MATHS

BOOKS - CBSE COMPLEMENTARY MATERIAL MATHS (HINGLISH)

APPLICATION OF DERIVATIVES

1 Mark Questions

1. Find an angle `theta,0

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2. Find te slope of the normal to the curve $x = a\cos^2\theta$ and $y = a\sin^3\theta$ at $\theta = \frac{\pi}{4}$

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3. A balloon which always remains spherical has a variable radius. Find the rate at which its volume is increasing with respect to its radius when the radius is 7 cm.

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4. Write the interval for which the function $f(x) = \cos x, 0 \leq x \leq 2\pi$ is decreasing

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5. For what values of x is the rate of increase of $x^3 - 5x^2 + 5x + 8$ is twice the rate of increase of x?

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6. Find the point on the curve $y = x^2 - 2x + 3$, where the tangent is parallel to x-axis.

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7. Write the maximum value of $f(x) = \frac{\log x}{x}$, if it exists.

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8. Find the least value of $f(x) = ax + \frac{b}{x}$, where $a > 0,\ \ b > 0$ and $x > 0$.

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9. The interval of increase of the function $f(x) = x - e^x + \tan\left(\frac{2\pi}{7}\right)$ is

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10. Find the value of a for which the function $f(x) = x^2 - 2ax + 6, x > 0$ is strictly increasing.

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11. Find the minimum value of sin x + cos x.

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12. Which of the following functions are decreasing on $(0, \pi/2)$? (i) $\cos x$ (ii) $\cos 2x$ (iii) $\tan x$ (iv) $\cos 3x$

A. sin 2x

B. cos 3x

C. tan x

D. cos 2x

Answer: D

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13. Find the absolute maximum of $x^{40} - x^{20}$ on the interval [0,1].

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14. The angle between $y^2 = x$ and $x^2 = y$ at the origin is

A. $2\tan^{-1}\frac{3}{4}$

B. $\tan^{-1}\frac{4}{3}$

C. $\frac{\pi}{2}$

D. $\frac{\pi}{4}$

Answer: c

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15. Find local minimum value of the function f given by $f(x) = 3 + |x|, x \in R.$

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16. The distance covered by a particle in t sec is given by $x = 3 + 8t - 4t^2$. What will be its velocity after 1 second.

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17. The rate of change of volume of a sphere is equal to the rate of change of its radius, then its radius is equal to (a) 1 unit (b) units (c) unit (d) unit

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2 Mark Questions

1. Find the coordinates of the point on the curve $y^2 = 3 - 4x$ where tangent is parallel to the line $2x + y - 2 = 0$.

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2. The sum of the two numbers is 8, what will be the minimum value of the sum of their reciprocals.

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3. Find the maximum value of $f(x) = 2x^3 - 24x + 107$ in the internal [1,3]

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4. If the rate of change of area of a circle is equal to the rate of change of its diameter, then its radius is equal to (a) unit (b) unit (c) units (d) units

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5. The sides of an equilateral triangle are increasing at the rate of 2 cm/sec. Find the rate at which the area increases, when the side is 10 cm.

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6. If there is an error of a% in measuring the edge of a cube, then percentage error in its

surface is (a) 2a% (b) $\frac{a}{2}\%$ (c) 3a% (d) none of these

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7. If an error of k% is made in measuring the radius of a sphere, then percentage error in its volume is (a) k% (b) 3k% (c) 2k% (d) $k/3\%$

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8. The point on the curve $y^2 = x$ where tangent makes 45° angle with x-axis, is

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9. The slope of the tangent to the curves $x = 3t^2 + 1, y = t^2 - 1$ at t=1 is

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10. If the curves $y = 2\,e^{x}$ and $y = ae^{-x}$ intersect orthogonally, then $a =$ $1/2$ (b) $-1/2$ (c) 2 (d) $2e^{2}$

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11. Find the point on the curve $y^{2}\cdot 8x\cdot$ for which the abscissa and ordinate change at the same rate.

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12. Prove that the function $f(x) = \tan x - 4x$ is strictly decreasing on $(-\pi/3, \pi/3)$.

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13. Find the point on the curve $y = x^2$, where the slope of the tangent is equal to the x co-ordinate of the point.

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14. Use differentials to approximate the cube root of 66.

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15. Find the maimum and minimum values of the function f(x)= sin (sin x)

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16. Find the local maxima and minima of the function $f(x) = 2x^3 - 21x^2 + 36x - 20$.

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17. If $y = a \log x + bx^2 + x$ has its exteme values at x=-1 and x=2, then find a and b.

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18. Find the equation of the tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point (x_0, y_0).

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19. If the radius of a circle increases from 5 cm to 5.1 cm, find the increase in area.

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20. Find the equation of the normal to the curve $y = 2x^3 + 3\sin x \quad \text{at} \quad x = 0.$

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4 Mark Questions

1. A balloon which always remains spherical, is being inflated by pumping in 900 cubic centimetres of gas per second. Find the rate at

which the radius of the balloon is increasing when the radius is 15 cm.

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2. An inverted cone has a depth of 10cm and a base of radius 5cm. Water is poured into it at the rate of 3/2 c.c. per minute. Find the rate at which the level of water in the cone is rising when the depth is 4cm.

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3. The volume of a cube is increasing at a constant rate. Prove that the increase in surface area varies inversely as the length of the edge of the cube.

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4. A kite is moving horizontally at a height of $151.5m$. If the speed of the kite is $10\frac{m}{s}$, how fast is the string being let out, when the kite is 250 m away from the boy who is flying the

kite? The height of the boy is 1.5 m. (A) 8 m/s (B) 12 m/s (C) 16 m/s (D) 19 m/s

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5. A swimming pool is to be drained by cleaning. If L represents the number of litres of water in the pool t seconds after the pool has been plugged off to drain and $L = 2000(10 - t)^2$. How fast is the water ruining out at the end of 5 seconds? What is

the average rate at which the water flows out during the first 5 seconds?

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6. A man of 2 metres height walks at a uniform speed of 6 km/hr away from a lamp post of 6 metres high. Find the rate at which the length of his shadow increases.

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7. A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lowermost. Its semi-vertical angle is $\tan^{-1}(0.5)$. Water is poured into it at a constant rate of 5 cubic metre per hour. Find the rate at which

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8. A spherical ball of salt is dissolving in water in such a manner that the rate of decrease of

volume at any instant is proportional to the surface. Prove that the radius is decreasing at a constant rate.

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9. A conical vessel whose height is 10 meters and the radius of whose base is half that of the height is being filled with a liquid at a uniform rate of $1.5m^3/\min.$ Find the rate which the level of the water in the vessel is

rising when it is 3m below the top of the vessel.

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10. $xandy$ are the sides of two squares such that $y = x - x^2$. Find the rate of the change of the area of the second square with respect to the first square.

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11. The length of a rectangle is increasing at the rate 3.5 cm/sec. and its breadth is decreasing at the rate of 3 cm/sec. Find the rate of change of the area of the rectangle when length is 12 cm and breadth is 8 cm.

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12. If the area of circle increases at a uniform rate, then prove that the perimeter varies inversely as the radius.

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13. Show that $f(x) = x^3 - 6x^2 + 18x + 5$ is an increasing function for all $x \in R$, Findits value when the rate increases of f(x) is least.

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14. Determine whether the following function is increasing or decreasing in the given interval :

$$f(x) = \cos\left(2x + \frac{\pi}{4}\right), \frac{3\pi}{8} \leq x \leq \frac{5\pi}{8}.$$

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15. Determine for which values of x, the function $y = x^4 - \frac{4x^3}{3}$ is increasing and for which it is decreasing.

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16. Find the interval of increasing and decreasing of the function $f(x) = \frac{\log x}{x}$

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17. Find the intervals in which function f(x) = $\sin x - \cos x, 0 < x < 2\pi$ is (i) increasing, (ii) decreasing.

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18. Show that $f(x) = x^2 e^{-x}, 0 \leq x \leq 2$ is increasing in the indicated interval.

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19. Prove that $y = \dfrac{4\sin\theta}{(2+\cos\theta)-\theta}$ is an increasing function of θ in $\left[0, \dfrac{\pi}{2}\right]$.

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20. Find the intevals in which the following functionis decreasing.

$f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$

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21. Find the interval in which the function $f(x) = 5x^{\frac{3}{2}} - 3x^{\frac{5}{2}}, x > 0$ is strictly decreasing.

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22. Show that $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function on the interval $(0, \pi/4)$.

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23. Find the interval in which the function $f(x) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ is increasing or decreasing.

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24. Find intervals in which the function given by $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5} + 11$is

(a) strictly increasing (b) strictly decreasing.

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25. Find the equation of the tangent to the curve $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point $\left(\sqrt{2}a, b\right)$.

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26. Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$which is(a) parallel to the line $2xy + 9 = 0$(b) perpendicular to the line $5y15x = 13$.

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27. Find the required point be $P(x_1, y_1)$. The tangent to the curve $\sqrt{x} + \sqrt{y} = 4$ at which tangent is equally inclined to the axes.

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28. Find a point on the parabola $y = (x - 3)^2$, where the tangent is parallel to the chord joining (3, 0) and (4, 1).

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29. Find the equation of the normal to the curve $y = e^{2x} + x^2$ at $x = 0$. Also find the distance from origin to the line.

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30. The line $\frac{x}{a} + \frac{y}{b} = 1$ touches the curve $y = be^{-x/a}$ at the point

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31. At what point on the circle $x^2 + y^2 - 2x - 4y + 1 = 0$ the tangent is parallel to

(1) X-axis

(2) Y-axis

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32. Show that the equation of the normal at any point $'\theta'$ on the curvex =

$3\cos\theta - \cos^3\theta, y = 3\sin\theta - \sin^3\theta$ is

$4\left(y\cos^3\theta - x\sin^3\theta\right) = 3\sin 4\theta.$

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33. Show that the curves

$xy = a^2 and x^2 + y^2 = 2a^2$ touch each other

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34. For the curve $y = 5x - 2x^3$, if x increases at the rate of 2 units/sec, then how fast is the

slope of the curve changing when $x = 3$?

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35. Find the condition for the curve $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $xy = c^2$ to interest orthogonally.

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36. Show that the curves $y = a^x$ and $y = b^x, a > b > 0$ intersect at

an angle of $\tan^{-1}\left(\left|\frac{\log\left|\frac{a}{b}\right|}{1+\log a \log b}\right|\right)$

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37. Find the equation of the normal to the curve $ay^2 = x^3$ at the point $\left(am^2, am^3\right)$.

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38. Find the equation of the normal at a point on the curve $x^2 = 4y$, which passes through

the point (1,2). Also find the equation of the corresponding tangent.

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39. Find the point on the curve $9y^2 = x^3$, where the normal to the curve makes equal intercepts on the axes.

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40. Show that the tangents to the curve $y = 2x^3 - 3$ at the points where $x = 2$ and $x = -2$ are parallel.

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41. Using differentials, find the approximate value of $(66)^{1/3}$

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42. $\sqrt{401}$

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43. Using differentials, find the approximate value of $\sqrt{0.037}$.

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44. $\sqrt{25.3}$

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45. Using differentials, find the approximate value of $(3.968)^{\frac{3}{2}}$

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46. $(26.57)^{1/3}$

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47. Find the value of $\log_{10}(10.1)$ given that $\log_{10} e = 0.4343$.

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48. If the radius of a circle increases from 5 cm to 5.1 cm, find the increase in area.

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49. If the side of a cube be increased by 0.1% find the corresponding increase in the volume of the cube.

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50. Find the approximate value of f(2.01) where $f(x) = x^3 - 4x + 7$.

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51. Find approximate value of $\frac{1}{\sqrt{25.1}}$ using differentials.

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52. The radius of a sphere shrinks from 10 to 9.8 cm. Find approximately the decrease in its volume.

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53. Find the maximum and minimum value of $f(x) = \sin x + \frac{1}{2}\cos 2\xi n\left[0, \frac{\pi}{2}\right]$.

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54. Find the absolute maximum value and absolute minimum value of the following question

$f(x) = \left(\frac{1}{2} - x\right)^2 + x^3 \text{ in } [-2, 2.5]$

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55. Find the maximum and minimum values of $f(x) = x^{50} - x^{20}$ in the interval $[0, 1]$.

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56. Find the absolute maximum and absolute minimum value of $f(x) = (x-2)\sqrt{x-1}$ in [1,9]

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57. Find the difference between the greatest and least values of the function $f(x) = \sin 2x - x$ on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

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6 Mark Questions

1. Prove that the least perimeter of an isosceles triangle in which a circle of radius r can be inscribed is $6\sqrt{3}r$.

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2. If the sum of lengths of hypotenuse and a side of a right angled triangle is given, show that area of triangle is maximum, when the angle between them is $\frac{\pi}{3}$.

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3. Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\frac{\cos^{-1} 1}{\sqrt{3}}$

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4. The sum of the surface areas of the rectangular parallelopiped with sides x, $2x$ and $\frac{x}{3}$ and a sphere is given to be constant. Prove that the sum of the volumes is minimum, if x is equal to three times the radius of the sphere. Also, find the minimum value of the sum of their volumes.

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5. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

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6. If a cone of maximum volume is inscribed in a given sphere, then the ratio of the height of the cone to the diameter of the sphere is $3/4$ (b) $1/3$ (c) $1/4$ (d) $2/3$

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7. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

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8. Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and semi vertical

angle is one-third that of the cone and the greatest volume of cylinder is $\frac{4}{27}\pi h^3 \tan^2 \alpha$.

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9. Find the point on the curve $y^2 = 4x$ which is nearest to the point $(2,\ 1)$.

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10. The shortest distance between line y-x=1 and curve $x = y^2$ is

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11. A wire of length 36m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the lengths of the two pieces, so that the combined area of the square and the circle is minimum?

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12. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$.

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13. Find the area of the greatest rectangle that can be inscribed in an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

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