MATHS

BOOKS - CENGAGE MATHS (ENGLISH)

LIMITS AND DERIVATIVES

Others

1. Evaluate the limit: $(\lim)(x \rightarrow 0)\frac{\sin ax}{\sin bx}$

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2. If $(\lim)_{x\vec{1}}\frac{a\sin(x-1)+b\cos(x-1)+4}{x^2-1} = -2, then|a+b|$ is______

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3. Evaluate the limit $(\lim)_{x \to 0} \frac{\sin 3x}{x}$

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4. The integer n for which $\lim_{x \to 0} \frac{(\cos x - 1)\left(\cos x - e^x\right)}{x^n}$ is finite nonzero number is _______

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5. Evaluate the limit: $(\lim)_{n \to \infty} \left(\frac{1^2 - 2^2 + 3^3 - 4^2 + 5^2 + nterms}{n^2} \right.$

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6. Let $(\lim)_{x\vec{1}} \frac{x^a - ax + a - 1}{(x - 1)^2} = f(a)$ Then the value of $f(4)$ is _______

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7. Evaluate the limit: $(\lim)_{x \to a} \frac{\sqrt{3x - a} - \sqrt{x + a}}{x - a}$

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8. $f(x) = \frac{3x^2 + ax + a + 1}{x^2 + x - 2}$ and $\lim_{x \to -2} f(x)$ exists. Then the value of $(a - 4)$ is________

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9. Evaluate the limit: $(\lim)_{x \to \infty}\left[\sqrt{a^2x^2 + ax + 1} - \sqrt{a^2x^2 + 1}\right]$

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10. $\lim_{x \to 0} \frac{1 - \sqrt{\cos 2x} \cdot \sqrt[3]{\cos 3x} \ldots . \sqrt[n]{\cos nx}}{x^2}$ has value 10 then value of n equal to

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11. Evaluate the limit: $(\lim)_{x\vec{1}} \frac{\sum_{k=1}^{100} x^k - 100}{x - 1}$

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12. Let $S_n = 1 + 2 + 3 + \ + n$ and $P_n = \frac{S_2}{S_2 - 1} \frac{\dot{S_3}}{S_3 - 1} \frac{\dot{S_4}}{S_4 - 1} \cdots \frac{S_n}{S_n - 1}$

Where $n \in N, (n \geq 2)^{\cdot}$ Then $(\lim)_{n \to \infty} P_n = \_ \ \_\_$

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13. If $a_1 = 1 and a_{n+1} = \frac{4+3a_n}{3+2a_n}, n \geq 1, and$ if $(\lim)_{n \vec{\infty}} a_n = a,$ then find the value of a

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14. If $L = (\lim)_{x \vec{\infty}} \left\{x - x^2 (\log)_e \left(1 + \frac{1}{x}\right)\right\},$ then the value of $8L$ is_____

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15. Evaluate the limit: $(\lim)_{n \to \infty} \cos\left(\pi\sqrt{n^2+n}\right)$ when n is an integer

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16. Evaluate: $\lim_{x \to a} \frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}}, (a \neq 0)$

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17. Evaluate the limit: $(\lim)_{x\vec{0}} \frac{\sqrt{2} - \sqrt{1 + \cos x}}{\sin^2 x}$

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18. Let f^x be continuous at $x = 0$ If $(\lim)_{x\vec{0}}\left(2f(x) - 3a\frac{f(2x) + bf(8x)}{\sin^2 x}\right.$ exists and $f(0) \neq 0, f'(0) \neq 0,$ then the value of $\frac{3a}{b}$ is____

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19. Evaluate the limit: $(\lim)_{h\vec{0}}\left[\frac{1}{h(8 + h)^{\frac{1}{3}}} - \frac{1}{2h}\right]$

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20. Evaluate: $\lim_{x \to 0} \frac{e - (1 + x)^{\frac{1}{x}}}{x}$

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21. Using $\lim_{\theta \to 0}\left(\frac{\sin\theta}{\theta}\right) = 1$ prove that the area of circle of radius R is πR^2 (Figure)

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22. Evaluate: $(\lim)_{x\vec{1}}\sec\left(\frac{\pi}{2^x}\right)\log x^{\cdot}$

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23. Evaluate: $(\lim)_{x\vec{0}}\frac{\sin x}{x}$, $where[.]$ represents the greatest integer function.

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24. Let $f(x) = \lim_{m \to \infty} \left\{ \lim_{n \to \infty} \cos^{2m}(n!\pi x) \right\}$, where $x \in R$. Then prove that $f(x) = \{1$, if x is rational and 0, if x is irrational

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25. Evaluate: $\lim_{x \to \frac{\pi}{2}} \frac{1 + \cos 2x}{(\pi - 2x)^2}$

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26. Evaluate

$\lim_{n \to \infty} n^{-n^2}\left[\left(n + 2^0\right)\left(n + 2^{-1}\right)\left(n + 2^{-2}\right)\ldots\left(n + 2^{-n+1}\right)\right]^n.$

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27. Solve: $\lim \{x \rightarrow \infty\}2^{x-1}\tan\left(\frac{a}{2^x}\right)$

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28. Evaluate $\lim_{x \rightarrow \pi/2}\frac{\sin x - (\sin x)^{\sin x}}{1 - \sin x + \log_e \sin x}$.

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29. Evaluate: $\lim_{x \rightarrow 2}\frac{x^2 - x - 2}{x^2 - 2x - \sin(x - 2)}$

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30. Evaluate: $\lim_{x \rightarrow 0}\left(\frac{1}{x^2} - \frac{1}{\sin^2 x}\right)$

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31. Evaluate: $\lim_{x \to \infty} x\left(\tan^{-1}\left(\frac{x+1}{x+4}\right) - \frac{\pi}{4}\right)$

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32. Evaluate the value of

$(\lim)_{n\vec{\frac{\pi}{2}}} \tan^2 x \sqrt{\left(2\sin^2 x + 3\sin x + 4\right) - \sqrt{\sin^2 x + 6\sin x + 2}}$

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33. Evaluate the limit: $(\lim)_{x\vec{1}} \frac{\sin(\log x)}{\log x}$

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34. If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then the set of possible values of a is

(1) $(-3, 3)$ (2) $(-3, \infty)$ (3) $(3, \infty)$ (4) $(-\infty, -3)$

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35. Evaluate: $(\lim)_{x\vec{0}} \frac{1}{x} \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

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36. At the endpoint and midpoint of a circular are AB, tangent lines are drawn, and the points, A and B are jointed with a chord. Prove that the ratio of the areas of the triangles thus formed tends to 4 as the arc AB decreases infinitely.

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37. Evaluate: $\lim_{x \to 0} \frac{1 - \cos 2x}{x^2}$

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38. Evaluate $(\lim)_{x \to 0} \frac{x - \sin x}{x^3}$. (Do not use either LHospitals rule or series expansion for $\sin x$) Hence, evaluate $(\lim)_{n \to 0} \frac{\sin x - x\cos x + x^2 \cot x}{x^5}$

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39. Evaluate: $(\lim)_{x \to 0} \frac{\tan x - \sin x}{x^3}$

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40. The value of $(\lim)_{n \to 0} \left[\frac{1}{n} + \frac{e^{\frac{1}{n}}}{n} + \frac{e^{\frac{2}{n}}}{n} + + \frac{e^{\frac{n-1}{n}}}{n}\right]$ is

A. 1

B. 0

C. $e - 1$

D. $e + 1$

Answer: null

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41. Find the values of a and b in order that $\lim_{x \to 0} \frac{x(1 + a\cos x) - b\sin x}{x^3} = 1$ [using L' Hospital' srule].

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42. $(\lim)_{x \to 1} \frac{nx^{n+1} - (n+1)x^n + 1}{\left(e^x - e\right)\sin \pi x}$, where $n = 100$, is equal to :

A. $\frac{5050}{\pi e}$

B. $\frac{100}{\pi e}$

C. $-\frac{5050}{\pi e}$

D. $-\frac{4950}{\pi e}$

Answer: null

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43. Find the integral value of n for which

$\lim_{x \to 0} \frac{\cos^2 x - \cos x - e^x \cos x + e^x - \frac{x^3}{2}}{x^n}$ is a finite nonzero number

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44. If $\lim_{x \to \infty} \left(\frac{x^2 + x + 1}{x + 1} - ax - b\right) = 4,$ then

A. $a = 1, b = 4$

B. $a = 1, b = -4$

C. $a = 2, b = -3$

D. $a = 2, b = 3$

Answer: null

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45. Evaluate: $\lim_{x \to 0} \frac{\log \cos x}{x}$

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46. Let $L = \lim_{x \to 0} \frac{a - \sqrt{a^2 - x^2} - \frac{x^2}{4}}{x^4}, a > 0$. If L is finite ,then`

A. $a = 2$

B. $a = 1$

C. $L = \frac{1}{64}$

D. $L = \frac{1}{32}$

Answer: null

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47. Evaluate: $(\lim)_{x \to 0} \frac{2^x - 1}{(1 + x)^{\frac{1}{2}} - 1}$

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48. The largets value of non negative integer for which

$$\lim_{x \to 1} \left\{ \frac{-ax + \sin(x - 1) + a}{x + \sin(x - 1) - 1} \right\}^{\frac{1-x}{1-\sqrt{x}}} = \frac{1}{4}$$

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49. Evaluate: $\lim_{x \to 0} x^x$

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50. Let m and n be two positive integers greater than 1.If $\lim_{\alpha \to 0} \frac{e^{\cos \alpha^n} - e}{\alpha^m} = -\left(\frac{e}{2}\right)$ then the value of $\frac{m}{n}$ is

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51. Evaluate: $(dy/dx) of \log \sin x + \tan x$

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52. The integer n for which $\lim_{x \to 0} \frac{(\cos x - 1)\left(\cos x - e^x\right)}{x^n}$ is finite nonzero number is _______

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53. If $m, n \in I_0 and (\lim)_{x \to 0} \frac{\tan 2x - n \sin x}{x^3}$ = some integer, then find the value of n and also the value of limit.

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54. If $(\lim)_{x \to 0} \frac{\{(a - n)nx - \tan x\}\sin nx}{x^2} = 0$, where n is nonzero real number, the a is 0 (b) $\frac{n+1}{n}$ (c) n (d) $n + \frac{1}{n}$

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55. If $(\lim)_{x \to 0} \frac{\cos 4x + a \cos 2x + b}{x^4}$ is finite, find a and b using expansion formula.

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56. The value of $\lim_{x \to 0}\left((\sin x)^{\frac{1}{x}} + \left(\frac{1}{x}\right)^{\sin x}\right)$, where $x > 0$, is (a)0 (b) -1 (c) 1 (d) 2

A. 0

B. -1

C. 1

D. 2

Answer: null

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57. If $\lim_{x \to a}\frac{a^x - x^a}{x^x - a^a} = -1$ and $a > 0$, then find the value of a.

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58. If $\lim_{x \to 0}\left[1 + x\ln\left(1 + b^2\right)\right]^{\frac{1}{x}} = 2b\sin^2\theta, b > 0$,where $\theta \in (-\pi, \pi]$, then the value of θ is (a)$\pm\frac{\pi}{4}$ (b) $\pm\frac{\pi}{3}$ (c) $\pm\frac{\pi}{6}$ (d) $\pm\frac{\pi}{2}$

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59. If $L = \lim_{x \to 0}\frac{\sin 2x + a\sin x}{x^3}$ is finite, then find the value of a and L

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60. Evaluate: $(\lim)_{x \to \infty}\left(\frac{a_1^{\frac{1}{x}} + a_2^{\frac{1}{x}} + \ldots + a_n^{\frac{1}{x}}}{n}\right)^{nx}$

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61. Evaluate: $(\lim)_{x \to 0}\frac{\sin x^0}{x}$

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62. Evaluate: $(\lim)_{x \to 0} \dfrac{1 - \cos(1 - \cos x).}{x^4}$

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63. Evaluate: $\left[(\lim)_{x \to 0} \dfrac{\tan^{-1} x}{x}\right]$, where [.]represent the greatest integer function

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64. Let $f(x) = \{x + 1, x > 0n2 - x, x \le 0$ and $g(x) = \{x + 3, x < 1, x^2 - 2x - 2, 1 \le x < 2x - 5, x \ge 2$ Find the LHL and RHL of $g(f(x))$ at $x = 0$ and, hence, fin $\lim_{x \to 0} g(f(x))$.

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65. Evaluate: $(\lim)_{x \to \frac{\pi}{4}} \frac{\sqrt{2}\cos x - 1}{\cot x - 1}$

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66. $\lim_{x \to 0} \left[\left(1 - e^x\right) \frac{\sin x}{|x|} \right] is(where[.]$ represents the greatest integer function). (a)- 1 (b) 1 (c) 0 (d) does not exist

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67. Evaluate: $(\lim)_{x \vec{0}} \frac{1 - \cos mx}{1 - \cos nx}$

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68. $\lim_{x \to \infty} \frac{\cot^{-1}\left(x^{-a}\log_a x\right)}{\sec^{-1}\left(a^x \log_x a\right)}, (a > 1)$is equal to

(a)2

(b)1

(c) $(\log)_a 2$

(d) 0

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69. Evaluate: $\lim_{x \to 0} \frac{\tan 2x - x}{3x - \sin x}$

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70. The value of $(\lim)_{x\vec{a}}\sqrt{a^2 - x^2}\cot\left(\frac{\pi}{2}\right)\sqrt{\frac{a - x}{a + x}}$ is (a) $\frac{2a}{\pi}$ (b) $-\frac{2a}{\pi}$ (c) $\frac{4a}{\pi}$

(d) $-\frac{4a}{\pi}$

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71. Evaluate: $(\lim)_{x\vec{0}}\frac{\cot 2x - \operatorname{cos}ec2x}{x}$

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72. $\lim_{x \to 0}\frac{\log\left(1 + x + x^2\right) + \log\left(1 - x + x^2\right)}{\sec x - \cos x}$ = (a)-1 (b) 1 (c) 0 (d) 2

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73. Evaluate: $\left(\lim_{n \to \infty} n\cos\left(\frac{\pi}{4n}\right)\sin\left(\frac{\pi}{4n}\right)\right)$

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74. The value of $\lim_{n \to \infty} \left[\frac{2n}{2n^2 - 1} \frac{\cos(n+1)}{2n - 1} - \frac{n}{1 - 2n} \frac{n(-1)^n}{n^2 + 1} \right] is$ 1 (b) -1 (c) 0 (d) none of these

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75. Evaluate: $(\lim)_{h\vec{0}} \frac{2\left[\sqrt{3}\sin\left(\frac{\pi}{6} + h\right) - \cos\left(\frac{\pi}{6} + h\right)\right]}{\sqrt{3}h\left(\sqrt{3}\cosh - \sinh\right)}$

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76. Evaluate:

$$(\lim)_{x \to 0} \frac{8}{x^8}\left\{1 - \cos\left(\frac{x^2}{2}\right) - \cos\left(\frac{x^2}{4}\right) + \cos\left(\frac{x^2}{2}\right)\cos\left(\frac{x^2}{4}\right)\right\}$$

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77. Evaluate: $(\lim)_{x\vec{0}} \frac{\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)}{\sin^{-1}x}$

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78. Evaluate:

$$\lim_{n \to \infty} n\left\{\sqrt{\left(1-\cos\left(\frac{1}{n}\right)\right)\sqrt{\left(1-\cos\left(\frac{1}{n}\right)\right)\sqrt{\left(1-\cos\left(\frac{1}{n}\right)\right)\ldots\ldots.\infty}}}\right\}.$$

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79. Evaluate: $(\lim)_{x \to 0, y \to 0} \frac{y^2+\sin x}{x^2+\sin y^2}$ where $(x, y) \to (0, 0)$ along the curve $x = y^2$

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80. Evaluate $(\lim)_{n \to \infty}\left\{\cos\left(\frac{x}{2}\right)\cos\left(\frac{x}{4}\right)\cos\left(\frac{x}{8}\right)...\cos\left(\frac{x}{2^n}\right)\right\}$

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81. Find the value of α so that $\lim_{x \to 0}\frac{1}{x^2}\left(e^{\alpha x} - e^x - x\right) = \frac{3}{2}$

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82. If x_1 and x_2 are the real and distinct roots of $ax^2 + bx + c = 0$, then prove that $\lim_{n \to x_1}\left\{1 + \sin\left(ax^2 + bx + c\right)\right\}^{\frac{1}{x - x_1}} = e^{a\left(x_1 - x_2\right)}$

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83. If $\lim_{x \to 0}\left(1 + ax + bx^2\right)^{\frac{2}{x}} = e^3$, then find the value of a and b.

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84. Evaluate: $(\lim)_{n \to \infty} x\left[\tan^{-1}\left(\frac{x+1}{x+2}\right) - \tan^{-1}\left(\frac{x}{x+2}\right)\right]$

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85. If $(\lim)_{x \vec{\infty}}\left\{\frac{x^2+1}{x+1} - (ax+b)\right\} = 0,$ then find the value of $a and b$.

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86. Evaluate: $(\lim)_{x \to 0} \frac{e^x - 1 - x}{x^2},$ without using LHospitals rule and expansion f the series.

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87. If $(\lim)_{x \to 0} \frac{ae^x - b}{x} = 2,$ then find the value of $a and b$.

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88. Evaluate: $(\lim)_{x\vec{1}}\frac{\sin\{x\}}{\{x\}}$ if exists, where $\{x\}$ is the fractional part of

x.

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89. Evaluate: $(\lim)_{x\vec{-}2^+}\frac{x^2-1}{2x+4}$

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90. Evaluate: $(\lim)_{n\vec{0}}\left(1^{1/\sin^2 x}+2^{1/\left(\sin^2 x\right)}+\ +n^{1/\sin^2 x}\right)^{\sin^2 x}$

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91. Let $f(x) = \{\cos[x], x \geq 0|x| + a, x < 0$ The find the value of a, so that $(\lim)_{x \to 0} f(x)$ exists, where $[x]$ denotes the greatest integer function less than or equal to x .

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92. If $f(x) = |x - 1| - [x], where\ [x]$ is the greatest integer less then or equal to x , then

(A)$f(1 + 0) = -1, f(1 - 0) = 0$

(B) $f(1 + 0) = 0 = f(1 - 0)$

(C) $(\lim)_{x \to 1} f(x)$ exists

(D) $(\lim)_{x \to 1} f(x)$does not exist

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93. If $y = 2^{-2\left(\frac{1}{1-x}\right)}$, then find $\lim_{x \to 1^+} y$

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94. Let $f(x) = \begin{cases} 1 + \frac{2x}{a}, 0 \le x < 1 \text{ and } ax, 1 \le x < 2 \end{cases}$ If $\lim_{x \to 1} f(x)$ exists ,then a is (a)1 (b) -1 (c) 2 (d) -2

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95. Evaluate $(\lim)_{x \to 0} \frac{\sin x - 2}{\cos x - 1}$

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96. $(\lim)_{x \to 0} \left(\frac{\sin\left(\pi \cos^2 x\right)}{x^2}\right)$ is equal to (a) $-\pi$ (b) π (c) $\frac{\pi}{2}$ (d) 1

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97. Evaluate $\lim_{x \to 0^-} \dfrac{x^2 - 3x + 2}{x^3 - 2x^2}$

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98. For $x \in R$, $\lim_{x \to \infty} \left(\dfrac{x - 3}{x + 2}\right)^x$ is equal to (a) e (b) e^{-1} (c) e^{-5} (d) e^5

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99. Evaluate $\lim_{n \to \infty} \left[\sum_{r=1}^{n} \dfrac{1}{2^r}\right]$, where [.] denotes the greatest integer function.

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100. $(\lim)_{x \vec{\infty}}\left\{\frac{1}{1-n^2}+\frac{2}{1-n^2}+\ldots\ldots\frac{n}{1-n^2}\right\} isequa < o$ 0 (b) $-\frac{1}{2}$ (c) $\frac{1}{2}$

(d) none of these

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101. Evaluate $(\lim)_{x \to \frac{5\pi}{4}}[\sin x+\cos x]$, where [.] denotes the greatest integer function.

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102. If $G(x)=-\sqrt{25-x^2}$, *then* $\lim_{x \to 1}\frac{G(x)-G(1)}{x-1}$ *is* (a) $\frac{1}{24}$ (b) $\frac{1}{5}$ (c) $-\sqrt{24}$ (d) none of these

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103. Evaluate the left-and right-hand limits of the function defined by

$f(x) = \begin{cases} 1 + x^2 & 0 \le x < 1 \\ 2 - x & x > 1 \end{cases}$ at $x = 1$ Also, show that $\lim_{x \to 1} f(x)$ does not exist

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104. If $f(x) = \sqrt{\dfrac{x - \sin x}{x + \cos^2 x}}$, then $\lim (x \to \infty) f(x)$ is

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105. Evaluate the left-and right-hand limits of the function

$f(x) = \left\{ \dfrac{|x - 4|}{x - 4}, x \ne 4, 0, x = 4, atx = 4 \right.$

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106. If $(\lim)_{x \to a}[f(x)g(x)]$ exists, then both $(\lim)_{x \to a}f(x) and (\lim)_{x \to a}g(x)$ exist.

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107. If $\alpha_1, , \alpha_2, , \alpha_n$ are the roots of equation $x^n + nax - b = 0,$ show that $\left(\alpha_1 - \alpha_2\right)\left(\alpha_1 - \alpha_3\right)\left(\alpha_1 - \alpha_n\right) = n\left(\alpha_1^n - 1 + a\right)$

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108. $\lim\ x \to 0\left(\frac{x\tan 2x - 2x\tan x}{(1 - \cos 2x)^2}\right)$ is equal to

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109. Evaluate: $(\lim)_{x\vec{0}} \frac{\sin^{-1}x - \tan^{-1}x}{x^3}$

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110. $\lim_{x \to 1} \frac{\sqrt{1 - \cos 2(x - 1)}}{x - 1}$ a. exists and its equals $\sqrt{2}$ b. exists and its equals $\sqrt{-2}$ c.does not exist because $x - 1 \to 0$ d. L.H.L not equal R.H.L

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111. $\lim_{x \to 0} \sin^2\left(\frac{\pi}{2 - px}\right)^{\sec^2\left(\left(\frac{\pi}{2 - px}\right)\right)}$

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112. The value of $\lim_{x \to 0} \frac{\sqrt{\frac{1}{2}(1 - \cos 2x)}}{x}$ *is* (a)1 (b) -1 (c) 0 (d) none of these

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113. Evaluate: $(\lim)_{x \to \frac{7}{2}}\left(2x^2 - 9x + 8\right)^{\cot(2x-7)}$

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114. If $f(x) = \begin{cases} \frac{\sin[x]}{[x]}, & \text{for } [x] \neq 0 \\ 0, & \text{for } [x] = 0 \end{cases}$ where $[x]$ denotes the greatest integer less than or equal to x. Then find $\lim_{x \to 0} f(x)$.

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115. Let $f(a) = g(a) = k$ and their nth derivatives exist and are not equal for some n. If. $(\lim)_{x\vec{a}} \frac{f(a)g(x) - f(a) - g(a)f(x) + g(a)}{g(x) - f(x)} = 4$. Find the value of k.

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116. If $f(x) = \begin{cases} x^n \sin\left(\frac{1}{x^2}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$, $(n \in I)$, then (a) $\lim_{x \to 0} f(x)$ exists for $n > 1$ (b) $\lim_{x \to 0} f(x)$ exists for $n < 0$ (c) $\lim_{x \to 0} f(x)$ does not exist for any value of n (d) $\lim_{x \to 0} f(x)$ cannot be determined

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117. Let $f(x)$ be a twice-differentiable function and $f(0) = 2$. The evaluate: $(\lim)_{x \vec{0}} \frac{2f(x) - 3f(2x) + f(4x)}{x^2}$

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118. The value of $\lim_{x \to 1} \left(\frac{p}{1 - x^p} - \frac{q}{1 - x^q}\right), p, q, \in N,$ equal (a) $\frac{p + q}{2}$ (b) $\frac{pq}{2}$ (c) $\frac{p - q}{2}$ (d) $\sqrt{\frac{p}{q}}$

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119. Evaluate : $\lim x \to 0(\log)_{\tan^2 x}\left(\tan^2 2x\right)^{\cdot}$

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120. $(\lim)_{x\vec{0}}\frac{\sin\left(x^2\right)}{1n\left(\cos\left(2x^2 - x\right)\right)}$ is equal to (a) 2(b) -2 (c) 1 (d) -1

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121. If the graph of the function $y = f(x)$ has a unique tangent at the point $(a, 0)$ through which the graph passes, then evaluate $(\lim)_{x\vec{a}}\frac{(\log)_e\{1 + 6f(x)\}}{3f(x)}$

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122. $\lim_{x \to -1} \frac{1}{\sqrt{|x| - \{-x\}}}$ (where $\{x\}$ denotes the fractional part of (x)) is equal to (a)does not exist (b) 1 (c)∞ (d) $\frac{1}{2}$

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123. Evaluate: $\lim_{x \to 0} x^m (\log x)^n, m, n \in N$.

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124. Let $f(x) = (\lim)_{n \to \infty} \frac{1}{\left(\frac{3}{\pi}\tan^{-1}2x\right)^{2n} + 5}$. Then the set of values of x for which $f(x) = 0$ is (a)$|2x| > \sqrt{3}$ (b) `|2x|

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125. Evaluate $\left(\lim x \to \infty\right)\left(x(\log)_e\left\{\frac{\sin\left(a + \frac{1}{x}\right)}{\sin a}\right\}\right), 0 < a < \frac{\pi}{2}$

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126. $\lim x \to 0\left\{(1 + x)^{\frac{2}{x}}\right\}$ (where {.} denotes the fractional part of x (a) $e^2 - 7$ (b) $e^2 - 8$ (c) $e^2 - 6$ (d) none of these

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127. Evaluate: $(\lim)_{x\vec{0}}\frac{2^x - 1}{\sqrt{1 + x} - 1}$

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128. If $f(n+1)=\frac{1}{2}\left\{f(n)+\frac{9}{f(n)}\right\}, n \in N,$ and $f(n)>0$ for all $n \in N,$

then find $\lim n \to \infty f(n)$

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129. Evaluate $\lim x \to 1 \frac{a^{x-1}-1}{\sin \pi x}$.

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130. Find $(\lim)_{x \vec{\infty}} \frac{5x+2\cos x}{3x+14}$ using sandwitch theorem

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131. Evaluate: $\left[(\lim)_{x \to 0} \frac{\tan x}{x}\right]$ where [.] represents the greatest

integer function

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132. If $\lim_{n \to \infty} \frac{1}{\left(\sin^{-1}x\right)^{n} + 1} = 1$, *then* find the value of x.

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133. Evaluate: $(\lim)_{x \to 0} \frac{\tan x}{x}$ where [.] represents the greatest integer function

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134. Evaluate $\lim_{x \to 0} \frac{e^{x} - e^{x\cos x}}{x + \sin x}$.

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135. Evaluate: $(\lim)_{x\vec{2}} \frac{x - 2}{(\log)_a(x - 1)}$

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136. Evaluate: $\lim_{n \to 0} \frac{e^{\sin x} - (1 + \sin x)}{\left\{\tan^{-1}(\sin x)\right\}^2}$

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137. Evaluate: $(\lim)_{x \to a} \frac{\log x - \log a}{x - a}$

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138. Evaluate: $(\lim)_{x \to \frac{3\pi}{4}} \frac{1 + (\tan x)^{\frac{1}{3}}}{1 - 2\cos^2 x}$

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139. Evaluate: $\lim_{x \to 0} \frac{10^x - 2^x - 5^x + 1}{x\tan x}$

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140. $(\lim)_{x\vec{1}} \frac{\left(\frac{x^4 + x^2 + x + 1}{x^2 - x + 1}\right)^{1 - \cos(x+1)}}{(x+1)^2} isequa < o\ 1$ (b) $\left(\frac{2}{3}\right)^{\frac{1}{2}}$ (c) $\left(\frac{3}{2}\right)^{\frac{1}{2}}$

(d) $e^{\frac{1}{2}}$

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141. Evaluate: $\lim_{x \to 0} \frac{3^{2x} - 2^{3x}}{x}$

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142. $(\lim)_{x\vec{2}}\left(\left(\frac{x^3-4x}{x^3-8}\right)^{-1}-\left(\frac{x+\sqrt{2x}}{x-2}-\frac{\sqrt{2}}{\sqrt{x}-\sqrt{2}}\right)^{-1}\right) isequa < o\ \frac{1}{2}$ (b) 2

(c) 1 (d) none of these

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143. Evaluate: $(\lim)_{x\vec{0}}\frac{\log(5+x)-\log(5-x)}{x}$

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144. Each question contains statements given in two columns which have to be matched. Statements a,b,c,d in column I have to be matched with statements p,q,r,s in column II. If the correct match are a-p, a-s, b-r, c-p, c-q, and d-s, then the correctly bubbled 4 x 4 matrix should be as follows: fig Column I, Column II If

$L=(\lim)_{x\vec{-}1}\frac{(7-x)^{\frac{1}{3}}-2}{(x+1)}, then 12L =$, p. -2 If

$L = (\lim)_{x \to \frac{\pi}{4}} \frac{\tan^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$, then $- \frac{L}{4} =$, q. 2 If

$L = (\lim)_{x \to 1} \frac{(2x - 3)\left(\sqrt{x} - 1\right)}{2x^2 + x - 3}$, then $20L =$, r. 1 If

$L = (\lim)_{x \to \infty} \frac{\log x^n - [x]}{[x]}$, where $n \in N$, ($[x]$ denotes greatest integer less than or equal to x), then $-2L =$, s. -1

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145. Let $p_n = a^{P_{n-1}} - 1, \forall n = 2, 3, \ldots$, and let $P_1 = a^x - 1$, where $a \in R^+$.

Then evaluate $(\lim)_{x \to 0} \frac{P_n}{x}$

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146. Column I ([.] denotes the greatest integer function), Column II

$(\lim)_{x \to 0}\left(\left[100 \frac{\sin x}{x}\right] + \left[100 \frac{\tan x}{x}\right]\right)$, p. 198

$(\lim)_{x\vec{0}}\left(\left[100\frac{x}{\sin x}\right]+\left[100\frac{\tan x}{x}\right]\right)$, q. 199

$(\lim)_{x\vec{0}}\left(\left[100\frac{\sin^{-1}x}{x}\right]+\left[100\frac{\tan^{-1}x}{x}\right]\right)$, r. 200

$(\lim)_{x\vec{0}}\left(\left[100\frac{x}{\sin^{-1}x}\right]+\left[100\frac{\tan^{-1}x}{x}\right]\right)$, s. 201

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147. Let $f(x) = \{x + 1, x > 0, 2 - x, x \leq 0$ and $g(x) = \{x + 3, x < 1, x^2 - 2x - 2, 1 \leq x < 2, x - 5, x \geq 2$ Find the LHL and RHL of $g(f(x))$ at $x = 0$ and, hence, fin $\lim_{x \to 0} g(f(x))$.

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148. Evaluate: $\lim_{x \to \infty} \frac{x + 7\sin x}{-2x + 13}$ using sandwich theorem.

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149. If $f(x) = \begin{cases} \frac{x - |x|}{x}, x \neq 0, 2, x = 0 \end{cases}$, show that $\lim_{x \to 0} f(x)$ does not exist.

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150. The reciprocal of the value of: $(\lim)_{n \to \infty} \left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\left(1 - \frac{1}{4^2}\right)\left(1 - \frac{1}{n^2}\right) is$

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151. Show that $(\lim)_{x \to 0} \frac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1} does \neg e\xi st$

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152. If $f(x) = \begin{cases} x^2 + 2 & x \geq 2 \\ 1 - x & x < 2 \end{cases}$; $g(x) = \begin{cases} 2x & x > 1 \\ 3 - x & x \leq 1 \end{cases}$ then the value of $\lim_{x \to 1} f(g(x))$ is

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153. Evaluate : $(\lim)_{x \to 2^+} \frac{[x - 2]}{\log(x - 2)}$, where [.] represents the greatest integer function.

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154. The value of the limit $(\lim)_{x\vec{0}} \frac{a^{\sqrt{x}} - a^{1\sqrt{x}}}{a^{\sqrt{x}} + a^{1\sqrt{x}}}, a > 1, is$ (a) 4 (b) 2 (c) -1 (d) 0

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155. Evaluate: $(\lim)_{x \to 0} \frac{\sin[\cos x]}{1 + [\cos x]}$([.] denotes the greatest integer function).

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156. $\lim_{x \to 0} \left(\frac{1^x + 2^x + \ldots\ldots\ldots. + n^x}{n}\right)^{\frac{1}{x}}$ is equal to

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157. Consider the following graph of the function $y = f(x)^{\cdot}$ Which of the following is/are correct? fig. 1. $(\lim)_{x\vec{1}} f(x)$ does not exist. 2. $(\lim)_{x\vec{2}} f(x)$ does not exist. 3. $(\lim)_{x\vec{3}} f(x) = 3$. 4. $(\lim)_{x\overrightarrow{1.99}} f(x)$= exists

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158. If $\lim_{x \to 0} \frac{x^n - \sin x^n}{x - \sin^n x}$ is non-zero finite, then n must be equal to

(a) 4 (b) 1 (c) 2 (d) 3

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159. If $(\lim)_{x \to a}[f(x) + g(x)] = 2 and\ (\lim)_{x \to a}[f(x) - g(x)] = 1$, then find the value of $(\lim)_{x \to a} f(x)g(x)$.

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160. Among (i) $\lim_{x \to \infty} \sec^{-1}\left(\frac{x}{\sin x}\right)$ and $(ii) \lim_{x \to \infty} \sec^{-1}\left(\frac{\sin x}{x}\right)$. which of the following limit exists?

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161. Evaluate $\lim_{x \to 0} \frac{3x + |x|}{7x - 5|x|}$

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162. The value of $\lim_{x \to 0} \frac{1 + \sin x - \cos x + \log(1 - x)}{x^3} is$

$\frac{1}{2}$

(b) $-\frac{1}{2}$

(c) 0

(d) none of these

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163. $\lim_{x \vec{\infty}} \frac{e^{\frac{1}{x^2}} - 1}{2\tan^{-1}\left(x^2\right) - \pi}$ is equal to (a) 1(b)-1(c)1/2\` (d)-1/2

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164. Evaluate: $\lim x \to \infty \left(1 + \frac{1}{a + bx}\right)^{c + dx}$, where a, b, c and dare positive

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165. If $x_1 = 3$ and $x_{n+1} = \sqrt{2 + x_n}, n \geq 1, then(\lim)_{x \to \infty} x_n$ is (a) -1 (b) 2 (c) $\sqrt{5}$ (d) 3

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166. Evaluate: $(\lim)_{x \to \infty}\left(1 + \frac{1}{x}\right)^x$

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167. $(\lim)_{x\vec{0}} \frac{\cos(\tan x) - \cos x}{x^4} isequa < o \frac{1}{6}$ (b) - $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 1

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168. Evaluate: $(\lim)_{x \to 0}\left(\frac{a^x + b^x + c^x}{3}\right)^{\frac{2}{x}}; (a, b, c > 0)$

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169. $(\lim)_{x \vec{\infty}}\{x + 5)\tan^{-1}(x + 5) - (x + 1)\tan^{-1}(x + 1)\}$ is equal to π (b) 2π (c) $\frac{\pi}{2}$ (d) none of these

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170. If $f(n) = \lim_{x \to 0}\left\{\left(1 + \sin\frac{x}{2}\right)\left(1 + \sin\frac{x}{2^2}\right)...\left(1 + \sin\frac{x}{2^n}\right)\right\}^{\frac{1}{x}}$ then find $\lim_{n \to \infty} f(n)$.

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171. If $\lim_{x \to 2^-} \frac{ae^{\frac{1}{|x+2|}} - 1}{2 - e^{\frac{1}{|x+2|}}} = \lim_{x \to 2^+} \sin\left(\frac{x^4 - 16}{x^5 + 32}\right)$, then a is

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172. The population of a country increases by 2% every year. If it increases k times in a century, then prove that $[k] = 7, where[.]$ represents the greatest integer function.

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173. $\lim_{x \to o} \left[\frac{x^4 \sin\left(\frac{1}{x}\right) + x^2}{1 + |x|^3}\right] = \_ \_ \_ \_$

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174. Evaluate the limit: $(\lim)_{x \vec{\infty}}\left(\frac{x+2}{x+1}\right)^{x+3}$

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175. ABC is an isosceles triangle inscribed in a circle of radius r. If $AB = AC$ and h is the altitude from A to BC, then triangle ABC has perimeter $P = 2\left(\sqrt{2hr - h^2} + \sqrt{2hr}\right)$ and area A= __________ and = _________ and also $(\lim)_{x \vec{0}} \frac{A}{P^3} =$ _ — _ —

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176. Evaluate: $(\lim)_{x \to \frac{\pi}{2}}(\cos x)^{\cos x}$

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177. $(\lim)_{x \to 0}\left(x^4\left(\frac{\cot^4 x - \cot^2 x + 1}{\tan^4 x - \tan^2 x + 1}\right)\right)$ is equals to (a)1 (b) 0 (c) 2 (d) none of these

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178. Evaluate: $(\lim)_{x \to 0}(1 + x)^{\operatorname{cosec} x}$

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179. $(\lim)_{x \to \infty}\left(\frac{1}{e} - \frac{x}{1 + x}\right)^x$ is equal to (a) $\frac{e}{1 - e}$ (b) 0 (c) $\frac{e}{e^{1 - e}}$ (d) does not exist

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180. Evaluate: $(\lim)_{x \to 0}(\cos x)^{\cot x}$

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181. $\lim_{x \to 1} \frac{1 - x^2}{\sin 2\pi x}$ is equal to (a) $\frac{1}{2\pi}$ (b) $-\frac{1}{\pi}$ (c) $\frac{-2}{\pi}$ (d) none of these

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182. Evaluate $(\lim)_{x\vec{0}}\left(\frac{\sin x}{x}\right)^{\left(\frac{\sin x}{x - \sin x}\right)}$

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183. $\lim_{x \vec{\infty}}\left(\frac{x^2 + 2x - 1}{2x^2 - 3x - 2}\right)^{\frac{2x + 1}{2x - 1}}$ is equal to (a) 0 (b) ∞ (c) $\frac{1}{2}$ (d) none of these

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184. Evaluate: $(\lim)_{x\vec{0}}\frac{(1+x)^{\frac{1}{x}}-e+\frac{1}{2}ex}{x^2}$

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185. $(\lim)_{x\vec{0}}\frac{x^a\sin^b x}{\sin\left(x^c\right)}$, where a , b , c in R ~{0},exists and has non-zero value. Then, $a+c=$ (a) b (b) -1 0 (d) none of these

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186. $(\lim)_{x\vec{0}}\frac{5\sin x-7\sin 2x+3\sin 3x}{x^2\sin x}$

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187. $(\lim)_{x \to \infty}\left\{\frac{x^3+1}{x^2+1} - (ax+b)\right\} = 2, then$ (a)$a = 1, b = 1$ (b) $a = 1, b = 2$ (c)$a = 1, b = -2$ (d) none of these

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188. Evaluate the following limits using sandwich theorem: $(\lim)_{x \vec{\infty}}\frac{(\log)_e x}{x}$

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189. The value of $\lim x \to 1(2-x)^{\tan\left(\frac{\pi x}{2}\right)}$ is (a) $e^{-\frac{2}{\pi}}$ (b) $e^{\frac{1}{\pi}}$ (c) $e^{\frac{2}{\pi}}$ (d) $e^{-\frac{1}{\pi}}$

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190. Evaluate the following limits using sandwich theorem: $\lim_{x \to \infty} \frac{[x]}{x}$, *where*[.] represents greatest integer function.

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191. $\lim_{x \to 0} \frac{(\sin x)^n}{(\sin x)^m}, (m < n)$, is equal to (a)1 (b) 0 (c) n/m (d) none of these

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192. If $\frac{x^2 + x - 2}{x + 3} \leq \frac{f(x)}{x^2} \leq \frac{x^2 + 2x - 1}{x + 3}$ hold for a certain interval containing the point $x = -1$ and $\lim_{x \to -1} f(x)$ then find the value of $\lim_{x \to -1} f(x)$

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193. $\lim_{x \to \infty} \frac{2\sqrt{x} + 3\sqrt[3]{x} + 4\sqrt[4]{x} + ... + n\sqrt[n]{x}}{\sqrt{(2x - 3)} + \sqrt[3]{2x - 3} + ... + \sqrt[n]{2x - 3}}$ is equal to (a) 1 (b) ∞ (c) $\sqrt{2}$ (d) none of these

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194. Let $f:(1, 2)\vec{R}$ satisfies the inequality $\frac{\cos(2x - 4) - 33}{2} < f(x) < \frac{x^2|4x - 8|}{x - 2}\ \forall x \in (1, 2)$. Then find $\lim_{x \to 2^-} f(x)$.

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195. $(\lim)_{x\vec{0}} \frac{(x + y)\sec(x + y) - x\sec x}{y}\ isequa < o$ $\sec x(x\tan x + 1)$ (b) $x\tan x + \sec x$ $x\sec x + \tan x$ (d) none of these

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196. Evaluate : $(\lim)_{x \to \infty} \frac{1}{1+n^2} + \frac{2}{2+n^2} + \ldots + \frac{n}{n+n^2}$

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197. If $L = \lim_{x \to 2} \frac{(10-x)^{\frac{1}{3}} - 2}{x-2}$, then the value of $\left|\frac{1}{4L}\right|$ is

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198. Suppose that f is a function such that $2x^2 \leq f(x) \leq x\left(x^2+1\right)$ for all x that are near to 1 but not equal to 1. Show that this fact contains enough information for us to find $(\lim)_{x \to 1} f(x)$. Also, find this limit.

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199. If $L = \lim_{x \to 0} \frac{e^{-\left(\frac{x^2}{2}\right)} - \cos x}{x^3 \sin x}$, then the value of $\frac{1}{3L}$ is

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200. If [.] denotes the greatest integer function, then find the value of $\lim_{x \to 0} \frac{[x] + [2x] + \quad + [nx]}{n^2}$

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201. If $\lim_{x \to \infty} f(x)$ exists and is finite and nonzero and if $\lim_{x \to \infty} \left\{ f(x) + \frac{3f(x) - 1}{f^2(x)} \right\} = 3$,then the value of $\lim_{x \to \infty} f(x)$ is

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202. If $3-\left(\frac{x^2}{12}\right) \leq f(x) \leq 3+\left(\frac{x^3}{9}\right)$ for all $x \neq 0$, then find the value of

$(\lim)_{x\vec{0}} f(x)$

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203. If

$f(x)=\{x-1, x \geq 12x^2-2, x<1, g(x)=\{x+1, x>0-x^2+1, x \leq 0, and h(x)$

$=|x|, then \lim_{x \to 0} f(g(h(x)))$ is___

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204. Evaluate the limits using the expansion formula of functions

$$(\lim)_{x\vec{0}}\left\{\frac{\sin x - x + \frac{x^3}{6}}{x^5}\right\}$$

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205. $\lim_{x \to \infty}$, *where* $\frac{2x - 3}{x} < f(x) < \frac{2x^2 + 5x}{x^2}$, *is*

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206. Evaluate the limits using the expansion formula of functions

$$\lim_{x \to 0} \frac{\sin x + \log(1 - x)}{x^2}$$

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207. If $\lim_{x \to 0}\left[1 + x + \frac{f(x)}{x}\right]^{\frac{1}{x}} = e^3$, then find the value of

$1n\left(\lim_{x \to 0}\left[1 + \frac{f(x)}{x}\right]^{\frac{1}{x}}\right) is\_\_$

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208. Evaluate the limits using the expansion formula of functions $(\lim)_{x\vec{0}}\frac{e^x - 1 - x}{x^2}$

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209. The value of $(\lim)_{n \to \infty}\left[(n+1)^23 - (n-1)^23\right]$ is_____

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210. Evaluate : $(\lim)_{x\vec{2}}\frac{x^2 - 5x + 6}{x^2 - 4}$

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211. If: $(\lim)_{x \to 1}\left(1 + ax + bx^2\right)^{\frac{c}{(x-1)}} = e^3$, then the value of bc is___

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212. Evaluate : $(\lim)_{x\vec{1}}\left(\frac{2}{1-x^2}+\frac{1}{x-1}\right)$

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213. $(\lim)_{x\vec{0}}\left(\frac{1+5x^2}{1+3x^2}\right)^1/x^2 = \_-$

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214. Evaluate : $(\lim)_{x \to 1}\frac{x^2+x(\log)_e x-(\log)_e x-1}{\left(x^2-1\right)}$

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215. If $L = (\lim)_{n \to \infty} \dfrac{\left(2x3^2x2^3x3^4x2^{n-1}x3^n\right)^1}{\left(n^{2+1}\right)}$, then the value of L^4 is

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216. Evaluate $(\lim)_{x \to \frac{\pi}{4}} \dfrac{1 - \sin 2x}{1 + \cos 4x}$

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217. The value of $\lim_{x \to \infty} \left((\log)_e \dfrac{(\log)_e x}{e^{\sqrt{x}}} \right.$ *is* _ _ _ _ _ _

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218. Evaluate : $(\lim)_{x \to \frac{\pi}{4}} \dfrac{1 - \cot^3 x}{2 - \cot x - \cot^3 x}$

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219. $\lim_{y \to 0}\frac{(x+y)\sec(x+y) - x\sec x}{y}$ is equal to (a) $\sec x(x\tan x + 1)$ (b) $x\tan x + \sec x$ (c) $x\sec x + \tan x$ (d) none of these

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220. Evaluate $(\lim)_{x \to 0}\frac{\sqrt{2+x} - \sqrt{2}}{x}$

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221. The value of $\lim_{m \to \infty}\left(\cos\left(\frac{x}{m}\right)\right)^{m}$ is 1 (b) e (c) e^{-1} (d) none of these

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222. Find the value of $\lim_{x \to 0} \frac{\sin x + \log_e\left(\sqrt{1 + \sin^2 x} - \sin x\right)}{\sin^3 x}$.

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223. The value of $\lim_{h \to 0} \frac{In(1 + 2h) - 2\ln(1 + h)}{h^2}$, is

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224. Evaluate : $(\lim)_{x \to \infty} \frac{ax^2 + bx + c}{dx^2 + ex + f}$

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225. The value of $\lim_{x \to 1}(2 - x)^{\tan\left(\frac{\pi x}{2}\right)}$ is $e^{-\frac{2}{\pi}}$ (b) $e^{\frac{1}{\pi}}$ (c) $e^{\frac{2}{\pi}}$ (d) $e^{-\frac{1}{\pi}}$

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226. Evaluate: $(\lim)_{x \to 2} \frac{\sqrt{(x+7)} - 3\sqrt{(2x-3)}}{(x+6)^{\frac{1}{3}} - 2(3x-5)^{\frac{1}{3}}}$

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227. $\lim_{x \to 0} \frac{\sin x^n}{(\sin x)^m}, (m < n),$ is equal to (a) 1 (b) 0 (c) n/m (d) none of these

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228. Evaluate: $(\lim)_{x \vec{\infty}} \frac{\sqrt{3x^2 - 1} - \sqrt{2x^2 - 1}}{4x + 3}$

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229. $\lim_{x \to 0}\left(x^4\frac{\cot^4 x - \cot^2 x + 1}{\tan^4 x - \tan^2 x + 1}\right)$ is equal to (a) 1 (b) 0 (c) 2 (d) none of these

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230. Evaluate: $\lim_{n \to \infty}\sin^n\left(\frac{2\pi n}{3n+1}\right), n \in N$

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231. $\lim_{x \to \infty}\left(\frac{1}{e} - \frac{x}{1+x}\right)^x$ is equal to (a) $\frac{e}{1-e}$ (b) 0 (c) $\frac{e}{e^{1-e}}$ (d) does not exist

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232. Evaluate: $\lim_{x \to 1} \frac{\sqrt{x} + \sqrt{\sqrt{x}} + \sqrt{\sqrt{\sqrt{x}}} + \sqrt{\sqrt{\sqrt{\sqrt{x}}}} - 4}{x - 1}$

Watch Video Solution

233. $\lim_{x \to 1} \frac{1 - x^2}{\sin 2\pi x}$ is equal to (a) $\frac{1}{2\pi}$ (b) $-\frac{1}{\pi}$ (c) $\frac{-2}{\pi}$ (d) none of these

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234. Evaluate $(\lim)_{x\vec{2}} \frac{x^{10} - 1024}{x^5 - 32}$

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235. $(\lim)_{x\vec{0}} \frac{1}{x} \cos^{-1}\left(\frac{1 - x^2}{1 + x^2}\right) isequa < o$ 1 (b) 0 (c) 2 (d) none of these

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236. If $(\lim)_{x \to 2} \frac{x^n - 2^n}{x - 2} = 80$ and $n \in N$,then find the value of n.

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237. $\lim_{x \to \infty} \left(\frac{x^2 + 2x - 1}{2x^2 - 3x - 2}\right)^{\frac{2x+1}{2x-1}}$ is equal to (a) 0 (b) ∞ (c) $\frac{1}{2}$ (d) none of these

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238. Evaluate: $\lim_{x \to a} \frac{(x+2)^{\frac{5}{3}} - (a+2)^{\frac{5}{3}}}{x - a}$

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239. $\lim_{x \to \infty} \dfrac{2\sqrt{x} + 3\sqrt[3]{x} + 4\sqrt[4]{x} + \ldots\ldots. + n\sqrt[n]{x}}{\sqrt{(2x-3)} + \sqrt[3]{2x-3} + \ldots\ldots. + \sqrt[n]{2x-3}}$ is equal to (a) 1 (b) ∞ (c) $\sqrt{2}$ (d) none of these

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240. Evaluate: $\lim_{x \to \infty} \left(\dfrac{\sqrt{x^2+1} - \sqrt[3]{x^2+1}}{4\sqrt{x^4+1} - \sqrt[5]{x^4+1}}\right)$

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241. The value of $\lim_{x \to 1} \dfrac{1 - \sqrt{x}}{\left(\cos^{-1}x\right)^2}$ is 4 (b) $\dfrac{1}{2}$ (c) 2 (d) $\dfrac{1}{4}$

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242. Evaluate : $(\lim)_{x \vec{\infty}} \sqrt{x}\left(\sqrt{x+c} - \sqrt{x}\right)$

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243. $\lim_{x \to \infty} \frac{(x+1)^{10} + (x+2)^{10} + \ldots + (x+100)^{10}}{x^{10} + 10^{10}}$ is equal to (a) 0 (b) 1 (c) 10 (d) 100

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244. Evaluate: $(\lim)_{x\vec{0}} \frac{e^x \wedge 2 - \cos x}{x^2}$

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245. $\lim_{x \to \infty} \left[\sqrt{x + \sqrt{x + \sqrt{x}}} - \sqrt{x}\right]$ is equal to (a)0 (b) $\frac{1}{2}$ (c) log 2 (d) e^4

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246. Evaluate: $\lim_{x \to 2} \frac{\sin\left(e^{x-2} - 1\right)}{\log(x - 1)}$

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247. $\lim_{n \to \infty} \frac{n(2n + 1)^2}{(n + 2)\left(n^2 + 3n - 1\right)}$ is equal to (a)0 (b) 2 (c) 4 (d) ∞

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248. Evaluate: $(\lim)_{x\vec{0}} \frac{x2^x - x}{1 - \cos x}$

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249. $\lim_{x \to 0} \frac{x\left(e^x - 1\right)}{1 - \cos x}$ is equal to

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250. Evaluate: $(\lim)_{x \to \infty}\left[x\left(a^{\frac{1}{x}} - 1\right)\right], a > 1$

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251. If $f(x) = \frac{2}{x - 3}, g(x) = \frac{x - 3}{x + 4}$, and $h(x) = -\frac{2(2x + 1)}{x^2 + x - 12}$ then $\lim_{x \to 3}[f(x) + g(x) + h(x)]$ is

A. -2

B. -1

C. $-\frac{2}{7}$

D. 0

Answer: C

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252. Evaluate: $(\lim)_{x \vec{0}} \frac{\left(1 - 3^x - 4^x + 12^x\right)}{\sqrt{(2\cos x + 7)} - 3}$

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253. $\lim_{x \to \infty} \left(\frac{x^3}{3x^2 - 4} - \frac{x^2}{3x + 2}\right)$ is equal to (a) does not exist (b) $1/3$ (c) 0 (d) $\frac{2}{9}$

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254. Evaluate: $(\lim)_{x \vec{0}} \frac{a^{\tan x} - a^{\sin x}}{\tan x - \sin x}, a > 0$

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255. $\lim_{x \to \infty} \frac{(2x + 1)^{40}(4x - 1)^5}{(2x + 3)^{45}}$ is equal to (a)16 (b) 24 (c) 32 (d) 8

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256. Evaluate: $(\lim)_{x\vec{a}} \frac{\log(x - a)}{\log\left(e^x - e^a\right)}$

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257. The value of $(\lim)_{x\vec{2}} \frac{\sqrt{1 + \sqrt{2 + x}} - \sqrt{3}}{x - 2} is$ (a) $\frac{1}{8\sqrt{3}}$ (b) $\frac{1}{4\sqrt{3}}$ (c) 0 (d) none of these

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258. Evaluate: $(\lim)_{x\vec{0}} \frac{e^x + e^{-x} - 2}{x^2}$

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259. $\lim_{x \to \infty} n^2\left(x^{\frac{1}{n}} - x^{\frac{1}{(n+1)}}\right), x > 0$, is equal to (a)0 (b) e^x (c) $(\log)_e x$

(d) none of these

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260. Evaluate: $(\lim)_{x - \infty} x^{\frac{1}{x}}$

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261. The value of $(\lim)_{x \to \pi} \frac{1 + \cos^3 x}{\sin^2 x} is \frac{1}{3}$ (b) $\frac{2}{3}$ (c) $-\frac{1}{4}$ (d) $\frac{3}{2}$

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262. Evaluate: $\lim_{x \to 0} \frac{(729)^x - (243)^x - (81)^x + 9^x + 3^x - 1}{x^3}$

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263. If $f(x) = \frac{3x^2 + ax + a + 1}{x^2 + x - 2}$, then which of the following can be correct $(\lim)_{x \vec{1}} f(x) e \xi s t s a = -2$ $(\lim)_{x \vec{-} 2} f(x) e \xi s t s a = 13$ $(\lim)_{x \vec{1}} f(x) = \frac{4}{3}$ $(\lim)_{x \vec{-} 2} f(x) = -\frac{1}{3}$

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264. Evaluate: $\lim n \rightarrow \infty(-1)^{n-1} \sin\left(\pi\sqrt{n^2 + 0.5n + 1}\right)$,where $n \in N$

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265. $(\lim)_{x \vec{\infty}} \frac{1}{1 + n\sin^2 nx} isequal \rightarrow (a)-1(b)0(c)1(d)$oo`

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266. Let the sequence $\left\{b_n\right\}$ real numbers satisfies the recurrence relation $b_{n+1} = \frac{1}{3}\left(2b_n + \frac{125}{b_{n^2}}\right), b_n \neq 0$. Then find the $(\lim)_{n \to \infty} b_n$.

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267. Which of the following true ({.} denotes the fractional part of the function)? $(\lim)_{x \vec{\infty}} \frac{(\log)_e x}{\{x\}} = \infty$ (b) $(\lim)_{x \vec{2}^+} \frac{x}{x^2 - x - 2} = \infty$ $(\lim)_{x \vec{1}^-} \frac{x}{x^2 - x - 2} = -\infty$ (d) $(\lim)_{x \vec{\infty}} \frac{(\log)_{0.5} x}{\{x\}} = \infty$

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268. Evaluate: $(\lim)_{n \to \infty} \left(4^n + 5^n\right)^{\frac{1}{n}}$

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269. If $(\lim)_{x \to 1}(2 - x + a[x - 1] + b[1 + x])$ exists, then a and b can take the values of (where [.] denotes the greatest integer function).

(a)$a = \frac{1}{3}, b = 1$ (b) $a = 1, b = -1$ (c) $a = 9, b = -9$ (d)$a = 2, b = \frac{2}{3}$

A. $a = \frac{1}{3}, b = 1$

B. $a = 1, b = -1$

C. $a = 9, b = -9$

D. $a = 2, b = \frac{2}{3}$

Answer: null

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270. Evaluate: $(\lim)_{n \to \infty} \frac{n^p \sin^2(n!)}{n + 1}$

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271. $(\lim)_{n \to \infty}\left(an - \frac{1 + n^2}{1 + n}\right) = b,$ where a is a finite number, then

$a = 1$ (b) $a = 0$ (c) $b = 1$ (d) $b = -1$

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272. Evaluate: $(\lim)_{x \vec{\infty}} x^3\left\{\sqrt{x^2 + \sqrt{1 + x^4}} - x\sqrt{2}\right\}$

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273. If $m, n \in N, (\lim)_{x \vec{0}} \frac{\sin x^n}{(\sin x)^m} is$ (a) 1, if n=m (b) 0, if n>m

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274. For any two complex numbers z1,z2 and any real numbers

$a and b, \left|az_1 - bz_2\right|^2 + \left|bz_1 + az_2\right|^2 =$

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275. $\lim_{n \to \infty} \frac{-3n + (-1)^n}{4n - (-1)^n}$ is equal to a. $-\frac{3}{4}$ b. 0 if n is even c. $-\frac{3}{4}$ if n is odd d. none of these

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276. Evaluate : $(\lim)_{x \vec{\infty}} \left(\sqrt{25x^2 - 3x} + 5x\right)$

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277. Given a real-valued function f such that $f(x) = \left\{ \frac{\tan^2\{x\}}{\left(x^2 - [x]^2\right)\sqrt{\{x\}\cot\{x\}}}, f \text{ or } x < 0, fx > 0 \right.$ Where $[x]$ is the integral part and $\{x\}$ is the fractional part of x, then $(\lim)_{x \vec{0}^+} f(x) = 1$, $(\lim)_{x \vec{0}^-} f(x) = \cot 1$, $\cot^{-1}\left((\lim)_{x \vec{0}^-} f(x)\right)^2 = 1$, $\tan^{-1}\left((\lim)_{x \vec{0}^+} f(x)\right) = \frac{\pi}{4}$

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278. Evaluate: $\lim_{x \to \infty}\left(\frac{x^2 + x - 1}{3x^2 + 2x + 4}\right)^{\frac{3x^2 + x}{x - 2}}$

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279. $L = (\lim)_{x \to a}\frac{|2\sin x - 1|}{2\sin x - 1}\cdot$ $Then$ limit does not exist when (a) $a = \frac{\pi}{6}$ (b) $L = -1 when a = \pi$ (c) $L = 1 when a = \frac{\pi}{2}$ (d) $L = 1 when a = 0$

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280. If $[x]$ denotes the greatest integer less than or equal to $x,$ then evaluate $(\lim)_{n \to \infty}\frac{1}{n^3}\left\{\left[1^2x\right] + \left[2^2x\right] + \left[3^2x\right] + \right\}\left[n^2x\right]\}$

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281. $f(x) = (\lim)_{n \to \infty} \frac{x}{x^{2n} + 1}$. $Then,$ A. $f\left(1^{+}\right) + f\left(1^{-1}\right) = 0$ B. $f\left(1^{+}\right) + f\left(1^{-}\right) + f(1) = \frac{3}{2}$ C. $f\left(-1^{+}\right) + f\left(-1^{-}\right) = -1$ D. $f\left(1^{+}\right) + f\left(-1^{-}\right) = 0$

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282. Evaluate the limit: $\lim_{x \to 1} \frac{(2x - 3)\left(\sqrt{x} - 1\right)}{2x^2}$

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283. $(\lim)_{x \to \infty} \sum_{x=1}^{20} \cos^{2n}(x - 10)$ is equal to (a) 0 (b) 1 (c) 19 (d) 20

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284. $\lim_{x \to 1} \frac{1 + \sin\left(\frac{3\pi x}{1+x^2}\right)}{1 + \cos\pi x}$ is (a) 0 (b) 1 (c) 2 (d) 3

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285. $f(x) = \frac{1n\left(x^2 + e^x\right)}{1n\left(x^4 + e^{2x}\right)}.$ *Then* $\lim_{x \to \infty} f(x)$ is equal to (a) 1 (b) $\frac{1}{2}$ (c) 2

(d) none of these

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286. $(\lim)_{n\vec{\infty}}\{\left(\frac{n}{n+1}\right)^{\alpha} + \sin\left(\frac{1}{n}\right)]^{n}$(when $\alpha \in Q$) is equal to (a) $e^{-\alpha}$

(b) $-\alpha$ (c) $e^{1-\alpha}$ (d) $e^{1+\alpha}$

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287. The value of $\lim_{x \to 2} \frac{2^x + 2^{3-x} - 6}{\sqrt{2^{-x}} - 2^{1-x}}$ is

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288. If $\lim (n \to \infty) \frac{n \cdot 3^n}{n(x-2)^n + n \cdot 3^{n+1} - 3^n} = \frac{1}{3}$ then the range of x is (where $n \varepsilon N$)

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289. If $(\lim)_{x \to a}[f(x)g(x)]$ exists, then both $(\lim)_{x \to a} f(x) and (\lim)_{x \to a} g(x)$ exist.

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290. If $f(x) = \lim n \rightarrow \infty n\left(x^{\frac{1}{n}} - 1\right), thenf$ or $x > 0, y > 0, f(xy)$ is equal to : (a)$f(x)f(y)$ (b) $f(x) + f(y)$ (c)$f(x) - f(y)$ (d) none of these

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291. $\lim n \rightarrow \infty \left(\frac{n^2 - n + 1}{n^2 - n - 1}\right)^{n(n-1)}$ is

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292. $(\lim)_{x \rightarrow 1}\left[\operatorname{cosec}\frac{\pi x}{2}\right]^{\frac{1}{(1-x)}}$ $(where[.]$ represents the GIF) is equal to $(a)0(b)1(c)$oo` (d) does not exist

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293. Given $(\lim)_{x\vec{0}}\frac{f(x)}{x^2} = 2, \textit{where}\ [.]$ denotes the greatest integer function, then (A) $(\lim)_{x\vec{0}}[f(x)] = 0$ (B) $(\lim)_{x\vec{0}}[f(x)] = 1$ (C)$(\lim)_{x\vec{0}}\left[\frac{f(x)}{x}\right]$ does not exist (D)$(\lim)_{x\vec{0}}\left[\frac{f(x)}{x}\right]$ exists

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294. Let $f(x) = \frac{x^2 - 9x + 20}{x - [x]}$ (where $[x]$ is the greatest integer not greater than x $\dot{}$ Then (A)$(\lim)_{x \to 5}f(x) = 1$ (B) $(\lim)_{x \to 5}f(x) = 0$ (C) $(\lim)_{x \to 5}f(x)$ does not exist. (D)none of these

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295. Use formula $\lim_{x \to 0}\frac{a^x - 1}{x} = \log(a)$ to find $\lim_{x \to 0}\frac{2^x - 1}{(1 + x)^{\frac{1}{2}} - 1}$

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296. Find $(\lim)_{x\vec{0}}\left\{\tan\left(\frac{\pi}{4}+x\right)\right\}^{1/x}$

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297. $f(x)$ is the integral of $\frac{2\sin x - \sin 2x}{x^3}, x \neq 0.$ Find $\lim_{x \to 0} f'(x)\left[where f'(x) = \frac{df}{dx}\right]$

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298. Evaluate $\lim_{h \to 0}\frac{(a+h)^2\sin(a+h) - a^2\sin a}{h}.$

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299. $\lim_{x \to \infty}\left(x\frac{\log(x)^3}{1+x+x^2}\right)$ equals 0 (b) -1 (c) 1 (d) none of these

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300. $(\lim)_{x\vec{0}}\frac{\left(2^m+x\right)^{\frac{1}{m}}-\left(2^n+x\right)^{\frac{1}{n}}}{\xi}sequa<o$ $\frac{1}{m2^m}-\frac{1}{n2^n}$ (b) $\frac{1}{m2^m}+\frac{1}{n2^n}$ $\frac{1}{m2^{-m}}-\frac{1}{n2^{-n}}$ (d) $\frac{1}{m2^{-m}}+\frac{1}{n2^{-n}}$

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301. $(\lim)_{x\vec{1}}(1-x)\frac{\tan(\pi x)}{2}=\_-\_$

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302. If $f(x)=\begin{cases}\sin x & x\neq n\pi \text{ and } n\in I_2\\ 2 & x=n\pi\end{cases}$ and $g(x)=\begin{cases}x^2+1 & x\neq 0\\ 4 & x=0\\ 5 & x=2\end{cases}$

then $\lim_{x\to 0} g\{f(x)\}$ is

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303. $(\lim)_{x\vec{0}}\left[\min\left(y^2-4y+11\right)\frac{\sin x}{x}\right](where[.]de\neg esthe$ greatest integer function is 5 (b) 6 (c) 7 (d) does not exist

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304. $\lim_{x\to\pi/2}\frac{\sin(x\cos x)}{\cos(x\sin x)}$ is equal to

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305. If $\lim_{x\to 0}\left(x^{-3}\sin 3x+ax^{-2}+b\right)$ exists and is equal to 0, then (a) $a=-3$ and $b=\frac{9}{2}$ (b) $a=3$ and $b=\frac{9}{2}$ (c) $a=-3$ and $b=-\frac{9}{2}$ (d) $a=3$ and $b=-\frac{9}{2}$

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306. If $\lim x \to 0 \frac{x^n - \sin x^n}{x - \sin^n x}$ is non-zero finite, then n must be equal to 4

(b) 1 (c) 2 (d) 3

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307. $(\lim)_{x\vec{1}} \frac{(1-x)\left(1-x^2\right)...\left(1-x^{2n}\right)}{\left\{(1-x)\left(1-x^2\right)...\left(1-x^n\right)\right\}^2}, n \in N, equals \quad \wedge 2nP_n$ (b)

$\wedge 2nC_n$ (c) $(2n)!$ (d) none of these

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308. The value of $\lim x \to 0 \left(\left[\frac{100x}{\sin x}\right] + \left[\frac{99\sin x}{x}\right]\right)$ (where [.] represents the greatest integral function) is (a)199 (b) 198 (c)0 (d) none of these

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309. The value of $(\lim)_{x \to \frac{1}{\sqrt{2}}}\left(\frac{x - \cos\left(\sin^{-1}x\right)}{1 - \tan\left(\sin^{-1}x\right)}\right)$ is (a) $-\frac{1}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\sqrt{2}$ (d) $-\sqrt{2}$

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310. The value of $\lim_{x \to \infty} \frac{\left(2^{x^n}\right)^{\frac{1}{e^x}} - \left(3^{x^n}\right)^{\frac{1}{e^x}}}{x^n}$ (where $n \in N$) is (a) $\log n\left(\frac{2}{3}\right)$ (b) 0 (c) $n \log n\left(\frac{2}{3}\right)$ (d) none of defined

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311. Let $\lim_{x \to 0} \frac{[x]^2}{x^2} = l$ and $\lim_{x \to 0} \frac{\left[x^2\right]}{x^2} = m,$ where [.] denotes greatest integer. Then (a)l exists but m does not (b)m exists but l does not (c)both l and m exist (d) neither $\ln$ or m exists

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312. $(\lim)_{x\vec{1}}\frac{x\sin(x-[x])}{x-1}, where[.]$ denotes the greatest integer function is equal to 0 (b) -1 (c) non-existent (d) none of these

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313. $(\lim)_{x\to 0}\left[\frac{\sin(sgn(x))}{(sgn(x))}\right],$ where[.] denotes the greatest integer function, is equal to (a)0 (b) 1 (c) -1 (d) does not exist

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314. $\lim_{x\to 0}\frac{2+2x+\sin 2x}{(2x+\sin 2x)e^{\sin x}}$ is

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315. If $f(x) = \dfrac{\cos x}{(1 - \sin x)^{\frac{1}{3}}}$ then (a) $(\lim)_{x \to \frac{\pi}{2}} f(x) = - \infty$ (b) $(\lim)_{x \to \frac{\pi}{2}} f(x) = \infty$ (c) $(\lim)_{x \to \frac{\pi}{2}} f(x) = \infty$ (d) none of these

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316. $\lim_{x \to -\infty} \dfrac{x^2 \cdot \tan\left(\frac{1}{x}\right)}{\sqrt{8x^2 + 7x + 1}}$ is

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317. T_1 is an isosceles triangle in circle C. Let T_2 be another Ãsosceles triangle inscribed in C whose base is one of the equal sides of T_1 and which overlaps the interior of T_1. Similarly, create isosceles triangle T_3 from T_2; T_4 and T_5, and so on. Prove that the triangle T_n, approaches an equilateral triangle as $n \to \infty$,

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318. If $f(x) = 0$ is a quadratic equation such that $f(-\pi) = f(\pi) = 0$ and $f\left(\frac{\pi}{2}\right) = -\frac{3\pi^2}{4}$, then $\lim_{x \to -\pi} \frac{f(x)}{\sin(\sin x)}$ is equal to (a)0 (b) π (c) 2π (d) none of these

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319. $(\lim)_{x \vec{\infty}} \left[\left(\frac{e}{1-e}\right)\left(\frac{1}{e} - \frac{x}{1+x}\right)\right]^{\xi} s\, e^{(1-e)}$ (b) $e^{\left(\frac{1-e}{e}\right)}$ (c) $e^{\left(\frac{e}{1-e}\right)}$ (d) $e^{\left(\frac{1+e}{e}\right)}$

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320. $(\lim)_{x \to 0}\left(\frac{1+\tan x}{1+\sin x}\right)^{\operatorname{cosec} x}$ is equal to (a)e (b) $\frac{1}{e}$ (c) 1 (d) none of these

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321. $(\lim)_{x \to \infty} \frac{\sin^4 x - \sin^2 x + 1}{\cos^4 x - \cos^2 x + 1}$ is equal to (a) 0 (b) 1 (c) $\frac{1}{3}$ (d) $\frac{1}{2}$

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322. Find $\frac{dy}{dx}$ if $y = \tan^{-1}\left(\frac{4x}{1 + 5x^2}\right) + \tan^{-1}\left(\frac{2 + 3x}{3 - 2x}\right)$

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323. $F i n d \frac{dy}{dx} if y = \tan^{-1}\left(\frac{\sqrt{1 + x^2} - 1}{x}\right)$, where $x \neq 0$

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324. If $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$, then find $\frac{dy}{dx}$

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325. If $y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right), -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$, then find $\frac{dy}{dx}$

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326. Find $\frac{dy}{dx}$ for the functions: $y = \frac{x + \sin x}{x + \cos x}$

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327. Find the derivative of the function given by $f(x) = (1+x)\left(1+x^2\right)\left(1+x^4\right)\left(1+x^8\right)$ and, hence, find $f'(1)$

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328. Find $\frac{dy}{dx}$ if $y = \sec^{-1}\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right) + \sin^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right)$

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329. Let $f(x) = x\sin\pi x, x > 0$. Then for all natural numbers $n, f'(x)$ vanishes at (A)a unique point in the interval $\left(n, n+\frac{1}{2}\right)$ (B)a unique point in the interval $\left(n+\frac{1}{2}, n+1\right)$ (C)a unique point in the interval $(n, n+1)$ (D)two points in the interval $(n, n+1)$

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330. Find $\frac{dy}{dx}$ if $y = \tan^{-1}\left(\frac{4x}{1+5x^2}\right) + \tan^{-1}\left(\frac{2+3x}{3-2x}\right)$

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331. If $y = \sec^{-1}\left(\frac{1}{2x^2 - 1}\right); 0 < x < (\sqrt{2})$, ="" then="" find="" $\frac{dy}{dx}$

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332. $\frac{d^2x}{dy^2}$ equals: (1) $\left(\frac{d^2y}{dx^2}\right)^{-1}$ (2) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$ (3) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-2}$ (4) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{3}$

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333. Find $\frac{dy}{dx}$ if $y = \log\left\{e^x\left(\frac{x - 2}{x + 2}\right)^{\frac{3}{4}}\right\}$

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334. let $f(x) = 2 + \cos x$ for all real x Statement 1: For each real t, there exists a pointc in $[t, t + \pi]$ such that $f'(c) = 0$ Because statement 2: $f(t) = f(t + 2\pi)$ for each real t

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335. Differentiate the function with respect to x using the first principle : $\tan^{-1}x$

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336. Match the following: List - I, List - II Let $y(x) = \cos\left(3\cos^{-1}x\right)$ $x \in [-1, 1], x \neq \pm\frac{\sqrt{3}}{2}$ Then $\frac{1}{y(x)}\left\{\left(x^2 - 1\right)\frac{d^2y(x)}{dx^2} + \frac{dy(x)}{dx}\right\}$ equals, 1 Let $A_1, A_2, A_n (n > 2)$ be the vertices of a regular polygon of n sides with its centre at the origin. Let $\vec{a}_k$ be the position vector of the point $A_k, k = 1, 2, n$ If $\left|\sum_{k=1}^{n-1}\left(\vec{a}_k x \vec{a}_{k+1}\right)\right| = \left|\sum_{k=1}^{n-1}\left(\vec{a}_k \vec{a}_{k+1}\right)\right|$, then the minimum value of n is, 2 If the normal from the point $P(h, 1)$ on the ellipse $\frac{x^2}{6} + \frac{y^2}{3} = 1$ is perpendicular to the line $x + y = 8$, then the value of h is, 8 Number of positive solutions satisfying the equation

$\tan^{-1}\left(\frac{1}{2x} + 1\right) + \tan^{-1}\left(\frac{1}{4x + 1}\right) = \tan^{-1}\left(\frac{2}{x^2}\right)$ is, 9

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337. Differentiate the function with respect to x using the first principle : $\cos^2 x$

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338. Differentiate the following functions with respect to x from first principles: $\sqrt{\sin x}$

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339. If $f(x)$ is differentiable and strictly increasing function, then the value of $(\lim)_{x\vec{0}}\frac{f\left(x^2\right) - f(x)}{f(x) - f(0)}$ is 1 (b) 0 (c) - 1 (d) 2

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340. If $f(x) = [2x]\sin 3\pi x$ then prove that $f'\left(k^+\right) = 6k\pi(-1)^k$, (where [.] denotes the greatest integer function and $k \in N$).

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341. $\lim h \to 0\frac{f\left(2h + 2 + h^2\right) - f(2)}{f\left(h - h^2 + 1\right) - f(1)}$ given that $f'(2) = 6$ and $f'(1) = 4$ then (a) limit does not exist (b) is equal to $-\frac{3}{2}$ (c) is equal to $\frac{3}{2}$ (d) is equal to 3

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342. Evaluate $(\lim)_{h\vec{0}}\frac{(a+h)^2\sin^{-1}(a+h)-a^2\sin^{-1}a}{h}$

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343. If $x^2+y^2=1, then$ (a)$yy''-2\left(y'\right)^2+1=0$ (b)$yy''+\left(y'\right)^2+1=0$ (c)$yy''+\left(y'\right)^{-2}-1=0$ (d)$yy''+2\left(y'\right)^2+1=0$

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344. Using the first principle, prove that $\frac{d}{dx}\left(\frac{1}{f(x)}\right)=\frac{-f'(x)}{[f(x)]^2}$

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345. If y is a function of x and $\log(x+y)-2xy=0$, then the value of $y'(0)$ is

(a)1 (b) -1 (c) 2 (d) 0

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346. Find the derivative of $\sqrt{4-x}$ w.r.t. x using the first principle.

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347. The slope of the tangent to the curve $\left(y-x^5\right)^2=x\left(1+x^2\right)^2$ at the point $(1, 3)$ is.

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348. If $f(x)=x\tan^{-1}x$, find $f'\left(\sqrt{3}\right)$ using the first principle.

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349. Let $f(\theta) = \sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$, where $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$ then the value of $\frac{d}{d(\tan\theta)} f(\theta)$ is

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350. Find the derivative of $e^{\sqrt{x}}$ w.r.t. x using the first principle.

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351. $f(x) = \tan^{-1}\left\{\frac{\log\left(\frac{e}{x^2}\right)}{\log\left(ex^2\right)}\right\} + \tan^{--1}\left(\frac{3 + 2\log x}{1 - 6\log x}\right)$ then find $\frac{d^n y}{dx^n}$

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352. If $f\left(\frac{x+2y}{3}\right)=\frac{f(x)+2f(y)}{3}\ \forall x, y \in R and f'(0)=1, f(0)=2,$ then find $f(x)$

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353. If $f(x)=\left|x^2-5x+6\right|$, then $f'(x)$ equals

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354. If graph of $y=f(x)$ is symmetrical about the point $(5, 0)$ and $f'(7)=3,$ then the value of $f'(3)$ is________

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355. Let $f\left(x^m y^n\right) = mf(x) + nf(y)$ for all $x, y \in R^+$ and for all $m, n \in R$.

If $f'(x)$ exists and has the value $\frac{e}{x}$, then find $(\lim)_{x\vec{0}} \frac{f(1+x)}{x}$

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356. Let $g : R\vec{R}$ be a differentiable function satisfying $g(x) = g(y)g(x - y) \forall x, y \in R$ and $g'(0) = a and g'(3) = b$. Then find the value of $g'(-3)$.

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357. If $f, g,$ and h are differentiable functions of x and $(\delta)=$ $\begin{vmatrix} f & g & h \\ (xf)' & (xg)' & (xh)' \\ \left(x^2 f\right)'' & \left(x^2 g\right)'' & \left(x^2 h\right)'' \end{vmatrix}$ prove that $\delta' =$

$$\begin{vmatrix} f & g & h \\ f' & g' & h \\ \left(x^3 f''\right)' & \left(x^3 g''\right)' & \left(x^3 h''\right)' \end{vmatrix}$$

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358. Let f be a function such that $f(x+y) = f(x) + f(y)$ for all $xandyandf(x) = \left(2x^2 + 3x\right)g(x)$ for all$x$, where $g(x)$ is continuous and $g(0) = 3$. Then find $f'(x)$

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359. If $f\left(\frac{x+y}{3}\right) = \frac{2 + f(x) + f(y)}{3}$ for all x, y $f'(2) = 2$ then find $f(x)$

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360. Prove that $\lim_{x \to 0} \frac{f(x+h) + f(x-h) - 2f(x)}{h^2} = f''(x)$ (without using L' Hospital srule).

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361. Let $f(x+y) = f(x) + f(y) + 2xy - 1$ for all real x and y and $f(x)$ be a differentiable function. If $f'(0) = \cos\alpha$, the prove that $f(x) > 0 \, \forall x \in R$.

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362. If $x = e^{y + e^{y + e^{y} \dots \infty}}$, where $x > 0$,then find $\frac{dy}{dx}$

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363. Let $f : R\vec{R}$ be a function satisfying condition $f\left(x + y^{3}\right) = f(x) + [f(y)]^{3}f$ or $all x, y \in R^{\cdot}$ If $f'(0) \geq 0$, find $f(10)^{\cdot}$

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364. If $xy = e^{(x-y)}$, then find $\frac{dy}{dx}$

Watch Video Solution

365. $\frac{dy}{dx}$ for $y = \tan^{-1}\left\{\sqrt{\frac{1 + \cos x}{1 - \cos x}}\right\}$,where $0 < x < \pi$, is

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366. If $y^{x} = x^{y}$, then find $\frac{dy}{dx}\cdot$

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367. Differentiate $(x\cos x)^x$ with respect to x.

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368. If $f(xy) = \frac{f(x)}{y} + \frac{f(y)}{x}$ holds for all real x and y greater than 0 and $f(x)$ is a differentiable function for all $x > 0$ such that $f(e) = \frac{1}{e}$, then find $f(x)$

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369. Find $\frac{dy}{dx}$ for $y = x^x$.

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370. If P_n is the sum of a $\ddot{GP}$ upto n terms $(n \geq 3)$, then prove that $(1 - r)\dfrac{dP_n}{dr} = (1 - n)P_n + nP_{n-1}$, where r is the common ratio of $\ddot{GP}$

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371. If $y = f\left(a^x\right) and f'(\sin x) = (\log)_e x, then find \left(\dfrac{dy}{dx}\right)$, if it exists, where pi/2 less than x less than pi

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372. if $x < 1$ then $\dfrac{1}{1+x} + \dfrac{2x}{1+x^2} + \dfrac{4x^3}{1+x^4} + \ldots\ldots\ldots\ldots.\infty$

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373. Find the derivative of $\frac{\sqrt{x}(x+4)^{\frac{3}{2}}}{(4x-3)^{\frac{4}{3}}}$

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374. If $(\lim)_{t\vec{x}} \frac{e^t f(x) - e^x f(t)}{(t-x)(f(x))^2} = 2 and f(0) = \frac{1}{2}$, then find the value of $f'(0)$.

4 (b) 2 (c) 0 (d) 1

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375. If $y = x^{x^{x^{x \dots \infty}}}$, find $\frac{dy}{dx}$

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376. If $y = e^{\sqrt{x}} + e^{-\sqrt{x}}$, then $\frac{dy}{dx}$ is equal to (a) $\frac{e^{\sqrt{x}}}{2\sqrt{x}}$ (b) $\frac{e^{\sqrt{x}} - e^{-\sqrt{x}}}{2\sqrt{x}}$ $\frac{1}{2\sqrt{x}}\sqrt{y^2 - 4}$ (d) $\frac{1}{2\sqrt{x}}\sqrt{y^2 + 4}$

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377. Differentiate $\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$ with respect to x

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378. If $f(x) = |\sin x - |\cos x||$, then the value of $f'(x)$ at $x = \frac{7\pi}{6}$ is

(a) *positive* (b) $\frac{1-\sqrt{3}}{2}$ (c)0 (d) none of these

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379. If $y = (\tan x)^{(\tan x)^{(\tan x)\dots\infty}}$,then find $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$

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380. If graph of $y = f(x)$ is symmetrical about the y-axis and that of $y = g(x)$ is symmetrical about the origin and if $h(x) = f(x)\dot{g}(x)$, then $\frac{d^3h(x)}{dx^3}$ at $x = 0$ is (a)cannot be determined (b) $f(0)\dot{g}(0)$ (c)0 (d) none of these

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381. Differentiate the function with respect to x using the first principle : $(\log)_e x$

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382. If $x = t^2, y = t^3$, then $\frac{d^2y}{dx^2}$ = (a)$\frac{3}{2}$ (b) $\frac{3}{(4t)}$ (c) $\frac{3}{2(t)}$ (d) $\frac{3t}{2}$

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383. Using the first principle, prove that: $\frac{d}{dx}(f(x)g(x)) = f(x)\frac{d}{dx}(g(x)) + g(x)\frac{d}{dx}(f(x))$

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384. If $y = x + e^x$, then $\frac{d^2x}{dy^2}$ is (a) e^x (b) $-\frac{e^x}{\left(1 + e^x\right)^3}$ (c) $-\frac{e^x}{\left(1 + e^x\right)^3}$ (d) $\frac{-1}{\left(1 + e^x\right)^3}$

Watch Video Solution

385. If $y = \left(1 + x^{\frac{1}{4}}\right)\left(1 + x^{\frac{1}{2}}\right)\left(1 - x^{\frac{1}{4}}\right)$, then find $\frac{dy}{dx}$.

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386. Let $f(x) = (\lim)_{h \to 0} \frac{(\sin(x+h))^{1n(x+h)} - (\sin x)^{1nx}.}{h}$ Then $f\left(\frac{\pi}{2}\right)$ equal to (a)0 (b) equal to 1 (c)In $\frac{\pi}{2}$ (d) non-existent

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387. If $f(x) = x|x|,$ then prove that $f'(x) = 2|x|$

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388. A function $f: R \to R$ satisfies $\sin x \cos y\left(f(2x+2y) - f(2x-2y) = \cos x \sin y(f(2x+2y) + f(2x-2y))^{.}\right.$ If $f'(0) = \frac{1}{2}, then$ (a)$f''(x) = f(x) = 0$ (b)$4f''(x) + f(x) = 0$ (c) $f''(x) + f(x) = 0$ (d)$4f''(x) - f(x) = 0$

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389. If $y = \sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}}, x \in \left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right)$, then find $\frac{dy}{dx}$.

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390. If $x = \log p$ and $y = \frac{1}{p}$,then (a) $\frac{d^2y}{dx^2} - 2p = 0$ (b) $\frac{d^2y}{dx^2} + y = 0$ (c) $\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$ (d) $\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$

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391. If $y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + + \frac{x^n}{n!}$, show that $\frac{dy}{dx} - y + \frac{x^n}{n!} = 0$.

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392. Let $y = 1n(1 + \cos x)^2$. Then the value of $\frac{d^2y}{dx^2} + \frac{2}{e^{\frac{y}{2}}}$ equal

A. 0

B. $\frac{2}{1+\cos x}$

C. $\frac{4}{1+\cos x}$

D. $\frac{-4}{(1+\cos x)^2}$

Answer: A

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393. Find $\frac{dy}{dx} \text{ for } y = x \sin x \log x$.

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394. If the function $f(x) = -4e^{\frac{1-x}{2}} + 1 + x + \frac{x^2}{2} + \frac{x^3}{3} \text{ and } g(x) = f^{-1}(x)$, then the reciprocal of $g'\left(\frac{-7}{6}\right)$ is________

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395. Differentiate $y = \dfrac{e^x}{1 + \sin x}$

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396. Suppose that $f(0) = 0 and f'(0) = 2,$ and let $g(x) = f(-x + f(f(x)))$.

The value of g' (0) is equal to _____

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397. If $y = \sqrt{\dfrac{1 - x}{1 + x}},$ prove that $\left(1 - x^2\right)\dfrac{dy}{dx} + y = 0$

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398. If $g(x) = \frac{f(x)}{(x-a)(x-b)(x-c)}$,where f(x) is a polynomial of degree < 3, then $\int g(x)dx = \begin{vmatrix} 1 & a & f(a)\log|x-a| \\ 1 & b & f(b)\log|x-b| \\ 1 & c & f(c)\log|x-c| \end{vmatrix} \div \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} + k$

$$\frac{dg(x)}{dx} = \begin{vmatrix} 1 & a & -f(a)(x-a)^{-2} \\ 1 & b & -f(b)(x-b)^{-2} \\ 1 & c & -f(c)(x-c)^{-2} \end{vmatrix} : - \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

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399. If $f(x) = \cos x \cos 2x \cos 4x \cos 8x \cos 16x$, then find $f'\left(\frac{\pi}{4}\right)$.

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400. $f(x) = e^{-\frac{1}{x}}, where x > 0,$ Let for each positive integer n, P_n be the polynomial such that $\frac{d^n f(x)}{dx^n} = P_n\left(\frac{1}{x}\right)e^{-\frac{1}{x}}$ for all $x > 0$. Show that

$$P_{n+1}(x) = x^2\left[P_n(x) - \frac{d}{dx}P_n(x)\right]$$

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401. If $a_i, b_i \in N$ for i 1,2,3, then coefficient of x in the determinant;

$$\begin{vmatrix} (1+x)^{a_1 b_1} & (1+x)^{a_1 b_2} & (1+x)^{a_1 b_3} \\ (1+x)^{a_2 b_1} & (1+x)^{a_2 b_2} & (1+x)^{a_2 b_3} \\ (1+x)^{a_3 b_1} & (1+x)^{a_3 b_2} & (1+x)^{a_3 b_3} \end{vmatrix}$$

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402. If $\frac{d}{dx}\left[\left(x^m - A_1 x^{m-1} + A_2 x^{m-2} - \ldots\ldots. + (-1)^m A_m\right)e^x\right] = x^m e^x$, find the value of A_r

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403. If $y = \begin{vmatrix} \sin x & \cos x & \sin x \\ \cos x & -\sin x & \cos x \\ x & 1 & 1 \end{vmatrix}$, find $\frac{dy}{dx}$

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404. Let $f(x) and g(x)$ be two function having finite nonzero third-order derivatives $f'''(x)$ and $g'''(x)$ for all $x \in R$. If $f(x).\,g(x) = 1$ for all $x \in R$, then prove that $\frac{f'''}{f'} - \frac{g'''}{g'} = 3\left(\frac{f''}{f} - \frac{g''}{g}\right)$.

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405. Differentiate log $\sin x$ w.r.t. $\sqrt{\cos x}$.

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406. If $\frac{\cos x}{2}\frac{\cos x}{2^2}\frac{\cos x}{2^3}\infty = \frac{\sin x}{x}$, then find the value of $\frac{1}{2^2}\frac{\sec^2 x}{2} + \frac{1}{2^4}\frac{\sec^2 x}{2^2} + \frac{1}{2^6}\frac{\sec^2 x}{2^3} + \infty$

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407. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right) w.r.t \tan^{-1}x, where x \neq 0.$

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408. Find the derivative of f(tan x) w.r.t. g (sec x) at $x = \frac{\pi}{4}$, where f'(1)=2 and $g'\left(\sqrt{2}\right) = 4$.

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409. If $f(x)=\cos^{-1}\left(\frac{1}{\sqrt{13}}\right)(2\cos x-3\sin x)+\sin^{-1}\left(\frac{1}{\sqrt{13}}\right)(2\cos x+3\sin x) wrt \sqrt{1+x^2}$, then find $\frac{df(x)}{dx}$ at $x=\frac{3}{4}$.

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410. Find the derivative of $\frac{\tan^{-1}(2x)}{1-x^2} w.r.t \frac{\sin^{-1}(2x)}{1+x^2}$

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411. If $\left|a_1\sin x+a_2\sin 2x+\quad+a_n\sin nx\right|\leq|\sin x|$ for $x\in R$, then prove that $\left|a_1+2a_2+3a_3+na_n\right|\leq 1$

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412. Find the derivative of $\sec^{-1}\left(\frac{1}{2x^2 - 1}\right)$ w.r.t $\sqrt{1 - x^2}$ at $x = \frac{1}{2}$

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413. If $y = \left(\frac{1}{2}\right)^{n-1}\cos\left(n\cos^{-1}x\right)$, then prove that y satisfies the differential equation $\left(1 - x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + n^2y = 0$

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414. If $y = f\left(x^3\right), z = g\left(x^5\right), f'(x) = \tan x, and g'(x) = \sec x,$ then find the value of $(\lim)_{x\vec{0}}\frac{\left(\frac{dy}{dz}\right)}{x}$

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415. If $x \in \left(0, \frac{\pi}{2}\right)$, *thenshowthat*

$$\frac{d}{dx}\cos^{-1}\left\{\frac{7}{2}(1+\cos 2x)+\left(\sqrt{\sin^2 x - 48\cos^2 x}\right)\sin x\right\}$$

$$= 1 + \frac{7\sin x}{\sqrt{\sin^2 - 48\cos^2 x}}$$

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416. If $f(x) = \left|x + a^2 abacabx + b^2 bcacbcx + c^2\right|$, *then* prove that $f'(x) = 3x^2 + 2x\left(a^2 + b^2 + c^2\right)$.

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417. If $y = \cos^{-1}\left(\frac{2x}{1+x^2}\right)$, *then* $\frac{dy}{dx}$ *is* (a) $\frac{-2}{1+x^2}$ for all x (b) $\frac{-2}{1+x^2}$ for all $|x| < 1$ (c) $\frac{2}{1+x^2}$ for $|x| > 1$ (d) none of these

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418. If $f(x), g(x)$ and $h(x)$ are three polynomials of degree 2, then prove that $\phi(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ f'(x) & g'(x) & h'(x) \\ f''(x) & g''(x) & h''(x) \end{vmatrix}$ is a constant polynomial.

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419. If $f(x-y), f(x)f(y)$ and $f(x+y)$ are in A.P. for all $x, y,$ and $f(0) \neq 0,$ then (a) $f(4) = f(-4)$ (b) $f(2) + f(-2) = 0$ (c) $f'(4) + f'(-4) = 0$ (d) $f'(2) = f'(-2)$

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420. If $x^3 + y^3 + 3axy = 0, find \frac{dy}{dx}$

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421. If 1 is the twice repeated root of the equation $ax^3 + bx^2 + bx + d = 0$, *then* (a) $a = b = d$ (b) $a + b = 0$ (c) $b + d = 0$ (d) $a = d$

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422. If $\log\left(x^2 + y^2\right) = 2\tan^{-1}\left(\frac{y}{x}\right)$, show that $\frac{dy}{dx} = \frac{x + y}{x - y}$

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423. If $y = b\tan^{-1}\left(\frac{x}{a} + \frac{\tan^{-1}y}{x}\right)$,find $\frac{dy}{dx}\cdot$

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424. Let $y = \sqrt{x + \sqrt{x + \sqrt{x + \infty}}}$, $\frac{dy}{dx}$ is equal to

(a) $\frac{1}{2y - 1}$ (b) $\frac{x}{x + 2y}$ (c) $\frac{1}{\sqrt{1 + 4x}}$ (d) $\frac{y}{2x + y}$

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425. If $y = \sqrt{\sin x + y}$, then find $\frac{dy}{dx}$

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426. $f(x) = \left|x^2 - 3\left|x\right| + 2\right|$ Then which of the following is/are true

$f'(x) = 2x - 3$ for x in $(0, 1) \cup (2, \infty)$

$f'(x) = 2x + 3$ for x in $(-\infty, -2) \cup (-1, 0)$

$f'(x) = -2x - 3$ for x in (-2,-1)`

None of these

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427. If $x\sqrt{1+y}+y\sqrt{1+x}=0$, prove that $\frac{dy}{dx}=-\frac{1}{(x+1)^2}$

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428. If $x^3-2x^2y^2+5x+y-5=0$ and $y(1)=1$, then (a)$y'(1)=\frac{4}{3}$ (b) $y'(1)=-\frac{4}{3}$ (c)$y''(1)=-8\frac{22}{27}$ (d) $y'(1)=\frac{2}{3}$

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429. If $x^y=e^{x-y}$, Prove that $\frac{dy}{dx}=\frac{\log x}{(1+\log x)^2}$

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430. Let $f(x)=\frac{\sqrt{x-2\sqrt{x-1}}}{\sqrt{x-1}-1}.x$ Then (a)$f'(10)=1$ (b)$f'\left(\frac{3}{2}\right)=-1$ (c)domain of f(x) is x ≥ 1 (d)range of $f(x) is(-2,-1)\cup(2,\infty)$

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431. If $\sqrt{x} + \sqrt{y} = 4, find \left(\frac{dx}{dy}\right) at (y = 1).$

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432. If $y = \frac{x^4 - x^2 + 1}{x^2 + \sqrt{3}x + 1}$ and $\frac{dy}{dx} = ax + b,$ then the value of a+b is (a) $\cot\left(\frac{\pi}{8}\right)$ (b) $\cot\left(\frac{5\pi}{12}\right)$ (c) $\tan\left(\frac{5\pi}{12}\right)$ (d) $\tan\left(\frac{5\pi}{8}\right)$

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433. If $xy + y^2 = \tan x + y$,then find $\frac{dy}{dx}$

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434. Which of the following is/are true?

$\frac{dy}{dx}f$ or $y = \sin^{-1}(\cos x), where x \in (0, \pi), is - 1$

$\frac{dy}{dx}f$ or $y = \sin^{-1}(\cos x), where x \in (0, 2\pi), is 1$

$\frac{dy}{dx}f$ or $y = \cos^{-1}(\sin x), where x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right), is - 1$

$\frac{dy}{dx}f$ or $y = \cos^{-1}(\sin x), where x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right), is - 1$

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435. If $y = \sqrt{x + \sqrt{y + \sqrt{x + \sqrt{y + \infty}}}}, then prove that \frac{dy}{dx} = \frac{y^2 - x}{2y^3 - 2xy - 1}$

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436. If $y = x^{(\log x)^{\log(\log x)}}, then \frac{dy}{dx} is$

(a)$\frac{y}{x}\left(1nx^{x-1}\right) + 21nx1n(1nx)\Big)$ (b)$\frac{y}{x}(\log x)^{\log(\log x)}(2\log(\log x) + 1)$ (c)

$\frac{y}{x1nx}\left[(1nx)^2 + 21n(1nx)\right]$ (d)$\frac{y}{x}\frac{\log y}{\log x}[2\log(\log x) + 1]$

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437. If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \to \infty}}}, provethat\frac{dy}{dx} = \frac{\cos x}{2y - 1}$

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438. The *nth* derivative of xe^x vanishes when (a)$x = 0$ (b) $x = -1$ (c) $x = -n$ (d) $x = n$

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439. If $x = a\left(\cos t + \frac{1}{2}\log\tan^2 t\right)$ and $y = a\sin t$ then find $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$

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440. The derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at $x = 0$ is

A. $\frac{1}{8}$

B. $\frac{1}{4}$

C. $\frac{1}{2}$

D. 1

Answer: B

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441. If $x^m y^n = (x+y)^{m+n}, provethat \frac{dy}{dx} = \frac{y}{x}.$

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442. Let $g(x) = f(x)\sin x, where f(x)$ is a twice differentiable function on $(-\infty, \infty)$ such that $f'(-\pi) = 1.$ The value of $g''(-\pi)$ equals ________

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443. If $x = a\cos^3\theta, y = a\sin^3\theta$, then find the value of d2y/dx2 at θ=π/6

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444. If $x = \sqrt{a^{\sin^{-1}t}}, y = \sqrt{a^{\cos^{-1}t}}$, $a > 0$ and $-1 < t < 1$, show that $\frac{dy}{dx} = -\frac{y}{x}$.

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445. If $\frac{x}{a} + \frac{y}{b} = \sqrt{2}$ touches the ellipses $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then find the ecentricity angle θ of point of contact.

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446. If $y = f\left(\frac{2x - 1}{x^2 + 1}\right)$ and $f'(x) = \sin x^2$, then find $\frac{dy}{dx}$

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447. Let $y = x^3 - 8x + 7$ and $x = f(t)$. $\frac{dy}{dx} = 2$ and $x = 3a$t=0, $thenf \in dthevalueo\frac{f(dx)}{dt}at$ t=0.`

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448. If $x = \frac{2t}{1 + t^2}, y = \frac{1 - t^2}{1 + t^2}, then$find$\frac{dy}{dx}$.

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449. If $f_r(x), g_r(x), h_r(x), r = 1, 2, 3$ are polynomials such that

$f_r(a) = g_r(a) = h_r(a), r = 1, 2, 3$ and $F(x) = \begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ g_1(x) & g_2(x) & g_3(x) \\ h_1(x) & h_2(x) & h_3(x) \end{vmatrix}$ then

$F'(x) at x = a$ is________________

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450. Find $\frac{dy}{dx}$ if $x = a(\theta - \sin\theta)$ and $y = a(1 - \cos\theta)^{\cdot}$

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451. If $f(x) = (\log)_x(\log x), then f'(x)$ at $x = e$ is equal to (a) $\frac{1}{e}$ (b) e (c) 1 (d) zero

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452. Differentiate $\frac{\tan^{-1}x}{1+\sqrt{\left(1-x^2\right)}}+\left\{2\tan^{-1}\sqrt{\left(\frac{1-x}{1+x}\right)}\right\}\sin wrtx$

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453. If $x = a\sec^3\theta$ and $y = a\tan^3\theta$, find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{3}$

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454. Differentiate $(\log x)^{\cos x}$ with respect to x

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455. Find the differential equation of the family of curves $y = Ae^{2x} + Be^{-2x}$, where A and B are arbitrary constants.

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456. If $f(x) = |x|^{|\sin x|}$, then find $f'\left(-\frac{\pi}{4}\right)$

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457. If $x = \operatorname{cosec}\theta - \sin\theta$ and $y = \operatorname{cosec}^n\theta - \sin^n\theta$, then show that $\left(x^2 + 4\right)\left(\frac{dy}{dx}\right)^2 = n^2\left(y^2 + 4\right)$.

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458. If $y = \sqrt{\log\left\{\sin\left(\frac{x^2}{3} - 1\right)\right\}}$, *then* find $\frac{dy}{dx}$.

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459. Find $\frac{dy}{dx}$ of $y = x^3$

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460. Find $\frac{dy}{dx}$ for $y = \sin\left(x^2 + 1\right)$.

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461. Let $f(x) = x + \cfrac{1}{2x + \cfrac{1}{2x + \cfrac{1}{2x}}}$ Compute the value of $f(5)$.

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462. $y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$. Find $\frac{dy}{dx}$.

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463. Find y' of $y = 5x^5$

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464. $y = \sin^{-1}\left(\frac{1}{\sqrt{1 + x^2}}\right) + \cos^{-1}\left(\frac{1}{\sqrt{1 + x^2}}\right)$. find $\frac{dy}{dx}$.

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465. If $x^2 + y^2 = R^2(where R > 0) and k = \frac{y''}{\left(1 + y'^2\right)^3}$ then find k in terms of R alone.

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466. $y = \tan^{-1}\left(\frac{\sqrt{1 + x^2} + \sqrt{1 - x^2}}{\sqrt{1 + x^2} - \sqrt{1 - x^2}}\right)$, where $-1 < x < 1$, find $\frac{dy}{dx}$

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467. Find the derivative of $\sec^{-1}\left(\frac{1}{2x^2-1}\right) \ddot{wrt} \sqrt{1-x^2}$ at $x = \frac{1}{2}$

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468. $y = \tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$, where $-\frac{\pi}{2} < x < \pi$and $\frac{a}{b}\tan x > -1$.Find $\frac{dy}{dx}$

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469. If $f(9) = 9, f'(9) = 4, then(\lim)_{x \to 9}\frac{\sqrt{f(x)}-3}{\sqrt{x}-3} =$ __________.

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470. Find the sum of the series $1 + 2x + 3x^2 + (n - 1)x^{n-2}$ using differentiation.

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471. If $f(x)$ satisfies the relation $f\left(\frac{5x - 3y}{2}\right) = \frac{5f(x) - 3f(y)}{2} \forall x, y \in R$, and $f(0) = 3 and f'(0) = 2$, then the period of $\sin(f(x))$ is (a)2π (b) π (c) 3π (d) 4π

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472. $y = 4x^2 + e^{3x}$ find y'

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473. Instead of the usual definition of derivative $Df(x)$, if we define a new kind of derivative $D^{\cdot}F(x)$ by the formula $D \cdot f(x) = \lim_{h \to 0} \frac{f^2(x+h) - f^2(x)}{h}, where f^2(x)$ mean $[f(x)]^2$ and if $f(x) = x\log x$,then $D^{\cdot} f(x)|_{x=e}$ has the value (A)e (B) 2e (c) 4e (d) none of these

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474. $y = \tan^{-1}\left(\frac{x}{1+\sqrt{1-x^2}}\right)$

Watch Video Solution

475. If $y = |\cos x| + |\sin x|, then \frac{dy}{dx} at x = \frac{2\pi}{3}$ is $\frac{1-\sqrt{3}}{2}$ (b) 0 (c) $\frac{1}{2}\left(\sqrt{3}-1\right)$ (d) none of these

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476. $y = \ln x + e^{2x}$ find y'

Watch Video Solution

477. If g is the inverse function of $f and f'(x) = \sin x, then g'(x)$ is (a) $\cos ec\{g(x)\}$ (b) $\sin\{g(x)\}$ (c)- $\frac{1}{\sin\{g(x)\}}$ (d) none of these

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478. Find $\frac{dy}{dx}$ for the function: $y = \sin x + \ln x$

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479. If $x = \varphi(t), y = \psi(t), then \frac{d^2y}{dx^2}$ is (a) $\frac{\varphi^{'}\psi^{''} - \psi'\varphi''}{\left(\varphi^{'}\right)^2}$ (b) $\frac{\varphi^{'}\psi^{''} - \psi'\varphi''}{\left(\varphi^{'}\right)^3}$

(c) $\frac{\varphi^{''}}{\psi^{''}}$ (d) $\frac{\psi^{''}}{\varphi^{''}}$

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480. If $y = (1 + x)\left(1 + x^2\right)\left(1 + x^4\right)\left(1 + x^{2n}\right)$, then find $\frac{dy}{dx} at x = 0.$

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481. $f(x) = e^x - e^{-x}$ then find $f'(x)$

Watch Video Solution

482. Find $\frac{dy}{dx}$ for the function: $y = \sqrt{\sin\sqrt{x}}$

Watch Video Solution

483. If $y = \frac{\sqrt{a+x} - \sqrt{a-x}}{\sqrt{a+x} + \sqrt{a-x}}$, then $\frac{dy}{dx}$ is equal to (a) $\frac{ay}{x\sqrt{a^2 - x^2}}$ (b) $\frac{ay}{\sqrt{a^2 - x^2}}$ (c) $\frac{ay}{x\sqrt{a^2 - x^2}}$ (d) none of these

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484. Find $\frac{dy}{dx}$ for the function: $y = e^x + \cos x$

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485. Find $\frac{dy}{dx}$ for the function: $y = x^3 + e^{2x}$

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486. Find $\frac{dy}{dx}$ for the function: $y = \log\sqrt{\sin\sqrt{e^x}}$

Watch Video Solution

487. Let $g(x)$ be the inverse of an invertible function $f(x)$, which is differentiable for all real x. Then $g''(f(x))$ equals. (a)- $\frac{f''(x)}{\left(f'(x)\right)^3}$ (b) $\frac{f'(x)f''(x)-\left(f'(x)\right)^3}{f'(x)}$ (c) $\frac{f'(x)f''(x)-\left(f'(x)\right)^2}{\left(f'(x)\right)^2}$ (d) none of these

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488. Find $\frac{dy}{dx}$ for the function: $y = x^{\frac{1}{2}} + \sin 2x$

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489. Find $\frac{dy}{dx}$ for the function: $y = \sin 5x$

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490. Differentiate the function $f(x) = x^{99}$ with respect to x.

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491. If $f(x) = x + \tan x$ and $g(x) is the$ inverseoff(x), $then$differentiation of g(x)$is(a)$1/(1+[g(x)-x]^2)(b) 1/(2-[g(x)+x]^2)(c)1/(2+[g(x)-x]^2)(d) none of these`

Watch Video Solution

492. Find $\frac{dy}{dx}$ for $y = \cos 55x$

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493. Find $\frac{dy}{dx}$ for $y = e^{6x}$

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494. Let f: $R \to R$ be a one-one onto differentiable function, such that $f(2) = 1$ and $f'(2) = 3$. Then, find the value of $\left(\frac{d}{dx}\left(f^{-1}(x)\right)\right)_{x=1}$

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495. If $f''(x) = -f(x)$ and $g(x) = f'(x)$ and $F(x) = \left(f\left(\frac{x}{2}\right)\right)^2 + \left(g\left(\frac{x}{2}\right)\right)^2$ and given that $F(5) = 5$, then $F(10)$ is (a)5 (b) 10 (c) 0 (d) 15

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496. Find $\frac{dy}{dx}$ for the function: $y = \sin 4x - \left(\frac{1}{x^4}\right)$

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497. Find $\frac{dy}{dx}$ for the function: $y = \sin 2x - x^4 + e^{-3x}$

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498. If $x = a\cos\theta, y = b\sin\theta$, then prove that $\frac{d^3y}{dx^3} = \frac{3b}{a^3}\operatorname{cosec}^4\theta\cot\theta$.

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499. If $y = x\log\left(\frac{x}{a + bx}\right), then x^3\frac{d^2y}{dx^2} =$ (a)$x\frac{dy}{dx} - y$ (b) $\left(x\frac{dy}{dx} - y\right)^2$ $y\frac{dy}{dx} - x$ (d) $\left(y\frac{dy}{dx} - x\right)^2$

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500. A function $f: R \rightarrow R$ satisfies the equation $f(x + y) = f(x)f(y)$ for all $x, y \in R$ and $f(x) \neq 0$ for all $x \in R$. If $f(x)$ is differentiable at

$x = 0 and f'(0) = 2$, then prove that $f'(x) = 2f(x)$

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501. If $u = x^2$ and $x = s + 3t, y = 2s - t$, then $\frac{d^2u}{ds^2}$ is (a) $\frac{5}{2}t$ (b) $20t^8$ (c) $\frac{5}{16t^6}$ (d) non of these

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502. If $f(x) = (1 - x)^n$, then the value of $f(0) + f'(0) + \frac{f''(0)}{2!} + \frac{f'''(0)}{3!} + \ldots\ldots \frac{f^n(0)}{n!}$.

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503. If $f(x) = \sin x + e^x$, then $f''(x)$

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504. If $y^2 = ax^2 + bx + c,$ then $y^3 \frac{d^2y}{dx^2}$ is

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505. If $f(x) = \left| x^{\cap} \ !2\cos x \frac{\cos(n\pi)}{2} 4\sin x \frac{\sin(n\pi)}{2} 8 \right|$ then find the value of $\frac{d^n}{dx^n}([f(x)])_{x=0} n \in \dot{z}$

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506. Let $f\left(\frac{x+y}{2}\right) = \frac{f(x) + f(y)}{2}$ for all real x and y. If $f'(0)$ exits and equals -1 and $f(0) = 1$, then find $f(2)$.

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507. If $y^{\frac{1}{m}} = \left(x + \sqrt{1 + x^2}\right), then \left(1 + x^2\right)y_2 + xy_1$ is (where y_r represents the rth derivative of y w.r.t. x (a)m^2y (b) my^2 (c) m^2y^2 (d) none of these

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508. Suppose the function $f(x) - f(2x)$ has the derivative 5 at $x = 1$ and derivative 7 at $x = 2$. The derivative of the function $f(x) - f(4x)$ at x=1` has the value equal to (a) 19 (b) 9 (c) 17 (d) 14

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509. $f(x) + f(y) = f\left(\frac{x + y}{1 - xy}\right)$,for all$x, y \in R.(xy \neq 1)$,and $\lim_{x \to 0} \frac{f(x)}{x} = 2$.Find$f\left(\frac{1}{\sqrt{3}}\right)$and $f'(1)$.

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510. Let $f: R \rightarrow R$ satisfying $|f(x)| \leq x^2, \forall x \in R$ be differentiable at $x = 0$. Then find $f'(0)$.

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511. If $f(x) = \sin^{-1}\cos x$, then the value of $f(10) + f'(10)$ is (a)$11 - \frac{\pi}{2}$ (b) $\frac{\pi}{2} - 11$ (c)$\frac{5\pi}{2} - 11$ (d) none of these

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512. If $(\sin x)(\cos y) = \frac{1}{2}$, then $\frac{d^2y}{dx^2}$ at $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$ is

A. -4

B. -2

C. -6

D. 0

Answer: A

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513. Suppose $p(x) = a_0 + a_1x + a_2x^2 + ... + a_nx^n$. If $|p(x)| \leq \left|e^{x-1} - 1\right|$ for all $x \geq 0$, prove that $\left|a_1 + 2a_2 + ... + na_n\right| \leq 1.$

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514. Let $f(x + y) = f(x)f(y)$ for all $x and y$. Suppose $f(5) = 2 and f'(0) = 3.$ Find $f'(5)$.

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515. A function f satisfies the condition $f(x) = f'(x) + f''(x) + f'''(x).... + \in f \in ity, wheref(x)$ is a differentiable function indefinitely and dash denotes the order of derivative. If $f(0) = 1, thenf(x)$ is $e^{\frac{x}{2}}$ (b) e^x (c) e^{2x} (d) e^{4x}

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516. Let $f(xy) = f(x)f(y)\,\forall x, y \in Randf$ is differentiable at $x = 1$ such that $f'(1) = 1.$ Also, $f(1) \neq 0, f(2) = 3.$ Then find $f'(2)$

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517. Let $f(x)$ be a polynomial of degree 3 such that $f(3) = 1, f'(3) = -1, f''(3) = 0, andf'''(3) = 12.$ Then the value of $f'(1)$ is (a)12(b)23 (c) -13 (d) none of these

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518. Find $\frac{dy}{dx}$ for the functions: $y = x^3 e^x \sin x$

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519. $\frac{d}{dx}\left[\tan^{-1}\left(\frac{\sqrt{x}(3-x)}{1-3x}\right)\right] is$ (a) $\frac{1}{2(1+x)\sqrt{x}}$ (b) $\frac{3}{(1+x)\sqrt{x}}$ (c) $\frac{2}{(1+x)\sqrt{x}}$ (d) $\frac{3}{2(1+x)\sqrt{x}}$

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520. If

$$y = \tan^{-1}\left(\frac{1}{1+x+x^2}\right) + \tan^{-1}\left(\frac{1}{x^2+3x+3}\right) + \tan^{-1}\left(\frac{1}{x^2+5x+7}\right) + +$$

upto n terms, then find the value of $y'(0)$.

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521. Let $g(x)$ be the inverse of an invertible function $f(x)$ which is differentiable at $x = c$. Then $g'(f(x))$ equal. (a)$f'(c)$ (b) $\frac{1}{f'(c)}$ (c) $f(c)$ (d) none of these

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522. If $\sqrt{1 - x^6} + \sqrt{1 - y^6} = a\left(x^3 - y^3\right)$, then prove that $\frac{dy}{dx} = \frac{x^2}{y^2}\sqrt{\frac{1 - y^6}{1 - x^6}}$

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523. Let $f: R \to R$ be such that $f(1) = 3$ and $f'(1) = 6$. Then $\lim_{x \to 0}\left(\frac{f(1 + x)}{f(1)}\right)^{1/x}$ = (a)1 (b) $e^{\frac{1}{2}}$ (c) e^2 (d) e^3

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524. If $\cos y = x\cos(a + y)$, with $\cos a \neq \pm 1$, prove that $\frac{dy}{dx} = \frac{\cos^2(a + y)}{\sin a}$.

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525. Let $f(x) = \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$, where p is a constant. Then $\frac{d^3}{dx^3}(f(x))$ at $x = 0$ is

(a)p (b) $p - p^3$ (c)$p + p^3$ (d) independent of p

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526. Find $\frac{dy}{dx}$ for $y = \sin^{-1}(\cos x)$, where $x \in (0, 2\pi)$.

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527. The derivative of an even function is always an odd function.

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528. If $y=\sqrt{(a-x)(x-b)}-(a-b)\tan^{-1}\sqrt{\frac{a-x}{x-b}}, thend\frac{dy}{dx}isequal \rightarrow '$

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529. Let $F(x)=f(x)g(x)h(x)$ for all real $x, wheref(x), g(x), andh(x)$ are differentiable functions. At some point $x_0, F'\left(x_0\right)=21F\left(x_0\right), f'\left(x_0\right)=4f\left(x_0\right), g'\left(x_0\right)=-7g\left(x_0\right),$ and $h'\left(x_0\right)=kh\left(x_0\right)$. Then k is_______

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530. Find $\frac{dy}{dx}$ for $y=\tan^{-1}\left\{\frac{1-\cos x}{\sin x}\right\}, -\pi<x<\pi.$

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531. If $x = y + \sin^2 x$ then at $x = 0, \frac{dy}{dx} =$

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532. Differentiate $\sin^{-1}\left(2x\sqrt{1 - x^2}\right)$ with respect to x.

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533. If $f(x) = |x - 2| and g(x) = f[f(x)], then g'(x) =$ ______ for x>20

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534. Find $\frac{dy}{dx}$ for $y = \tan^{-1}\sqrt{\frac{\sec^2 x}{\cos e}c^2 x}$

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535. If $y=(\sin x)^{\tan x}, then \frac{dy}{dx}=$ (a)$(\sin x)^{\tan x}\left(1+\sec^2 x\log\sin x\right)$ (b) $\tan x(\sin x)^{\tan x-1}\cos x$ (c)$(\sin x)^{\tan x}$ (d)$\sec^2 x\log\sin x\ \tan x(\sin x)^{\tan x-1}$

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536. If $y=\sin^{-1}\left(\sqrt{1-x^2}\right)$ and $0<x<1,$ then find $\frac{dy}{dx}$

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537. Suppose that $f(x)$ is a quadratic expresson positive for all real x.

If $g(x)=f(x)+f'(x)+f''(x),$ then for any real x (where $f'(x)$ and $f''(x)$ represent 1st and 2nd derivative, respectively).

(a)$g(x)<0$ (b). $g(x)>0$ (c). $g(x)=0$ (d). $g(x)\geq 0$

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538. If $y = \cos^{-1}x$,find $\frac{d^2y}{dx^2}$.

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539. If $y^2 = P(x)$, then $2\frac{d}{dx}\left(y^3\left(d^2\frac{y}{dx^2}\right)\right)$

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540. If $g(x) = \begin{vmatrix} f(x+c) & f(x+2c) & f(x+3c) \\ f(c) & f(2c) & f(3c) \\ f'(c) & f'(2c) & f'(3c) \end{vmatrix}$, where c is a constant,

then $\lim_{x \to 0}\frac{g(x)}{x}$ is equal to

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541. If $f(a) = 2, f'(a) = 1, g(a) = -1, g'(a) = 2,$ then the value of $\lim_{x \to a} \frac{g(x)f(a) - g(a)f(x)}{x - a}$ is (a)-5 (b) $\frac{1}{5}$ (c) 5 (d) none of these

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542. If $x = a(\cos t + t\sin t)$ and $y = a(\sin t - t\cos t)$, find $\frac{d^2y}{dx^2}$

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543. Let $y = \frac{2^{(\log)_{2}\frac{1}{4}x} - 3^{(\log)_{27}\left(x^2+1\right)^3} - 2x}{7^{4(\log)_{49}x} - x - 1}$ and $\frac{dy}{dx} = ax + b,$ then the value of $a + b$ is______.

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544. $(\lim)_{h\vec{0}} \frac{(e+h)^{1n(e+h)} - e}{h}$ is _–

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545. If $(x-a)^2+(y-b)^2=c^2$, for some $c>0$, prove that $\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$ is a constant independent of a and b.

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546. if $y=\left(x^2-1\right)^m$, then the $(2m)th$ differential coefficient of y is

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547. If function f satisfies the relation $f(x)\cdot f'(-x)=f(-x)\cdot f'(x)$ for all x ,and f(0)=3, and if f(3)=3, then the value of $f(-3)$ is ____________

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548. If $y = \frac{a + bx^{\frac{3}{2}}}{x^{\frac{5}{4}}} and y' = 0 at x = 5$, then the value of $\frac{a^2}{b^2}$ is_______

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549. If $y = e^{a\cos^{-1}x}$, $-1 \leq x < 1, show that \left(1 - x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$

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550. Let $f(x) = (x - 1)(x - 2)(x - 3) - - (x - n), n \in N, and f(n) = 5040.$

Then the value of n is_______

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551. If $y = \frac{(ax + b)}{\left(x^2 + c\right)}$, then find $\frac{dy}{dx}$.

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552. If $y = x\log\{\frac{x}{(a + bx)}]$, then show that $x^3\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^{2\cdot}$

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553. y=f(x), where f satisfies the relation $f(x + y) = 2f(x) + xf(y) + y\sqrt{f(x)}$,$\forall x, y \rightarrow R$ and f'(0)=0.Then f(6)is equal _____

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554. If $x^2 + y^2 = t - \frac{1}{t}$ and $x^4 + y^4 = t^2 + \frac{1}{t^2}$, then $x^3y\frac{dy}{dx} =$

(a) 0 (b) 1(c) -1 (d) none of these

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555. Prove that $\frac{d^n}{dx^n}\left(e^{2x}+e^{-2x}\right)=2^n\left[e^{2x}+(-1)^n e^{-2x}\right]$

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556. If $e^y(x+1)=1$, show that $\frac{d^2y}{dx^2}=\left(\frac{dy}{dx}\right)^2$

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557. If $e^x=\frac{\sqrt{1+t}-\sqrt{1-t}}{\sqrt{1+t}+\sqrt{1-t}}a$ and $\frac{\tan y}{2}=\sqrt{\frac{1-t}{1+t}}$,then $\frac{dy}{dx}$ at $t=\frac{1}{2}$ is

(a)-$\frac{1}{2}$ (b) $\frac{1}{2}$ (c) 0 (d) none of these

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558. If $y=\cos^{-1}(\cos x)$,then $\frac{dy}{dx}$ at x=$\frac{5\pi}{4}$ is equal to

(a)1 (b)-1 (c)0 (d)4

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559. If $y = \cos^{-1}(\cos x)$, *then* $\frac{dy}{dx}$ at x=$\frac{5\pi}{4}$ is equal to (a) 1 (b) -1 (c) 0 (d) 4

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560. If $\sin^{-1}\left(\frac{x^2 - y^2}{x^2 + y^2}\right) = \log a$, *then* $\frac{dy}{dx}$ is equal to (a) $\frac{x}{y}$ (b) $\frac{y}{x^2}$ (C) $\frac{x^2 - y^2}{x^2 + y^2}$ (d) $\frac{y}{x}$

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561. If $y = \tan^{-1}\sqrt{\frac{x + 1}{x - 1}}$, *then* $\frac{dy}{dx}$ *is* (a) $\frac{-1}{2|x|\sqrt{x^2 - 1}}$ (b) $\frac{-1}{2x\sqrt{x^2 - 1}}$ (c) $\frac{1}{2x\sqrt{x^2 - 1}}$ (d) none of these

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562. If $y=\cos^{-1}\left(\frac{5\cos x - 12\sin x}{13}\right)$, $where x \in \left(0, \frac{\pi}{2}\right)$, then $\frac{dy}{dx}$ is. (a)1 (b) -1 (c) 0 (d) none of these

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563. The derivative of $y=(1-x)(2-x)....(n-x)$ at $x=1$ is (a)0 (b) $(-1)(n-1)!$ (c)$n!-1$ (d) $(-1)^{n-1}(n-1)!$

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564. If $y=\sqrt{\frac{1-x}{1+x}}$, prove that $\frac{\left(1-x^2\right)dy}{dx}+y=0$

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565. $\frac{d^{20}}{dx^{20}}(2\cos x\cos 3x)$is equal to (a)$2^{20}\left(\cos 2x - 2^{20}\cos 3x\right)$ (b) $2^{20}\left(\cos 2x + 2^{20}\cos 4x\right)$ (c)$2^{20}\left(\sin 2x + 2^{20}\sin 4x\right)$ (d) $2^{20}\left(\sin 2x - 2^{20}\sin 4x\right)$

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566. Differentiate $x^2 + 1$ with respect to x .

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567. Let $g(x) = \begin{cases} \frac{x^2 + x\tan x - x\tan 2x}{ax + \tan x - \tan 3x}, x \neq 0 \\ 0, x = 0 \end{cases}$ If $g'(0)$ exists and is equal to nonzero value b, then $52\frac{b}{a}$ is equal to _____

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568. Given $y = \sin 2x + x^3, \frac{dy}{dx}$

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569. Find the derivative of $f(x) = e^{4x} + \cos 3x$

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570. A nonzero polynomial with real coefficient has the property that $f(x) = f'(x)f''(x)$. If a is the leading coefficient of $f(x)$, then the value of $\frac{1}{2a}$ is____

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571. Suppose $f(x) = e^{ax} + e^{bx}$, where $a \neq b$, and that $f''(x) - 2f'(x) - 15f(x) = 0$ for all x. Then the value of $\frac{|ab|}{3}$ is ___

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572. Let $z=(\cos x)^5 and y=\sin x^{\cdot}$ Then the value of $2\frac{d^2z}{dy^2}at x=\frac{2\pi}{9}$ is__________.

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573. A function is reprersented parametrically by the equations $x=\frac{1+t}{t^3}; y=\frac{3}{2t^2}+\frac{2}{t}^{\cdot} Thenthevalueof\left|\frac{dy}{dx}-x\left(\frac{dy}{dx}\right)^3\right|$ is_________

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574. Let f be a twice differentiable function such that $f''(x)=-f(x), andf'(x)=g(x), h(x)=[f(x)]^2+[g(x)]^{2^{\cdot}}$ Find $h(10)$ if $h(5)=11$

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575. Let $y = e^{x\sin x^3} + (\tan x)^x$ Find $\frac{dy}{dx}$

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576. If $ax^2 + 2hxy + by^2 = 1, then \frac{d^2y}{dx^2}$ is (a) $\frac{h^2 - ab}{(hx + by)^2}$ (b) $\frac{ab - h^2}{(hx + by)^2}$ $\frac{h^2 + ab}{(hx + by)^2}$ (d) none of these

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577. If $y = \sin px$ and y_n is the nth derivative of y, then $\begin{vmatrix} y & y_1 & y_2 \\ y_3 & y_4 & y_5 \\ y_6 & y_7 & y_8 \end{vmatrix} is$

(a)1 (b) 0 (c) -1 (d) none of these

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578. A function $f,$ defined for all positive real numbers, satisfies the equation $f\left(x^2\right) = x^3$ for every $x > 0$. Then the value of $f'(4)$ is (a)12(b) 3 (c) $3/2$ (d) cannot be determined

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579. If $y = x - x^2,$ then the derivative of y^2 with respect to x^2 is (a) $1 - 2x$ (b) $2 - 4x$ (c)$3x - 2x^2$ (d) $1 - 3x + 2x^2$

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580. The first derivative of the function $\left[\cos^{-1}\left(\sin\sqrt{\frac{1 + x}{2}}\right) + x^x\right]$ with respect to x at $x = 1$ is (a)$3/4$ (b) 0 (c)$1/2$ (d) $-1/2$

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581. The function $f(x)=e^{x}+x$, being differentiable and one-to-one, has a differentiable inverse $f^{-1}(x)$. The value of $\frac{d}{dx}\left(f^{-1}\right)$ at the point $f(\log 2)$ is (a) $\frac{1}{1n2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) none of these

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582. Let $h(x)$ be differentiable for all x and let $f(x)=\left(kx+e^{x}\right)h(x)$, where k is some constant. If $h(0)=5, h'(0)=-2, and f'(0)=18,$ then the value of k is (a)5 (b) 4 (c) 3 (d) 2.2.

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583. Suppose $y=e^{ax}+e^{bx}$, find y''

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584. If $y = \frac{(a - x)\sqrt{a - x} - (b - x)\sqrt{x - b}}{\left(\sqrt{a - x} + \sqrt{x - b}\right)}$,then $\frac{dy}{dx}$ wherever it is defined is

(a) $\frac{x + (a + b)}{\sqrt{(a - x)(x - b)}}$ (b) $\frac{2x - a - b}{2\sqrt{a - x}\sqrt{x - b}}$ (c)- $\frac{(a + b)}{2\sqrt{(a - x)(x - b)}}$ (d) $\frac{2x + (a + b)}{2\sqrt{(a - x)(x - b)}}$

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585. If $y = 7x^5$, then $\frac{dy}{dx}$

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586. The nth derivative of the function $f(x) = \frac{1}{1 - x^2}$ [where $\in (- 1, 1)$ at the point $x = 0$ where n is even is (a) 0 (b) $n!$ (c) n^nC_2 (d) 2^nC_2

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587. Let $u(x)$ and $v(x)$ be differentiable functions such that $\frac{u(x)}{v(x)} = 7$.

If (u'(x))/(v'(x))=p and ((u'(x))/(v'(x)))=q,$then$(p+q)/(p-q)$hasthevalueof \rightarrow$

(a) 1 (b) 0 (c) 7 (d) -7

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588. Statement 1: Let $f: R \rightarrow R$ be a real-valued function $\forall x, y \in R$ such that $|f(x) - f(y)| \leq |x - y|^3$. Then $f(x)$ is a constant function. Statement 2: If the derivative of the function w.r.t. x is zero, then function is constant.

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589. Statement 1: For $f(x) = \sin x, f'(\pi) = f'(3\pi)$ Statement 2: For $f(x) = \sin x, f(\pi) = f(3\pi)$

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590. $f: R^{+} \rightarrow R$ is a continuous function satisfying $f\left(\frac{x}{y}\right) = f(x) - f(y) \forall x, y \in R^{+}$.If f'(1)=1,then (a)f is unbounded (b) $\lim_{x \rightarrow 0} f\left(\frac{1}{x}\right) = 0$ (c) $\lim_{x \rightarrow 0} \frac{f(1+x)}{x} = 1$ (d) $\lim_{x \rightarrow 0} x. f(x) = 0$

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591. $f_n(x) = e^{f_{n-1}(x)}$ for all $n \in N and f_0(x) = x, then \frac{d}{dx}\left\{f_n(x)\right\}$ is (a) $\frac{f_n(x)d}{dx}\left\{f_{n-1}(x)\right\}$ (b) $f_n(x)f_{n-1}(x)$ (c)$f_n(x)f_{n-1}(x)\ldots\ldots. f_2(x)f_1(x)$ (d)none of these

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592. Suppose $f$and g are functions having second derivative f'' and g'' everywhere. If $f(x)g(x) = 1$ for all x and f' and g' are never zero, then $\frac{f''(x)}{f'(x)} - \frac{g''(x)}{g'(x)}$ is equal (a) $\frac{-2f'(x)}{f}$ (b) $\frac{2g'(x)}{g(x)}$ (c) $\frac{-f'(x)}{f(x)}$ (d) $\frac{2f'(x)}{f(x)}$

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593. If $y = e^{-x}\cos x and y_n + k_n y = 0, where y_n = \frac{d^n y}{dx^n} and k_n$ are constants $\forall n \in N$, then $k_4 = 4$ (b) $k_8 = -16$ $k_{12} = 20$ (d) $k_{16} = -24$

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594. If a function is represented parametrically by the equations $x = \frac{1 + \log_e t}{t^2}, y = \frac{3 + 2\log_e t}{t}$, then which of the following statements are true ?

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595. Statement 1: Leg $f(x) = x[x] and [.]$ denotes the greatest integral function, when x is not an integer, then rule for $f'(x)$ is given by $[x]$. Statement 2: $f'(x)$ does not exist for any x integer. (a)Both the statements are TRUE and STATEMENT 2 is the correct explanation of STATEMENT 1. (b)Both the statements are TRUE but STATEMENT 2 is NOT the correct explanation of STATEMENT 1. (c)STATEMENT 1 is TRUE and STATEMENT 2 is FALSE. (d)STATEMENT 1 is FALSE and STATEMENT 2 is TRUE.

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596. Statement 1: If $f(x)$ is an odd function, then $f'(x)$ is an even function.

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597. If $y = ae^{mx} + be^{-mx}$, then $\frac{d^2y}{dx^2}$ is equals to

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598. If $y = \cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right], \left(0 < x < \frac{\pi}{2}\right)$, then $\frac{dy}{dx} =$

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599. If $y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \infty}}}$, *then* $\frac{dy}{dx}$ *is* (a) $\frac{x}{2y-1}$ (b) $\frac{x}{2y+1}$ (c) $\frac{1}{x(2y-1)}$ (d) $\frac{1}{x(1-2y)}$

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600. $\frac{d^n}{dx^n}(\log x) = ?$ (a) $\frac{(n-1)!}{x^n}$ (b) $\frac{n!}{x^n}$ (c) $\frac{(n-2)!}{x^n}$ (d) $(-1)^{n-1}\frac{(n-1)!}{x^n}$

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601. If $y = \sec\left(\tan^{-1}x\right), then \frac{dy}{dx}$ at x=1 is (a)$\cos\left(\frac{\pi}{4}\right)$ (b) $\sin\left(\frac{\pi}{2}\right)$ (c) $\sin\left(\frac{\pi}{6}\right)$ (d) $\cos\left(\frac{\pi}{3}\right)$

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602. The differential coefficient of $f\left((\log)_e x\right)$ with respect to x, where $f(x) = (\log)_e x$, is

(a)$\frac{x}{(\log)_e x}$ (b) $\frac{1}{x}(\log)_e x$ (c)$\frac{1}{x(\log)_e x}$ (d) none of these

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603. If $u = f\left(x^3\right), v = g\left(x^2\right), f'(x) = \cos x, and g'(x) = \sin x, then \frac{du}{dv}$ is `

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604. If $f'(x) = \sqrt{2x^2 - 1}$ and $y = f(x^2)$, then $\frac{dy}{dx}$ at $x = 1$ is (a)2 (b) 1 (c) -2

(d) none of these

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605. If $f(x) = \sqrt{1 - \sin 2x}$, then $f'(x)$ is equal to

(a)$-(\cos x + \sin x)$,for $x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

(b)$\cos x + \sin x$,for $x \in \left(0, \frac{\pi}{4}\right)$

(c)$-(\cos x + \sin x)$,for $x \in \left(0, \frac{\pi}{4}\right)$

(d)$\cos x - \sin x$,for $x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

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606. If $x = t\cos t, y = t + \sin t$. Then $\frac{d^2x}{dy^2}$ at $t = \frac{\pi}{2}$ is

(a) $\frac{\pi + 4}{2}$ (b) $-\frac{\pi + 4}{2}$ (c) -2 (d) none of these

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607. If $x^3 + 3x^2 - 9x + c$ is of the form $(x - \alpha)^2(x - \beta)$ then c is equal to

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608. If $y = f(x)$ is an odd differentiable function defined on $(-\infty, \infty)$ such that $f'(3) = -2, then \left|f'(-3)\right|$ equals________.

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609. $f'(x) = \phi(x)$ and $\phi'(x) = f(x)$ for all x. Also, $f(3) = 5$ and $f'(3) = 4$. Then the value of $[f(10)]^2 - [\phi(10)]^2$ is ____

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610. If $y = \cos^{-1}\left(\frac{2x}{1 + x^2}\right)$, *then* $\frac{dy}{dx}$ *is* (a) $\frac{-2}{1 + x^2}$ for all x (b) $\frac{-2}{1 + x^2}$ for all $|x| < 1$ (c) $\frac{2}{1 + x^2}$ for $|x| > 1$ (d) none of these

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611. Each question contains statements given in two columns which have to be matched. Statements a,b,c,d in column I have to be matched with statements p,q,r,s in column II. If the correct matches are a-p, q-s, b-q, r c-p, q and d-s, then the correctly bubbled $4x4$ matrix should be as follows: Figure Column I, Column II: Differential equation order 1, p. of all parabolas whose axis is the x-axis order 2, q. of family of curves $y = a(x + a)^2$, whre a is an arbitrary constant degree 1, r. $\left(1+3\frac{dy}{dx}\right)^{\frac{2}{3}}=\frac{4d^3y}{dx^3}$ degree 3, s. of family of curve $y^2=2c\left(x+\sqrt{c}\right)$, where $c>0$

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612. Suppose the function $f(x)$ satisfies the relation $f\left(x+y^3\right)=f(x)+f\left(y^3\right)\dot{\forall}x, y\in R$ and is differentiable for all $\dot{x}$ Statement 1: If $f'(2)=a, then f'(-2)=a$ Statement 2: $f(x)$ is an odd function.

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613. If for some differentiable function $f(\alpha)=0 and f'(\alpha)=0$, Statement 1: Then sign of $f(x)$ does not change in the neighbourhood of $x=\alpha$ Statement 2: α is repeated root of $f(x)=0$

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614. Statement 1: If differentiable function $f(x)$ satisfies the relation $f(x) + f(x - 2) = 0 \, \forall x \in R,$ and if $\left(\frac{d}{dx}f(x)\right)_{x=a} = b, then \left(\frac{d}{dx}f(x)\right)_{x=a+4000} = b.$ Statement 2: $f(x)$ is a periodic function with period 4.

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615. Let α be a repeated root of a quadratic equation $f(x) = 0 and A(x), B(x), C(x)$ be polynomials of degrees 3, 4, and 5, respectively, then show that $|A(x)B(x)C(x)A(\alpha)B(\alpha)C(\alpha)A'(\alpha)B'(\alpha)C'(\alpha)|$ is divisible by $f(x)$, where prime (′) denotes the derivatives.

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616. If $y=\left\{(\log)_{\cos x}\sin x\right\}\left\{(\log)_{\sin x}\cos x\right\}^{-1}+\sin^{-1}\left(\frac{2x}{1+x^2}\right)$, fin d $\frac{dy}{dx}atx=\frac{\pi}{4}$

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617. $\frac{dy}{dx}=\left(\sqrt{\frac{1-\sin 2x}{1+\sin 2x}}\right)$ is equal to, $0<x<\frac{\pi}{2}$

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618. If $y=\left(x+\sqrt{x^2+a^2}\right)^n$, $then\frac{dy}{dx}$ is (a)$\frac{ny}{\sqrt{x^2+a^2}}$ (b) $-\frac{ny}{\sqrt{x^2+a^2}}$ (c) $\frac{nx}{\sqrt{x^2+a^2}}$ (d) $-\frac{nx}{\sqrt{x^2+a^2}}$

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619. If $f(x)=\sqrt{1+\cos^2(x^2)}$, $then f'\left(\frac{\sqrt{\pi}}{2}\right)$ is (a) $\frac{\sqrt{\pi}}{6}$ (b) -$\sqrt{\pi/6}$ (c)$1/\sqrt{6}$

(d) $\pi/\sqrt{6}$

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620. $\frac{d}{dx}\cos^{-1}\sqrt{\cos x}, 0 < x < \frac{\pi}{2}$ is equal to

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621. If $f(0)=0, f'(0)=2,$ then the derivative of $y=f(f(f(x)))$ at $x=0$ is

2 (b) 8 (c) 16 (d) 4

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622. If $y = ax^{n+1} + bx^{-n}, then x^2 \frac{d^2y}{dx^2}$ is equal to (a) $n(n-1)y$ (b) $n(n+1)y$ (c) ny (d) n^2y

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623. If $y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!}, then \frac{dy}{dx}$ is equal to (a) y (b) $y + \frac{x^n}{n!}$ (c) $y - \frac{x^n}{n!}$ (d) $y - 1 - \frac{x^n}{n!}$

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624. If $y = a\sin x + b\cos x$, then $y^2 + \left(\frac{dy}{dx}\right)^2$ is a (a)function of x (b) function of$y$ (c)function of x and y (d) constant

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625. If $y = (\log)_{\sin x}(\tan x)$, *then* $\left(\left(\frac{dy}{dx}\right)\right)_{\frac{\pi}{4}}$ is equal to (a) $\frac{4}{\log 2}$ (b) $-4\log 2$ (c) $\frac{-4}{\log 2}$ (d) none of these

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626. If $y = \frac{\sin^{-1}x}{\sqrt{1-x^2}}$, *then* $\frac{\left(1-x^2\right)dy}{dx}$ is equal to (a)$x+y$ (b) $1+xy$ (c) $1-xy$ (d) $xy-2$

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627. If $(\lim)_{x\vec{}0}\frac{ae^x + b\cos x + ce^{-x}}{e^{2x} - 2e^x + 1} = 4$, *then* a.$a = 2$ b. b=-4 c. $c = 2$ d. $a + b + c = -8$

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628. If $\lim x \to \infty x\log_e\left(\begin{vmatrix} \alpha/x & 1 & \gamma \\ 0 & 1/x & \beta \\ 1 & 0 & 1/x \end{vmatrix}\right) = -5.$ where α, β, γ are finite real numbers, then

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629. The value of $(\lim)_{x \to \frac{3\pi}{4}} \frac{1 + \tan x^{\frac{1}{3}}}{1 - 2\cos^2 x}$ is (a)-$1/2$ (b.) -$2/3$ (c). -$3/2$ (d).-$1/3$

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630. Let $f: R \to R$ be a differentiable function at x = 0 satisfying f(0) = 0 and f'(0) = 1, then the value of $\lim x \to 0\frac{1}{x}. \sum^{\infty} n=1(-1)^n. f\left(\frac{x}{n}\right)$, is

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631. The value of $\lim_{x \to 0}\left(\frac{1+2x}{1+3x}\right)^{\frac{1}{x^2}} e^{\frac{1}{x}}$ is $e^{\frac{5}{2}}$ b. e^2 c. e^{-2} d. 1

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632. $(\lim)_{x\vec{0}^{+}} \frac{1}{x\sqrt{x}}\left(a\tan^{-1}\frac{\sqrt{x}}{a} - b\frac{\tan^{-1}(\sqrt{x})}{b}\right)$ has the value equal to

$\frac{a-b}{3}$ b. 0 c. $\frac{(a^2-b^2)}{6a^2b^2}$ d. $\frac{a^2-b^2}{3a^2b^2}$

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633. If $f'(a) = \frac{1}{4}, then (\lim)_{h\vec{0}} \frac{f(a+2h^2) - f(a-2h^2)}{f(a+h^3-h^2) - f(a-h^3+h^2)} = 0$ b. 1 c.

-2 d. none of these

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634. The value of $(\lim)_{x\to 0}\frac{e^{x^2}-e^x+x}{1-\cos 2x}$ is

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635. Evalaute $\lim_{x\to 1}\frac{1+\log x-x}{1-2x+x^2}$

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636. The value of $(\lim)_{x\vec{1}}\frac{x^{\frac{1}{13}}-x^{\frac{1}{7}}}{\left(x^{\frac{1}{5}}-x^{\frac{1}{3}}\right)}$ is $\frac{44}{91}$ b. $\frac{45}{91}$ c. $\frac{45}{89}$ d. $\frac{40}{93}$

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637. Number of integral values of λ for which $(\lim)_{x\to 1}\sec^{-1}\left(\frac{\lambda^2}{(\log)_e x}-\frac{\lambda^2}{x-1}\right)$ does not exist is a. 1 b. 2 c. 3 d. 4

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638. If $f(x) = \left\{\frac{x}{s \in x}, x > 02 - x, x \le 0\right.$ and $g(x) = \left\{x + 3, x < 1x^2 - 2x - 2, 1 \le x < 2x - 5, x \ge 2\right.$ Then the value of $(\lim)_{x \vec{0}} g(f(x))$ a. is -2 b. is -3 c. is 1 d. does not exist

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639. If $k \in I$ sucht that $(\lim)_{x \vec{\infty}} \left(\cos\frac{k\pi}{4}\right)^{2n} - \left(\cos - \frac{k\pi}{6}\right)^{2n} = 0,$ then (a) k must not be divisible by 24 (b) k is divisible by 24 of k is divisible neither by 4 nor by 6 (c)k must be divisible by 12 but not necessarily by 24 (d) none of these

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640. $\lim_{x \to \infty} \dfrac{\sum_{r=1}^{10} (x+r)^{2010}}{\left(x^{1006}+1\right)\left(2x^{1004}+1\right)} =$

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641. If $\lim_{x \to 0} \dfrac{f(x)}{x^2} = a$ and $\lim_{x \to 0} \dfrac{f(1 - \cos x)}{g(x)\sin^2 x} = b$ (where $b \neq 0$), then $\lim_{x \to 0} \dfrac{g(1 - \cos 2x)}{x^4}$ is

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642. The value of $(\lim)_{\vec{x}\pi/4} \dfrac{\tan^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$ is 8 b. 4 c. -8 d. -2

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643. $(\lim)_{x \to \frac{\pi}{2}} \frac{(1 - \sin x)\left(8x^3 - \pi^3\right)\cos x}{(\pi - 2x)^4}$

a. $\frac{\pi^2}{6}$ b. $\frac{3\pi^2}{16}$ c. $\frac{\pi^2}{16}$ d. $-\frac{3\pi^2}{16}$

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644. The value of $\lim_{x \to \frac{\pi}{4}} \frac{\sqrt{1 - \sqrt{\sin 2x}}}{\pi - 4x}$ is

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645. The value of $(\lim)_{x \to \infty}\left(e^{\sqrt{x^{4+1}}} - e^{x^{2+1}}\right)$ is (a) 0 b. e c. $1/e$ d. $-\infty$

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646. If a_n and b_n are positive integers and $a_n + \sqrt{2}b_n = \left(2 + \sqrt{2}\right)^n, then(\lim)_{n \to \infty}\left(\frac{a_n}{b_n}\right) =$ a. 2 b. $\sqrt{2}$ c. $e^{\sqrt{2}}$ d. e^2

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647. If$f(x) = \begin{cases} x + \frac{1}{2}, x < 0 \\ 2x + \frac{3}{4}, x \geq 0 \end{cases}$, then $\left[(\lim)_{x \to 0} f(x)\right] =$

(where [.] denotes the greatest integer function)

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648. $(\lim)_{X \to (-7)} \frac{[x]^2 + 15[x] + 56}{\sin(x+7)\sin(x+8)} =$ (where [.] denotes the greatest integer function) a. 0 b. 1 c. -1 d. does not exist

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649. Let $L_1 = (\lim)_{x\vec{4}}(x - 6)^x$ and $L_2 = (\lim)_{x\vec{4}}(x - 6)^4$. Which of the following is true? Both $L_1 and L_2$ exists Neither $L_1 and L_2$ exists L_1 exists but L_2 does not exist L_2 exists but L_1 does not exist

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650. If $f: R \rightarrow R$ is defined by $f(x) = [x - 3] + |x - 4|$ for $x \in R$, then $\lim_{x \rightarrow 3} f(x)$ is equal to (where [.] represents the greatest integer function)

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651. If [.] denotes the greatest integer function, then $(\lim)_{x\vec{0}} \frac{x}{a}\left[\frac{b}{x}\right]$

a. $\frac{b}{a}$ b. 0 c. $\frac{a}{b}$ d. does not exist

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652. $\lim_{x \to \frac{-1}{3}} \frac{1}{x}\left[\frac{-1}{x}\right] =$ (where [.] denotes the greatest integer function)

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653. $(\lim)_{x \to \infty} x^2 \sin\left((\log)_e \sqrt{\frac{\cos \pi}{x}}\right)$ a. 0 b. $\frac{\pi^2}{2}$ c. $\frac{\pi^2}{4}$ d. $\frac{\pi^2}{8}$

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654. $\lim_{x \to 0} \frac{1}{x^2}\begin{vmatrix} 1 - \cos 3x & \log_e(1 + 4x) \\ \sin^{-1}\left(x^x -\right) & \tan^{-1}(2x) \end{vmatrix}$ is equal to

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655. If graph of the function $y = f(x)$ is continuous and passes through point $(3, 1)$ then $(\lim)_{x \vec{3}} \frac{(\log)_e(3f(x) - 2)}{2(1 - f(x))}$ is equal $\frac{3}{2}$ b. $\frac{1}{2}$ c. $-\frac{3}{2}$ d. $-\frac{1}{2}$

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656. Set of all values of x such that $\lim_{n \to \infty} \frac{1}{1 + \left(\frac{4\tan^{-1}(2\pi x)}{\pi}\right)^{4n}}$ is non-zero and finite number when $n \in N$ is

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657. $\lim_{x \vec{\infty}} \left[x - \log_e\left(\frac{e^x + e^{-x}}{2}\right)\right] =$

a)$(\log)_e 4$ b. 0 c. ∞ d. $(\log)_e 2$

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658. The value of $(\lim)_{x\vec{0}} \frac{1 + \sin x - \cos x + \log(1 - x)}{x^3} is \frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 0 (d) none of these

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659. If $\lim_{x \to \infty} \left(\frac{x + c}{x - c}\right)^x = 4$ then the value of e^c is

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660. If

$$f(x) = \lim_{n \to \infty} \frac{\left(x^2 + ax + 1\right) + x^{2n}\left(2x^2 + x + b\right)}{1 + x^{2n}} \text{ and } \lim_{x \to \pm 1} f(x)$$

exists, then

The value of b is

A. -1

B. 1

C. 0

D. 2

Answer: null

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661. If $\lim x \to 0\left[1 + x + \frac{f(x)}{x}\right]^{\frac{1}{x}} = e^3$, then find the value of

$1n\left(\lim x \to 0\left[1 + \frac{f(x)}{x}\right]^{\frac{1}{x}}\right) is \_\_$

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662. Let $f(x)$ be the fourth degree polynomial such that $f^{'}(0) - 6, f(0) = 2and(\lim)_{x\vec{1}}\frac{f(x)}{(x - 1)^2} = 1$ The value of $f(2)$ is 3 b. 1 c. 0

d.2

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663. Let $f(x)$ be the fourth degree polynomial such that $f'(0) - 6, f(0) = 2 and (\lim)_{x \vec{1}} \frac{f(x)}{(x - 1)^2} = 1$ The value of $f(2)$ is 3 b. 1 c. 0 d.2

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664. $(\lim)_{x \to 0} \left(\frac{\sqrt{1 + x \sin x} - \sqrt{\cos 2x}}{\tan^2(x/2)} \right)$ is equal to $\frac{1}{6}$ b. 6 c.3 d. 2

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665. The value of $\lim_{x \to \infty} x^2 \left(1 - \cos . \frac{1}{x} \right)$ is

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666. $\lim x \to 0 \frac{\cos x - 1}{x}$ is equal to

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667. $\lim x \to \left(2^{+}\right)\{x\}\frac{\sin(x-2)}{(x-2)^2} =$ (where [.] denotes the fractional part function) a. 0 b. 2 c. 1 d. does not exist

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668. The value of $(\lim)x \to 0\frac{\sqrt{1 - \cos x^2}}{1 - \cos x}$ is a. $\frac{1}{2}$ b. 2 c. $\sqrt{2}$ d. none of these

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669. If

$$f(x) = \lim_{n \to \infty} \frac{\left(x^2 + ax + 1\right) + x^{2n}\left(2x^2 + x + b\right)}{1 + x^{2n}} \text{ and } \lim_{x \to \pm 1} f(x)$$

exists, then

The value of b is

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670. $\lim_{x \to 0} \frac{e^{5x} - 1}{3x}$

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671. $\lim_{x \to \left(\frac{\pi}{2}\right)} (1 - \sin x)\tan x =$

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672. The value of $\left(\lim\right)_{x \vec{3}} \frac{\left(x^3 + 27\right)(\log)_e(x - 2)}{x^2 - 9}$ is a. 9 b. 18 c. 27 d. 1/3

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673. If $\left(\lim\right)_{x \vec{0}} \frac{e^{ax} - e^x - x}{x^2} = b$ (finite), then $a = 2, b = 0$ b. $a = 0, b = \frac{3}{2}$ c. $a = 2, b = \frac{3}{2}$ d. $a = 0, b = 2$

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674. If $\lim_{x \to 0} \frac{x^3}{\sqrt{a + x}(bx - \sin x)} = 1, a > 0$, then $a + b$ is equal to

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675. $\lim_{x \to 0}\left[\frac{\sin^{-1}x}{\tan^{-1}x}\right] =$ (where [.] denotes the greatest integer function)

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676. The value of $\lim_{x \to \frac{\pi}{4}}\frac{\sqrt{1-\sqrt{\sin 2x}}}{\pi - 4x}$ is

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677. $\lim_{x \to 0}\left(\frac{\sin x}{x}\right)$

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678. If $f(x) = x\left(\frac{e^{|x| + [x]} - 2}{|x| + [x]}\right)$ then (where [.] represents the greatest integer function) $(\lim)_{x\vec{0}^{+}} f(x) = -1$ b. $(\lim)_{x\vec{0}^{-}} f(x) = 0$ c. $(\lim)_{x\vec{0}\square} f(x) = -1$ d. $(\lim)_{x\vec{0}\square} f(x) = 0$

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679. If $A = \lim_{x \to 0} \frac{\sin^{-1}(\sin x)}{\cos^{-1}(\cos x)}$ and $B = \lim_{x \to 0} \frac{[|x|]}{x}$, then

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680. Let $f(x) = \lim_{m \to \infty} \left\{ \lim_{n \to \infty} \cos^{2m}(n!\pi x) \right\}$, where $x \in R$.

Then prove that $f(x) = \{1$, if x is rational and 0, if x is irrational

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681. $\lim_{x \to \infty} \left\{ \left(e^x + \pi^x\right)^{\frac{1}{x}} \right\} =$ where {.} denotes the fractional part of x is equal to

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682. Let $f(x) = (\lim)_{x \to \infty} \frac{\tan^{-1}(\tan x)}{1 + \left((\log)_e x\right)^n}, x \neq (2n+1)\frac{\pi}{2}$ then `AA1 e ,f(x)` is a constant function

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683. Assume that $\lim_{\theta \to -1} f(\theta)$ exists and $\frac{\theta^2 + \theta - 2}{\theta + 3} \leq \frac{f(\theta)}{\theta^2} \leq \frac{\theta^2 + 2\theta - 1}{\theta + 3}$ holds for certain interval containing the point $\theta = -1$ then $\lim_{\theta \to -1} f(\theta)$

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684. $\lim n \to \infty \left(\frac{\sqrt{n^2 + n} - 1}{n}\right)^{2\sqrt{n^2+n}-1}$

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685. Let $f: R\vec{R}$ be such that $f(a) = 1, f'(a) = 2.$ Then $(\lim)_{x\vec{0}}\left(\frac{f^2(a + x)}{f(a)}\right)^{1/x}$ is a. e^2 b. e^4 c. e^{-4} d. $1/e$

A. e^2

B. e^4

C. e^-4

D. 1/e

Answer: null

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686. $\lim_{x \to \infty}\left(1 - x + x . e^{\frac{1}{n}}\right)^{n}$

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687. If $f(n) = \lim_{x \to 0}\left\{\left(1 + \sin\frac{x}{2}\right)\left(1 + \sin\frac{x}{2^2}\right)...\left(1 + \sin\frac{x}{2^n}\right)\right\}^{\frac{1}{x}}$ then find $\lim_{n \to \infty} f(n)$.

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688. If $a > 0, b > 0$ then $(\lim)_{n \to \infty}\left(\frac{a - 1 + b^{\frac{1}{n}}}{a}\right)^{n} = b^{1/a}$ b. $a^{\frac{1}{b}}$ c. a^{b} d. b^{a}

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689. $(\lim)_{x \to \frac{\pi}{2}} \left[1 + (\cos x)^{\cos x}\right]^2 =$

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690. $(\lim)_{x \to 0} \frac{\log\left(e^x \wedge 2 + 2\sqrt{x}\right)}{tan\sqrt{x}}$ is equal to 0 b. 1 c. 2 d. e^2

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691. If $f(x) = (\lim)_{n \to \infty} \left(\cos\left(\frac{x}{\sqrt{n}}\right)\right)^n$, then the value of $(\lim)\left(x \to 0\right) \frac{f(x) - 1}{x}$ is 0 b. 1 c. 2 d. 3/2

A. 0

B. 1

C. 2

D. 3/2

Answer: null

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