)^5$ then prove that $Re(z) = Im(z)$, where $a, b \in R$.

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607. If a and b are positive numbers and eah of the equations $x^2 + ax + 2b = 0$ and $x^2 + 2bx + a = 0$ has real roots, then the smallest possible value of $(a + b)$ is________.

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608. The real values of $xandy$ for which the following equation is satisfied: $\frac{(1 + i)(x - 2i)}{3 + i} + \frac{(2 - 3i)(y + i)}{3 - i} = i$ $x = 3, y = 1$ b. $x = 3, y = -1$ c. $x = -3, y = 1$ d. $x = -3, y = -1$

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609. The three angular points of a triangle are given by $Z = \alpha, Z = \beta, Z = \gamma, where\alpha, \beta, \gamma$ are complex numbers, then prove that the perpendicular from the angular point $Z = \alpha$ to the opposite side is given by the equation $Re\left(\frac{Z - \alpha}{\beta - \gamma}\right) = 0$

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610. Suppose a, b, c are the roots of the cubic $x^3 - x^2 - 2 = 0$. Then the value of $a^3 + b^3 + c^3$ is ____.

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611. Prove that $x^3 + x^2 + x$ is factor of $(x + 1)^n - x^n - 1$ where n is odd integer greater than 3, but not a multiple of 3.

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612. If $\alpha, \beta, \gamma, \delta$ are four complex numbers such that $\frac{\gamma}{\delta}$ is real and $\alpha\delta - \beta\gamma \neq 0$ then z = $\frac{\alpha + \beta t}{\gamma + \delta t}$ where t is a rational number, then it represents:

A. A. Circle

B. B. Parabola

C. C. Ellipse

D. D, Straight line

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613. If $ax^2 + (b - c)x + a - b - c = 0$ has unequal real roots for all $c \varepsilon R$, then

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614. If $z_1^2 + z_2^2 - 2z_1 . z_2 . \cos\theta = 0$ prove that the points represented by z_1, z_2, and the origin form an isosceles triangle.

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615. Prove that the circles $z\bar{z} + z\left(\bar{a}_1\right) + \bar{z}\left(a_1\right) + b_1 = 0, b_1 \in R$ and $z\bar{z} + z\left(\bar{a}_2\right) + \bar{z}a_2 + b_2 = 0,$

$b_2 \in R$ will intersect orthogonally if $2Re\left(a_1\bar{a}_2\right) = b_1 + b_2$

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616. If a, b, c real in G.P., then the roots of the equation $ax^2 + bx + c = 0$ are in the ratio a. $\frac{1}{2}\left(-1 + i\sqrt{3}\right)$ b. $\frac{1}{2}\left(1 - i\sqrt{3}\right)$ $c\frac{1}{2}\left(-1 - i\sqrt{3}\right)$ d. $\frac{1}{2}\left(1 + i\sqrt{3}\right)$

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617. If z_0 is the circumcenter of an equilateral triangle with vertices z_1, z_2, z_3 then $z_1^2 + z_2^2 + z_3^2$ is equal to

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618. Two different non-parallel lines cut the circle $|z| = r$ at points a, b, c and d, respectively. Prove that these lines meet at the point z given by $\frac{a^{-1} + b^{-1} - c^{-1} - d^{-1}}{a^{-1}b^{-1} - c^{-1}d^{-1}}$

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619. If the equations $x^2 + px + q = 0$ and $x^2 + p'x + q' = 0$ have a common root, then it must be equal to a. $\frac{p' - p'q}{q - q'}$ b. $\frac{q - q'}{p' - p}$ c. $\frac{p' - p}{q - q'}$ d. $\frac{pq' - p'q}{p - p'}$

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620. Prove that $|z - z_1|^2 + |z - z_2|^2 = a$ will represent a real circle on the Agrand plane if $2a \geq |z_1 - z_2|^2$

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621. Complex numbers z_1, z_2, z_3 are the vertices A, B, C respectively of an isosceles right angled trianglewith right angle at C and $(z_1 - z_2)^2 = k(z_1 - z_3)(z_3 - z_2)$, then find k.

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622. Given that α, γ are roots of the equation $Ax^2 - 4x + 1 = 0$, and $\beta, \delta 1$ the equation of $Bx^2 - 6x + 1 = 0$, such that α, β, γ and δ are in H.P., then

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623. Show that the area of the triangle on the Argand diagram formed by the complex number $z, izandz + iz$ is $\frac{1}{2}|z|^2$

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624. Intercept made by the circle $z\bar{z} + \bar{a} + a\bar{z} + r = 0$ on the real axis on complex plane is $\sqrt{(a + \bar{a}) - r}$ b. $\sqrt{(a + \bar{a})^2 - r}$ c. $\sqrt{(a + \bar{a})^2 - 4r}$ d. $\sqrt{(a + \bar{a})^2 - 4r}$

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625. The graph of the quadratic trinomial $u = ax^2 + bx + c$ has its vertex at (4, -5) and two x-intercepts, one positive and one negative. Which of the following holds good? a. $a > 0$ b. $b < 0$ c. $c < 0$ d. $8a = b$

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626. Show that if $iz^3 + z^2 - z + i = 0$, then $|z| = 1$

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627. Show that the equation of a circle passing through the origin and having intercepts a and b on real and imaginary axes, respectively, on the argand plane is given by $z\bar{z} = a(Rez) + b(Imz)$

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628. The function $f(x) = ax^2 + bx^2 + cx + d$ has three positive roots. If the sum of the roots of $f(x)$ is 4, the larget possible inegal values of c/a is

__________.

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629. Let $z_1 = 10 + 6i$ and $z_2 = 4 + 6i$. If z is any complex number such that the argument of $\frac{(z - z_1)}{(z - z_2)}$ is $\frac{\pi}{4}$, then prove that $|z - 7 - 9i| = 3\sqrt{2}$.

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630. Let vertices of an acute-angled triangle are $A(z_1), B(z_2), andC(z_3)$. If the origin O is he orthocentre of the triangle, then prove that

$z_1\bar{z}_2 + \bar{z}_1 z_2 = z_2\bar{z}_3 + \bar{z}_2 z_3 = z_3\bar{z}_1 + \bar{z}_3 z_1$

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631. If $\left(18x^2 + 12x + 4\right)^n = a_0 + a_{1x} + a2x2 + \ + a_{2n}x^{2n}$, prove that $a_r = 2^n 3^r\left(\wedge (2n)C_r +^n C_1^{2n-2}C_r +^n C_2^{2n-4}C_r + \ \right)$.

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632. If $z = z_0 + A\left(\bar{z} - \left(\bar{z}_0\right)\right)$, $whereA$ is a constant, then prove that locus of z is a straight line.

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633. If $(\sin\alpha)x^2 - 2x + b \geq 2$ for all real values of $x \leq 1and\alpha \in \left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right)$, then the possible real values of b is/are 2 (b) 3 (c) 4 (d) 5

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634. If z_1, z_2, z_3 are three complex numbers such that $5z_1 - 13z_2 + 8z_3 = 0,$

then prove that $\begin{bmatrix} z_1 & (\bar{z})_1 & 1 \\ z_2 & (\bar{z})_2 & 1 \\ z_3 & (\bar{z})_3 & 1 \end{bmatrix} = 0$

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635. If one root $x^2 - x - k = 0$ is square of the other, then $k =$ a.$2 \pm \sqrt{5}$ b. $2 \pm \sqrt{3}$ c. $3 \pm \sqrt{2}$ d. $5 \pm \sqrt{2}$

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636. If $2z_1/3z_2$ is a purely imaginary number, then find the value of $|\left(z_1 - z_2)/\left(z_1 + z_2\right)|$.

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637. If $\alpha, and\beta$ be t roots of the equation $x^2 + px - 1/2p^2 = 0, wherep \in R$

Then the minimum value of $\alpha^4 + \beta^4$ is $2\sqrt{2}$ b. $2 - \sqrt{2}$ c. 2 d. $2 + \sqrt{2}$

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638. If z_1, z_2, z_3 are complex numbers such that $\left(2/z_1\right) = \left(1/z_2\right) + \left(1/z_3\right)$, then show that the points represented by $z_1, z_2()_, z_3$ lie one a circle passing through e origin.

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639. Find the range of

(a) $f(x) = \dfrac{x^2 + 34x - 71}{x^2 + 2x - 7}$

(b) $f(x) = \dfrac{x^2 - x + 1}{x^2 + x + 1}$

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640. If $\left(\frac{3-z_1}{2-z_1}\right)\left(\frac{2-z_2}{3-z_2}\right)=k(k>0)$, then prove that points $A(z_1), B(z_2), C(3), andD(2)$ (taken in clockwise sense) are concyclic.

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641. $x^2-xy+y^2-4x-4y+16=0$ represents a. a point b. a circle c. a pair of straight line d. none of these

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642. If $(x+iy)^5=p+iq,$ then prove that $(y+ix)^5=q+ip$

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643. If α, β are the nonzero roots of $ax^2+bx+c=0$ and α^2, β^2 are the roots of $a^2x^2+b^2x+c^2=0$, then a, b, c are in (A) G.P. (B) H.P. (C) A.P. (D) none of these

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644. Find real value of θ for which $\frac{3 + 2i\sin\theta}{1 - 2i\sin\theta}$ is purely real.

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645. If the roots of the equation $ax^2 + bx + c = 0$ are of the form $(k + 1)/kand(k + 2)/(k + 1), then(a + b + c)^2$ is equal to $2b^2 - ac$ b. $a62$ c. $b^2 - 4ac$ d. $b^2 - 2ac$

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646. Prove that $\tan\left(i\log_e\left(\frac{a - ib}{a + ib}\right)\right) = \frac{2ab}{a^2 - b^2}$ (where $a, b \in R^+$)

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647. If α, β are the roots of $ax^2 + bx + c = 0 and \alpha + h, \beta + h$ are the roots of $px^2 + qx + r = 0 then h =$ $-\frac{1}{2}\left(\frac{a}{b} - \frac{p}{q}\right)$ b. $\left(\frac{b}{a} - \frac{q}{p}\right)$ c. $\frac{1}{2}\left(\frac{b}{q} - \frac{q}{p}\right)$ d. none of these

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648. Find the real part of $(1 - i)^{-i}$

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649. The equation $\left(x^2 + x + 1\right)^2 + 1 = \left(x^2 + x + 1\right)\left(x^2 - x - 5\right)$ for $x \in (-2, 3)$ will have number of solutions. 1 b. 2 c. 3 d. 0

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650. Convert of the complex number in the polar form: $1 - i$

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651. If α, β re the roots of $ax^2 + c = bx,$ then the equation $(a + cy)^2 = b^2y$ in y has the roots a.$\alpha\beta^{-1}, \alpha^{-1}\beta$ b. α^{-2}, β^{-2} c. α^{-1}, β^{-1} d. α^2, β^2

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652. If $z = re^{i\theta}$, then prove that $\left|e^{iz}\right| = e^{-r\sin\theta}$

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653. If the roots of the equation $x^2 + 2ax + b = 0$ are real and distinct and they differ by at most $2m$,then b lies in the interval a.$\left(a^2, a^2, + m^2\right)$ b. $\left(a^2 - m^2, a62\right)$ c. $\left[a^2 - m^2, a^2\right)$ d. none of these

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654. $Z_1 \neq Z_2$ are two points in an Argand plane. If $a|Z_1| = b|Z_2|$, then prove that $\frac{aZ_1 - bZ_2}{aZ_1 + bZ_2}$ is purely imaginary.

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655. If the ratio of the roots of $ax^2 + 2bx + c = 0$ is same as the ratios of roots of $px^2 + 2qx + r = 0$, then a. $\frac{2b}{ac} = \frac{q^2}{pr}$ b. $\frac{b}{ac} = \frac{q^{\square}}{pr}$ c. $\frac{b^2}{ac} = \frac{q^2}{pr}$ d. none of these

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656. Find real value of $xandy$ for which the complex numbers $-3 + ix^2yandx^2 + y + 4i$ are conjugate of each other.

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657. Show that $\frac{(x+b)(x+c)}{(b-a)(c-a)} + \frac{(x+c)(x+a)}{(c-b)(a-b)} + \frac{(x+a)(x+b)}{(a-c)(b-c)} = 1$ is an identity.

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658. Show that $e^{2mi\theta}\left(\frac{i\cot\theta + 1}{i\cot\theta - 1}\right)^{m} = 1.$

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659. A certain polynomial $P(x), x \in R$ when divided by $x - a, x - b$ and $x - c$ leaves remainders$a, b,$ and c , resepectively. Then find remainder when $P(x)$ is divided by $(x - a)(x - b)(x - c)wherea, b, c$ are distinct.

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660. It is given that complex numbers z_1 and z_2 satisfy $|z_1| = 2$and $|z_2| = 3.$ If the included angled of their corresponding vectors is 60^0 , then find the value of $\left|\frac{z_1 + z_2}{z_1 - z_2}\right|$.

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661. If c, d are the roots of the equation $(x - a)(x - b) - k = 0$, prove that a, b are roots of the equation $(x - c)(x - d) + k = 0$.

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662. If θ is real and z_1, z_2 are connected by $z_1 2 + z_2 2 + 2z_1 z_2 \cos\theta = 0$, then prove that the triangle formed by vertices $O, z_1 and z_2$ is isosceles.

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663. If $\left(a^2 - 1\right)x^2 + (a - 1)x + a^2 - 4a + 3 = 0$ is identity in x, then find the value of a.

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664. Show that a real value of x will satisfy the equation $(1 - ix)/(1 + ix) = a + ib$ if $a^2 + b^2 = 1, where a, b$ real.

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665. Prove that the roots of the equation $\left(a^4 + b^4\right)x^2 + 4abcdx + \left(c^4 + d^4\right) = 0$ cannot be different, if real.

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666. If $a + ib = \dfrac{(x + i)^2}{2x^2 + 1}$, prove that $a^2 + b^2 = \dfrac{\left(x^2 + 1\right)^2}{\left(2x^2 + 1\right)^2}$

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667. If the roots of the equation $x^2 - 8x + a^2 - 6a = 0$ are real distinct, then find all possible value of a.

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668. Solve : $z^2 + |z| = 0$.

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669. If roots of equation $x^2 - 2cx + ab = 0$ are real and unequal, then prove that the roots of $x^2 - 2(a + b)x + a^2 + b^2 + 2c^2 = 0$ will be imaginary.

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670. Find the range of real number α for which the equation $z + \alpha|z - 1| + 2i = 0$ has a solution.

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671. If the roots of the equation $a(b-c)x^2 + b(c-a)x + c(a-b) = 0$ are equal, show that $2/b = 1/a + 1/c$.

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672. If $\frac{(1+i)^2}{3-i}$, then $Re(z) =$

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673. Find the quadratic equation with rational coefficients whose one root is $1/\left(2+\sqrt{5}\right)$.

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674. Let z be a complex number satisfying the equation $\left(z^3+3\right)^2 = -16$, then find the value of $|z|$.

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675. If $f(x) = ax^2 + bx + c, g(x) = -ax^2 + bx + c$,where ac $\neq 0,$ then prove that $f(x)g(x) = 0$ has at least two real roots.

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676. Find the real numbers x and y if $(x - iy)(3 + 5i)$ is the conjugate of $-6 - 24i$.

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677. If x is real, then $x/\left(x^2 - 5x + 9\right)$ lies between $-1 and - 1/11$ b. $1 and - 1/11$ c. $1 and 1/11$ d. none of these

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678. Find the least positive integer n such that $\left(\frac{2i}{1+i}\right)^n$ is a positive integer.

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679. Set of all real value of a such that $f(x) = \frac{(2a-1)x^2 + 2(a+1)x + (2a-1)}{x^2 - 2x + 40}$ is always negative is a. $(-\infty, 0)$ b. $(0, \infty)$ c. $\left(-\infty, \frac{1}{2}\right)$ d. none

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680. Find the real part of $e^e \wedge (i\theta)$

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681. If α, β and γ are the roots of $x^3 - x^2 - 1 = 0$, then value of $\frac{1+\alpha}{1-\alpha} + \frac{1+\beta}{1-\beta} + \frac{1+\gamma}{1-\gamma}$ is

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682. Prove that $z = i^i$, where $i = \sqrt{-1}$, is purely real.

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683. If $\alpha, \beta, \gamma, \delta$ are the roots of the equation $x^4 - Kx^3 + Kx^2 + Lx + m = 0$, $whereK, L, andM$ are real numbers, then the minimum value of $\alpha^2 + \beta^2 + \gamma^2 + \delta^2$ is 0 b. -1 c. 1 d. 2

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684. In $ABC, A(z_1), B(z_2), andC(z_3)$ are inscribed in the circle $|z| = 5$. If $H(z_n)$ be the orthocenrter of triangle ABC, then find z_n

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685. Suppose that $f(x)$ is a quadratic expresson positive for all real x If $g(x) = f(x) + f'(x) + f''(x)$, then for any real $x(where f'(x) and f''(x)$ represent 1st and 2nd derivative, respectively). a. $g(x) < 0$ b. $g(x) > 0$ c. $g(x) = 0$ d. $g(x) \geq 0$

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686. about to only mathematics

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687. Let $f(x) = ax^2 - bx + c^2, b \neq 0$ and $f(x) \neq 0$ for all $x \in R$. Then (a) $a + c^2 < b$ (b) $4a + c^2 > 2b$ (c) $a - 3b + c^2 < 0$ (d) none of these

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688. If n is n odd integer that is greater than or equal to 3 but not a multiple of 3, then prove that $(x+1)^n - x^n - 1$ is divisible by $x^3 + x^2 + x$

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689. If $a, b \in R, a \neq 0$ and the quadratic equation $ax^2 - bx + 1 = 0$ has imaginary roots, then $(a+b+1)$ is a. positive b. negative c. zero d. Dependent on the sign of b

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690. Find the complex number ω satisfying the equation $z^3 = 8i$ and lying in the second quadrant on the complex plane.

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691. If the expression $[mx - 1 + (1/x)]$ is non-negative for all positive real x, then the minimum value of m must be $-1/2$ b. 0 c. $1/4$ d. $1/2$

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692. When the polynomial $5x^3 + Mx + N$ is divided by $x^2 + x + 1$, the remainder is 0. Then find the value of $M + N$

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693. x_1 and x_2 are the roots of $ax^2 + bx + c = 0$ and $x_1x_2 < 0$. Roots of $x_1(x - x_2)^2 + x_2(x - x_1)^2 = 0$ are: (a) real and of opposite sign b. negative c. positive d. none real

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694. if$\omega and \omega^2$ are the nonreal cube roots of unity and $[1/(a+\omega)]+[1/(b+\omega)]+[1/(c+\omega)]=2\omega^2$ and $\left[1/(a+\omega)^2\right]+\left[1/(b+\omega)^2\right]+\left[1/(c+\omega)^2\right]=2\omega$, then find the value of $[1/(a+1)]+[1/(b+1)]+[1/(c+1)]^{.}$

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695. If a, b, c, d are four consecutive terms of an increasing A.P., then the roots of the equation $(x-a)(x-c)+2(x-b)(x-d)=0$ are a. non-real complex b. real and equal c. integers d. real and distinct

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696. Find the relation if z_1, z_2, z_3, z_4 are the affixes of the vertices of a parallelogram taken in order.

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697. Let a and b are the roots of the equation $x^2 - 10xc - 11d = 0$ and those of $x^2 - 10ax - 11b = 0$, are c and d then find the value of `a+b+c+d

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698. If z_1, z_2, z_3 are three nonzero complex numbers such that $z_3 = (1 - \lambda)z_1 + \lambda z_2$ where $\lambda \in R - \{0\}$, then prove that points corresponding to z_1, z_2 and z_3 are collinear .

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699. Fill in the blanks The coefficient of x^{99} in the polynomial $(x - 1)(x - 2)\ldots\ldots\ldots.(x - 100) is \_ \_ \_ \_$

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700. Let z_1, z_2, z_3 be three complex numbers and a, b, c be real numbers not all zero, such that $a + b + c = 0$ and $az_1 + bz_2 + cz_3 = 0$. Show that z_1, z_2, z_3 are collinear.

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701. Fill in the blanks If $2 + i\sqrt{3}$ is a root of the equation $x^2 + px + q = 0, wherepandq$ are real, then $(p, q) = \left(\_\_\_\_,\_\_\_\_ \right)$.

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702. Prove that the triangle formed by the points $1, \frac{1+i}{\sqrt{2}}, andi$ as vertices in the Argand diagram is isosceles.

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703. Fill in the blanks. If the product of the roots of the equation $x^2 - 3kx + 2e^{2\log k} - 1 = 0$ is 7, then the roots are real for__________.

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704. Solve for $z : z^2 - (3 - 2i)z = (5i - 5)$

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705. If the equations $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ have one common root. Then find the numerical value of a+b.

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706. Find all possible values of $\sqrt{i} + \sqrt{-i}$.

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707. Fill in the blanksIf $x < 0, y < 0, x + y + (x/y) = (1/2) and (x + y)(x/y) = -(1/2), then x = \_ \_ and y = \_$

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708. Find the square roots of the following: (i) $7 - 24i$

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709. True or false The equation$2x^2 + 3x + 1 = 0$ has an irrational root.

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710. If $z \neq 0$ is a complex number, then prove that $Re(z) = 0 \Rightarrow Im\left(z^2\right) = 0.$

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711. If l, m, n are real and $l \neq m$, then the roots of the equation $(l - m)x^2 - 5(l + m)x - 2(l - m) = 0$ are a) real and equal b) Complex c) real and unequal d) none of these

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712. Let z be a complex number satisfying the equation $z^2 - (3 + i)z + m + 2i = 0$,where $m \in R$. Suppose the equation has a real root. Then the value of m is-

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713. If x, y and z are real and different and $u = x^2 + 4y^2 + 9z^2 - 6yz - 3zx - 2xy$,then u is always (a). non-negative b. zero c. non-positive d. none of these

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714. If $(x + iy)^3 = u + iv$, then show that $\frac{u}{x} + \frac{v}{y} = 4\left(x^2 - y^2\right)$

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715. Let $a > 0, b > 0$ and $c > 0$. Then, both the roots of the equation $ax^2 + bx + c = 0$: a. are real and negative b. have negative real parts c. have positive real parts d. None of the above

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716. If the sum of square of roots of the equation $x^2 + (p + iq)x + 3i = 0$ is 8, then find the value of p and q, where p and q are real.

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717. Column I, Column II $y = \frac{x^2 - 2x + 4}{x^2 + 2x + 4}, x \in R, thenycanbe$, p. 1 $y = \frac{x^2 - 3x - 2}{2x - 3}, x \in R, thenycanbe$, q. 4 $y = \frac{2x^2 - 2x + 4}{x^2 - 4x + 3}, x \in R, thenycanbe$

, r. $-3x^2-(a-3)x+2<0,\ \forall,\ x\in(-2,3),\ thanacanbe$, s. -10

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718. If $\sqrt{x+iy}=\pm(a+ib)$, then find $\sqrt{x-iy}$.

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719. Match the following for the equation $x^2+a\mid x1=0, wherea$ is a parameter. Column I, Column II No real roots, p. $a<-2$ Two real roots, q. φ Three real roots, r. $a=-2$ Four distinct real roots, s. $a\geq 0$

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720. Find the ordered pair (x,y) for which $x^2-y^2-i(2x+y)=2i$

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721. If a, b, c are non zero complex numbers of equal modlus and satisfy $az^2 + bz + c = 0$, hen prove that $\left(\sqrt{5} - 1\right)/2 \leq |z| \leq \left(\sqrt{5} + 1\right)/2.$

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722. Column I, Column II If $a, b, c, andd$ are four zero real numbers such that $(d + a - b)^2 + (d + b - c)^2 = 0$ and he root of the equation $a(b - c)x^2 + b(c - a)x = c(a - b) = 0$ are real and equal, then, p. $a + b + c = 0$ If the roots the equation $\left(a^2 + b^2\right)x^2 - 2b(a + c)x + \left(b^2 + c^2\right) = 0$ are real and equal, then, q. $a, b, care \in \ddot{AP}$ If the equation $ax^2 + bx + c = 0andx^3 - 3x^2 + 3x - 0$ have a common real root, then, r. $a, b, care \in \ddot{GP}$ Let a, b, c be positive real number such that the expression $bx^2 + \left(\sqrt{(a + b)^2 + b^2}\right)x + (a + c)$ is non-negative $\forall x \in R$, then, s. $a, b, care \in \ddot{HP}$

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723. Let z be not a real number such that $\left(1+z+z^2\right)/\left(1-z+z^2\right) \in R$, then prove tha $|z| = 1$.

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724. Let a is a real number satisfying $a^3 + \frac{1}{a^3} = 18$. Then the value of $a^4 + \frac{1}{a^4} - 39$ is ____.

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725. Find non zero integral solutions of $|1-i|^x = 2^x$.

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726. If $(1+i)(1+2i)(1+3i)......(1+ni) = (x+iy)$, show that $2.\ 5.\ 10......\left(1+n^2\right) = x^2+y^2$.

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727. If $ax^2 + bx + c = 0, a, b, c \in R$ has no real zeros, and if $c < 0$, then which of the following is true? (a) $a < 0$ (b) $a + b + c > 0$ (c)$a > 0$

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728. If ω is a cube root of unity, then find the value of the following: $\frac{a + b\omega + c\omega^2}{c + a\omega + b\omega^2} + \frac{a + b\omega + c\omega^2}{b + c\omega + a\omega^2}$

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729. If $f(x) = \sqrt{x^2 + ax + 4}$ is defined for all x, then find the values of a

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730. If ω is a cube root of unity, then find the value of the following:

$(1-\omega)\left(1-\omega^2\right)\left(1-\omega^4\right)\left(1-\omega^8\right)$

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731. Find the domain and range of $f(x)=\sqrt{x^2-4x+6}$

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732. Prove that $\left|z_1+z_2\right|^2=\left|z_1\right|^2+\left|z_2\right|^2, \quad \text{if} \quad z_1/z_2$ is purely imaginary.

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733. Find the range of the function $f(x)=6^x+3^x+6^{-x}+3^{-x}+2.$

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734. If ω is a cube root of unity, then find the value of the following: $\left(1+\omega-\omega^2\right)\left(1-\omega+\omega^2\right)$

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735. If α, β are the roots of the equation $2x^2+2(a+b)x+a^2+b^2=0$ then find the equation whose roots are$(\alpha+\beta)^2$ and $(\alpha-\beta)^2$

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736. Let z_1, z_2, z_3, z_n be the complex numbers such that $|z_1|=|z_2|=|z_n|=1.$ If $z=\left(\sum_{k=1}^{n} z_k\right)\left(\sum_{k=1}^{n}\frac{1}{z_k}\right)$ then proves that z is a real number

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737. If $a, b, \in R$ such that $a + b = 1 and (1 - 2ab)\left(a^3 + b^3\right) = 12$. The value of $\left(a^2 + b^2\right)$ is equal to____.

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738. If $|z| \leq 4$, then find the maximum value of $|iz + 3 - 4i|$

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739. Find the range of $f(x) = x^2 - x - 3$.

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740. If the fraction $\dfrac{x^3 + (a - 10)x^2 - x + a - 6}{x^3 + (a - 6)x^2 - x + a - 10}$ reduces to a quotient of two functions then a equals

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741. The polynomial $f(x) = x^4 + ax^3 + bx^3 + cx + d$ has real coefficients and $f(2i) = f(2 + i) = 0$. Find the value of $(a + b + c + d)$

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742. Find the value of $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ for all $n \in N$

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743. If the quadratic equation $ax^2 + bx + c = 0(a > 0)$ has $\sec^2\theta and \operatorname{cosec}^2\theta$ as its roots, then which of the following must hold good? (a.) $b + c = 0$ (b.) b^2 - $4ac \geq 0$ (c.) c $\geq 4a$ (d.) $4a + b \geq 0$

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744. Find the value of $1 + i^2 + i^4 + i^6 + \ + i^{2n}$

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745. Let $x, y, z \in R$ such that $x + y + z = 6andxy + yz + zx = 7$. Then find the range of values of $x, y, andz$

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746. Show that the polynomial $x^{4p} + x^{4q+1} + x^{4r+2} + x^{4s+3}$ is divisible by $x^3 + x^2 + x + 1, wherep, q, r, s \in n$.

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747. if $ax^2 + bx + c = 0$ has imaginary roots and $a + c < b$ then prove that $4a + c < 2b$

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748. Solve: $ix^2 - 3x - 2i = 0,$

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749. Let $a, b, andc$ be distinct nonzero real numbers such that $\frac{1 - a^3}{a} = \frac{1 - b^3}{b} = \frac{1 - c^3}{c}$. The value of $\left(a^3 + b^3 + c^3\right)$ is ___________.

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750. Express each one of the following in the standard form $a + ib \cdot \frac{5 + 4i}{4 + 5i}$

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751. If the cubic $2x^3 - 9x^2 + 12x + k = 0$ has two equal roots then minimum value of $|k|$ is_____.

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752. If $z = 4 + i\sqrt{7}$, then find the value of $z^2 - 4z^2 - 9z + 91$.

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753. If the quadratic equation$4x^2 - 2(a + c - 1)x + ac - b = 0(a > b > c)$ (a)Both roots se greater than a (b)Both roots are less than c (c)Both roots lie between $\frac{c}{2}$ and $\frac{a}{2}$ (d)Exactly one of the roots lies between $\frac{c}{2}$ and $\frac{a}{2}$

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754. If $(a + b) - i(3a + 2b) = 5 + 2i$, then find $aandb$

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755. If the equation $x^2 = ax + b = 0$ has distinct real roots and $x^2 + a|x| + b = 0$ has only one real root, then which of the following is true? $b = 0, a > 0$ b. $b = 0, a < 0$ c. $b > 0, a < 0$ d. $b\ 0, a\ 0$

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756. Given that x, $y \in R$. Solve: $\frac{x}{1 + 2i} + \frac{y}{3 + 2i} = \frac{5 + 6i}{8i - 1}$

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757. If the equation $\left|x^2 + bx + c\right| = k$ has four real roots, then

A. $b^2 - 4c > 0 \text{ and } 0 < k < \frac{4c - b^2}{4}$

B. $b^2 - 4c < 0 \text{ and } 0 < k < \frac{4c - b^2}{4}$

C. $b^2 - 4c > 0 \text{ and } k > \frac{4c - b^2}{4}$

D. none of these

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758. If P(x) is a polynomial with integer coefficients such that for 4 distinct integers $a, b, c, d, P(a) = P(b) = P(c) = P(d) = 3$, if $P(e) = 5$, (e is an integer) then 1. e=1 , 2. e=3 , 3. e=4 , 4. No integer value of e

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759. Let x, y, z, t be real numbers $x^2 + y^2 = 9, z^2 + t^2 = 4,$ and $xt - yz = 6$

Then the greatest value of $P = xz$ is a. 2 b. 3 c. 4 d. 6

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760. If a, b, c are distinct positive numbers, then the nature of roots of the equation $1/(x - a) + 1/(x - b) + 1/(x - c) = 1/x$ is a. all real and is distinct b. all real and at least two are distinct c. at least two real d. all non-real

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761. If $\left(b^2 - 4ac\right)^2\left(1 + 4a^2\right) < 64a^2, a < 0$, then maximum value of quadratic expression $ax^2 + bx + c$ is always less than a. 0 b. 2 c. -1 d. -2

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762. For $x^2 - (a+3)|x| + 4 = 0$ to have real solutions, the range of a is a. $(-\infty, -7] \cup [1, \infty)$ b. $(-3, \infty)$ c. $(-\infty, -7)$ d. $[1, \infty)$

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763. The number of integral value of x satistying $\sqrt{x^2 + 10x - 16} < x - 2$ is

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764. If $x^2 + ax - 3x - (a+2) = 0$ has real and distinct roots, then minimum value of $\left(a^2+1\right)/\left(a^2+2\right)$ is

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765. Let $\alpha + i\beta; \alpha, \beta \in R$, be a root of the equation $x^3 + qx + r = 0; q, r \in R$ A real cubic equation, independent of $\alpha \& \beta$, whose one root is 2α is (a)$x^3 + qx - r = 0$ (b) $x^3 - qx + 4 = 0$ (c) $x^3 + 2qx + r = 0$ (d) None of these

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766. In equation $x^4 - 2x^3 + 4x^2 + 6x - 21 = 0$ if two its roots are equal in magnitude but opposite in sign, find all the roots.

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767. If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0,$ then find he value of $\left(\alpha - \frac{1}{\beta\gamma}\right)\left(\beta - \frac{1}{\gamma\alpha}\right)\left(\gamma - \frac{1}{\alpha\beta}\right)$.

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768. Equations $x^3 + 5x^2 + px + q = 0 and x^3 + 7x^2 + px + r = 0$ have two roots in common. If the third root of each equation is $x_1 and x_2$, respectively, then find the ordered pair $\left(x_1, x_2\right)^{.}$

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769. If α, β, γ are the roots of he euation $x^3 + 4x + 1 = 0$, then find the value of $(\alpha + \beta)^{-1} + (\beta + \gamma)^{-1} + (\gamma + \alpha)^{-1}$.

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770. If the roots of the equation $x^3 + Px^2 + Qx - 19 = 0$ are each one more that the roots of the equation $x^3 - Ax^2 + Bx - C = 0, whereA, B, C, P, andQ$ are constants, then find the value of $A + B + C$

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771. If a, b, p, q are non zero real numbers, then how many comman roots would two equations: $2a^2x^2 - 2abx + b^2 = 0$ and $p^2x^2 + 2pqx + q^2 = 0$ have?

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772. If $x^2+px+q=0 and x^2+qx+p=0, (p \ne q)$ have a common roots, show that $1+p+q=0$. Also, show that their other roots are the roots of the equation $x^2+x+pq=0$.

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773. a,b,c are positive real numbers forming a G.P. ILf $ax^2+2bx+c=0$and $x^2+2ex+f=0$ have a common root, then prove that $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

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774. If equations $x^2+ax+12=0. x^2+bx+15=0 and x^2+(a+b)x+36=0,$ have a common positive root, then find the values of $a and b$

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775. If x is real and the roots of the equation $ax^2 + bx + c = 0$ are imaginary, then prove tat $a^2x^2 + abx + ac$ is always positive.

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776. Solve $\left(x^2 + 2\right)^2 + 8x^2 = 6x\left(x^2 + 2\right)$

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777. Find the value of $2 + \dfrac{1}{2 + \dfrac{1}{2 + \dfrac{1}{2 + \infty}}}$

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778. If both the roots of $ax^2 + ax + 1 = 0$ are less than 1, then find the exhaustive range of values of a

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779. If both the roots of $x^2 + ax + 2 = 0$ lies in the interval (0, 3), then find the exhaustive range of value of a.

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780. Solve $\frac{x^2 + 3x + 2}{x^2 - 6x - 7} = 0$.

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781. Solve $\sqrt{x - 2} + \sqrt{4 - x} = 2$.

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782. Solve $\sqrt{x - 2}\left(x^2 - 4x - 5\right) = 0$.

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783. Solve the equation $x(x+2)\left(x^2-1\right) = -1.$

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784. The number of distinct real roots of $x^4 - 4x^3 + 12x^2 + x - 1 = 0$ is :

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785. Prove that graphs of $y = x^2 + 2andy = 3x - 4$ never intersect.

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786. In how many points the line $y + 14 = 0$ cuts the curve whose equation is $x\left(x^2 + x + 1\right) + y = 0$?

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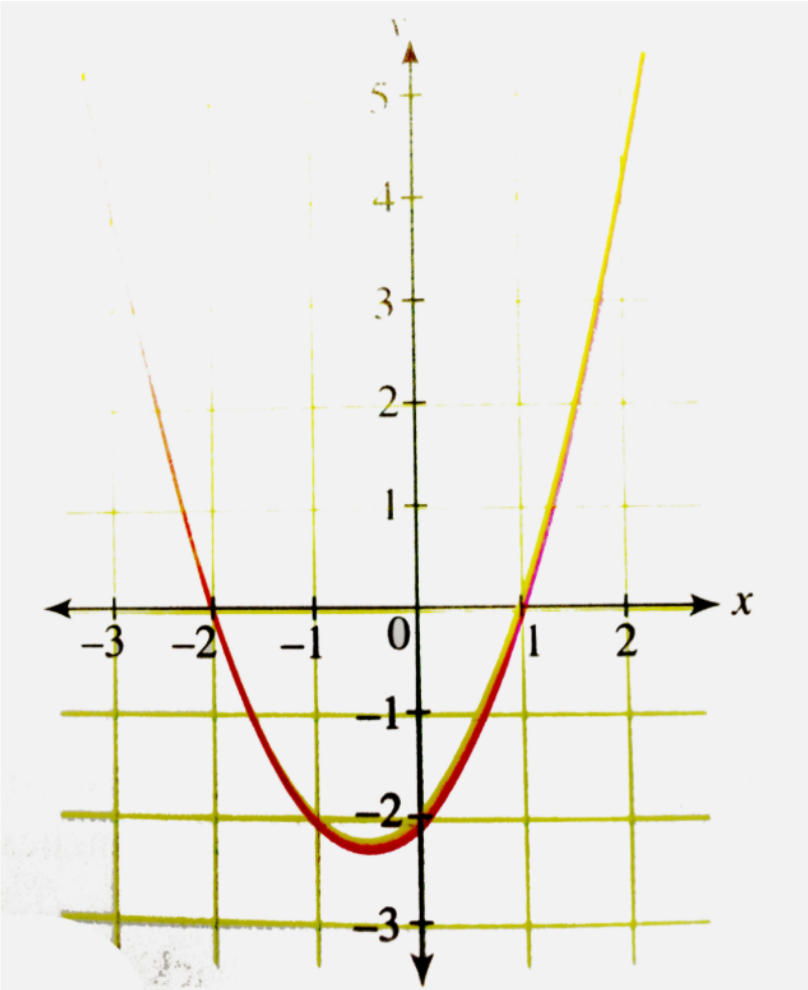
787. Consider the graph of y = f(x) as shown in the following figure.



(i) Find the sum of the roots of the equation f (x) = 0.

(ii) Find the product of the roots of the equation f(x) = 4.

(iii) Find the absolute value of the difference of the roots of the equation

f(x) = x+2 .

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788. If $x^2 + px - 444p = 0$ has integral roots where p is prime number, then find the value of p .

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789. The equation $ax^2 + bx + c = 0$ has real and positive roots. Prove that the roots of the equation $a^2x^2 + a(3b - 2c)x + (2b - c)(b - c) + ac = 0$ re real and positive.

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790. If the roots of the equation $x^2 - ax + b = 0$y are real and differ b a quantity which is less than $c(c > 0)$, then show that b lies between $\frac{a^2 - c^2}{4}$ and $\frac{a^2}{4}$.

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791. If $\left(ax^2 + bx + c\right)y + \left(a'x^2 + b'x^2 + c'\right) = 0$ and x is a rational function of y, then prove that $\left(ac' - a'c\right)^2 = \left(ab' - a'b\right) \times \left(bc' - b'c\right)^{.}$

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792. Show that the minimum value of $(x + a)(x + b)/(x + c)$ where $a > c, b > c,$ is $\left(\sqrt{a - c} + \sqrt{b - c}\right)^2$ for real values of $x > -c.$

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793. Let $a, b \in N$ and $a > 1.$ Also p is a prime number. If $ax^2 + bx + c = p$ for any intergral values of $x,$ then prove that $ax^2 + bx + c \neq 2p$ for any integral value of $x^{.}$

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794. If $2x^2 - 3xy - 2y^2 = 7$, then prove that there will be only two integral pairs (x, y) satisfying the above relation.

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795. If a and c are odd prime numbers and $ax^2 + bx + c = 0$ has rational roots , where $b \in I$, prove that one root of the equation will be independent of $a, b, \dot{c}$

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796. If $f(x) = x^3 + bx^2 + cx + d$ and $f(0), f(-1)$ are odd integers, prove that $f(x) = 0$ cannot have all integral roots.

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797. If x is real, then the maximum value of $y = 2(a - x)\left(x + \sqrt{x^2 + b^2}\right)$

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798. If equation $x^4 - (3m + 2)x^2 + m^2 = 0(m > 0)$ has four real solutions which are in A.P., then the value of m is_____.

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799. Number of positive integers x for which $f(x) = x^3 - 8x^2 + 20x - 13$ is a prime number is_____.

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800. If set of values a for which $f(x) = ax^2 - (3 + 2a)x + 6, a \neq 0$ is positive for exactly three distinct negative integral values of x is $(c, d]$, then the value of $\left(c^2 + 4|d|\right)$ is equal to _______.

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801. Polynomial $P(x)$ contains only terms of aodd degree. when $P(x)$ is divided by $(x - 3)$, the ramainder is 6. If $P(x)$ is divided by $\left(x^2 - 9\right)$ then remainder is $g(x)$. Then find the value of $g(2)$.

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802. If the equation $2x^2 + 4xy + 7y^2 - 12x - 2y + t = 0,$ where t is a parameter has exactly one real solution of the form (x, y) , then the sum of $(x + y)$ is equal to _____.

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803. Let α_1, β_1 be the roots $x^2 - 6x + p = 0 and$ α_2, β_2 be the roots $x^2 - 54x + q = \dot{0}$ If $\alpha_1, \beta_1, \alpha_2, \beta_2$ form an increasing G.P., then sum of the digits of the value of $(q - p)$ is _________.

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804. If $\sqrt{\sqrt{\sqrt{x}}} = \left(3x^4 + 4\right)^{\frac{1}{64}}$, then the value of x^4 is____.

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805. Let $P(x) = x^4 + ax^3 + bx^2 + cx + d$ be a polynomial such that $P(1) = 1, P(2) = 8, P(3) = 27, P(4) = 64$ then the value of $152 - P(5)$ is__________.

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806. If the equation $x^2 + 2(\lambda + 1)x + \lambda^2 + \lambda + 7 = 0$ has only negative roots, then the least value of λ equals________.

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807. Given $\alpha and \beta$ are the roots of the quadratic equation $x^2 - 4x + k = 0 (k \neq 0)$. If $\alpha\beta, \alpha\beta^2 + \alpha^2\beta, \alpha^3 + \beta^3$ are in geometric progression, then the value of $7k/2$ equals________.

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808. If $\frac{x^2 + ax + 3}{x^2 + x + a}$ takes all real values for possible real values of x, then

a.$a^3 - 9a + 12 \leq 0$ b. $4a^5 + 39 \geq 0$ c. $a \geq \frac{1}{4}$ d. $a < \frac{1}{4}$

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809. If $\cos^4\theta + \alpha$ and $\sin^4\theta + \alpha$ are the roots of the equation $x^2 + 2bx + b = 0$ and $\cos^2\theta + \beta, \sin^2\theta + \beta$ are the roots of the equation $x^2 + 4x + 2 = 0$, then values of b are a. 2 b. -1 c. -2 d. 1

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810. If the roots of the equation $x^2 + ax + b = 0 arec$ and d, then roots of the equation $x^2 + (2c + a)x + c^2 + ac + b = 0$ are a c b. $d - c$ c. $2c$ d. 0

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811. If $a, b, c \in R$ *and* $abc < 0$, then equation $bcx^2 + (2b + c - a)x + a = 0$ *has* (a). both positive roots (b). both negative roots (c). real roots (d) one positive and one negative root

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812. For the quadratic equation $x^2 + 2(a + 1)x + 9a - 5 = 0$, which of the following is/are true? (a) If $2 < a < 5$, then roots are opposite sign (b)If $a < 0$, then roots are opposite in sign (c) if $a > 7$ then both roots are negative (d) if $2 \leq a \leq 5$ then roots are unreal

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813. Let $P(x) = x^2 + bx + cwherebandc$ are integer. If $P(x)$ is a factor of both $x^4 + 6x^2 + 25and3x^4 + 4x^2 + 28x + 5, then$ a.$P(x) = 0$ has imaginary roots b. $P(x) = 0$ has roots of opposite c.$P(1) = 4$ d .$P(1) = 6$

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814. If $\left|ax^2 + bx + c\right| \leq 1$ for all x in [0, 1], then

a. $|a| \leq 8$

b. $|b| > 8$

c. $|c| \leq 1$

d. $|a| + |b| + |c| = 19$

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815. Let $f(x) = ax^2 + bx + \cdot$ Consider the following diagram. Then Fig $c < 0$

$b > 0$ $a + b - c > 0$ $abc < 0$

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816. If roots of $ax^2 + bx + c = 0$ are $\alpha and \beta and 4a + 2b + c > 0, 4a, -2b + c > 0, and c < 0,$ then possible values /values of $[\alpha] + [\beta]$ is/are (where [.] represents greatest integer function)

a.-2 b.-1c. 0d. 1

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817. The equation $\left(\frac{x}{x+1}\right)^2 + \left(\frac{x}{x-1}\right)^2 = a(a-1)$ has

a. Four real roots if $a > 2$

b.Four real roots if $a < -1$

c. Two real roots if $1 < a < 2$

d . No real roots if $a < -1$

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818. Find the complete set of values of a such that $\left(x^2 - x\right)/(1 - ax)$ attains all real values.

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819. If α, β are roots of $x^2 + px + 1 = 0 and \gamma, \delta$ are the roots of $x^2 + qx + 1 = 0$, then prove that $q^2 - p^2 = (\alpha - \gamma)(\beta - \gamma)(\alpha + \delta)(\beta + \delta)$.

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820. If he roots of the equation $12x^2 - mx + 5 = 0$ are in the ratio 2:3 then find the value of m.

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821. If $\alpha and \beta$ are the roots of $x^2 - a(x - 1) + b = 0$ then find the value of $1/\left(\alpha^2 - a\alpha\right) + 1/\left(\beta^2 - \beta\right) + 2/a + b$.

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822. The equation formed by decreasing each root of $ax^2 + bx + c = 0$ by 1 is $2x^2 + 8x + 2 = 0$ then

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823. If the sum of the roots of an equation is 2 and the sum of their cubes is 98, then find the equation.

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824. If x is real and $\left(x^2 + 2x + c\right) / \left(x^2 + 4x + 3c\right)$ can take all real values, of then show that $0 \le c \le 1$.

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825. If α, β are the roots of the equation $2x^2 + 2(a + b)x + a^2 + b^2 = 0$, then find the equation whose roots are $(\alpha + \beta)^2 and (\alpha - \beta)^2$

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826. If $x^2 + ax + bc = 0 and x^2 + bx + ca = 0 (a \ne b)$ have a common root, then prove that their other roots satisfy the equation $x^2 + cx + ab = 0$.

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827. Let α, β are the roots of $x^2 + bx + 1 = 0$. Then find the equation whose roots are $(\alpha + 1/\beta) and (\beta + 1/\alpha)$.

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828. Find the greatest value of a non-negative real number λ for which both the equations $2x^2 + (\lambda - 1)x + 8 = 0 and x^2 - 8x + \lambda + 4 = 0$ have real roots.

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829. If $a, b, c \in R$ such that $a + b + c = 0 and a \neq c$, then prove that the roots of $(b + c - a)x^2 + (c + a - b)x + (a + b - c) = 0$ are real and distinct.

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830. If the fraction $\frac{x^3 + (a - 10)x^2 - x + a - 6}{x^3 + (a - 6)x^2 - x + a - 10}$ reduces to a quotient of two functions, then a equals_____.

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831. If the equation $(a - 5)x^2 + 2(a - 10)x + a + 10 = 0$ has roots of opposite sign, then find the value of $\dot{a}$

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832. If $\alpha and \beta$ are the roots of $ax^2 + bx + c = 0 and S_n = \alpha^n + \beta^n$, then $aS_{n+1} + bS_n + cS_{n-1} = 0$ and hence find $\dot{S_5}$

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833. If α is a root of the equation $4x^2 + 2x - 1 = 0$, then prove that $4\alpha^3 - 3\alpha$ is the other root.

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834. If both the roots of $x^2 - ax + a = 0$ are greater than 2, then find the value of a.

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835. If $\left(y^2 - 5y + 3\right)\left(x^2 + x + 1\right) < 2x$ for all $x \in R,$ then fin the interval in which y lies.

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836. The values of 'a' for which $4^x - (a - 4)2^x + \frac{9a}{4} < 0 \, \forall x \in (1, 2)$ is

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837. Find the number of positive integral values of k for which $kx^2 + (k - 3)x + 1 < 0$ for atleast one positive x.

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838. If $x^2 + 2ax + a < 0 \forall x \in [1, 2]$ then find set of all possible values of a

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839. Given that a, b, c are distinct real numbers such that expressions $ax^2 + bx + c, bx^2 + cx + a and cx^2 + ax + b$ are always non-negative. Prove that the quantity $\left(a^2 + b^2 + c^2\right)/(ab + bc + ca)$ can never lie inn $(-\infty, 1)$ ∪ $[4, \infty)$.

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840. Find the number of quadratic equations, which are unchanged by squaring their roots.

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841. Solve the following: $\left(\sqrt{x^2 - 5x + 6} + \sqrt{x^2 - 5x + 4}\right)^{\frac{x}{2}} + \left(\sqrt{x^2 - 5x + 6} - \sqrt{x^2 - 5x + 4}\right)^{x/2} = 2^{\frac{x+4}{4}}$

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842. Show that the equation $A^2/(x - a) + B^2/(x - b) + C^2/(x - c) + \ldots + H^2/(x - h) = k$ has no imaginary root, where $A, B, C, \ldots., Handa, b, c, \ldots\ldots., handk \in R$

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843. Find the values of a if $x^2 - 2(a - 1)x + (2a + 1) = 0$ has positive roots.

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844. If $\alpha and \beta, \alpha and y, \alpha and \delta$ are the roots of the equations $ax^2 + 2bx + c = 0, 2bx^2 + cx + a = 0 adn cx^2 + ax + 2b = 0,$ respectively, where a, b, and c are positive real numbers, then $\alpha + \alpha^2 =$ a.abc b. $a + 2b + c$ c. -1 d. 0

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845. If $\alpha\beta$ the roots of the equation $x^2 - x - 1 = 0$, then the quadratic equation whose roots are $\frac{1 + \alpha}{2 - \alpha}, \frac{1 + \beta}{2 - \beta}$

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846. If $a(p + q)^2 + 2bpq + c = 0$ and $a(p + r)^2 + 2bpr + c = 0(a \neq 0)$, then which one is correct? a) $qr = p^2$ b) $qr = p^2 + \frac{c}{a}$ c) none of these d) either a) or b)

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847. If α_1, α_2 are the roots of equation $x^2 - px + 1 = 0 and \beta_1, \beta_2$ are those of equation $x^2 - qx + 1 = 0$ and vector $\alpha_1\hat{i} + \beta_1\hat{j}$ is parallel to $\alpha_2\hat{i} + \beta_2\hat{j}$, then $p = a. \ \pm q$ b. $p = \ \pm 2q$ c. $p = 2q$ d. none of these

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848. Suppose A, B, C are defined as $A = a^2b + ab^2 - a^2c - ac^2, B = b^2c + bc^2 - a^2b - ab^2,$ and $C = a^2c + ' ac^2 - b^{2\prime} c -$ and the equation $Ax^2 + Bx + C = 0$ has equal roots, then a, b, c are in $\ddot{AP}$ b. $\ddot{GP}$ c. $\ddot{HP}$ d. $\dddot{AGP}$

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849. The integral value of m for which the root of the equation $mx^2 + (2m - 1)x + (m - 2) = 0$ are rational are given by the expression [where n is integer]

(A)n^2

(B) $n(n + 2)$

(C) $n(n + 1)$

(D) none of these

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850. If $b_1 . b_2 = 2\left(c_1 + c_2\right)$ then at least one of the equation $x^2 + b_1 x + c_1 = 0$ and $x^2 + b_2 x + c_2 = 0$ has a) imaginary roots b) real roots c) purely imaginary roots d) none of these

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851. If the root of the equation $(a - 1)\left(x^2 - x + 1\right)^2 = (a + 1)\left(x^4 + x^2 + 1\right)$ are real and distinct, then the value of $a \in$ a.$(- \infty, 3]$ b. $(- \infty, - 2) \cup (2, \infty)$ c. $[- 2, 2]$ d. $[- 3, \infty)$

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852. If α and β are roots of the equation $ax^2 + bx + c = 0,$ then the roots of the equation $a(2x + 1)^2 + b(2x + 1)(x - 3) + c(x - 3)^2 = 0$ are a. $\frac{2\alpha + 1}{\alpha - 3}, \frac{2\beta + 1}{\beta - 3}$ b. $\frac{3\alpha + 1}{\alpha - 2}, \frac{3\beta + 1}{\beta - 2}$ c. $\frac{2\alpha - 1}{\alpha - 2}, \frac{2\beta + 1}{\beta - 2}$ d. none of these

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853. If $a, b, c, d \in R,$ then the equation $\left(x^2 + ax - 3b\right)\left(x^2 - cx + b\right)\left(x^2 - dx + 2b\right) = 0$ has a. 6 real roots b. at least 2 real roots c. 4 real roots d. none of these

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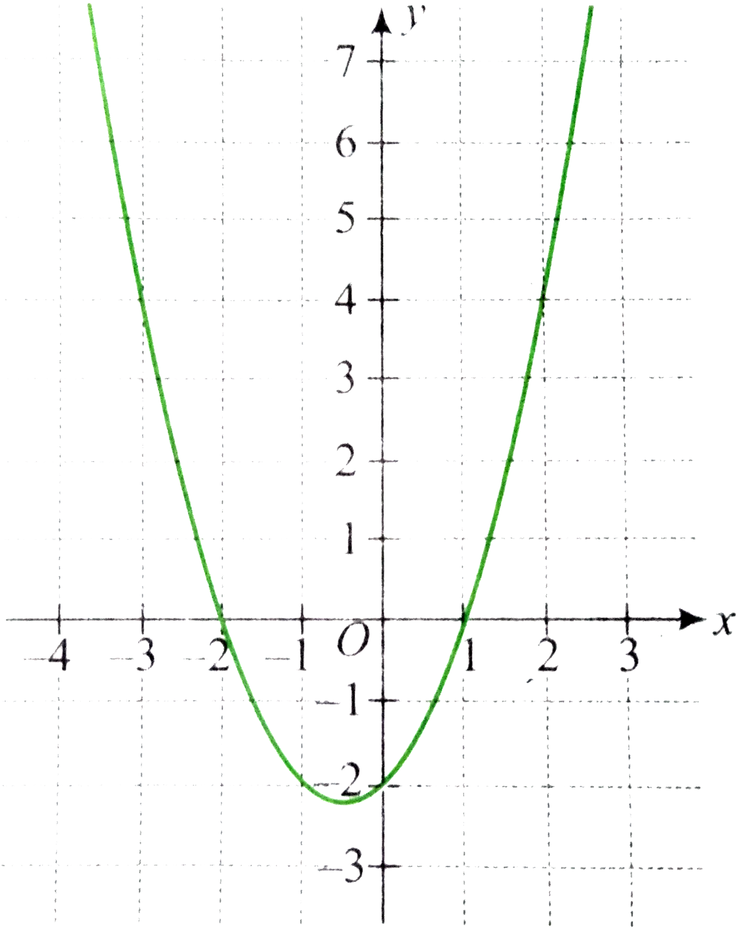
854. Graph of $y = f(x)$ is as shown in the following figure.



Find the roots of the following equations

$f(x) = 0$

$f(x) = 4$

$f(x) = x + 2$

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855. In how many points graph of $y = x^3 - 3x^2 + 5x - 3$ intersect the x-axis?

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856. The quadratic polynomial $p(x)$ has the following properties:$p(x) \geq 0$ for all real numbers, $p(1) = 0$ and $p(2) = 2$. Find the value of $p(3)$ is_________.

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857. If $(1 - p)$ is a root of quadratic equation $x^2 + px + (1 - p) = 0$, then find its roots.

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858. A polynomial in x of degree 3 vanishes when $x = 1$ and $x = -2$, ad has the values 4 and 28 when $x = -1$ and $x = 2$, respectively. Then find the value of polynomial when $x = 0$.

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859. Let $f(x) = a^2 + bx + c$ where a ,b , c in R and $a \neq 0.$ It is known that $f(5) = -3f(2)$ and that 3 is a root of $f(x) = 0.$ Then find the other of $f(x) = 0.$

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860. If $x = 1$ and $x = 2$ are solutions of equations $x^3 + ax^2 + bx + c = 0$ and $a + b = 1,$ then find the value of b.

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861. If $x \in R, and a, b, c$ are in ascending or descending order of magnitude, show that $(x - a)(x - c)/(x - b)(where x \neq b)$ can assume any real value.

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862. Prove that graphs $y = 2x - 3$ and $y = x^2 - x$ never intersect.

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863. Which of the following pair of graphs intersect? $y = x^2 - x$a and $y = 1$ $y = x^2 - 2x$ and y=sinxy=x^2-x+1 and y=x-4`

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864. If $\alpha and \beta$ are the rootsof he equations $x^2 - ax + b = 0 and A_n = \alpha^n + \beta^n$, then which of the following is true? a.$A_{n+1} = aA_n + bA_{n-1}$ b.

$A_{n+1} = bA_{n-1} + aA_n$ c. $A_{n+1} = aA_n - bA_{n-1}$ d. $A_{n+1} = bA_{n-1} - aA_n$

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865. If α, β are the roots of $x^2 + px + q = 0$ and γ, δ are the roots of $x^2 + px + r = 0$, then $\frac{(\alpha - \gamma)(\alpha - \delta)}{(\beta - \gamma)(\beta - \delta)} =$

(a) 1 (b) q (c) r (d) $q + r$

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866. If the equations $ax^2 + bx + c = 0$ and $x^3 + 3x^2 + 3x + 2 = 0$ have two common roots, then a. $a = b = c$ b. $a = b \neq c$ c. $a = -b = c$ d. none of these.

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867. The value m for which one of the roots of $x^2 - 3x + 2m = 0$ is double of one of the roots of $x^2 - x + m = 0$ is a. -2 b. 1 c. 2 d. none of these

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868. Let $p(x) = 0$ be a polynomial equation of the least possible degree, with rational coefficients having $\sqrt[3]{7} + \sqrt[3]{49}$ as one of its roots. Then product of all the roots of $p(x) = 0$ is

a. 56 b. 63 c. 7 d. 49

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869. The number of values of a for which equations $x^3 + ax + 1 = 0$ and $x^4 + ax^2 + 1 = 0$ have a common root is a) 0 b) 1 c) 2 d) Infinite

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870. If $\left(m_r, \frac{1}{m_r}\right)$ where r=1,2,3,4, are four pairs of values of x and y that satisfy the equation $x^2 + y^2 + 2gx + 2fy + c = 0$, then the value of

$m_1 . m_2 . m_3 . m_4$ is a. 0 b. 1 c. -1 d. none of these

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871. If $\alpha, \beta, \gamma, \sigma$ are the roots of the equation $x^4 + 4x^3 - 6x^2 + 7x - 9 = 0$, then the value of $\left(1 + \alpha^2\right)\left(1 + \beta^2\right)\left(1 + \gamma^2\right)\left(1 + \sigma^2\right)$ is a. 9 b. 11 c. 13 d. 5

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872. If $\tan\theta_1, \tan\theta_2, \tan\theta_3$ are the real roots of the $x^3 - (a + 1)x^2 + (b - a)x - b = 0, where \theta_1 + \theta_2 + \theta_3 \in (0, \pi)$, then $\theta_1 + \theta_2 + \theta_3$, is equal to $\pi/2$ b. $\pi/4$ c. $3\pi/4$ d. π

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873. If roots of an equation $x^n - 1 = 0 are 1, a_1, a_2, \ldots . a_{n-1}$, then the value of $\left(1 - a_1\right)\left(1 - a_2\right)\left(1 - a_3\right)\left(1 - a_{n-1}\right)$ will be n b. n^2 c. n^n d. 0

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874. If α, β are the roots of $ax^2 + bx + c = 0, (a \neq 0)$ and $\alpha + \delta, \beta + \delta$ are the roots of $Ax^2 + Bx + C = 0, (A \neq 0)$ for some constant δ then prove that $\frac{b^2 - 4ac}{a^2} = \frac{B^2 - 4AC}{A^2}$

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875. Let $f(x) = Ax^2 + Bx + c, whereA, B, C$ are real numbers. Prove that if $f(x)$ is an integer whenever x is an integer, then the numbers $2A, A + B, andC$ are all integer. Conversely, prove that if the number $2A, A + B, andC$ are all integers, then $f(x)$ is an integer whenever x is integer.

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876. Let S be a square of unit area. Consider any quadrilateral which has one vertex on each side of S. If a, b, c and d denote the lengths of sides of the quadrilateral, prove that $2 \leq a_2 + b_2 + c_2 + d_2 \leq 4$

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877. The real numbers x_1, x_2, x_3 satisfying the equation $x^3 - x^2 + bx + \gamma = 0$ are in A.P. Find the intervals in which β and γ lie.

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878. Let a, b, c be real. If $ax^2 + bx + c = 0$ has two real roots $\alpha and \beta, where \alpha \ -1 and \beta\ 1$, then show that $1 + \frac{c}{a} + \left|\frac{b}{a}\right| < 0$

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879. For a $a \leq 0,$ determine all real roots of the equation $x^2 - 2a|x - a| - 3a^2 = 0.$

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880. Solve for x: $\left(5+2\sqrt{6}\right)^{x^2-3}+\left(5-2\sqrt{6}\right)^{x^2-3}=10.$

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881. If one root of the equation $ax^2+bx+c=0$ is equal to the n^{th} power of the other, then $\left(ac^n\right)^{\frac{1}{n+1}}+\left(a^n c\right)^{\frac{1}{n+1}}+b$ is equal to

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882. If $a, b, c \in R$ and equations $ax^2+bx+c=0$ and $x^2+2x+3=0$ have a common root, then find $a:b:c$

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883. Find the condition that the expressions $ax^2-bxy+cy^2 and a_1x^2+b_1xy+c_1y^2$ may have factors $y-mx and my-x$, respectively.

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884. If $x^2 + (a - b)x + (1 - a - b) = 0. where a, b \in R,$ then find the values of a for which equation has unequal real roots for all values of b.

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885. Let a, b, c be real numbers with $a \neq 0$ and α, β be the roots of the equation $ax^2 + bx + c = 0.$ Express the roots of $a^3x^2 + abcx + c^3 = 0$ in terms of α, β.

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886. If the product of the roots of the equation $(a + 1)x^2 + (2a + 3)x + (3a + 4) = 0 is 2,$ then find the sum roots.

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