# MATHS

# BOOKS - CBSE COMPLEMENTARY MATERIAL

# MATHS (HINGLISH)

# RELATIONS AND FUNCTIONS

## One Mark Questions

**1.** If A is the set of students of a school then write, which of following relations are Universal, Empty or neither of the two.

$R_1 = \{(a, b) : a, b \text{ are ages of students and} |a - b| > 0\}$

$R_2 = \{(a, b) : a, b \text{ are weights of students, and } |a - b| < 0\}$

$R_3 = \{(a, b) : a, b \text{ are students studying in same class}\}$

A.

B.

C.

D.

**Answer:** $R_1$ **: is universal relation.**

$R_2$**: is empty relation.**

$R_3$**: is neither universal nor empty**

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**2.** Is the relation R in the set $A = \{1, 2, 3, 4, 5\}$ defined as

$R = \{(a, b) : b = a + 1\}$ reflexive ?

A.

B.

C.

D.

**Answer: No, R is not reflexive.**

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**3.** If R, is a relation in set N given by

$R = \{(a, b) : a = b - 3, b > 5\}$,

then does element $(5, 7) \in R$?

A.

B.

C.

D.

**Answer:** $(5, 7) \not\in R$

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**4.** If $f: \{1, 3\} \to \{1, 2, 5\}$and $g: \{1, 2, 5\} \to \{1, 2, 3, 4\}$ be given by $f = \{(1, 2), (3, 5)\}, g = \{(1, 3), (2, 3), (5, 1)\}$, write g of.

A.

B.

C.

D.

**Answer:** $gof = \{(1, 3), (3, 1)\}$

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**5.** Let $g, f: R \to R$ be defined by

$g(x) = \frac{x + 2}{3}, f(x) = 3x - 2$. Write fog (x)

A.

B.

C.

D.

**Answer:** $(fog)(x) = x \, \forall x \in R$

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**6.** If $f: R \to R$ defined by

$$f(x) = \frac{2x - 1}{5}$$

be an invertible function, write $f^{-1}(x)$.

A.

B.

C.

D.

**Answer:** $f^{-1}(x) = \frac{5x + 1}{2}$

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**7.** If $f(x) = \log x$ and $g(x) = e^x$. Find fog and gof , $x > 0$.

A.

B.

C.

D.

**Answer:** $gof(x) = x, fog, (x) = x$

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**8.** If $n(A) = n(B) = 3$, then how many bijective functions from A to B can be formed?

A.

B.

C.

D.

**Answer: 6**

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**9.** Is $f: N \to N$ given by $f(x) = x^2$, one-one? Give reason.

A.

B.

C.

D.

**Answer: Yes, f is one-one** $\because \forall x_1, x_2 \in N \to x_1^2 = x_2^2$.

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**10.** If $f: R \to A$, given by

$f(x) = x^2 - 2x + 2$ is onto function, find set A.

A.

B.

C.

D.

**Answer:** $A = [1, \infty)$ **because** $R_f = [1, \infty)$

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**11.** If $f: A \to B$ is bijective function such that $n(A) = 10$,

then $n(B) = ?$

A.

B.

C.

D.

**Answer:** $n(B) = 10$

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**12.** If $f: R \to R$ defined by $f(x) = \frac{x-1}{2}$, find $(fof)(x)$

A.

B.

C.

D.

**Answer:** $(fof)(x) = \frac{x-3}{4}$

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**13.** $R = \{(a, b) : a, b \in N, a \neq b \text{ and a divides b}\}$. Is R reflexive ? Give reason

A.

B.

C.

D.

**Answer: No, R is not reflexive** $\because (a, a) \not\in R \forall a \in N$

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**14.** Is $f: R \to R$, given by $f(x) = |x - 1|$ one-one? Give reason

A.

B.

C.

D.

**Answer: f is not one-one function**

$\because f(3) = f(-1) = 2$

$3 \neq -1$ **i.e. distinct elements have same images.**

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**15.** $f: R \to B$ given by $f(x) = \sin x$ is onto function, then write set B.

A.

B.

C.

D.

**Answer:** $B = [-1, 1]$

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**16.** If $f(x) = \log\left(\frac{1+x}{1-x}\right)$ show that $f\left(\frac{2x}{1+x^2}\right) = 2(f(x))$

A.

B.

C.

D.

**Answer:**

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**17.** State the reason for the relation $R$ on the set {1, 2, 3} given by $R = \{(1, 2), (2, 1)\}$ not to be transitive.

A.

B.

C.

D.

**Answer:** $(1, 2) \in R$ **and** $(2, 1) \in R$ **but** $(1, 1) \notin R$

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**18.** If $R = \{(x, y) : x + 2y = 8\}$ is a relation on $N$ , then write the range of $R$ .

A.

B.

C.

D.

**Answer: Range** $= \{1, 2, 3\}$

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**19.** Let $A = \{(0, 1, 2, 3\}$ and define a relation R on A as follows:

$R = \{(0, 0), (0, 1), (0, 3), (1, 0), (1, 1), (2, 2), (3, 0), (3, 3)\}$

, Is R reflexive? Symmetive? Transitive?

A.

B.

C.

D.

**Answer: Reflexive and symmetric but not transitive.**

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**20.** Write the smallest equivalence relation on the set $A = \{1,\ 2,\ 3\}$.

A.

B.

C.

D.

**Answer:** $\{(1, 1), (2, 2), (3, 3)\}$

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**21.** Let the relation R be defined in N by aRb, if 2a + 3b = 30.

Then R = ..... .

A.

B.

C.

D.

**Answer:** $\{(3, 8), (6, 6), (9, 4), \{12, 2\}$

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**22.** If $f: R \to R$ be defined by $f(x) = \frac{x}{\sqrt{1 + x^2}}$, then $(fofof)(x) = \ldots\ldots\ldots\ldots$

A.

B.

C.

D.

**Answer:** $(fofof)(x) = \frac{x}{\sqrt{3x^2+1}}$

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**23.** If $f(x) = \left[4-(x-7)^3\right]$, then $f^{-1}(x) = \ldots\ldots\ldots\ldots$

A.

B.

C.

D.

**Answer:** $f^{-1}(x) = 7 + (4-x)^{1/3}$

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**24.** Let $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$and let $f = \{(1, 4), (2, 5), (3, 6)\}$be a function from A to B. Show that f is one-one.

A.

B.

C.

D.

**Answer: Yes**

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**25.** If $A = \{1, 2, 3, 4\}$ and $B = \{-1, 3\}$, then what is the number of onto functions from A to B?

A.

B.

C.

D.

**Answer: 14**

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**26.** If $A = \{-1, 2, 3\}$ and $B = \{0, 3, 5\}$ then what is the number of bijections from A to B?

A.

B.

C.

D.

**Answer: 6**

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**27.** If $A = \{-1, 2, 3\}$ and $B = \{0, 3, 5, 7\}$ then what is the number of bijections from A to B?

A.

B.

C.

D.

**Answer: 24**

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## Two Mark Questions

**1.** Check the following functions for one-one and onto

(a) $f: R \to R,\ f(x) = \dfrac{2x-3}{7}$

(b) $f: R \to R,\ f(x) = |x+1|$

(c) $f: R - \{2\} \to R,\ f(x) = \dfrac{3x-1}{x-2}$

(d) $f: R - \{-1, 1\},\ f(x) = \sin^2 x.$

A.

B.

C.

D.

**Answer: (a) Bijective (one-one, onto)**

**(b) Neither one-one nor onto**

**(c) One-one but not onto**

**(d) Neither one-one nor onto**

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**2.** Let $f, g: R \overrightarrow{R}$ be two functions defined as $f(x) = |x| + x$ and $g(x) = |x| - x$ , for all $xR$. Then find fog and gof.

A.

B.

C.

D.

**Answer:** $gof(x) = 0 \forall x \in R$

$$fog(x) = \begin{cases} 0, & x \geq 0 \\ -4x, & x > 0 \end{cases}$$

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**3.** If $f: [1, \infty) \to [2, \infty)$ is defined by $f(x) = x + \frac{1}{x}$, find $f^{-1}(x)$

A.

B.

C.

D.

**Answer:** $f^{-1}(x) = \dfrac{x + \sqrt{x^2 - 4}}{2}$

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**4.** Let $A = \{1, 2, 3\}$ and define $R = \{(a, b) : a - b = 12\}$.

Show that R is empty relation on Set A.

A.

B.

C.

D.

**Answer:**

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**5.** Let $A = \{1, 2, 3\}$ and define $R = \{(a, b) : a + b > 0\}$.

Show that R is a universal relation on Set A.

A.

B.

C.

D.

**Answer:**

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**6.** Let $A = \{a, b, c\}$. How may relation can be defined in the set ? How may of these are reflexive ?

A.

B.

C.

D.

**Answer: 512, 64**

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**7.** Let $f: R \to R$ be defined by $f(x) = x^2 + 1$, find the pre image of $17$ and $-3$, respectively, are

A.

B.

C.

D.

**Answer: $\pm 4$, pre image of $-3$ does not exist.**

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**8.** If $f: R \to R$, $g: R \to R$, given by $f(x) = [x]$, $g(x) = [x]$, then find fog $\left(-\frac{2}{3}\right)$ and gof $\left(-\frac{2}{3}\right)$.

A.

B.

C.

D.

**Answer:**

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**9.** Let Q be the set of rational number and R be the relation on Q defined by $R = \{(x, y) : x, y \in Q, x^2 + 3y^2 = 4xy\}$ check whether R is reflexive, symmetric and transitive.

A.

B.

C.

D.

**Answer: R is reflexive, not symmetric, not transitive.**

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**10.** Let $A = \{2, 4, 6, 8\}$ and R be the relation "is greater than" on the set A. Write R as a set of order pairs. Is this relation

(i) reflexive? (ii) symmetic? (iii) equivalance relation?

Justify your answer.

A.

B.

C.

D.

**Answer:** $R = \{(8, 6), (8, 4), (8, 2), (6, 4), (6, 2), (4, 2)\}$
**(i) Not reflexive (ii) Not symmetric (iii) Not equivalence relation**

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**11.** Let N be the set of natural numbers and relation R on N be defined by $R = \{(x, y) : x, y \in N, x + 4y = 10\}$ check whether R is reflexive , symmetric and transitive.

A.

B.

C.

D.

**Answer:**

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## Four Mark Questions

**1.** Let $f: R - \left\{\frac{-4}{3}\right\} \to R - \left\{\frac{4}{3}\right\}$ be a function given by $f(x) = \frac{4x}{3x+4}$

Show that f is invertible with $f^{-1}(x) = \frac{4x}{4-3x}$

A.

B.

C.

D.

**Answer:**

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**2.** Let R be the relation on set $A = \{x: x \in Z, 0 \leq x \leq 10\}$ given by $R = \{(a, b): (a-b) \text{is divisible by } 4\}$. Show that

R is an equivalence relation.

Also, write all elements related to 4.

A.

B.

C.

D.

**Answer: Elements related to 4 are 0, 4, 8.**

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**3.** Show that function $f: A \to B$ defined as $f(x) = \frac{3x+4}{5x-7}$ where $A = R - \left\{\frac{7}{5}\right\}, B = R - \left\{\frac{3}{5}\right\}$ is invertible and hence find $f^{-1}$.

A.

B.

C.

D.

**Answer:** $f^{-1}(x) = \dfrac{7x+4}{5x-3}$

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**4.** Prove that the relation $R$ on the set $N \times N$ defined by $(a,\ b)R\,(c,\ d) a + d = b + c$ for all $(a,\ b),\ (c,\ d) \in N \times N$ is an equivalence relation. Also, find the equivalence classes [(2, 3)] and [(1, 3)].

A.

B.

C.

D.

**Answer:**

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**5.** Show that $f: R_+ \to R_+$ defined by $f(x) = \frac{1}{2x}$ is bijective, where $R_+$ is the set of all non-zero positive real numbers.

A.

B.

C.

D.

**Answer:**

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**6.** Let $A = \{1, 2, 3, ....., 12\}$ and R be a relation in $A \times A$ defined by $(a, b)R(c, d)$ if $ad = bc \forall (a, b), (c, d) \in A \times A$. Prove that R is an equivalence relation. Also obtain the equivalence class $[3 \quad 4]$.

A.

B.

C.

D.

**Answer:** $[3 \quad 4] = \{(3, 4), (6, 8), (9, 12)\}$

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**7.** Let $A = \{1, 2, 3, , 9\}$ and $R$ be the relation on $A \times A$ defined by $(a, b)R(c, d)$ if $a + d = b + c$ for all $(a, b), (c, d) \in A \times A$ . Prove that $R$ is an equivalence relation and also obtain the equivalence class [(2, 5)].

A.

B.

C.

D.

**Answer:** Equivalence class

$[2 \quad 5] = \{(1,4),(2,5),(3,6),(4,7),(5,8),(6,9)\}.$

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**8.** Show that $f: N \to N$ given by

$$f(x) = \begin{cases} x+1, \text{if x is odd} \\ x-1, \text{if x is even} \end{cases}$$

is both one-one and onto.

A.

B.

C.

D.

**Answer:**

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**9.** Consider $f : R_+ \overrightarrow{4, \infty}$ given by $f(x) = x^2 + 4$. Show that $f$ is invertible with the inverse $f^{-1}$ of $f$ given by $f^{-1}(y) = \sqrt{y-4}$, where $R_+$ is the set of all non-negative real numbers.

A.

B.

C.

D.

**Answer:**

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**10.** Let $A = R - \{2\}$ and $B = R - \{1\}$ if $f: A \to B$ is a function defined by $f(x) = \dfrac{x-1}{x-2}$ show that f is one-one and onto. Hence find $f^{-1}$.

A.

B.

C.

D.

**Answer:** $f^{-1}(x) = \dfrac{2x-1}{x-1}$

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**11.** Let $f: R\overrightarrow{R}$ be a function given by $f(x) = ax + b$ for all $x \in R$. Find the constants $aandb$ such that $fof = I_R$.

A.

B.

C.

D.

**Answer:** $a = 1$ **and** $b = 0$ **or** $a = -1$ **and b can be any real number.**

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**12.** Prove that the relation $R$ in the set $A = \{5, 6, 7, 8, 9\}$ given by $R = \{(a, b): |a - b|,$ is divisible by 2$\}$, is an

equivalence relation. Find all elements related to the element 6.

A.

B.

C.

D.

**Answer: {6,8}**

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**13.** Let $f: W \to W$ be defined as $f(n) = n - 1$, if is odd and $f(n) = n + 1$, if n is even. Show that f is invertible. Find the inverse of f. Here, W is the set of all whole numbers.

A.

B.

C.

D.

**Answer:** $f^{-1}(y) = \begin{cases} y+1, \text{if y is even} \\ y-1, \text{if y is odd} \end{cases}$

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## Six Mark Questions

**1.** Let $N$ denote the set of all natural numbers and R be the relation on $NxN$ defined by

$(a, b)R(c, d)ad(b + c) = bc(a + d)$. Check whether R is an equivalence relation on $NxN$.

A.

B.

C.

D.

**Answer:**

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**2.** Let $f: N \to R$ be a function defined as $f(x) = 4x^2 + 12x + 15$.

Show that $f: N \to S$, where S is the range of f, is invertible.

Also find the inverse of f. Hence find $f^{-1}(31)$.

A.

B.

C.

D.

**Answer:** $f^{-1}(y) = \frac{\sqrt{y-6}-3}{2}, f^{-1}(31) = 1$

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**3.** If the function $f: R \to R$ be defined by $f(x) = 2x - 3$ and $g: R \to R$ by $g(x) = x^3 + 5$, then show that fog is invertible. Also find $(fog)^{-1}(x)$, hence find $(fog)^{-1}(9)$.

A.

B.

C.

D.

**Answer:** $(fog)^{-1}(x) = \left(\frac{x+7}{2}\right)^{1/3}, (fog)^{-1}(9) = 2$

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**4.** Test whether, $R_3$ on $R$ defined by $(a,\ b) \in R_3 a^2 - 4\,ab + 3b^2 = 0\,.$

A.

B.

C.

D.

**Answer: Reflexive, not symmetric, not transitive**

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**5.** Let $A = \{(1, 2, 3, 4\}, B = (3, 5, 7, 9\}$ and $C = (7, 23, 47, 79\}$ and $f = A \to B, g: B \to C$ be defined by $f(x) = 2x + 1 \forall x \in A$ and $g(x) = x^2 - 2 \forall x \in B$.. Find $(gof)^{-1}$ and $f^{-1}og^{-1}$ at sets of ordered pairs. Is $(gof)^{-1} = f^{-1}og^{-1}$?

A.

B.

C.

D.

**Answer:** $(gof)^{-1} = \{7, 1\}, (23, 2), (47, 3), (79, 4)\}$

$f^{-1}of^{-1} = \{(7, 1), (23, 2), (47, 3), (79, 4),$ **Yes**

$(gof)^{-1} = f^{-1}og^{-1}.$

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**6.** Let $A = \{-1, 0, 1, 2\}$, $B = \{-4, -2, 0, 2\}$ and $f, g: A \to B$ be functions defined by $f(x) = x^2 - x, x \in A$ and $g(x) = 2\left|x - \left(\frac{1}{2}\right)\right| - 1, x \in A$ . Are f and g equal? Justify your answer. (Hint: One may note that two functio

A.

B.

C.

D.

**Answer:** $(gof)(x) = 2\left|x^2 - x - \frac{1}{2}\right| - 1$

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**7.** Consider $f: R_+ \to [-9, \infty)$ given by $f(x) = 5x^2 + 6x - 9$, where $R_+$ is the set of all non-negative real numbers. Prove that f is invertible. Also find the inverse of f. Hence find $f^{-1}(2)$ and $f^{-1}(18)$.

A.

B.

C.

D.

**Answer:**

$$f^{-1}(x) = \frac{\sqrt{54 - 5y} - 3}{5} f^{-1}(2) = 1, f^{-1}(18) = \frac{9}{5}$$

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