# ©゙doubtnut 

## MATHS

## BOOKS - CENGAGE MATHS (ENGLISH)

## APPLICATION OF DERIVATIVES

## Concept Application Exercise 51

1. Find the equation of the tangent to the curve $\left(1+x^{2}\right) y=2-x$, where it crosses the x -axis.

## - Watch Video Solution

2. Show that the tangent to the curve $3 x y^{2}-2 x^{2} y=1 a t(1,1)$ meets the curve again at the point $\left(-\frac{16}{5},-\frac{1}{20}\right)$.
3. Find the equation of tangent and normal to the curve $x=\frac{2 a t^{2}}{\left(1+t^{2}\right)}, y=\frac{2 a t^{3}}{\left(1+t^{2}\right)}$ at the point for which $t=\frac{1}{2}$.

## ( Watch Video Solution

4. Find the normal to the curve $x=a(1+\cos \theta), y=a \sin \theta a \mathrm{~h} \eta$. Prove that it always passes through a fixed point and find that fixed point.

## - Watch Video Solution

5. Find the equation of the normal to the following curve 1) $y=x^{3}+2 x+6$ which are parallel to the line $x+14 y+4=0.2$ ) $x^{3}+y^{3}=8 x y$ at the point where it meets the curve $y^{2}=4 x$ other than the origin.

## - Watch Video Solution

6. If the curve $y=a x^{2}-6 x+b$ pass through $(0,2)$ and has its tangent parallel to the x -axis at $x=\frac{3}{2}$, then find the values of $a a n d b$.

## - Watch Video Solution

7. Find the value of $n \in N$ such that the curve $\left(\frac{x}{a}\right)^{n}+\left(\frac{y}{b}\right)^{n}=2$ touches the straight line $\frac{x}{a}+\frac{y}{b}=2$ at the point $(a, b)$.

## - Watch Video Solution

8. If the tangent to the curve $x y+a x+b y=0$ at $(1,1)$ is inclined at an angle $\tan ^{-1} 2$ with $x$-axis, then find $a a n d b$ ?

## - Watch Video Solution

9. Does there exists line/lines which is/are tangent to the curve $y=\sin x$ at $\left(x_{1}, y_{1}\right)$ and normal to the curve at $\left(x_{2}, y_{2}\right)$ ?
10. Find the condition that the line $A x+B y=1$ may be normal to the curve $a^{n-1} y=x^{n}$.

## - Watch Video Solution

11. In the curve $x^{a} y^{b}=K^{a+b}$, prove that the potion of the tangent intercepted between the coordinate axes is divided at its points of contact into segments which are in a constant ratio. (All the constants being positive).

## - Watch Video Solution

## Concept Application Exercise 52

1. Find the angle of intersection of $y=a^{x} a n d y=b^{x}$
2. Find the angle of intersection of the curves $x y=a^{2} a n d x^{2}+y^{2}=2 a^{2}$

## - Watch Video Solution

3. Find the angle at which the curve $y=K e^{K x}$ intersects the $y$-axis.

## - Watch Video Solution

4. Find the angle between the curves $x^{2}-\frac{y^{2}}{3}=a^{2}$ and $a x^{3}=c$.

## - Watch Video Solution

5. Find the angle at which the two curves $x^{3}-3 x y^{2}+2=0$ and $3 x^{2} y-y^{3}+3=0$ intersect each other.
6. If the curves ay $+x^{2}=7$ and $y=x^{3}$ cut orthogonally at ( 1,1 ), then the value of $a$ is

## - Watch Video Solution

## Concept Application Exercise 53

1. Find the length of the tangent for the curve $y=x^{3}+3 x^{2}+4 x-1$ at point $x=0$.

## - Watch Video Solution

2. For the curve $y=a 1 n\left(x^{2}-a^{2}\right)$, show that the sum of length of tangent and sub-tangent at any point is proportional to product of coordinates of point of tangency.
3. For a curve (length of normal)^ $2 /(\text { length of tangent) })^{\wedge} 2$ is equal to

## Watch Video Solution

4. If the sub-normal at any point on $y^{1-n} x^{n}$ is of constant length, then find the value of $n$.

## - Watch Video Solution

## Concept Application Exercise 54

1. Minimum integral value of k for which the equation $e^{x}=k x^{2}$ has exactly three real distinct solution,

## - Watch Video Solution

2. Find the point on the curve $3 x^{2}-4 y^{2}=72$ which is nearest to the line $3 x+2 y+1=0$.

## - Watch Video Solution

3. Find the possible values of 'a' such that the inequality $3-x^{2}>|x-a|$ has atleast one negative solution

## - Watch Video Solution

4. Tangents are drawn from the origin to curve $y=\sin x$. Prove that points of contact lie on $y^{2}=\frac{x^{2}}{1+x^{2}}$

## - Watch Video Solution

5. Find the distance of the point on $y=x^{4}+3 x^{2}+2 x$ which is nearest to the line $y=2 x-1$
6. The graph $y=2 x^{3}-4 x+2 a n d y=x^{3}+2 x-1$ intersect in exactly 3 distinct points. Then find the slope of the line passing through two of these points.

## - Watch Video Solution

## Concept Application Exercise 55

1. The distance covered by a particle moving in a straight line from a fixed point on the line is $s$, where $s^{2}=a t^{2}+2 b t+c$. Then prove that acceleration is proportional to $s^{-3}$.

## - Watch Video Solution

2. Two cyclists start from the junction of two perpendicular roads, there velocities being $3 u m / m \in$ and $4 u m / m \in$, respectively. Find the rate at which the two cyclists separate.

## - Watch Video Solution

3. A sphere of 10 cm radius has a uniform thickness of ice around it. Ice is melting at rate $50 \mathrm{~cm}^{3} / \mathrm{min}$ when thickness is 5 cm then rate of change of thickness

## - Watch Video Solution

4. $x a n d y$ are the sides of two squares such that $y=x-x^{2}$. Find the rate of the change of the area of the second square with respect to the first square.
5. Two men PandQ start with velocity $u$ at the same time from the junction of two roads inclined at $45^{0}$ to each other. If they travel by different roads, find the rate at which they are being separated.

## - Watch Video Solution

6. Sand is pouring from a pipe at the rate of $12 \mathrm{~cm}^{3} / \mathrm{s}$. The falling sand forms a cone on the ground in such a way that the height of the cone is always $1 / 6$ th of the radius of the base. How fast does the height of the sand cone increase when the height in 4 cm ?

## - Watch Video Solution

7. A swimming pool is to be drained by cleaning. If $L$ represents the number of litres of water in the pool $t$ seconds after the pool has been plugged off to drain and $L=2000(10-t)^{2}$. How fast is the water ruining out at the end of 5 seconds? What is the average rate at which the water flows out during the first 5 seconds?
8. An aeroplane is flying horizontally at a height of $\frac{2}{3} k m$ with a velocity of $15 \mathrm{~km} / \mathrm{h}$. Find the rate at which it is receding from a fixed point on the ground which it passed over 2 min ago.

## - Watch Video Solution

Concept Application Exercise 56

1. Find the approximate value of $(26)^{\frac{1}{3}}$.

## - Watch Video Solution

2. Find the approximate value of $(1.999)^{6}$.

## - Watch Video Solution

3. If $1^{0}=\alpha$ radians, then find the approximate value of $\cos 60^{\circ} 1^{\prime}$.

## - Watch Video Solution

4. Find the approximate value of $f(3.02)$, where $f(x)=3 x^{2}+5 x+3$.

## - Watch Video Solution

5. If the radius of a sphere is measured as 9 cm with an error of 0.03 cm , then find the approximate error in calculating its volume.

## - Watch Video Solution

Concept Application Exercise 57

1. Let $0<a<b<\frac{\pi}{2} . \operatorname{Iff}(x)=\left|\begin{array}{lll}\tan x & \tan a & \tan b \\ \sin x & \sin a & \sin b \\ \cos x & \cos a & \cos b\end{array}\right|$, then find the minimum possible number of roots of $f^{\prime}(x)=0$ in $(a, b)$.

## - Watch Video Solution

2. Find the condition if the equation $3 x^{2}+4 a x+b=0$ has at least one root in $(0,1)$.

## - Watch Video Solution

3. Let $f(x) \operatorname{andg}(x)$ be differentiable for $0 \leq x \leq 2$ such that $f(0)=2, g(0)=1, \operatorname{andf}(2)=8$. Let there exist a real number $c$ in $[0,2]$ such that $f^{\prime}(c)=3 g^{\prime}(c)$. Then find the value of $g(2)$.

## - Watch Video Solution

4. Prove that if $2 a 02<15 a$, all roots of $x^{5}-a_{0} x^{4}+3 a x^{3}+b x^{2}+c x+d=0$ cannot be real. It is given that $a_{0}, a, b, c, d \in R$.

## - Watch Video Solution

5. Let $f(x)$ be continuous on $[a, b]$, differentiable in $(a, b)$ and $f(x) \neq 0$ for all $x \in[a, b]$. Then prove that there exists one
$c \in(a, b)$ such that $\frac{f^{\prime}(c)}{f(c)}=\frac{1}{a-c}+\frac{1}{b-c}$.

## - Watch Video Solution

6. Let $f$ and $g$ be function continuous in $[a, b]$ and differentiable on $[a, b]$ If $f(a)=f(b)=0$ then show that there is a point $c \in(a, b)$ such that $g^{\prime}(c) f(c)+f^{\prime}(c)=0$.
7. If $\phi(x)$ is a differentiable function $\forall x \in R$ and $a \in R^{+}$such that $\phi(0)=\phi(2 a), \phi(a)=\phi(3 a)$ and $\phi(0) \neq \phi(a)$, then show that there is at least one root of equation $\phi^{\prime}(x+a)=\phi^{\prime}(x) \operatorname{in}(0,2 a)$.

## - Watch Video Solution

8. Let $f$ is continuous on $[a, b]$ and differentiable on $(a, b)$ s.t. $t^{2}(a)-t^{2}(b)=a^{2}-b^{2}$. Show that $\ldots f(x) f^{\prime}(x)=x$ has atleast one root in $(a, b)$.

## - Watch Video Solution

## Concept Application Exercise 58

1. Find $c$ of Lagranges mean value theorem for the function $f(x)=3 x^{2}+5 x+7$ in the interval $[1,3]$.
2. If $f(x)$ is continuous in $[a, b]$ and differentiable in (a,b), then prove that there exists at least one $c \in(a, b)$ such that $\frac{f^{\prime}(c)}{3 c^{2}}=\frac{f(b)-f(a)}{b^{3}-a^{3}}$

## - Watch Video Solution

3. If $a, b \in R$ and $a<b$, then prove that there exists at least one real number $c \in(a, b)$ such that $\frac{b^{2}+a^{2}}{4 c^{2}}=\frac{c}{a+b}$.

## - Watch Video Solution

4. If $f(x) \operatorname{andg}(x)$ are continuous functions in $[a, b]$ and are differentiable in $(a, b)$ then prove that there exists at least one $c \in(a, b)$ for which. $|f(\mathrm{a}) \mathrm{f}(\mathrm{b}) \mathrm{g}(\mathrm{a}) \mathrm{g}(\mathrm{b})|=(\mathrm{b}-\mathrm{a}) \mid \mathrm{f}(\mathrm{a}) \mathrm{f}^{\wedge}($ prime $)(\mathrm{c}) \mathrm{g}(\mathrm{a}) \mathrm{g}^{\wedge}($ prime $)(\mathrm{c}) \mid$, w h e r ea

## - Watch Video Solution

5. Prove that $\left|\tan ^{-1} x-\tan ^{-1} y\right| \leq|x-y| \forall x, y \in R$.

## - Watch Video Solution

6. Using Lagranges mean value theorem, prove that $\frac{b-a}{b}<\log \left(\frac{b}{a}\right)<$ $\frac{b-a}{a}$,where $0<a<b$.

## - Watch Video Solution

7. If $a>b>0$, with the aid of Lagranges mean value theorem, prove that $n b^{\wedge}(n-1)(a-b)>1 . n b^{\wedge}(n-1)(a-b)>a^{\wedge} n-b^{\wedge} n>n a \wedge(n-1)(a-b)$, if0

## - Watch Video Solution

8. Let $f(x) \operatorname{andg}(x)$ be two functions which are defined and differentiable for all $x \geq x_{0}$. If $f\left(x_{0}\right)=g\left(x_{0}\right)$ and $f^{\prime}(x)>g^{\prime}(x)$ for all $x>x_{0}$, then prove that $f(x)>g(x)$ for all $x>x_{0}$.
9. If $f(x)$ is differentiate in $[a, b]$, then prove that there exists at least one $c \in(a, b)$ such that $\left(a^{2}-b^{2}\right) f^{\prime}(c)=2 c(f(a)-f(b))$.

## - Watch Video Solution

## Exercises

1. The number of tangents to the curve $x^{\frac{3}{2}}+y^{\frac{3}{2}}=2 a^{\frac{3}{2}}, a>0$, which are equally inclined to the axes, is 2 (b) 1 (c) 0 (d) 4
A. 2
B. 1
C. 0
D. 4
2. The angle made by any tangent to the curve $x=a(t+\sin t \cos t), y=(1+\sin t)^{2}$ with $x$-axis is:
A. $\frac{1}{4}(\pi+2 t)$
B. $\frac{1-\sin t}{\cos t}$
C. $\frac{1}{4}(2 t-\pi)$
D. $\frac{1+\sin t}{\cos 2 t}$

## Answer: A

## - Watch Video Solution

3. If $m$ is the slope of a tangent to the curve $e^{y}=1+x^{2}$, then (a)
$|m|>1$ (b) $m>1$ (c) $m \geq-1$ (d) $|m| \leq 1$
A. $|m|>1$
B. $m>1$
C. $m>-1$
D. $|m| \leq 1$

## Answer: D

## - Watch Video Solution

4. If at each point of the curve $y=x^{3}-a x^{2}+x+1$, the tangent is inclined at an acute angle with the positive direction of the $x$-axis, then
(a) $a>0$ (b) $a<-\sqrt{3}$ (c) $-\sqrt{3}<a<\sqrt{3}$ (d) noneofthese
A. $a>0$
B. $a \leq \sqrt{3}$
C. $-\sqrt{3} \leq a \leq \sqrt{3}$
D. none of these

## Answer: C

5. The slope of the tangent to the curve $y=\sqrt{4-x^{2}}$ at the point where the ordinate and the abscissa are equal is (a) -1 (b) 1 (c) 0 (d) none of these
A. -1
B. 1
C. 0
D. none of these

## Answer: A

## - Watch Video Solution

6. The curve given by $x+y=e^{x y}$ has a tangent parallel to the $y-a \xi s$ at the point $(0,1)(b)(1,0)(c)(1,1)$ (d) none of these
A. $(0,1)$
B. $(1,0)$
C. $(1,1)$
D. none of these

## Answer: B

## - Watch Video Solution

7. Find value of $c$ such that line joining the points $(0,3)$ and (5, -2) becomes tangent to curve $y=\frac{c}{x+1}$
A. 1
B. -2
C. 4
D. none of these
8. A differentiable function $y=f(x)$ satisfies $f^{\prime}(x)=(f(x))^{2}+5$ and $f(0)=1$. Then the equation of tangent at the point where the curve crosses $y$-axis, is
A. $x-y+1=0$
B. $x-2 y+1=0$
C. $6 x-y+1=0$
D. $x-2 y-1=0$

## Answer: C

## - Watch Video Solution

9. The distance between the origin and the tangent to the curve $y=e^{2 x}+x^{2}$ drawn at the point $x=0$ is (a) $\left(\frac{1}{\sqrt{5}}\right)$ (b) $\left(\frac{2}{\sqrt{5}}\right)$

$$
\left(-\frac{1}{\sqrt{5}}\right) \text { (d) }\left(\frac{2}{\sqrt{3}}\right)
$$

A. $\frac{1}{\sqrt{5}}$
B. $\frac{2}{\sqrt{5}}$
C. $\frac{-1}{\sqrt{5}}$
D. $\frac{2}{\sqrt{3}}$

## Answer: A

## Watch Video Solution

10. The point on the curve $3 y=6 x-5 x^{3}$ the normal at Which passes through the origin, is
A. $(1,1 / 3)$
B. $(-1,-1 / 3)$
C. $(2,-28 / 3)$
D. none of these

## D Watch Video Solution

11. The normal to the curve $2 x^{2}+y^{2}=12$ at the point $(2,2)$ cuts the curve again at (a) $\left(-\frac{22}{9},-\frac{2}{9}\right)$ (b) $\left(\frac{22}{9}, \frac{2}{9}\right)(-2,-2)$ (d) none of these
A. $\left(-\frac{22}{9},-\frac{2}{9}\right)$
B. $\left(\frac{22}{9}, \frac{2}{9}\right)$
C. $(-2,-2)$
D. none of these

## Answer: A

12. At what point of curve $y=\frac{2}{3} x^{3}+\frac{1}{2} x^{2}$, the tangent makes equal angle with the axis?
(a) $\left(\frac{1}{2}, \frac{5}{24}\right) \operatorname{and}\left(-1,-\frac{1}{6}\right)$
$\left(\frac{1}{2}, \frac{4}{9}\right) \operatorname{and}(-1,0)$
(c) $\quad\left(\frac{1}{3}, \frac{1}{7}\right)$ and $\left(-3, \frac{1}{2}\right)$
$\left(\frac{1}{3}, \frac{4}{47}\right)$ and $\left(-1,-\frac{1}{3}\right)$
A. $\left(\frac{1}{2}, \frac{4}{24}\right)$ and $\left(-1,-\frac{1}{6}\right)$
B. $\left(\frac{1}{2}, \frac{4}{9}\right)$ and $(-1,0)$
C. $\left(\frac{1}{3}, \frac{1}{7}\right)$ and $\left(-3, \frac{1}{2}\right)$
D. $\left(\frac{1}{3}, \frac{4}{47}\right)$ and $\left(-1,-\frac{1}{3}\right)$

## Answer: A

## - Watch Video Solution

13. The equation of tangent to the curve $y=b e^{-x / a}$ at the point where it crosses $Y$-axis is
A. $\frac{x}{a}-\frac{y}{b}=1$
B. $a x+b y+1$
C. $a x-b y=1$
D. $\frac{x}{a}+\frac{y}{b}=1$

## Answer: D

## - Watch Video Solution

14. Then angle of intersection of the normal at the point $\left(-\frac{5}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right)$ of the curves $x^{2}-y^{2}=8$ and $9 x^{2}+25 y^{2}=225$ is 0
(b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$
A. 0
B. $\frac{\pi}{2}$
C. $\frac{\pi}{3}$
D. $\frac{\pi}{4}$
15. A function $\mathrm{y}=\mathrm{f}(\mathrm{x})$ has a second-order derivative $f^{\prime \prime}(x)=6(x-1)$. If its graph passed through the point $(2,1)$ and at that point tangent to the graph is $y=3 x-5$, then the value of $\mathrm{f}(0)$ is
A. 1
B. -1
C. 2
D. 0

## Answer: B

## - Watch Video Solution

16. $x+y-\ln (x+y)=2 x+5$ has a vertical tangent at the point $(\alpha, \beta)$ then $\alpha+\beta$ is equal to
A. -1
B. 1
C. 2
D. -2

## Answer: B

## - Watch Video Solution

17. A curve is difined parametrically by $x=e^{\sqrt{t}}, y=3 t-\log _{e}\left(t^{2}\right)$, where $t$ is a parameter. Then the equation of the tangent line drawn to the curve at $\mathrm{t}=1$ is
A. $y=\frac{2}{e} x+1$
B. $y=\frac{2}{e} x-1$
C. $y=\frac{e}{2} x+1$
D. $y=\frac{e}{2} x-1$

## D Watch Video Solution

18. If $x+4 y=14$ is a normal to the curve $y^{2}=\alpha x^{3}-\beta$ at $(2,3)$, then the value of $\alpha+\beta$ is
A. 9
B. -5
C. 7
D. -7

## Answer: A

## D Watch Video Solution

19. In the curve represented parametrically by the equations $x=2 \ln \cot t+1$ and $y=\tan t+\cot t$, (A) tangent and normal
intersect at the point (2,1).(B) normal at $\mathrm{t}=\pi 4$ is parallel to the y -axis. (C) tangent at $\mathrm{t}=\pi 4$ is parallel to the line $\mathrm{y}=\mathrm{x}$.(D) tangent at $\mathrm{t}=\pi 4$ is parallel to the $x$-axis.
A. tangent and normal intersect at the point $(2,1)$
B. normal at $t=\pi / 4$ is parallel to the $y$-axis
C. tangent at $t=\pi / 4$ is parallel to the line $\mathrm{y}=\mathrm{x}$
D. tangent at $t=\pi / 4$ is parallel to the $x$-axis

## Answer: D

## - Watch Video Solution

20. The abscissas of point $\operatorname{PandQ}$ on the curve $y=e^{x}+e^{-x}$ such that tangents at PandQ make $60^{0}$ with the $x$-axis are. )a)
$1 n\left(\frac{\sqrt{3}+\sqrt{7}}{7}\right) \operatorname{and} 1 n\left(\frac{\sqrt{3}+\sqrt{5}}{2}\right)$
(b) $\quad \ln \left(\frac{\sqrt{3}+\sqrt{7}}{2}\right)$
$1 n\left(\frac{\sqrt{7}-\sqrt{3}}{2}\right)$ (d) $\pm 1 n\left(\frac{\sqrt{3}+\sqrt{7}}{2}\right)$
A. $\ln \left(\frac{\sqrt{3}+\sqrt{7}}{7}\right)$ and $\ln \left(\frac{\sqrt{3}+\sqrt{5}}{2}\right)$
B. $\left(\frac{\sqrt{3}+\sqrt{7}}{2}\right)$
C. $\ln \left(\frac{\sqrt{7}-\sqrt{3}}{2}\right)$
D. $\pm \ln \left(\frac{\sqrt{3}+\sqrt{7}}{2}\right)$

## Answer: B

## - Watch Video Solution

21. If a variable tangent to the curve $x^{2} y=c^{3}$ makes intercepts $a$, bonx - andy - axes, respectively, then the value of $a^{2} b$ is $27 c^{3}$ (b) $\frac{4}{27} c^{3}$ (c) $\frac{27}{4} c^{3}$ (d) $\frac{4}{9} c^{3}$
A. $27 c^{3}$
B. $\frac{4}{27} c^{3}$
C. $\frac{27}{4} c^{3}$
D. $\frac{4}{9} c^{3}$

## - Watch Video Solution

22. Let $C$ be the curve $y=x^{3}$ (where $x$ takes all real values). The tangent at $A$ meets the curve again at $B$. If the gradient at $B$ is $K$ times the gradient at $A$, then $K$ is equal to (a) 4 (b) 2 (c) -2 (d) $\frac{1}{4}$
A. 4
B. 2
C. -2
D. $\frac{1}{4}$

Answer: A

## - Watch Video Solution

23. The equation of the line tangent to the curve $\mathrm{x} \sin \mathrm{y}+\mathrm{y} \sin \mathrm{x}=\pi$ at the point $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ is
A. $3 x+y=2 \pi$
B. $x-y=0$
C. $2 x-y=\pi / 2$
D. $x+y=\pi$

## Answer: D

## (D) Watch Video Solution

24. The $x$-intercept of the tangent at any arbitrary point of the curve $\frac{a}{x^{2}}+\frac{b}{y^{2}}=1$ is proportional to square of the abscissa of the point of tangency square root of the abscissa of the point of tangency cube of the abscissa of the point of tangency cube root of the abscissa of the point of tangency
A. square of the abscissa of the point of tangency
B. square root of the absciss of the point of tangency
C. cube of the abscissa of the point of tangency
D. cube root of the abscissa of the point of tangency

## Answer: C

## (D) Watch Video Solution

25. At any point on the curve $2 x^{2} y^{2}-x^{4}=c$, the mean proportional between the abscissa and the difference between the abscissa and the sub-normal drawn to the curve at the same point is equal to (a)Ordinate (b) radius vector (c)x-intercept of tangent (d) sub-tangent
A. ordinate
B. radius vector
C. $x$-intercect of tangent
D. sub-tangent

## - Watch Video Solution

26. Given $\mathrm{g}(\mathrm{x}) \frac{x+2}{x-1}$ and the line $3 x+y-10=0$. Then the line is
A. tangent to $g(x)$
B. normal to $g(x)$
C. chord ofg(x)
D. none of these

## Answer: A

## D Watch Video Solution

27. If the length of sub-normal is equal to the length of sub-tangent at any point $(3,4)$ on the curve $y=f(x)$ and the tangent at $(3,4)$ to
$y=f(x)$ meets the coordinate axes at $\operatorname{AandB}$, then the maximum area of the triangle $O A B$, where $O$ is origin, is $45 / 2$ (b) $49 / 2$ (c) $25 / 2$ (d) $81 / 2$
A. $45 / 2$
B. $49 / 2$
C. $25 / 2$
D. $81 / 2$

## Answer: B

## - Watch Video Solution

28. The number of point in the rectangle $\{(x, y)\}-12 \leq x \leq 12 a n d-3 \leq y \leq 3\}$ which lie on the curve $y=x+\sin x$ and at which in the tangent to the curve is parallel to the $x$-axis is 0 (b) 2 (c) 4 (d) 8
A. 0
B. 2
C. 4
D. 8

## Answer: A

## - Watch Video Solution

29. Tangent of acute angle between the curves $y=\left|x^{2}-1\right|$ and $y=\sqrt{7-x^{2}}$ at their points of intersection is $\frac{5 \sqrt{3}}{2}$ (b) $\frac{3 \sqrt{5}}{2} \frac{5 \sqrt{3}}{4}$ (d) $\frac{3 \sqrt{5}}{4}$
A. $\frac{5 \sqrt{3}}{2}$
B. $\frac{3 \sqrt{5}}{2}$
C. $\frac{5 \sqrt{3}}{4}$
D. $\frac{3 \sqrt{5}}{4}$

## Answer: C

30. The line tangent to the curves $y^{3}-x^{2} y+5 y-2 x=0$ and $x^{2}-x^{3} y^{2}+5 x+2 y=0$ at the origin intersect at an angle $\theta$ equal to
(a) $\frac{\pi}{6}$
(b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$
(d) $\frac{\pi}{2}$
A. $\frac{\pi}{6}$
B. $\frac{\pi}{4}$
C. $\frac{\pi}{3}$
D. $\frac{\pi}{2}$

## Answer: D

## - Watch Video Solution

31. The two curves $x=y^{2}, x y=a^{3}$ cut orthogonally at a point. Then $a^{2}$ is equal to $\frac{1}{3}$ (b) 3 (c) 2 (d) $\frac{1}{2}$
A. $\frac{1}{3}$
B. 3
C. 2
D. $\frac{1}{2}$

## Answer: D

## - Watch Video Solution

32. The tangent to the curve $y=e^{k x}$ at a point $(0,1)$ meets the $x$-axis at $(\mathrm{a}, \mathrm{0})$ where $a \in[-2,-1]$, then $\mathrm{k} \in:$
A. $[-1 / 2,0]$
B. $[-1,-1 / 2]$
C. $[0,1]$
D. $[1 / 2,1]$
33. The curves $4 x^{2}+9 y^{2}=72$ and $x^{2}-y^{2}=5 a t(3,2)$ Then (a) touch each other (b) cut orthogonally intersect at $45^{\circ}$ (d) intersect at $60^{\circ}$
A. touch each other
B. cut orthogonally
C. intersect at $45^{\circ}$
D. intersect at $60^{\circ}$

## Answer: B

## - Watch Video Solution

34. The coordinates of a point on the parabola $y^{2}=8 x$ whose distance from the circle $x^{2}+(y+6)^{2}=1$ is minimum is (a) $(2,4)$ (b) $(2,-4)$ (c) $(18,-12)(\mathrm{d})(8,8)$
A. $(2,4)$
B. $(2,-4)$
C. $(18,-12)$
D. $(8,8)$

## Answer: B

## - Watch Video Solution

35. At the point $P\left(a, a^{n}\right)$ on the graph of $y=x^{n}(n \in N)$ in the first quadrant at normal is drawn. The normal intersects the $Y$-axis at the point $(0, b)$. If $\lim _{a \rightarrow 0} b=\frac{1}{2}$, then $n$ equals
A. 1
B. 3
C. 2
D. 4

## Answer: C

36. Let $f$ be a continuous, differentiable, and bijective function. If the tangent to $y=f(x) a t x=a$ is also the normal to $y=f(x) a t x=b$, then there exists at least one $c \in(a, b)$ such that $f^{\prime}(c)=0$ (b) $f^{\prime}(c)>0 f^{\prime}(c)<0$ (d) none of these
A. $f^{\prime}(c)=0$
B. $f^{\prime}(c)>0$
C. $f^{\prime}(c)<0$
D. none of these

## Answer: A

## - Watch Video Solution

37. A point on the parabola $y^{2}=18 x$ at which the ordinate increases at twice the rate of the abscissa is (a) $(2,6)$ (b) $(2,-6)\left(\frac{9}{8},-\frac{9}{2}\right)$
A. $(2,6)$
B. $(2,-6)$
C. $\left(\frac{9}{8}, \frac{9}{2}\right)$
D. $\left(\frac{9}{8}, \frac{9}{2}\right)$

## Answer: D

## - Watch Video Solution

38. Find the rate of change of volume of a sphere with respect to its surface area when the radius is 2 cm .
A. 1
B. 2
C. 3
D. 4

## - Watch Video Solution

39. If there is an error of $k \%$ in measuring the edge of a cube, then the percent error in estimating its volume is $k$ (b) $3 k \frac{k}{3}(\mathrm{~d})$ none of these
A. $k$
B. 3 k
C. $\frac{k}{3}$
D. none of these

## Answer: B

## D Watch Video Solution

40. A lamp of negligible height is placed on the ground $l_{1}$ away from a wall. A man $l_{2} m$ tall is walking at a speed of $\frac{l_{1}}{10} m / s$ from the lamp to
the nearest point on the wall. When he is midway between the lamp and the wall, the rate of change in the length of this shadow on the wall is $-\frac{5 l_{2}}{2} m / s$ (b) $-\frac{2 l_{2}}{5} m / s-\frac{l_{2}}{2} m / s$ (d) $-\frac{l_{2}}{5} m / s$
A. $-\frac{5 l_{2}}{2} m / s$
B. $-\frac{2 l_{2}}{5} m / s$
C. $-\frac{l_{2}}{2} m / s$
D. $-\frac{l_{2}}{5} m / s$

## Answer: B

## - Watch Video Solution

41. The function $f(x)=x(x+3) e^{-\left(\frac{1}{2}\right) x}$ satisfies the conditions of Rolle's theorem in $(-3,0)$. The value of $c$, is
A. -2
B. -1
C. 0
D. 3

## Answer: A

## D Watch Video Solution

42. The radius of a right circular cylinder increases at the rate of 0.1 $\mathrm{cm} / \mathrm{min}$, and the height decreases at the rate of $0.2 \mathrm{~cm} / \mathrm{min}$. The rate of change of the volume of the cylinder, in $\mathrm{cm}^{2} / m \in$, when the radius is $2 c m$ and the height is 3 cm is (a) $-2 p$ (b) $-\frac{8 \pi}{5}-\frac{3 \pi}{5}$ (d) $\frac{2 \pi}{5}$
A. $-2 \pi$
B. $-\frac{8 \pi}{5}$
C. $16 / 6$
D. $-8 / 15$
43. A cube of ice melts without changing its shape at the uniform rate of $4 \frac{\mathrm{~cm}^{3}}{\mathrm{~min}}$. The rate of change of the surface area of the cube, in $\frac{\mathrm{cm}^{2}}{\mathrm{~min}}$, when the volume of the cube is $125 \mathrm{~cm}^{3}$, is (a) -4 (b) $-\frac{16}{5}$ (c) $-\frac{16}{6}$ (d) $-\frac{8}{15}$
A. -4
B. $-16 / 5$
C. $-16 / 6$
D. $-8 / 15$

## Answer: B

## - Watch Video Solution

44. The radius of the base of a cone is increasing at the rate of $3 \mathrm{~cm} / \mathrm{min}$ and the altitude is decreasing at the rate of $4 \mathrm{~cm} / \mathrm{min}$. The rate of
change of lateral surface when the radius is 7 cm and altitude is 24 cm is
(a) $108 \pi \mathrm{~cm}^{2} / \mathrm{min}$
(b) $7 \pi \mathrm{~cm}^{2} / \mathrm{min}$
(c) $27 \pi \mathrm{~cm}^{2} / \mathrm{min}$
(d) none of these
A. $108 \pi \mathrm{~cm}^{2} / \mathrm{min}$
B. $7 \pi \mathrm{~cm}^{2} / \mathrm{min}$
C. $27 \pi \mathrm{~cm}^{2} / \mathrm{min}$
D. none of these

## Answer: A

## - Watch Video Solution

45. If $f(x)=x^{3}+7 x-1$, then $f(x)$ has a zero between $x=0$ and $x=1$. The theorem that best describes this is (a) mean value theorem (b) maximum-minimum value theorem (c) intermediate value theorem (d) none of these
A. mena value theorem
B. maximum-minimum value theorem
C. intermediate value theorem
D. none of these

## Answer: C

## - Watch Video Solution

46. Consider the function $f(x)= \begin{cases}x \frac{\sin (\pi)}{x} & \text { for } x>0 \\ 0 & \text { for } x=0\end{cases}$

Then, the number of points in $(0,1)$ where the derivative $f^{\prime}(x)$ vanishes is
A. 0
B. 1
C. 2
D. infinite

## Answer: D

47. Let $f(x) \operatorname{andg}(x)$ be differentiable for $0 \leq x \leq 1$, such that $f(0)=0, g(0)=0, f(1)=6$. Let there exists real number $c$ in $(0,1)$ such that $f^{\prime}(c)=2 g^{\prime}(c)$. Then the value of $g(1)$ must be (a) 1 (b) 3 (c) -2 (d) -1
A. 1
B. 3
C. -2
D. 1 -

## Answer: B

## - Watch Video Solution

48. If $3(a+2 c)=4(b+3 d)$, then the equation $a x^{3}+b x^{2}+c x+d=0$ will have (a) no real solution (b) at least one
real root in $(-1,0)$ (c) at least one real root in $(0,1)$ (d) none of these
A. no real solution
B. at least one real root in ( $-1,0$ )
C. at least one real root in $(0,1)$
D. none of these

## Answer: B

## - Watch Video Solution

49. A value of c for which the conclusion of Mean value theorem holds for the function $f(x)=\log _{e} x$ on the interval $[1,3]$ is
A. $\frac{1}{2} \log _{e} 3$
B. $\log _{3} e$
C. $\log _{e} 3$
D. $2 \log _{3} e$

## D Watch Video Solution

50. Let $f(x)$ be a twice differentiable function for all real values of $x$ and satisfies $f(1)=1, f(2)=4, f(3)=9$. Then which of the following is
definitely true?
(a)
$f^{\prime \prime}=2 \forall x \in(1,3)$
$f^{\prime \prime}=f(x)=5 f$ or somex $\in(2,3)$
(c) $f^{\prime \prime}=3 \forall x \in(2,3)$
$f^{\prime \prime}=2 f$ or somex $\in(1,3)$
A. $f^{\prime \prime}(x)=2 \forall x \in(1,3)$
B. $f^{\prime \prime}(x)=f(x) 5$ for some $x \in(2,3)$
C. $f^{\prime \prime}(x)=3 \forall x \in(2,3)$
D. $f^{\prime \prime}(x)=2$ for some $x \in(1,3)$

## Answer: D

## D Watch Video Solution

51. The value of $c$ in Largrange's theorem for the function $f(x)=\log _{e} \sin x$ in the interval $[\pi / 6,5 \pi / 6]$ is
A. $\pi / 4$
B. $\pi / 2$
C. $2 \pi / 3$
D. none of these

## Answer: B

## - Watch Video Solution

52. In which of the following function Rolle's theorem is applicable ?
A. $f(x)=\left\{\begin{array}{ll}x & 0 \leq x<1 \\ 0 & x=1\end{array}\right.$ on $[0,1]$
B. $f(x)=\left\{\begin{array}{ll}\frac{\sin x}{x} & -\pi \leq x<0 \\ 0 & x=0\end{array}\right.$ on $[-\pi, 0]$
C. $f(x) \frac{x^{2}-x-6}{x-1}$ on $[-2,3]$
D. $f(x)=\left\{\begin{array}{ll}\frac{x^{3}-2 x^{3}-5 x+6}{x-1} & \text { if } x \neq 1 \\ -6 & \text { if } x=1\end{array}\right.$ on $[-2,3]$

## Answer: D

## - Watch Video Solution

53. Let $f^{\prime}(x)=e^{x \wedge} 2$ and $f(0)=10$. If $A<\mathrm{f}(1)<\mathrm{B}$ can be concluded from mean value theorem then the value of $A-B$ will be
A. e
B. $1-e$
C. $e-1$
D. $1+e$

## Answer: B

54. If $f(x) \operatorname{and} g(x)$ are differentiable functions for $0 \leq x \leq 1$ such that $f(0)=10, g(0)=2, f(1)=2, g(1)=4$, then in the interval $(0,1) \cdot(\mathrm{a})$ $f^{\prime}(x)=0 f$ or allx (b) $f^{\prime}(x)+4 g^{\prime}(x)=0$ for at least one $x$ (c) $f(x)=2 g^{\prime}(x)$ for at most one $x(\mathrm{~d})$ none of these
A. $f(x)=0$ for all x
B. $f(x)+4 g^{\prime}(x)=0$ for at least one x
C. $f(x)=2 g^{\prime}(x)$ for at most one x
D. none of these

## Answer: B

## - Watch Video Solution

55. A continuous and differentiable function $y=f(x)$ is such that its graph cuts line $y=m x+c$ at $n$ distinct points. Then the minimum number of points at which $f^{\prime \prime}(x)=0$ is/are
A. $n-1$
B. $n-3$
C. $n-2$
D. cannot say

## Answer: C

## - Watch Video Solution

56. Given $f^{\prime}(1)=1$ and $\frac{d}{d x}(f(2 x))=f^{\prime}(x) \forall x>0.1 f \quad f^{\prime}(x)$ is differentiable then there exits a number $c \in(2,4)$ such that $f^{\prime \prime}(c)$ equals
A. $\frac{1}{4}$
B. $\frac{-1}{2}$
C. $-1 \frac{1}{4}$
D. $-\frac{1}{8}$

Answer: D

## D Watch Video Solution

57. If $(\mathrm{x})$ is differentiable in $[a, b]$ such that $f(a)=2, f(b)=6$, then there exists at least one $c, a<c \leq b$, such that $\left(b^{3}-a^{3}\right) f^{\prime}(c)=$
A. $c^{2}$
B. $2 c^{2}$
C. $-3 c^{2}$
D. $12 c^{2}$

## Answer: D

## - Watch Video Solution

Multiple Correct Answer Type

1. Points on the curve $f(x)=\frac{x}{1-x^{2}}$ where the tangent is inclined at an angle of $\frac{\pi}{4}$ to the $x$-axis are (a) ( 0,0 ) (b) $\left(\sqrt{3},-\frac{\sqrt{3}}{2}\right)$
$\left(-2, \frac{2}{3}\right)$ (d) $\left(-\sqrt{3}, \frac{\sqrt{3}}{2}\right)$
A. $(0,0)$
B. $\left(\sqrt{3},-\frac{\sqrt{3}}{2}\right)$
C. $\left(-2, \frac{2}{3}\right)$
D. $\left(\sqrt{3},-\frac{\sqrt{3}}{2}\right)$

## Answer: A::B::D

## - Watch Video Solution

2. For the curve $y=c e^{x / a}$, which one of the following is incorrect?
A. sub-tangent is constant
B. sub-normal varies as the square of the ordinate
C. tangent at $\left(x_{1}, y_{1}\right)$ on the curve intersects the x -axis at a distance of $\left(x_{1}-a\right)$ from the origin
D.equaltion of the normal at the point where the curve cuts $y-$ axis is $c y+a x=c^{2}$

## Answer: A::B::C::D

## - Watch Video Solution

3. Let the parabolas $y=x(c-x)$ andy $=x^{2}+a x+b$ touch each other at the point (1,0). Then (a) $a+b+c=0$ (b) $a+b=2$ (c) $b-c=1$ (d) $a+c=-2$
A. $a+b+c=0$
B. $a+b=2$
C. $b-c=1$
D. $a+c=-2$

## D Watch Video Solution

4. The angle formed by the positive $y-a \xi s$ and the tangent to
$y=x^{2}+4 x-17 a t\left(\frac{5}{2},-\frac{3}{4}\right) \quad$ is $\quad \tan ^{-1}(9) \quad$ (b) $\quad \frac{\pi}{2}-\tan ^{-1}(9)$ $\frac{\pi}{2}+\tan ^{-1}(9)(\mathrm{d})$ none of these
A. $\tan ^{-1}(9)$
B. $\frac{\pi}{2}-\tan ^{-1}(9)$
C. $\frac{\pi}{2}+\tan ^{-1}(9)$
D. none of these

## Answer: B::C

5. Which of the following pair(s) of curves is/are ortogonal
$y^{2}=4 a x, y=e^{-x / 2 a}$
(b) $y^{2}=4 a x, x^{2}=4 a y a t(0,0)$
$x y=a^{2}, x^{2}-y^{2}=b^{2}$ (d) $y=a x, x^{2}+y^{2}=c^{2}$
A. $y^{2}=4 a x, y=e^{-x / 2 a}$
B. $y^{2}=4 a x, x^{2}=4 a y a t(0,0)$
C. $x y=a^{2}, x^{2}-y^{2}=b^{2}$
D. $y=a x, x^{2}+y^{2}=c^{2}$

## Answer: A::B::C::D

## - Watch Video Solution

6. The coordinates of the point(s) on the graph of the function $f(x)=\frac{x^{3}}{x}-\frac{5 x^{2}}{2}+7 x-4$, where the tangent drawn cuts off intercepts from the coordinate axes which are equal in magnitude but opposite in sign, are (a) $\left(2, \frac{8}{3}\right)$ (b) $\left(3, \frac{7}{2}\right)$ (c) $\left(1, \frac{5}{6}\right)$ (d) none of these
A. $(2,8 / 3)$
B. $(3,7 / 2)$
C. $(1,5 / 6)$
D. none of these

## Answer: A: B

## - Watch Video Solution

7. The abscissa of a point on the curve $x y=(a+x)^{2}$, the normal which cuts off numerically equal intercepts from the coordinate axes, is $-\frac{1}{\sqrt{2}}$ (b) $\sqrt{2} a$ (c) $\frac{a}{\sqrt{2}}$ (d) $-\sqrt{2} a$
A. $-\frac{a}{\sqrt{2}}$
B. $\sqrt{2} a$
C. $\frac{a}{\sqrt{2}}$
D. $-\sqrt{2} a$

## D Watch Video Solution

8. The angle between the tangents at any point $P$ and the line joining $P$ to the orgin, where $P$ is $a$ point on the curve In $\left(x^{2}+y^{2}\right)=k \tan ^{1-} \frac{y}{x}, c$ is a constant, is
A. independent of $x$
B. independent of $y$
C. independent of $x$ but dependent on $y$
D. independent of $y$ but dependent on $x$

## Answer: A::B

## - Watch Video Solution

9. If OT and ON are perpendiculars dropped from the origin to the tangent and normal to the curve $x=a \sin ^{3} t, y=a \cos ^{3} t$ at an arbitrary point, then
A. $4 O T^{2}+O N^{2}=a^{2}$
B. $\left|\frac{y}{\cos t}\right|$
C. the length of the normal is $\left|\frac{y}{\sin t}\right|$
D. none of these

## Answer: A::B::C

## - Watch Video Solution

10. Let $C_{1}: y=x^{2} \sin 3 x, C_{2}: y=x^{2}$ and $C_{3}: y=-y^{2}$, then
A. $C_{1}$ touches $C_{2}$ at infinite points
B. $C_{1}$ touches $C_{3}$ at infinite points
C. $C_{1}$ and $C_{2}$ and $C_{1}$ and $C_{3}$ meet at alternate points
D. none of these

## Answer: A::B

## D Watch Video Solution

11. If the line $x \cos \theta+y \sin \theta=P$ is the normal to the curve $(x+a) y=1$, then $\theta$ maylie in
A. I quadrant
B. Il quadrant
C. III quadrant
D. IV quadrant

## Answer: B::D

12. Common tagent (s) to $y=x^{3}$ and $x=y^{3}$ is/are
A. $x-y=\frac{1}{\sqrt{3}}$
B. $x-y=-\frac{1}{\sqrt{3}}$
C. $x-y=\frac{2}{3 \sqrt{3}}$
D. $x-y=\frac{-2}{3 \sqrt{3}}$

## Answer: C::D

## - Watch Video Solution

13. Given $f(x)=4-\left(\frac{1}{2}-x\right)^{\frac{2}{3}}, g(x)=\left\{\frac{\tan [x]}{x}, x \neq 01, x=0\right.$ $h(x)=\{x\}, k(x)=5^{(\log )_{2}(x+3)}$ Then in [0,1], lagranges mean value theorem is not applicable to (where [.] and \{.\} represents the greatest integer functions and fractional part functions, respectively). $f$ (b) $g$ (c) $k$ (d) $h$
A. $f$
B. $g$
C. k
D. h

## Answer: A::B::D

## - Watch Video Solution

14. Let $f(x)=a_{5} x^{5}+a_{4} x^{4}+a_{3} x^{3}+a_{2} x^{2}+a_{1} x$, where $a_{i}{ }^{\prime} s$ are real and $f(x)=0$ has a positive root $\alpha_{0}$. Then $f^{\prime}(x)=0$ has a positive root $\alpha_{1}$ such that ${ }^{`} 0$
A. $\mathrm{f}^{\prime}(\mathrm{x})=0$ has a root $\alpha_{1}$ such that $<\alpha_{1}<\alpha_{0}$
B. $f^{\prime}(x)=0$ has at least one real root
C. $\mathrm{f}^{\prime \prime}(\mathrm{x})=0$ has at least one real root
D. none of these
15. Among the following, the function (s) on which LMVT theorem is applicable in the indecatd intervals is/are
A. $f(x)=x^{\frac{1}{3}}$ in $[-1,1]$
B. $f(x)=x+\frac{1}{x} \operatorname{in}\left[\frac{1}{2}, 3\right]$
C. $f(x)=(x-1)|(x-1)(x-2)| \operatorname{in}[-1,1]$
D. $f(x)=e^{|(x-1)(x-3)|} \operatorname{in}[1,3]$

## Answer: B::C::D

## - Watch Video Solution

16. Let $f(x)$ be a differentiable function and $f(\alpha)=f(\beta)=0(\alpha<\beta)$, then the interval $(\alpha, \beta)$
A. $f(x)+f^{\prime}(x)=0$ has at least one root
B. $f(x)-f^{\prime}(x)=0$ has at least one real root
C. $f(x) \times f^{\prime}(x)=0$ has at lease one real root
D. none of these

## Answer: A::B::C

## - Watch Video Solution

## Linked Comprehension Type

1. Tangent at a point $P_{1}$ [other than $(0,0)$ ] on the curve $y=x^{3}$ meets the curve again at $P_{2}$. The tangent at $P_{2}$ meets the curve at $P_{3} \&$ so on. Show that the abscissae of $P_{1}, P_{2}, P_{3}, \ldots \ldots \ldots P_{n}$, form a GP. Also find the ratio area of $A\left(P_{1} P_{2} P_{3}.\right)$ area of $\Delta\left(P_{2} P_{3} P_{4}\right)$
A. $1 / 4$
B. $1 / 2$
C. $1 / 8$
D. $1 / 16$

## Answer: D

## - Watch Video Solution

2. A spherical balloon is being inflated so that its volume increase uniformaly at the rate of $40 \mathrm{~cm}^{3} /$ minute. The rate of increase in its surface area when the radius is 8 cm , is
A. $8 \mathrm{~cm}^{2} / \mathrm{min}$
B. $10 \mathrm{~cm}^{2} / \mathrm{min}$
C. $20 \mathrm{~cm}^{2} / \mathrm{min}$
D. none of these

## Answer: B

3. A spherical balloon is being inflated so that its volume increase uniformly at the rate of $40 \frac{(\mathrm{~cm})^{3}}{\mathrm{~min}}$. How much the radius will increases during the next $1 / 2$ minute, if initial radius is 8 cm . ?
A. 0.025 cm
B. 0.050 cm
C. 0.075 cm
D. 0.01 cm

## Answer: A

## - Watch Video Solution

4. A conical paper cup 20 cm across the top and 15 cm deep is full of water. The cup springs a leak at the bottom and losses water at $5 \mathrm{cu} . \mathrm{cm}$ per minute.

How fast is the water level dropping at the instant when the water is exactly 7.5 cm deep?
A. $\frac{1}{\pi} \mathrm{~cm} / \mathrm{min}$
B. $\frac{1}{5 \pi} \mathrm{~cm} / \mathrm{min}$
C. $\frac{1}{2 \pi} \mathrm{~cm} / \min$
D. $\frac{2}{3 \pi} \mathrm{~cm} / \mathrm{min}$

## Answer: B

## - Watch Video Solution

5. A conical paper cup 20 cm across the top and 15 cm deep is full of water. The cup springs a leak at the bottom and losses water at $5 \mathrm{cu} . \mathrm{cm}$ per minute.

The amount of water (in $\mathrm{cm}^{3}$ ) when the hight of water is 3 cm is
A. $4 \pi$
B. $3 \pi$
C. $27 \pi$
D. $2 \pi$

## - Watch Video Solution

6. A conical paper cup 20 cm across the top and 15 cm deep is full of water. The cup springs a leak at the bottom and losses water at $5 \mathrm{cu} . \mathrm{cm}$ per minute.

The value of $\frac{d^{2} h}{d t^{2}}\left(\mathrm{incm} / \mathrm{min}^{2}\right)$ when the water is exactly
7.5 cm deep and $\frac{d^{2} V}{d t^{2}}=-\frac{4}{9} \mathrm{~cm}^{3} / \stackrel{2}{\min } i s$
A. $-\frac{2}{5}$
B. $\frac{-2}{125 \pi^{3}}$
C. $\frac{-2}{5 \pi^{3}}$
D. none of these

## Answer: D

7. Let $\mathrm{A}(0,0)$ and $\mathrm{B}(8,2)$ be two fixed points on the curve $y^{3}=|x|$ A point C (abscissa is less than 0 ) starts moving from origin along the curve such that rate of change in the ordinate is $2 \mathrm{~cm} / \mathrm{sec}$. After $t_{0}$ seconds, triangle $A B C$ becomes a right triangle.

After $t_{0}$ secods, tangent is drawn to teh curve at point C to intersect it again at $(\alpha, \beta)$. Then the value of $4 \alpha+3 \beta$ is
A. 1 sec
B. 2 sec
C. $\frac{1}{4} \mathrm{sec}$
D. $\frac{1}{2} \mathrm{sec}$

## Answer: C

## - Watch Video Solution

8. Let $\mathrm{A}(0,0)$ and $\mathrm{B}(8,2)$ be two fixed points on the curve $y^{3}=|x|$ A point

C (abscissa is less than 0 ) starts moving from origin along the curve such
that rate of change in the ordinate is $2 \mathrm{~cm} / \mathrm{sec}$. After $t_{0}$ seconds, triangle $A B C$ becomes a right triangle.

After $t_{0}$ secods, tangent is drawn to teh curve at point C to intersect it again at $(\alpha, \beta)$. Then the value of $4 \alpha+3 \beta$ is
A. $\frac{4}{3}$
B. $\frac{3}{4}$
C. 2
D. 1

## Answer: D

## - Watch Video Solution

## Numerical Value Type

1. There is a point ( $\mathrm{p}, \mathrm{q}$ ) on the graph of $f(x)=x^{2}$ and a point $(r, s)$ on the graph of $g(x)=\frac{-8}{x}$, wherep $>0 a n d r>0$. If the line through
$(p, q) a n d(r, s)$ is also tangent to both the curves at these points, respectively, then the value of $P+r$ is $\qquad$ .

## - Watch Video Solution

2. A curve is defined parametrically be equations $x=t^{2} a n d y=t^{3}$. A variable pair of perpendicular lines through the origin $O$ meet the curve of $\operatorname{PandQ}$. If the locus of the point of intersection of the tangents at $\operatorname{PandQ}$ is $a y^{2}=b x-1$, then the value of $(a+b)$ is $\qquad$

## - Watch Video Solution

3. If $d$ is the minimum distance between the curves $f(x)=e^{x} \operatorname{andg}(x)=(\log )_{e} x$, then the value of $d^{6}$ is

## - Watch Video Solution

4. Let $f(x 0$ be a non-constant thrice differentiable function defined on $(-\infty, \infty)$ such that $f(x)=f(6-x) \operatorname{and} f^{\prime}(0)=0=f^{\prime}(x)^{2}=f(5)$. If $n$ is the minimum number of roots of $\left(f^{\prime}(x)^{2}+f^{\prime}(x) f^{x}=0\right.$ in the interval $[0,6]$, then the value of $\frac{n}{2}$ is

## - Watch Video Solution

5. At the point $P\left(a, a^{n}\right)$ on the graph of $y=x^{n},(n \in N)$, in the first quadrant, a normal is drawn. The normal intersects the $y-a \xi s$ at the point $(0, b)$. If (lim) $\underset{a \overrightarrow{0}}{ } b=\frac{1}{2}$, then $n$ equals 1 (b) 3 (c) 2 (d) 4

## - Watch Video Solution

6. A curve is given by the equations $x=\sec ^{2} \theta, y=\cot \theta$. If the tangent at Pwhere $\theta=\frac{\pi}{4}$ meets the curve again at $Q$, then $[P Q]$ is, where [.] represents the greatest integer function, $\qquad$ .

## - Watch Video Solution

7. Water is dropped at the rate of $2 \mathrm{~m}^{3} / \mathrm{s}$ into a cone of semi-vertical angle is $45^{\circ}$. If the rate at which periphery of water surface changes when the height of the water in the cone is 2 m is d . Then the value of 5 d is _.__ $\mathrm{m} / \mathrm{sec}$

## - Watch Video Solution

8. If the slope of line through the origin which is tangent to the curve $y=x^{3}+x+16$ is $m$, then the value of $m-4$ is $\qquad$ .

## - Watch Video Solution

9. Let $y=f(x)$ be drawn with $f(0)=2$ and for each real number $a$ the line tangent to $y=f(x)$ at ( $a, f(a)$ ) has $x$-intercept ( $a-2$ ). If $f(x)$ is of the form of $k e^{p x}$ then $\frac{k}{p}$ has the value equal to

## - Watch Video Solution

10. Suppose $a, b, c$ are such that the curve $y=a x^{2}+b x+c$ is tangent to $y=3 x-3 a t(1,0)$ and is also tangent to $y=x+1 a t(3,4)$. Then the value of $(2 a-b-4 c)$ equals $\qquad$

## - Watch Video Solution

11. Let $C$ be a curve defined by $y=e^{a+b x^{2}}$. The curve $C$ passes through the point $P(1,1)$ and the slope of the tangent at $P$ is $(-2)$. Then the value of $2 a-3 b$ is $\qquad$ .

## - Watch Video Solution

12. If the curve $C$ in the $x y$ plane has the equation $x^{2}+x y+y^{2}=1$, then the fourth power of the greatest distance of a point on $C$ from the origin is $\qquad$ .

## - Watch Video Solution

13. If $a, b$ are two real numbers with $\mathrm{a}<\mathrm{b}$ then a real number $c$ can be found between $a$ and $b$ such that the value of $\frac{a^{2}+a b+b^{2}}{c^{2}} i s_{---}$

## - Watch Video Solution

14. Let $f:[1,3] \rightarrow[0, \infty)$ be continuous and differentiabl function. If $(f(3)-f(1))\left(f^{2}(3)+f^{2}(1)+f(3) f(1)\right)=k f^{2}(c) f^{\prime}(c)$ wherec $\in(1,3)$, then the value of $k$ is $\qquad$

## - Watch Video Solution

15. The x intercept of the tangent to a curve $\mathrm{f}(\mathrm{x}, \mathrm{y})=0$ is equal to the ordinate of the point of contact. Then the value of $\frac{d^{2} x}{d y^{2}}$ at the point $(1,1)$ on the curve is $\qquad$ .

## - Watch Video Solution

16. if $f(x)$ is differentiable function such that $f(1)=\sin 1, f(2)=\sin 4$ and $f(3)$ $=\sin 9$, then the minimum number of distinct roots of $f^{\prime}(x)=2 x \cos x^{2}$ in $(1,3)$ is $\qquad$

## - Watch Video Solution

17. Let $f(x)=x\left(x^{2}+m x+n\right)+2, \quad$ for all $x \neq R$ and $m, n \in R$. If Rolle's theorem holds for $f(x) a t x=4 / 3 x \in[1,2], \quad$ then $(m+n)$ equal $\qquad$ .

## - Watch Video Solution

18. If length of the perpendicular from the origin upon the tangent drawn to the curve $x^{2}-x y+y^{2}+\alpha(x-2)=4$ at $(2,2)$ is equal to 2 then $\alpha$ equals

## - Watch Video Solution

19. If $\mathrm{f}(\mathrm{x})=\left\{\begin{array}{ll}x \log _{e} x, & x>0 \\ 0, & x=0\end{array}\right.$,thenconclusion of LMVT holds at $x=1$ in the interval $[0, a]$ for $f(x)$, then $\left[a^{2}\right]$ is equal to (where [.] denotes the greatest integer) $\qquad$ .

## - Watch Video Solution

## Jee Previous Year

1. about to only mathematics
A. $\frac{3 \sqrt{2}}{8}$
B. $\frac{2 \sqrt{3}}{8}$
C. $\frac{3 \sqrt{2}}{5}$
D. $\frac{\sqrt{3}}{4}$

## Answer: A

2. The equation of the tangent to the curve $y=x+\frac{4}{x^{2}}$ that is parallel to the $X$-axis, is
A. $y=8$
B. $y=0$
C. $y=3$
D. $y=2$

## Answer: C

## - Watch Video Solution

3. Consider the function $f(x)=|x-2|+|x-5|, c \in R$.

Statement 1: $f^{\prime}(4)=0$
Statement 2: $f$ is continuous in $[2,5]$, differentiable in $(2,5)$,
A. Statement 1 is false, statement 2 is true.
B. Statement 1 is true, Statement 2 is true, statement 2 is correct explanation for Statement 1.
C. Statement 1 is true, Statement 2 is trur, Statement 2 is no a correct explanation for statement 1.
D. Statement 1 is true, Statement 2 is false.

## Answer: C

## - Watch Video Solution

4. If $f$ and $g$ are differentiable functions in $[0,1]$ satisfying

$$
\begin{equation*}
f(0)=2=g(1), g(0)=0 \text { and } f(1)=6, \text { then for some } c \in] 0,1[ \tag{1}
\end{equation*}
$$

$$
\begin{align*}
& 2 f^{\prime}(c)=g^{\prime}(c)  \tag{4}\\
& f^{\prime}(c)=2 g^{\prime}(c)
\end{align*}
$$

$$
\text { (2) } 2 f^{\prime}(c)=3 g^{\prime}(c)
$$

$$
\text { (3) } \quad f^{\prime}(c)=g^{\prime}(c)
$$

A. $\left.2 f^{\prime}(c)=g^{\prime}(c)\right)^{\prime}$
B. $2 f^{\prime}(c)=3 g^{\prime}(c)^{\prime}$
C. $f^{\prime}(c)=g^{\prime}(c)^{\prime}$
D. $f^{\prime}(c)=2 g^{\prime}(c)^{\prime}$

## Answer: D

## - Watch Video Solution

5. The normal to the curve $x^{2}+2 x y-3 y^{2}=0$, at (1,1)
A. does not meet the curve again.
B. meets the curve again in the second quadrant.
C. meets the curve again in the third quadrant.
D. meets the curve again in the fourth quadrant.

## Answer: D

6. Consider $f(x)=\tan ^{-1}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right), x \in\left(0, \frac{\pi}{2}\right)$. A normal to $y=f(x)$ at $x=\frac{\pi}{6}$ also passes through the point: (1) (0, 0) (2) $\left(0, \frac{2 \pi}{3}\right)$ (3) $\left(\frac{\pi}{6}, 0\right)(4)\left(\frac{\pi}{4}, 0\right)$
A. $\left(0, \frac{2 \pi}{3}\right)$
B. $\left(\frac{\pi}{6} 0,\right)$
C. $\left(\frac{\pi}{4}, 0\right)$
D. $(0,0)$

## Answer: A

## - Watch Video Solution

7. The normal to the curve $y(x-2)(x-3)=x+6$ at the point where the curve intersects the $y-a \xi s$, passes through the point :
$\left(\frac{1}{2},-\frac{1}{3}\right)$
(2) $\left(\frac{1}{2}, \frac{1}{3}\right)$
(3) $\left(-\frac{1}{2},-\frac{1}{2}\right)$
(4) $\left(\frac{1}{2}, \frac{1}{2}\right)$
A. $\left(\frac{1}{2}, \frac{1}{3}\right)$
B. $\left(-\frac{1}{2},-\frac{1}{2}\right)$
C. $\left(\frac{1}{2}, \frac{1}{2}\right)$
D. $\left(\frac{1}{2}, \frac{1}{3}\right)$

## Answer: C

## - Watch Video Solution

8. If the curves $y^{2}=6 x, 9 x^{2}+b y^{2}=16$ intersect each other at right angles then the value of $b$ is: (1) 6 (2) $\frac{7}{2}$ (3) 4 (4) $\frac{9}{2}$
A. $9 / 2$
B. 6
C. $7 / 2$
D. 4
9. Let $f, g:[-1,2] \vec{R}$ be continuous functions which are twice differentiable on the interval ( $-1,2$ ). Let the values of fandg at the points $-1,0$ and 2 be as given in the following table: , $x=-1, x=0$, $x=2 f(x), 3,6,0 g(x), 0,1,-1 \ln$ each of the intervals $(-1,0) \operatorname{and}(0,2)$ the function $(f-3 g)^{\prime \prime}$ never vanishes. Then the correct statement(s) is (are) $f^{\prime}(x)-3 g^{\prime}(x)=0$ has exactly three solutions in $(-1,0) \cup(0,2) \cdot f^{\prime}(x)-3 g^{\prime}(x)=0$ has exactly one solutions in $(-1,0) \cdot f^{\prime}(x)-3 g^{\prime}(x)=0$ has exactly one solutions in $(-1,2) \cdot f^{\prime}(x)-3 g^{\prime}(x)=0$ has exactly two solutions in $(-1,0)$ and exactly two solutions in $(0,2)$.
A. $f^{\prime}(x)-3 g^{\prime}(x)=0$ has exactly three solution in

$$
(-1,0) \cup(0,2)
$$

B. $f^{\prime}(x)-3 g^{\prime}(x)=0$ has exactly one solution in ( $-1,0$ )
C. $f^{\prime}(x)-3 g^{\prime}(x)=0$ has exactly one solution in ( 0,2 )
D. $f^{\prime}(x)-3 g^{\prime}(x)=0$ has excatly two solutions in $(-1,0)$ and exactly two solution in (0,2)

## Answer: B::C

## - Watch Video Solution

10. For every twice differentiable function $f: R \rightarrow[-2,2]$ with $(f(0))^{2}+\left(f^{\prime}(0)\right)^{2}=85$, which of the following statement(s) is (are) TRUE? There exist $r, s \in R$ where 'roo $) \mathrm{f}(\mathrm{x})=1(d)$ There $\exists$ alpha in $(-4, \backslash 4)$ sucht ${ }^{\wedge} \mathrm{f}(\mathrm{alpha})+\mathrm{f}$ "(alpha)=0 and $\mathrm{f}^{\wedge}($ prime $)\left(\right.$ alpha) $!=0^{`}$
A. There exist $\mathrm{r}, \mathrm{s} \in R$, where $r<s$, such that f is one-one on the open interval (r,s)
B. There exist $x_{0} \in(-4,0)$ such that $\left|f^{\prime}\left(x_{0}\right)\right| \leq 1$
C. $\lim _{x \rightarrow \infty} f(x)=1$
D. There exists $\alpha \in(-4,4)$ such that $f(\alpha)+f^{\prime \prime}(\alpha)=0$ and $f^{\prime}(\alpha) \neq 0$

## - Watch Video Solution

## Illustration

1. Find the total number of parallel tangents of $f_{1}(x)=x^{2}-x+1 \operatorname{and} f_{2}(x)=x^{3}-x^{2}-2 x+1$.

## - Watch Video Solution

2. Prove that the tangent drawn at any point to the curve $f(x)=x^{5}+3 x^{3}+4 x+8$ would make an acute angle with the $x$-axis.

## ( Watch Video Solution

3. Find the equation of tangent to the curve $y=\frac{\sin ^{-1}(2 x)}{1+x^{2}}$ atx $=\sqrt{3}$
4. The equation of the tangent tothe curve $y=\left\{x^{2} \sin \left(\frac{1}{x}\right), x \neq 0\right.$ and $0, x=0$ at the origin is

## - Watch Video Solution

5. Find the equation of normal line to the curve $y=x^{3}+2 x+6$ which is parallel to the line $x+14 y+4=0$.

## - Watch Video Solution

6. If the equation of the tangent to the curve $y^{2}=a x^{3}+b$ at point $(2,3) i s y=4 x-5$, then find the values of $a a n d b$.

## - Watch Video Solution

7. For the curve $y=4 x^{3}-2 x^{5}$, find all the points at which the tangents pass through the origin.

## - Watch Video Solution

8. Find the equation of tangent to the curve reprsented parametrically by the equations $x=t^{2}+3 t+1$ and $y=2 t^{2}+3 t-4$ at the point $M$ $(-1,10)$.

## - Watch Video Solution

9. For the curve $x y=c$, prove that the portion of the tangent intercepted between the coordinate axes is bisected at the point of contact.

## - Watch Video Solution

10. If the tangent at any point $\left(4 m^{2}, 8 m^{2}\right)$ of $x^{3}-y^{2}=0$ is a normal to the curve $x^{3}-y^{2}=0$, then find the value of $m$.

## - Watch Video Solution

11. Find all the tangents to the curve $y=\cos (x+y),-2 \pi \leq x \leq 2 \pi$ that are parallel to the line $x+2 y=0$.

## - Watch Video Solution

12. Find the equation of all possible normals to the parabola $x^{2}=4 y$ drawn from the point $(1,2)$.

## - Watch Video Solution

13. Find the equations of the tangents drawn to the curve $y^{2}-2 x^{3}-4 y+8=0$ from the point $(1,2)$.
14. From the point ( 1,1 ) tangents are drawn to the curve represented parametrically as $x=2 t-t^{2}$ and $y=t+t^{2}$. Find the possible points of contact.

## - Watch Video Solution

15. Show that the straight line $x \cos \alpha=p$ touches the curve $x y=a^{2}$, if $p^{2}=4 a^{2} \cos \alpha \sin \alpha$.

## - Watch Video Solution

16. Find the condition that the line $A x+B y=1$ may be normal to the curve $a^{n-1} y=x^{n}$.

Watch Video Solution
17. Find the acute angle between the curves $y=|x \hat{2}-1|$ and $y=\left|x^{2}-3\right|$ at their points of intersection.

## - Watch Video Solution

18. Find the angle between the curves $2 y^{2}=x^{3} a n d y^{2}=32 x$.

## - Watch Video Solution

19. Find the cosine of the angle of intersection of curves $f(x)=2^{x}(\log )_{e} \operatorname{xandg}(x)=x^{2 x}-1$.

## - Watch Video Solution

20. Find the angle between the curves $y^{2}=4 x$ and $y=e^{-x / 2}$.

## - Watch Video Solution

21. Find the value of $a$ if the curves $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{4}=1 a n d y^{3}=16 x$ cut orthogonally.

## - Watch Video Solution

22. Find the length of sub-tangent to the curve $y=e^{x / a}$

## - Watch Video Solution

23. Determine $p$ such that the length of the such-tangent and sub-normal is equal for the curve $y=e^{p x}+p x$ at the point $(0,1)$.

## - Watch Video Solution

24. Find the length of normal to the curve $x=a(\theta+\sin \theta), y=a(1-\cos \theta)$ at $\theta=\frac{\pi}{2}$.
25. In the curve $x^{m+n}=a^{m-n} y^{2 n}$, prove that the $m t h$ power of the sub-tangent varies as the $n t h$ power of the sub-normal.

## - Watch Video Solution

26. Find the possible values of $p$ such that the equation $p x^{2}=(\log )_{e} x$ has exactly one solution.

## - Watch Video Solution

27. Find the shortest distance between the line $y=x-2$ and the parabola $y=x^{2}+3 x+2$

## - Watch Video Solution

28. about to only mathematics
29. Prove that points of the curve $y^{2}=4 a\left\{x+a \sin \left(\frac{x}{a}\right)\right\}$ at which tangents are parallel to $x$-axis lie on the parabola.

## - Watch Video Solution

30. The tangent at any point on the curve $x=a \cos ^{3} \theta, y=a \sin ^{3} \theta$ meets the axes in $\operatorname{PandQ}$. Prove that the locus of the midpoint of $P Q$ is a circle.

## - Watch Video Solution

31. Displacement $s$ of a particle at time $t$ is expressed as $s=\frac{1}{2} t^{3}-6 t$.

Find the acceleration at the time when the vecity vanishes (i.e., velocity tends to zero).

## - Watch Video Solution

32. On the curve $x^{3}=12 y$, find the interval of values of $x$ for which the abscissa changes at a faster rate than the ordinate?

## - Watch Video Solution

33. एक आयत की लम्बाई $\mathrm{x}, 5 \mathrm{~cm} / \mathrm{min}$ की दर से घट रही है और चौड़ाई $\mathrm{y}, 4 \mathrm{~cm} / \mathrm{min}$ कि दर से बढ़ रही है जब $\mathrm{x}=8 \mathrm{~cm}$ और $\mathrm{y}=6 \mathrm{~cm}$ है तब आयत के (a) परिमाप (b) क्षेत्रफल की परिवर्तन की दर ज्ञात कीजिए

## - Watch Video Solution

34. Let $x$ be the length of one of the equal sides of an isosceles triangle, and let $\theta$ be the angle between them. If $x$ is increasing at the rate ( $1 / 12$ ) $\mathrm{m} / \mathrm{h}$, and $\theta$ is increasing at the rate of $\frac{\pi}{180}$ radius $/ \mathrm{h}$, then find the rate in $m^{3} / h$ at which the area of the triangle is increasing when $x=12$ mandth $\eta=\pi / 4$.

## - Watch Video Solution

35. A lamp is $50 f t$. above the ground. A ball is dropped from the same height from a point 30 ft . away from the light pole. If ball falls a distance $s=16 t^{2} f t$. in $t$ second, then how fast is the shadow of the ball moving along the ground $\frac{1}{2} s$ later?

## - Watch Video Solution

36. If water is poured into an inverted hollow cone whose semi-vertical angel is $30^{0}$, show that its depth (measured along the axis) increases at the rate of $1 \mathrm{~cm} / \mathrm{s}$. Find the rate at which the volume of water increases when the depth is 24 cm .

## - Watch Video Solution

37. A horse runs along a circle with a speed of $20 \mathrm{~km} / \mathrm{h}$. A lantern is at the centre of the circle. A fence is along the tangent to the circle at the point at which the horse starts. Find the speed with which the shadow of
the horse moves along the fence at the moment when it covers $1 / 8$ of the circle in $\mathrm{km} / \mathrm{h}$.

## - Watch Video Solution

38. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground,away from the wall at the rate of $10 \mathrm{~cm} / \mathrm{s}$. How fast is the angle between the ladder and the ground decreasing when the foot of the ladder is 2 m away from the wall?

## - Watch Video Solution

39. The radius of the base of a cone is increasing at the rate of $3 \mathrm{~cm} / \mathrm{min}$ and the altitude is decreasing at the rate of $4 \mathrm{~cm} / \mathrm{min}$. The rate of change of lateral surface when the radius is 7 cm and altitude is 24 cm is $108 \pi \mathrm{~cm}^{2} / \mathrm{min}$ (b) $7 \pi \mathrm{~cm}^{2} / \mathrm{min} 27 \pi \mathrm{~cm}^{2} / \mathrm{min}$ (d) none of these
40. Use differential to approximate $\sqrt{36.6}$

## - Watch Video Solution

41. Find the approximate value of $\sin 3$.

## - Watch Video Solution

42. Find the approximate volume of metal in a hollow spherical shell whose internal and external radii are 3 cm and 3.0005 cm , respectively.

## - Watch Video Solution

43. Find the approximate value of $f(5.001)$, where $f(x)=x^{3}-7 x^{2}+15$.

## - Watch Video Solution

44. Find the approximate change in the volume $V$ of a cube of side $x$ meters caused by increasing side by $1 \%$.

## - Watch Video Solution

45. Discuss the applicability of Rolles theorem for the following functions on $\quad$ the indicated $\quad$ intervals: $\quad f(x)=|x| \in[-1,1]$
$f(x)=3+(x-2)^{2 / 3} \quad$ in $\quad[1,3] \quad f(x)=\tan \xi n[0, \pi]$
$f(x)=\log \left\{\frac{x^{2}+a b}{x(a+b)}\right\}$ in `[a, b],w h e r e-

## - Watch Video Solution

46. If the function $f(x)=x^{3}-6 x^{2}+a x+b$ defined on $[1,3]$ satisfies Rolles theorem for $c=\frac{2 \sqrt{3}+1}{\sqrt{3}}$ then find the value of $a a n d b$

## - Watch Video Solution

47. Show that between any two roots of $e^{-x}-\cos x=0$, there exists at least one root of $\sin x-e^{-x}=0$

## - Watch Video Solution

48. How many roots of the equation $(x-1)(x-2)(x-3)+$ $(x-1)(x-2)(x-4)+(x-2)(x-3)(x-4)$
$(x-1)(x-3)(x-4)=0$ are positive?

## - Watch Video Solution

49. If $2 \mathrm{a}+3 \mathrm{~b}+6 \mathrm{c}=0$, then show that the equation $a x^{2}+b x+c=0$ has atleast one real root between 0 to 1 .

## - Watch Video Solution

50. Let $f(x)$ be differentiable function and $g(x)$ be twice differentiable function. Zeros of $f(x), g^{\prime}(x)$ be a,b respectively, '(a

## - Watch Video Solution

51. Let $f(x)$ be differentiable function and $g(x)$ be twice differentiable function. Zeros of $f(x), g^{\prime}(x)$ be a,b respectively, `(a

## - Watch Video Solution

52. Let $P(x)$ be a polynomial with real coefficients, Let $a, b \in R, a<b$, be two consecutive roots of $P(x)$. Show that there exists $c$ such that $a \leq c \leq b$ and $P^{\prime}(c)+100 P(c)=0$.

## - Watch Video Solution

53. The fucntion $f(x)=x^{3}-6 a x^{2}+5 x$ satisfies the conditions of Lagrange's mean value theorem over the interval $[1,2]$ and the value of $c$ (that of LMVT) is $\frac{7}{4}$, then find the value of a.

## - Watch Video Solution

54. If $f:[5,5] R$ is a differentiable function and if $f^{\prime}(x)$ does not vanish anywhere, then prove that $f(5) f(5)$.

## - Watch Video Solution

55. Let $f$ be differentiable for all $x$, If $f(1)=-2 a n d f^{\prime}(x) \geq 2$ for all $x \in[1,6]$, then find the range of values of $f(b)$.

## - Watch Video Solution

56. Let $f:[2,7] \overrightarrow{0, \infty}$ be a continuous and differentiable function. Then show that $(f(7)-f(2)) \frac{(f(7))^{2}+(f(2))^{2}+f(2) f(7)}{3}=5 f^{2}(c) f^{\prime}(c)$, where $c \in[2,7]$.

## - Watch Video Solution

57. Let $f(x) \operatorname{and} g(x)$ be differentiable function in $(a, b)$, continuous at $\operatorname{aandb}, \operatorname{and} g(x) \neq 0 \quad$ in $[a, b]$. Then prove that $\frac{g(a) f(b)-f(a) g(b)}{g(c) f^{\prime}(c)-f(c) g^{\prime}(c)}=\frac{(b-a) g(a) g(b)}{(g(c))^{2}}$

## - Watch Video Solution

58. Using Lagranges mean value theorem, prove that $|\cos a-\cos b|<|a-b|$.

## - Watch Video Solution

59. Using lagrange's mean value theorem, show that $\frac{\beta-\alpha}{1+\beta^{2}}<\tan ^{-1} \beta-\tan ^{-1} \alpha<\frac{\beta-\alpha}{1+\alpha^{2}}, \beta>\alpha>0$.

## ( Watch Video Solution

60. Prove that $\frac{1}{28}<(28)^{1 / 3}-3<\frac{1}{27}$

## - Watch Video Solution

61. Let $f(x) \operatorname{andg}(x)$ be two differentiable functions in $\operatorname{Randf}(2)=8, g(2)=0, f(4)=10, \operatorname{and} g(4)=8$. Then prove that $g^{\prime}(x)=4 f^{\prime}(x)$ for at least one $x \in(2,4)$.

## - Watch Video Solution

62. Let $f$ be continuous on $[a, b], a>0$, and differentiable on $(a, b)$. Prove that there exists $c \in(a, b)$ such that
$\frac{b f(a)-a f(b)}{b-a}=f(c)-c f^{\prime}(c)$

## - Watch Video Solution

## Solved Examples

1. Prove that the equation of the normal to $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$ is $y \cos \theta-x \sin \theta=a \cos 2 \theta$, where $\theta$ is the angle which the normal makes with the axis of $x$.

## - Watch Video Solution

2. If the area of the triangle included between the axes and any tangent to the curve $x^{n} y=a^{n}$ is constant, then find the value of $n$.

## - Watch Video Solution

3. Show that the segment of the tangent to the curve $y=\frac{a}{2} \operatorname{In}\left(\frac{a+\sqrt{a^{2}-x^{2}}}{a-\sqrt{a^{2}-x^{2}}}\right)-\sqrt{a^{2}-x^{2}}$ contained between the $\mathrm{y}=\mathrm{axis}$ and the point of tangency has a constant length.

## - Watch Video Solution

4. If the tangent at $\left(x_{1}, y_{1}\right)$ to the curve $x^{3}+y^{3}=a^{3}$ meets the curve again in $\left(x_{2}, y_{2}\right)$, then prove that $\frac{x_{2}}{x_{1}}+\frac{y_{2}}{y_{1}}=-1$

## - Watch Video Solution

5. Find the condition for the line $y=m x$ to cut at right angles the conic $a x^{2}+2 h x y+b y^{2}=1$.

## - Watch Video Solution

6. If two curves $a x^{2}+b y^{2}=1$ and $a^{\prime} x^{2}+b^{\prime} y^{2}=1$ intersect orthogonally,then show that $\frac{1}{a}-\frac{1}{b}=\frac{1}{a^{\prime}}-\frac{1}{b^{\prime}}$

## - Watch Video Solution

7. A man is moving away from a tower 41.6 m high at the rate of $2 \mathrm{~m} / \mathrm{sec}$.

Find the rate at which the angle of elevation of the top of tower is changing, when he is at a distance of 30 m from the foot of the tower.

Assume that the eye level of the man is 1.6 m from the ground.

## - Watch Video Solution

8. If $f$ is continuous and differentiable function and $f(0)=1, f(1)=2$, then prove that there exists at least one $c \in[0,1] f$ or which $f^{\prime}(c)(f(c))^{n-1}>\sqrt{2^{n-1}}$, where $n \in N$.

## - Watch Video Solution

9. Let $a, b, c$ be three real numbers such that $a<b<c . f(x)$ is continuous in $[a, c]$ and differentiable in ( $a, c$ ) Also, $f^{\prime}(x)$ is strictly increasing in $\quad(a, c)$ Prove that $(b-c) f(a)+(c-a) f(b)+(a-b) f(c)<0$.

## - Watch Video Solution

10. Use the mean value theorem to prove $e^{x} \geq 1+x \forall x \in R$

## - Watch Video Solution

11. Show that the square roots of two successive natural numbers greater than $N^{2}$ differ by less than $\frac{1}{2 N}$.

## - Watch Video Solution

12. Using Rolles theorem, prove that there is at least one root in $\left(45^{\frac{1}{100}}, 46\right)$ the equation.
$P(x)=51 x^{101}-2323(x)^{100}-45 x+1035=0$.

## - Watch Video Solution

13. Let $f(x 0$ be a non-constant thrice differentiable function defined on $(-\infty, \infty)$ such that $f(x)=f(6-x) \operatorname{and} f^{\prime}(0)=0=f^{\prime}(x)^{2}=f(5)$. If $n$ is the minimum number of roots of $\left(f^{\prime}(x)^{2}+f^{\prime}(x) f^{x}=0\right.$ in the interval $[0,6]$, then the value of $\frac{n}{2}$ is

## - Watch Video Solution

14. If $\mathrm{f}^{\prime \prime}(\mathrm{x})$ exists for all points in $[a, b]$ and
$\frac{f(c)-f(a)}{c-a}=\frac{f(b)-f(c)}{b-c}$, where $a<c<b$, then show that there exists a number ' $k$ ' such that $f^{\prime \prime}(k)=0$.

## - Watch Video Solution

15. Let $f$ defined on $[0,1]$ be twice differentiable such that $|f(x)| \leq 1$ for $x \in[0,1]$. if $f(0)=f(1)$ then show that $\mid f^{\prime}(x)<1$ for all $x \in[0,1]$

## - Watch Video Solution

## Matrix Match Type

1. Match the following lists:

| List I | List II |
| :--- | :--- |
| a. A circular plate is expanded by heat from <br> radius 6 cm to 6.06 cm . Approximate <br> increase in the area is | p. 5 |
| b. If an edge of a cube increases by $2 \%$, then <br> the percentage increase in the volume is | q. $0.72 \pi$ |

c. If the rate of decrease of $\frac{x^{2}}{2}-2 x+5$ is thrice the rate of decrease of $x$, then $x$ is equal to (rate of decrease is nonzero)
d. The rate of increase in the area of an equailateral triangle of side 30 cm , when each side increases at the rate of $0.1 \mathrm{~cm} / \mathrm{s}$, is
r. 6
s. $\frac{3 \sqrt{3}}{2}$
2. Match the following lists:

| List I: <br> Curves | List II: <br> Angle between the curves |
| :--- | :--- |
| a. $y^{2}=4 x$ and $x^{2}=4 y$ | p. $90^{\circ}$ |
| b. $2 y^{2}=x^{3}$ and $y^{2}=32 x$ | q. Any one of $\tan ^{-1} \frac{3}{4}$ or <br> $\tan ^{-1}\left(16^{1 / 3}\right)$ |
| c. $x y=a^{2}$ and $x^{2}+y^{2}=2 a^{2}$ | r. $0^{\circ}$ |
| d. $y^{2}=x$ and $x^{3}+y^{3}=3 x y$ at <br> other than origin | s. $\tan ^{-1} \frac{1}{2}$ |

## - Watch Video Solution

3. The curve $y=a x^{2}+b x^{2}+c x+5$ touches the $x$-axis at $P(-2,0)$ and cuts the $y$-axis at point $Q$, where its gradient is 3 . Now, match the
following lists and then choose the correct code.

| List I | List II |
| :--- | :--- |
| a. The value of $a$ is | p. 3 |
| b. The value of $b$ is | q. 0 |
| c. The value of $c$ is | r. $-\frac{3}{4}$ |
| d. The value of $y^{\prime}(1)$ is | s. $-\frac{1}{2}$ |

A. $\begin{array}{llll}a & b & c & d \\ s & r & q & p\end{array}$
$\begin{array}{llll}a & b & c & d\end{array}$
B.
$q \quad s \quad r \quad p$
C. $\begin{array}{llll}a & b & c & d\end{array}$
$\begin{array}{llll}s & r & q & p\end{array}$
D. $\begin{array}{llll}a & b & c & d\end{array}$
$s \quad p q r$

## Answer: C

