



MATHS

BOOKS - CENGAGE MATHS (ENGLISH)

APPLICATION OF DERIVATIVES

Concept Application Exercise 51

1. Find the equation of the tangent to the curve $ig(1+x^2ig)y=2-x,$

where it crosses the x-axis.

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2. Show that the tangent to the curve $3xy^2 - 2x^2y = 1at(1,1)$ meets the curve again at the point $\left(-\frac{16}{5}, -\frac{1}{20}\right)$.

3. Find the equation of tangent and normal to the curve
$$x = \frac{2at^2}{(1+t^2)}, y = \frac{2at^3}{(1+t^2)}$$
 at the point for which $t = \frac{1}{2}$.
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4. Find the normal to the curve $x = a(1 + \cos \theta), y = a \sin \theta a \mathbf{h} \eta$. Prove

that it always passes through a fixed point and find that fixed point.

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5. Find the equation of the normal to the following curve 1) $y = x^3 + 2x + 6$ which are parallel to the line x + 14y + 4 = 0.2) $x^3 + y^3 = 8xy$ at the point where it meets the curve $y^2 = 4x$ other than the origin.

6. If the curve $y = ax^2 - 6x + b$ pass through (0, 2) and has its tangent parallel to the x-axis at $x = \frac{3}{2}$, then find the values of aandb.

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7. Find the value of $n \in N$ such that the curve $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$ touches the straight line $\frac{x}{a} + \frac{y}{b} = 2$ at the point (a, b).

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8. If the tangent to the curve xy + ax + by = 0 at (1, 1) is inclined at an

angle $\tan^{-1} 2$ with x-axis, then find aandb?

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9. Does there exists line/lines which is/are tangent to the curve y=sinx at

 (x_1,y_1) and normal to the curve at (x_2,y_2) ?

10. Find the condition that the line Ax+By=1 may be normal to the curve $a^{n-1}y=x^n$.

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11. In the curve $x^a y^b = K^{a+b}$, prove that the potion of the tangent intercepted between the coordinate axes is divided at its points of contact into segments which are in a constant ratio. (All the constants being positive).

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Concept Application Exercise 5 2

1. Find the angle of intersection of $y = a^x andy = b^x$



6. If the curves ay $+x^2 = 7 \, ext{ and } y = x^3$ cut orthogonally at (1,1) , then

the value of a is

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Concept Application Exercise 5 3

1. Find the length of the tangent for the curve $y = x^3 + 3x^2 + 4x - 1$ at



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2. For the curve $y = a 1n (x^2 - a^2)$, show that the sum of length of tangent and sub-tangent at any point is proportional to product of coordinates of point of tangency.

3. For a curve (length of normal)²/(length of tangent)² is equal to



2. Find the point on the curve $3x^2 - 4y^2 = 72$ which is nearest to the

line 3x + 2y + 1 = 0.

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3. Find the possible values of 'a' such that the inequality $3-x^2>|x-a|$ has atleast one negative solution

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4. Tangents are drawn from the origin to curve $y = \sin x$. Prove that

points of contact lie on $y^2=rac{x^2}{1+x^2}$

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5. Find the distance of the point on $y=x^4+3x^2+2x$ which is nearest

to the line y = 2x - 1

6. The graph $y = 2x^3 - 4x + 2andy = x^3 + 2x - 1$ intersect in exactly

3 distinct points. Then find the slope of the line passing through two of these points.

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Concept Application Exercise 5 5

1. The distance covered by a particle moving in a straight line from a fixed point on the line is s, where $s^2 = at^2 + 2bt + c$. Then prove that acceleration is proportional to s^{-3} .

2. Two cyclists start from the junction of two perpendicular roads, there velocities being $3um/m \in$ and $4um/m \in$, respectively. Find the rate at which the two cyclists separate.



3. A sphere of 10cm radius has a uniform thickness of ice around it. Ice is melting at rate $50cm^3 / \min$ when thickness is 5cm then rate of change of thickness

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4. xandy are the sides of two squares such that $y = x - x^2$. Find the rate of the change of the area of the second square with respect to the first square.



5. Two men PandQ start with velocity u at the same time from the junction of two roads inclined at 45^0 to each other. If they travel by different roads, find the rate at which they are being separated.



6. Sand is pouring from a pipe at the rate of $12cm^3/s$. The falling sand forms a cone on the ground in such a way that the height of the cone is always 1/6th of the radius of the base. How fast does the height of the sand cone increase when the height in 4 cm?

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7. A swimming pool is to be drained by cleaning. If L represents the number of litres of water in the pool t seconds after the pool has been plugged off to drain and $L = 2000(10 - t)^2$. How fast is the water ruining out at the end of 5 seconds? What is the average rate at which the water flows out during the first 5 seconds?



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Concept Application Exercise 5 6

1. Find the approximate value of $(26)^{\frac{1}{3}}$.

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2. Find the approximate value of $(1.999)^6$.





5. If the radius of a sphere is measured as 9cm with an error of 0.03 cm,

then find the approximate error in calculating its volume.

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Concept Application Exercise 5 7

1. Let
$$0 < a < b < rac{\pi}{2}$$
. $Iff(x) = egin{bmatrix} \tan x & \tan a & \tan b \\ \sin x & \sin a & \sin b \\ \cos x & \cos a & \cos b \end{bmatrix}$, then find the

minimum possible number of roots of f'(x) = 0 in (a,b).



2. Find the condition if the equation $3x^2 + 4ax + b = 0$ has at least one root in (0, 1).

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3. Let f(x)andg(x) be differentiable for $0 \le x \le 2$ such that f(0) = 2, g(0) = 1, andf(2) = 8. Let there exist a real number c in [0, 2] such that f'(c) = 3g'(c). Then find the value of g(2).

4. Prove that if 2a02 < 15a, all roots of $x^5-a_0x^4+3ax^3+bx^2+cx+d=0$ cannot be real. It is given that $a_0,a,b,c,d\in R$.

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5. Let f(x)be continuous on [a,b], differentiable in (a,b) and $f(x) \neq 0$ for all $x \in [a, b]$. Then prove that there exists one $c \in (a, b)$ such that $\frac{f'(c)}{f(c)} = \frac{1}{a-c} + \frac{1}{b-c}$.

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6. Let f and g be function continuous in [a, b]and differentiable on [a, b].If f(a) = f(b) = 0 then show that there is a point $c \in (a, b)$ such that g'(c)f(c) + f'(c) = 0. 7. If $\phi(x)$ is a differentiable function $\forall x \in R$ and $a \in R^+$ such that $\phi(0) = \phi(2a), \phi(a) = \phi(3a)$ and $\phi(0) \neq \phi(a)$, then show that there is at least one root of equation $\phi'(x + a) = \phi'(x)in(0, 2a)$.

8. Let f is continuous on [a, b] and differentiable on $(a, b)s. t. t^2(a) - t^2(b) = a^2 - b^2$. Show that ...f(x)f'(x) = x has atleast one root in (a, b).

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Concept Application Exercise 5 8

1. Find c of Lagranges mean value theorem for the function $f(x)=3x^2+5x+7$ in the interval $[1,3]\cdot$

2. If f(x) is continuous in [a, b] and differentiable in (a,b), then prove that there exists at least one $c \in (a, b)$ such that $\frac{f'(c)}{3c^2} = \frac{f(b) - f(a)}{b^3 - a^3}$

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3. If $a, b \in R$ and a < b, then prove that there exists at least one real

number
$$c\in (a,b)$$
 such that $\displaystyle rac{b^2+a^2}{4c^2}=\displaystyle rac{c}{a+b}$

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4. If f(x)andg(x) are continuous functions in [a, b] and are differentiable in(a, b) then prove that there exists at least one $c \in (a, b)$ for which. $|f(a)f(b)g(a)g(b)|=(b-a)|f(a)f^{(prime)}(c)g(a)g^{(prime)}(c)|$,wher ea





7. If a > b > 0, with the aid of Lagranges mean value theorem, prove that `n b^(n-1)(a-b)>1.n b^(n-1)(a-b)> a^n-b^n > n a^(n-1)(a-b),if0

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8. Let f(x)andg(x) be two functions which are defined and differentiable for all $x \ge x_0$. If $f(x_0) = g(x_0)andf'(x) > g'(x)$ for all $x > x_0$, then prove that f(x) > g(x) for all $x > x_0$. 9. If f(x) is differentiate in [a,b], then prove that there exists at least one

$$c\in (a,b) ext{such that}ig(a^2-b^2ig)f'(c)=2c(f(a)-f(b)).$$

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Exercises

1. The number of tangents to the curve $x^{rac{3}{2}}+y^{rac{3}{2}}=2a^{rac{3}{2}},a>0,\,$ which are equally inclined to the axes, is 2 (b) 1 (c) 0 (d) 4

A. 2 B. 1 C. 0 D. 4

Answer: B

2. The angle made by any tangent to the curve $x = a(t + \sin t \cos t), y = (1 + \sin t)^2$ with x-axis is:

A.
$$\frac{1}{4}(\pi + 2t)$$

B.
$$\frac{1 - \sin t}{\cos t}$$

C.
$$\frac{1}{4}(2t - \pi)$$

D.
$$\frac{1 + \sin t}{\cos 2t}$$

Answer: A

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3. If m is the slope of a tangent to the curve $e^y=1+x^2,\,\,$ then (a)

|m|>1 (b) m>1 (c) $m\geq -1$ (d) $|m|\leq 1$

A. |m|>1

B. m>1C. m>-1D. $|m|\leq 1$

Answer: D

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4. If at each point of the curve $y = x^3 - ax^2 + x + 1$, the tangent is inclined at an acute angle with the positive direction of the x-axis, then (a)a > 0 (b) $a < -\sqrt{3}$ (c) $-\sqrt{3} < a < \sqrt{3}$ (d) noneof these

A. a > 0

B. $a \leq \sqrt{3}$

C. $-\sqrt{3} \leq a \leq \sqrt{3}$

D. none of these

Answer: C

5. The slope of the tangent to the curve $y = \sqrt{4 - x^2}$ at the point where the ordinate and the abscissa are equal is (a) -1 (b) 1 (c) 0 (d) none of these

- $\mathsf{A.}-1$
- B. 1
- C. 0

D. none of these

Answer: A



6. The curve given by $x+y=e^{xy}$ has a tangent parallel to the $y-a\xi s$ at the point (0,1) (b) (1,0) (c)(1,1) (d) none of these

A. (0, 1)

B. (1, 0)

C.(1,1)

D. none of these

Answer: B

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7. Find value of c such that line joining the points (0, 3) and (5, -2) becomes tangent to curve $y = rac{c}{x+1}$

A. 1

 $\mathsf{B.}-2$

C. 4

D. none of these

Answer: C

8. A differentiable function y = f(x) satisfies $f'(x) = (f(x))^2 + 5$ and f(0) = 1. Then the equation of tangent at the point where the curve crosses y-axis, is

- A. x y + 1 = 0
- B. x 2y + 1 = 0
- C. 6x y + 1 = 0
- D. x 2y 1 = 0

Answer: C

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9. The distance between the origin and the tangent to the curve $y=e^{2x}+x^2$ drawn at the point x=0 is (a) $\left(rac{1}{\sqrt{5}}
ight)$ (b) $\left(rac{2}{\sqrt{5}}
ight)$ (c)

$$\left(-\frac{1}{\sqrt{5}}\right) (d) \left(\frac{2}{\sqrt{5}}\right)$$
A. $\frac{1}{\sqrt{5}}$
B. $\frac{2}{\sqrt{5}}$
C. $\frac{-1}{\sqrt{5}}$
D. $\frac{2}{\sqrt{3}}$

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10. The point on the curve $3y = 6x - 5x^3$ the normal at Which passes through the origin, is

A. (1, 1/3)

B. (-1, -1/3)

C. (2, -28/3)

D. none of these



11. The normal to the curve $2x^2 + y^2 = 12$ at the point (2, 2) cuts the curve again at (a) $\left(-\frac{22}{9}, -\frac{2}{9}\right)$ (b) $\left(\frac{22}{9}, \frac{2}{9}\right)$ (-2, -2) (d) none of these

A.
$$\left(-\frac{22}{9}, -\frac{2}{9}\right)$$

B. $\left(\frac{22}{9}, \frac{2}{9}\right)$
C. $(-2, -2)$

D. none of these

Answer: A

12. At what point of curve
$$y = \frac{2}{3}x^3 + \frac{1}{2}x^2$$
, the tangent makes equal
angle with the axis? (a) $\left(\frac{1}{2}, \frac{5}{24}\right)and\left(-1, -\frac{1}{6}\right)$ (b)
 $\left(\frac{1}{2}, \frac{4}{9}\right)and(-1, 0)$ (c) $\left(\frac{1}{3}, \frac{1}{7}\right)and\left(-3, \frac{1}{2}\right)$ (d)
 $\left(\frac{1}{3}, \frac{4}{47}\right)and\left(-1, -\frac{1}{3}\right)$
A. $\left(\frac{1}{2}, \frac{4}{24}\right)$ and $\left(-1, -\frac{1}{6}\right)$
B. $\left(\frac{1}{2}, \frac{4}{9}\right)$ and $\left(-1, 0\right)$
C. $\left(\frac{1}{3}, \frac{1}{7}\right)$ and $\left(-3, \frac{1}{2}\right)$
D. $\left(\frac{1}{3}, \frac{4}{47}\right)$ and $\left(-1, -\frac{1}{3}\right)$

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13. The equation of tangent to the curve $y = be^{-x/a}$ at the point where

it crosses Y-axis is

A.
$$rac{x}{a} - rac{y}{b} = 1$$

 $\mathsf{B}.\,ax+by+1$

C.
$$ax - by = 1$$

D. $rac{x}{a} + rac{y}{b} = 1$

Answer: D

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14. Then angle of intersection of the normal at the point $\left(-\frac{5}{\sqrt{2}},\frac{3}{\sqrt{2}}\right)$ of the curves $x^2 - y^2 = 8$ and $9x^2 + 25y^2 = 225$ is 0 (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

A. 0

B. $\frac{\pi}{2}$ C. $\frac{\pi}{3}$

D.
$$\frac{\pi}{4}$$

Answer: B

15. A function y = f(x) has a second-order derivative f''(x) = 6(x - 1). If its graph passed through the point (2,1) and at that point tangent to the graph is y = 3x - 5, then the value of f(0) is

A. 1

B. - 1

C. 2

D. 0

Answer: B

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16. $x+y-\ln(x+y)=2x+5$ has a vertical tangent at the point (lpha,eta) then lpha+eta is equal to

 $\mathsf{A.}-1$

B. 1

C. 2

 $\mathsf{D.}-2$

Answer: B

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17. A curve is difined parametrically by $x = e^{\sqrt{t}}, y = 3t - \log_e(t^2)$, where t is a parameter. Then the equation of the tangent line drawn to the curve at t = 1 is

A.
$$y=rac{2}{e}x+1$$

B. $y=rac{2}{e}x-1$
C. $y=rac{e}{2}x+1$
D. $y=rac{e}{2}x-1$



18. If x+4y=14 is a normal to the curve $y^2=lpha x^3-eta$ at (2,3), then

the value of $\alpha + \beta$ is

A. 9

B.-5

C. 7

 $\mathsf{D.}-7$

Answer: A



19. In the curve represented parametrically by the equations $x = 2\ln \cot t + 1$ and $y = \tan t + \cot t$, (A) tangent and normal

intersect at the point (2,1).(B) normal at t= π 4 is parallel to the y-axis. (C) tangent at t= π 4 is parallel to the line y=x .(D) tangent at t= π 4 is parallel to the x-axis.

A. tangent and normal intersect at the point (2,1)

B. normal at $t = \pi/4$ is parallel to the y-axis

C. tangent at $t = \pi / 4$ is parallel to the line y = x

D. tangent at $t = \pi/4$ is parallel to the x-axis

Answer: D

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20. The abscissas of point PandQ on the curve $y = e^x + e^{-x}$ such that

tangents at PandQ make 60^0 with the x-axis are.)a) $1n\left(\frac{\sqrt{3}+\sqrt{7}}{7}\right)and1n\left(\frac{\sqrt{3}+\sqrt{5}}{2}\right)$ (b) $1n\left(\frac{\sqrt{3}+\sqrt{7}}{2}\right)$ (c) $1n\left(\frac{\sqrt{7}-\sqrt{3}}{2}\right)$ (d) $\pm 1n\left(\frac{\sqrt{3}+\sqrt{7}}{2}\right)$

A.
$$\ln\left(\frac{\sqrt{3}+\sqrt{7}}{7}\right)$$
 and $\ln\left(\frac{\sqrt{3}+\sqrt{5}}{2}\right)$
B. $\left(\frac{\sqrt{3}+\sqrt{7}}{2}\right)$
C. $\ln\left(\frac{\sqrt{7}-\sqrt{3}}{2}\right)$
D. $\pm \ln\left(\frac{\sqrt{3}+\sqrt{7}}{2}\right)$

Answer: B



21. If a variable tangent to the curve $x^2y = c^3$ makes intercepts a, bonx - andy - axes, respectively, then the value of a^2b is $27c^3$ (b) $\frac{4}{27}c^3$ (c) $\frac{27}{4}c^3$ (d) $\frac{4}{9}c^3$

A. $27c^{3}$ B. $\frac{4}{27}c^{3}$ C. $\frac{27}{4}c^{3}$ D. $\frac{4}{9}c^{3}$

Answer: C

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22. Let C be the curve $y = x^3$ (where x takes all real values). The tangent at A meets the curve again at B. If the gradient at B is K times the gradient at A, then K is equal to (a) 4 (b) 2 (c) -2 (d) $\frac{1}{4}$

A. 4

B. 2

 $\mathsf{C}.-2$

D.
$$\frac{1}{4}$$

Answer: A

23. The equation of the line tangent to the curve x siny + ysinx = π at the

point
$$\left(rac{\pi}{2},rac{\pi}{2}
ight)$$
 is
A. $3x+y=2\pi$
B. $x-y=0$
C. $2x-y=\pi/2$
D. $x+y=\pi$

Answer: D

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24. The x-intercept of the tangent at any arbitrary point of the curve $\frac{a}{x^2} + \frac{b}{y^2} = 1$ is proportional to square of the abscissa of the point of tangency square root of the abscissa of the point of tangency cube of the abscissa of the point of tangency cube root of the abscissa of the point of tangency

A. square of the abscissa of the point of tangency

B. square root of the absciss of the point of tangency

C. cube of the abscissa of the point of tangency

D. cube root of the abscissa of the point of tangency

Answer: C

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25. At any point on the curve $2x^2y^2 - x^4 = c$, the mean proportional between the abscissa and the difference between the abscissa and the sub-normal drawn to the curve at the same point is equal to (a)Ordinate (b) radius vector (c)x-intercept of tangent (d) sub-tangent

A. ordinate

B. radius vector

C. x-intercect of tangent

D. sub-tangent
Answer: A



26. Given g(x)
$$\frac{x+2}{x-1}$$
 and the line $3x + y - 10 = 0$. Then the line is

A. tangent to g(x)

B. normal to g(x)

C. chord ofg(x)

D. none of these

Answer: A



27. If the length of sub-normal is equal to the length of sub-tangent at any point (3,4) on the curve y = f(x) and the tangent at (3,4) to

y = f(x) meets the coordinate axes at AandB, then the maximum area of the triangle OAB, where O is origin, is 45/2 (b) 49/2 (c) 25/2 (d) 81/2

A. 45/2

B. 49/2

C. 25/2

D. 81/2

Answer: B

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28. The number of point in the rectangle $\{(x, y)\} - 12 \le x \le 12$ and $-3 \le y \le 3\}$ which lie on the curve $y = x + \sin x$ and at which in the tangent to the curve is parallel to the x-axis is 0 (b) 2 (c) 4 (d) 8

A. 0

B. 2

C. 4

D. 8

Answer: A

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29. Tangent of acute angle between the curves $y = |x^2 - 1|$ and $y = \sqrt{7 - x^2}$ at their points of intersection is $\frac{5\sqrt{3}}{2}$ (b) $\frac{3\sqrt{5}}{2} \cdot \frac{5\sqrt{3}}{4}$ (d) $\frac{3\sqrt{5}}{4}$ A. $\frac{5\sqrt{3}}{2}$ B. $\frac{3\sqrt{5}}{2}$ C. $\frac{5\sqrt{3}}{4}$ D. $\frac{3\sqrt{5}}{4}$

Answer: C

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30. The line tangent to the curves $y^3 - x^2y + 5y - 2x = 0$ and $x^2 - x^3y^2 + 5x + 2y = 0$ at the origin intersect at an angle θ equal to (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$ A. $\frac{\pi}{6}$ B. $\frac{\pi}{4}$ C. $\frac{\pi}{3}$ D. $\frac{\pi}{2}$

Answer: D

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31. The two curves $x=y^2, xy=a^3$ cut orthogonally at a point. Then a^2 is equal to $rac{1}{3}$ (b) 3 (c) 2 (d) $rac{1}{2}$

A. $\frac{1}{3}$

C. 2
D.
$$\frac{1}{2}$$

Answer: D

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32. The tangent to the curve $y = e^{kx}$ at a point (0,1) meets the x-axis at

(a,0) where $a \in [\,-2,\,-1], ext{ then } {\sf k} \ \in :$

A.
$$[-1/2, 0]$$

- B. [-1, -1/2]
- $\mathsf{C}.\left[0,1\right]$
- D. [1/2, 1]

Answer: D

33. The curves $4x^2 + 9y^2 = 72$ and $x^2 - y^2 = 5at(3,2)$ Then (a) touch each other (b) cut orthogonally intersect at 45^0 (d) intersect at 60^0

A. touch each other

B. cut orthogonally

C. intersect at 45°

D. intersect at 60°

Answer: B



34. The coordinates of a point on the parabola $y^2 = 8x$ whose distance from the circle $x^2 + (y+6)^2 = 1$ is minimum is (a)(2, 4) (b) (2, -4) (c) (18, -12) (d) (8, 8)

A. (2, 4)

B. (2, -4)

C.(18, -12)

D. (8, 8)

Answer: B

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35. At the point $P(a, a^n)$ on the graph of $y = x^n (n \in N)$ in the first quadrant at normal is drawn. The normal intersects the Y-axis at the point (0, b). If $\lim_{a \to 0} b = \frac{1}{2}$, then n equals

A. 1

B. 3

C. 2

D. 4

Answer: C

36. Let f be a continuous, differentiable, and bijective function. If the tangent to y = f(x)atx = a is also the normal to y = f(x)atx = b, then there exists at least one $c \in (a, b)$ such that f'(c) = 0 (b) f'(c) > 0 f'(c) < 0 (d) none of these

A. f'(c) = 0

B. f'(c) > 0

C. f'(c) < 0

D. none of these

Answer: A



37. A point on the parabola $y^2 = 18x$ at which the ordinate increases at twice the rate of the abscissa is (a) (2,6) (b) (2, -6) $\left(\frac{9}{8}, -\frac{9}{2}\right)$ (d)

$$\frac{9}{8}, \frac{9}{2}$$
A. (2, 6)
B. (2, -6)
C. $\left(\frac{9}{8}, \frac{9}{2}\right)$
D. $\left(\frac{9}{8}, \frac{9}{2}\right)$

Answer: D

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38. Find the rate of change of volume of a sphere with respect to its surface area when the radius is 2cm.

A. 1 B. 2

C. 3

D. 4

Answer: A



39. If there is an error of k % in measuring the edge of a cube, then the percent error in estimating its volume is k (b) $3k \frac{k}{3}$ (d) none of these

A. k

- B. 3k
- C. $\frac{k}{3}$

D. none of these

Answer: B



40. A lamp of negligible height is placed on the ground l_1 away from a wall. A man l_2m tall is walking at a speed of $\frac{l_1}{10}m/s$ from the lamp to

the nearest point on the wall. When he is midway between the lamp and the wall, the rate of change in the length of this shadow on the wall is $5l_0$ $2l_0$ l_0 l_0

$$-\frac{5l_2}{2}m/s \text{ (b)} - \frac{2l_2}{5}m/s - \frac{l_2}{2}m/s \text{ (d)} - \frac{l_2}{5}m/s$$

$$A. -\frac{5l_2}{2}m/s$$

$$B. -\frac{2l_2}{5}m/s$$

$$C. -\frac{l_2}{2}m/s$$

$$D. -\frac{l_2}{5}m/s$$

Answer: B

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41. The function $f(x) = x(x+3)e^{-\left(rac{1}{2}
ight)x}$ satisfies the conditions of

Rolle's theorem in (-3,0). The value of c, is

$$A. - 2$$

 $\mathsf{B.}-1$

C. 0

D. 3

Answer: A

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42. The radius of a right circular cylinder increases at the rate of 0.1 cm/min, and the height decreases at the rate of 0.2 cm/min. The rate of change of the volume of the cylinder, in $cm^2/m \in$, when the radius is 2cm and the height is 3cm is (a) -2p (b) $-\frac{8\pi}{5} - \frac{3\pi}{5}$ (d) $\frac{2\pi}{5}$

A. -2π B. $-\frac{8\pi}{5}$ C. 16/6

D. - 8/15

Answer: D



43. A cube of ice melts without changing its shape at the uniform rate of $4\frac{cm^3}{\min}$. The rate of change of the surface area of the cube, in $\frac{cm^2}{\min}$, when the volume of the cube is $125cm^3$, is (a) -4 (b) $-\frac{16}{5}$ (c) $-\frac{16}{6}$ (d) $-\frac{8}{15}$ A. -4B. -16/5C. -16/6D. -8/15

Answer: B



44. The radius of the base of a cone is increasing at the rate of 3 cm/min and the altitude is decreasing at the rate of 4 cm/min. The rate of

change of lateral surface when the radius is 7 cm and altitude is 24cm is (a) $108\pi cm^2 / \min$ (b) $7\pi cm^2 / \min$ (c) $27\pi cm^2 / \min$ (d) none of these

A. $108\pi cm^2 / \min$

B. $7\pi cm^2 / \min$

C. $27\pi cm^2 / \min$

D. none of these

Answer: A

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45. If $f(x) = x^3 + 7x - 1$, then f(x) has a zero between x = 0 and x = 1. The theorem that best describes this is (a) mean value theorem (b) maximum-minimum value theorem (c) intermediate value theorem (d) none of these

A. mena value theorem

- B. maximum-minimum value theorem
- C. intermediate value theorem
- D. none of these

Answer: C

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46. Consider the function
$$f(x) = egin{cases} x rac{\sin{(\pi)}}{x} & ext{for} x > 0 \ 0 & ext{for} x = 0 \end{cases}$$

Then, the number of points in (0, 1) where the derivative f'(x) vanishes is

A. 0

B. 1

C. 2

D. infinite

Answer: D

47. Let f(x)andg(x) be differentiable for $0 \le x \le 1$, such that f(0) = 0, g(0) = 0, f(1) = 6. Let there exists real number c in (0,1) such that f'(c) = 2g'(c). Then the value of g(1) must be (a) 1 (b) 3 (c) -2 (d) -1

A. 1

B. 3

C. -2

D. 1 –

Answer: B



48. If 3(a+2c)=4(b+3d), then the equation $ax^3+bx^2+cx+d=0$ will have (a) no real solution (b) at least one

real root in (-1,0) (c) at least one real root in (0,1) (d) none of these

A. no real solution

B. at least one real root in (-1, 0)

C. at least one real root in (0, 1)

D. none of these

Answer: B

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49. A value of c for which the conclusion of Mean value theorem holds for

the function $f(x) = \log_e x$ on the interval [1, 3] is

A.
$$\frac{1}{2}\log_e 3$$

 $B. \log_3 e$

 $C. \log_e 3$

D. $2\log_3 e$

Answer: D

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50. Let f(x) be a twice differentiable function for all real values of x and satisfies f(1) = 1, f(2) = 4, f(3) = 9. Then which of the following is (a) $f^{''}=2\,orall\,x\in(1,3)$ definitely true? (b) $f^{''}=f(x)=5f ext{ or } some x\in (2,3)$ (c) $f^{''}=3 ext{ } orall x\in (2,3)$ (d) $f^{''}=2f \,\, \mathrm{or} \,\, somex \in (1,3)$ A. $f''(x) = 2 \, \forall x \in (1,3)$ B. f''(x) = f(x)5for some $x \in (2, 3)$ C. $f''(x) = 3 \forall x \in (2,3)$ D. f''(x) = 2for some $x \in (1, 3)$

Answer: D

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51. The value of c in Largrange's theorem for the function $f(x)=\log_e\sin x$ in the interval $[\pi\,/\,6,\,5\pi\,/\,6]$ is

A. $\pi/4$

B. $\pi/2$

C. $2\pi/3$

D. none of these

Answer: B

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52. In which of the following function Rolle's theorem is applicable ?

$$egin{aligned} \mathsf{A.}\; f(x) &= egin{cases} x & 0 \leq x < 1 \ 0 & x = 1 \ \end{array} on[0,1] \ \mathsf{B.}\; f(x) &= egin{cases} rac{\sin x}{x} & -\pi \leq x < 0 \ 0 & x = 0 \ \end{array} on[-\pi,0] \ \mathsf{C.}\; f(x) rac{x^2 - x - 6}{x - 1} on[-2,3] \end{aligned}$$

$$\mathsf{D}.\,f(x) = egin{cases} rac{x^3-2x^3-5x+6}{x-1} & ext{if} \ x
eq 1 \ -6 & ext{if} \ x = 1 \end{pmatrix} on[-2,3]$$

Answer: D



53. Let $f'(x) = e^x \hat{\ } 2$ and f(0) = 10. If A < f(1) < B can be concluded from mean value theorem then the value of A-B will be____

A. e

- B.1 e
- C. e 1
- D.1 + e

Answer: B

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54. If f(x)andg(x) are differentiable functions for $0 \le x \le 1$ such that f(0) = 10, g(0) = 2, f(1) = 2, g(1) = 4, then in the interval (0, 1). (a) f'(x) = 0f or allx (b) f'(x) + 4g'(x) = 0 for at least one x (c) f(x) = 2g'(x) for at most one x (d) none of these

A.
$$f(x) = 0$$
 for all x

B. f(x) + 4g'(x) = 0 for at least one x

C. f(x) = 2g'(x) for at most one x

D. none of these

Answer: B

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55. A continuous and differentiable function y = f(x) is such that its graph cuts line y = mx + c at n distinct points. Then the minimum number of points at which $f^{''}(x) = 0$ is/are

A. n - 1

B. n - 3

 $\mathsf{C}.\,n-2$

D. cannot say

Answer: C

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56. Given f'(1) = 1 and $\frac{d}{dx}(f(2x)) = f'(x) \forall x > 0$. If f'(x) is differentiable then there exits a number $c \in (2, 4)$ such that f''(c) equals

A.
$$\frac{1}{4}$$

B. $\frac{-1}{2}$
C. $-1\frac{1}{4}$
D. $-\frac{1}{8}$

Answer: D



57. If (x) is differentiable in [a,b] such that f(a)=2, f(b)=6, then there exists at least one c, $a < c \leq b,$ such that $\left(b^3-a^3\right)f'(c)=$

A. c^2

 $\mathsf{B.}\,2c^2$

 $C. - 3c^2$

D. $12c^2$

Answer: D

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Multiple Correct Answer Type

1. Points on the curve $f(x) = \frac{x}{1-x^2}$ where the tangent is inclined at an angle of $\frac{\pi}{4}$ to the x-axis are (a) (0,0) (b) $\left(\sqrt{3}, -\frac{\sqrt{3}}{2}\right)$ (c) $\left(-2, \frac{2}{3}\right)$ (d) $\left(-\sqrt{3}, \frac{\sqrt{3}}{2}\right)$

A.
$$(0, 0)$$

$$\begin{array}{l} \mathsf{B.} \left(\sqrt{3}, \ -\frac{\sqrt{3}}{2}\right) \\ \mathsf{C.} \left(-2, \frac{2}{3}\right) \\ \mathsf{D.} \left(\sqrt{3}, \ -\frac{\sqrt{3}}{2}\right) \end{array}$$

Answer: A::B::D

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2. For the curve $y = ce^{x/a}$, which one of the following is incorrect?

A. sub-tangent is constant

B. sub-normal varies as the square of the ordinate

C. tangent at (x_1, y_1) on the curve intersects the x-axis at a distance

of $(x_1 - a)$ from the origin

D. equaltion of the normal at the point where the curve cuts

 $y - ext{axis is} cy + ax = c^2$

Answer: A::B::C::D

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3. Let the parabolas $y=x(c-x)andy=x^2+ax+b$ touch each other at the point (1,0). Then (a) a+b+c=0 (b) a+b=2 (c) b-c=1 (d) a+c=-2

A. a + b + c = 0

B. a + b = 2

C. b - c = 1

 $\mathsf{D}.\,a+c=\,-\,2$

Answer: A::C::D



4. The angle formed by the positive
$$y - a\xi s$$
 and the tangent to
 $y = x^2 + 4x - 17at\left(\frac{5}{2}, -\frac{3}{4}\right)$ is $\tan^{-1}(9)$ (b) $\frac{\pi}{2} - \tan^{-1}(9)$
 $\frac{\pi}{2} + \tan^{-1}(9)$ (d) none of these
A. $\tan^{-1}(9)$

B.
$$\frac{\pi}{2} - \tan^{-1}(9)$$

C. $\frac{\pi}{2} + \tan^{-1}(9)$

D. none of these

Answer: B::C

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5. Which of the following pair(s) of curves is/are ortogonal? (a)

$$y^2 = 4ax, y = e^{-x/2a}$$
 (b) $y^2 = 4ax, x^2 = 4ayat(0, 0)$ (c)
 $xy = a^2, x^2 - y^2 = b^2$ (d) $y = ax, x^2 + y^2 = c^2$
A. $y^2 = 4ax, y = e^{-x/2a}$
B. $y^2 = 4ax, x^2 = 4ayat(0, 0)$
C. $xy = a^2, x^2 - y^2 = b^2$
D. $y = ax, x^2 + y^2 = c^2$

Answer: A::B::C::D



6. The coordinates of the point(s) on the graph of the function $f(x) = \frac{x^3}{x} - \frac{5x^2}{2} + 7x - 4$, where the tangent drawn cuts off intercepts from the coordinate axes which are equal in magnitude but opposite in sign, are (a) $\left(2, \frac{8}{3}\right)$ (b) $\left(3, \frac{7}{2}\right)$ (c) $\left(1, \frac{5}{6}\right)$ (d) none of

these

A. (2, 8/3)

B. (3, 7/2)

C.(1,5/6)

D. none of these

Answer: A::B

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7. The abscissa of a point on the curve $xy = (a + x)^2$, the normal which cuts off numerically equal intercepts from the coordinate axes, is $-\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}a$ (c) $\frac{a}{\sqrt{2}}$ (d) $-\sqrt{2}a$ A. $-\frac{a}{\sqrt{2}}$ B. $\sqrt{2}a$ C. $\frac{a}{\sqrt{2}}$ D. $-\sqrt{2}a$

Answer: A::C



8. The angle between the tangents at any point P and the line joining P

to the orgin, where P is a point on the curve In $\left(x^2+y^2
ight)=k an^{1-}rac{y}{x},c$ is a constant, is

A. independent of x

B. independent of y

C. independent of x but dependent on y

D. independent of y but dependent on x

Answer: A::B



9. If OT and ON are perpendiculars dropped from the origin to the tangent and normal to the curve $x = a \sin^3 t$, $y = a \cos^3 t$ at an arbitrary point, then

A.
$$4OT^2 + ON^2 = a^2$$

B.
$$\left|\frac{y}{\cos t}\right|$$

C. the length of the normal is $\left|\frac{y}{\sin t}\right|$

D. none of these

Answer: A::B::C

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10. Let
$$C_1: y = x^2 \sin 3x, C_2: y = x^2$$
 and $C_3: y = -y^2$, then

A. C_1 touches C_2 at infinite points

B. C_1 touches C_3 at infinite points

C. C_1 and C_2 and C_1 and C_3 meet at alternate points

D. none of these

Answer: A::B



- 11. If the line x $\cos heta+y\sin heta=P$ is the normal to the curve
- (x+a)y=1, then heta may lie in

A. I quadrant

B. II quadrant

C. III quadrant

D. IV quadrant

Answer: B::D

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12. Common tagent (s) to $y=x^3 \, \, {
m and} \, \, x=y^3$ is/are

A.
$$x-y=rac{1}{\sqrt{3}}$$

B. $x-y=-rac{1}{\sqrt{3}}$
C. $x-y=rac{2}{3\sqrt{3}}$
D. $x-y=rac{-2}{3\sqrt{3}}$

Answer: C::D

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13. Given
$$f(x) = 4 - \left(\frac{1}{2} - x\right)^{\frac{2}{3}}$$
, $g(x) = \left\{\frac{\tan[x]}{x}, x \neq 01, x = 0 \ h(x) = \{x\}, k(x) = 5^{(\log)_2(x+3)}$ Then in [0,1], lagranges mean value theorem is not applicable to (where [.] and {.} represents the greatest integer functions and fractional part functions, respectively). f (b) g (c) k (d) h

A. f

B.g

C. k

D. h

Answer: A::B::D

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14. Let $f(x)=a_5x^5+a_4x^4+a_3x^3+a_2x^2+a_1x,\,$ where a_i 's are real and f(x)=0 has a positive root $lpha_0$. Then f'(x)=0 has a positive root $lpha_1$ such that '0

A. f'(x) = 0 has a root $lpha_1 {
m such that} < lpha_1 < lpha_0$

B. f' (x) = 0 has at least one real root

C. f''(x) = 0 has at least one real root

D. none of these

Answer: A::B::C

15. Among the following, the function (s) on which LMVT theorem is applicable in the indecatd intervals is/are

A.
$$f(x) = x^{\frac{1}{3}} in[-1, 1]$$

B. $f(x) = x + \frac{1}{x} in[\frac{1}{2}, 3]$
C. $f(x) = (x - 1)|(x - 1)(x - 2)|in[-1, 1]$
D. $f(x) = e^{|(x - 1)|(x - 3)|} in[1, 3]$

Answer: B::C::D

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16. Let f(x) be a differentiable function and $f(\alpha) = f(\beta) = 0(\alpha < \beta),$ then the interval (α, β)

A. f(x) + f'(x) = 0 has at least one root

B. f(x) - f'(x) = 0 has at least one real root

C. f(x) imes f'(x) = 0 has at lease one real root

D. none of these

Answer: A::B::C

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Linked Comprehension Type

1. Tangent at a point P_1 [other than (0,0)] on the curve $y = x^3$ meets the curve again at P_2 . The tangent at P_2 meets the curve at P_3 & so on. Show that the abscissae of $P_1, P_2, P_3, \dots, P_n$, form a GP. Also find the ratio area of $A(P_1P_2P_3)$.) area of $\Delta(P_2P_3P_4)$

- A. 1/4
- B. 1/2
- C.1/8

D. 1/16

Answer: D



2. A spherical balloon is being inflated so that its volume increase uniformaly at the rate of $40cm^3/\text{minute}$. The rate of increase in its surface area when the radius is 8 cm, is

A. $8cm^2 / \min$

B. $10cm^2 / \min$

 $\mathsf{C.}\,20cm^2\,/~\min$

D. none of these

Answer: B

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3. A spherical balloon is being inflated so that its volume increase uniformly at the rate of $40 \frac{(cm)^3}{\min}$. How much the radius will increases during the next 1/2 minute, if initial radius is 8cm. ?

 ${\rm A.}\, 0.025 cm$

 ${\rm B.}\, 0.050 cm$

 $\mathsf{C.}\,0.075cm$

 $D.\,0.01cm$

Answer: A

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4. A conical paper cup 20 cm across the top and 15 cm deep is full of water. The cup springs a leak at the bottom and losses water at 5 cu. cm per minute.

How fast is the water level dropping at the instant when the water is

exactly 7.5 cm deep ?

A.
$$\frac{1}{\pi}cm / \min$$

B. $\frac{1}{5\pi}cm / \min$
C. $\frac{1}{2\pi}cm / \min$
D. $\frac{2}{3\pi}cm / \min$

Answer: B



5. A conical paper cup 20 cm across the top and 15 cm deep is full of water. The cup springs a leak at the bottom and losses water at 5 cu. cm per minute.

The amount of water (in cm^3) when the hight of water is 3 cm is

A. 4π

B. 3π

 $\mathsf{C.}\ 27\pi$

D. 2π

Answer: A



6. A conical paper cup 20 cm across the top and 15 cm deep is full of water. The cup springs a leak at the bottom and losses water at 5 cu. cm per minute.

The value of $\frac{d^2h}{dt^2}(\operatorname{in cm}/\operatorname{min}^2)$ when the water is exactly 7.5cm deep and $\frac{d^2V}{dt^2} = -\frac{4}{9}cm^3/\operatorname{min}^2 is$

A.
$$-\frac{2}{5}$$

B. $\frac{-2}{125\pi^3}$
C. $\frac{-2}{5\pi^3}$

D. none of these

Answer: D

7. Let A (0,0) and B(8,2) be two fixed points on the curve $y^3 = |x|$ A point C (abscissa is less than 0) starts moving from origin along the curve such that rate of change in the ordinate is 2 cm/sec. After t_0 seconds, triangle ABC becomes a right triangle.

After t_0 secods, tangent is drawn to teh curve at point C to intersect it again at (α, β) . Then the value of $4\alpha + 3\beta$ is

A.1 sec

B. 2 sec

C.
$$\frac{1}{4}$$
 sec
D. $\frac{1}{2}$ sec

Answer: C



8. Let A (0,0) and B(8,2) be two fixed points on the curve $y^3 = |x|$ A point

C (abscissa is less than 0) starts moving from origin along the curve such

that rate of change in the ordinate is 2 cm/sec. After t_0 seconds, triangle ABC becomes a right triangle.

After t_0 secods, tangent is drawn to teh curve at point C to intersect it again at (α, β) . Then the value of $4\alpha + 3\beta$ is

A.
$$\frac{4}{3}$$

B. $\frac{3}{4}$
C. 2
D. 1

Answer: D

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Numerical Value Type

1. There is a point (p,q) on the graph of $f(x) = x^2$ and a point (r, s) on the graph of $g(x) = \frac{-8}{x}$, where p > 0 and r > 0. If the line through

(p,q)and(r,s) is also tangent to both the curves at these points, respectively, then the value of P+r is_____.

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2. A curve is defined parametrically be equations $x = t^2 andy = t^3$. A variable pair of perpendicular lines through the origin O meet the curve of PandQ. If the locus of the point of intersection of the tangents at PandQ is $ay^2 = bx - 1$, then the value of (a + b) is____

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3. If d is the minimum distance between the curves $f(x) = e^x andg(x) = (\log)_e x$, then the value of d^6 is

4. Let f(x0) be a non-constant thrice differentiable function defined on $(-\infty,\infty)$ such that $f(x) = f(6-x)andf'(0) = 0 = f'(x)^2 = f(5)$. If n is the minimum number of roots of $(f'(x)^2 + f'(x)f^x = 0$ in the interval [0,6], then the value of $\frac{n}{2}$ is____

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5. At the point $P(a, a^n)$ on the graph of $y = x^n$, $(n \in N)$, in the first quadrant, a normal is drawn. The normal intersects the $y - a\xi s$ at the point (0, b). If $(\lim_{a \to 0} b = \frac{1}{2}$, then n equals 1 (b) 3 (c) 2 (d) 4

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6. A curve is given by the equations $x = \sec^2 \theta$, $y = \cot \theta$. If the tangent at $Pwhere\theta = \frac{\pi}{4}$ meets the curve again at Q, then[PQ] is, where [.] represents the greatest integer function, _____.

7. Water is dropped at the rate of 2 m^3 /s into a cone of semi-vertical angle is 45° . If the rate at which periphery of water surface changes when the height of the water in the cone is 2m is d. Then the value of 5d is ____ m/sec

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8. If the slope of line through the origin which is tangent to the curve

 $y = x^3 + x + 16$ is m, then the value of m - 4 is____.

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9. Let y = f(x) be drawn with f(0) = 2 and for each real number a the line tangent to y = f(x) at (a, f(a)) has x-intercept (a - 2). If f(x) is of the form of ke^{px} then $\frac{k}{p}$ has the value equal to

10. Suppose a, b, c are such that the curve $y = ax^2 + bx + c$ is tangent to y = 3x - 3at(1, 0) and is also tangent to y = x + 1at(3, 4). Then the value of (2a - b - 4c) equals _____



11. Let C be a curve defined by $y = e^{a+bx^2}$. The curve C passes through the point P(1, 1) and the slope of the tangent at P is (-2). Then the value of 2a - 3b is____.

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12. If the curve C in the xy plane has the equation $x^2 + xy + y^2 = 1$, then the fourth power of the greatest distance of a point on C from the origin is___.



13. If a, b are two real numbers with a < b then a real number c can be

found between $a \; ext{and} \; b$ such that the value of $\displaystyle rac{a^2+ab+b^2}{c^2} is_{---}$

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14. Let $f:[1,3] \to [0,\infty)$ be continuous and differentiabl function. If $(f(3) - f(1))(f^2(3) + f^2(1) + f(3)f(1)) = kf^2(c)f'(c)where c \in (1,3),$ then the value of k is _____

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15. The x intercept of the tangent to a curve f(x,y) = 0 is equal to the ordinate of the point of contact. Then the value of $\frac{d^2x}{dy^2}$ at the point (1,1)

on the curve is _____.

16. if f(x) is differentiable function such that f(1) = sin 1, f (2)= sin 4 and f(3) = sin 9, then the minimum number of distinct roots of f'(x) = $2x \cos x^2$ in (1,3) is _____



18. If length of the perpendicular from the origin upon the tangent drawn to the curve $x^2-xy+y^2+lpha(x-2)=4$ at (2,2) is equal to 2 then lpha equals

19. If f (x) = $\begin{cases} x \log_e x, & x > 0\\ 0, & x = 0 \end{cases}$, then conclusion of LMVT holds at x = 1 in the interval [0, a] for f(x), then $[a^2]$ is equal to (where [.] denotes the greatest integer) _____.





1. about to only mathematics

A.
$$\frac{3\sqrt{2}}{8}$$

B.
$$\frac{2\sqrt{3}}{8}$$

C.
$$\frac{3\sqrt{2}}{5}$$

D.
$$\frac{\sqrt{3}}{4}$$

Answer: A

2. The equation of the tangent to the curve $y = x + rac{4}{x^2}$ that is parallel

to the X-axis, is

A. y = 8

B. y = 0

C. y = 3

D. y = 2

Answer: C

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3. Consider the function $f(x) = |x-2| + |x-5|, c \in R$.

Statement 1: f'(4) = 0

Statement 2: f is continuous in [2, 5], differentiable in (2, 5),

A. Statement 1 is false, statement 2 is true.

B. Statement 1 is true, Statement 2 is true, statement 2 is correct

explanation for Statement 1.

C. Statement 1 is true, Statement 2 is trur, Statement2 is no a correct

explanation for statement 1.

D. Statement 1 is true, Statement 2 is false.

Answer: C

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4. If f and g are differentiable functions in [0, 1] satisfying f(0) = 2 = g(1), g(0) = 0 and f(1) = 6, then for some $c \in]0, 1[$ (1) 2f'(c) = g'(c) (2) 2f'(c) = 3g'(c) (3) f'(c) = g'(c) (4) f'(c) = 2g'(c)

A. 2f'(c)=g'(c)`

B. 2f'(c)=3g'(c)`

C. f'(c)=g'(c)`

D. f'(c)=2g'(c)`

Answer: D



5. The normal to the curve $x^2+2xy-3y^2=0,\,\,$ at (1,1)

A. does not meet the curve again.

B. meets the curve again in the second quadrant.

C. meets the curve again in the third quadrant.

D. meets the curve again in the fourth quadrant.

Answer: D



6. Consider $f(x) = \tan^{-1}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right), x \in \left(0, \frac{\pi}{2}\right)$. A normal to y = f(x) at $x = \frac{\pi}{6}$ also passes through the point: (1) (0, 0) (2) $\left(0, \frac{2\pi}{3}\right)$ (3) $\left(\frac{\pi}{6}, 0\right)$ (4) $\left(\frac{\pi}{4}, 0\right)$ A. $\left(0, \frac{2\pi}{3}\right)$ B. $\left(\frac{\pi}{6}0, \right)$ C. $\left(\frac{\pi}{4}, 0\right)$

D.(0,0)

Answer: A

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7. The normal to the curve y(x-2)(x-3)=x+6 at the point where

the curve intersects the $y - a\xi s$, passes through the point : (1) $\left(\frac{1}{2}, -\frac{1}{3}\right)$ (2) $\left(\frac{1}{2}, \frac{1}{3}\right)$ (3) $\left(-\frac{1}{2}, -\frac{1}{2}\right)$ (4) $\left(\frac{1}{2}, \frac{1}{2}\right)$ A. $\left(\frac{1}{2}, \frac{1}{3}\right)$

B.
$$\left(-\frac{1}{2}, -\frac{1}{2}\right)$$

C. $\left(\frac{1}{2}, \frac{1}{2}\right)$
D. $\left(\frac{1}{2}, \frac{1}{3}\right)$

Answer: C

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8. If the curves $y^2 = 6x$, $9x^2 + by^2 = 16$ intersect each other at right angles then the value of b is: (1) 6 (2) $\frac{7}{2}$ (3) 4 (4) $\frac{9}{2}$

A. 9/2

B. 6

C.7/2

D. 4

Answer: A

9. Let $f, g: [-1, 2]\overrightarrow{R}$ be continuous functions which are twice differentiable on the interval (-1, 2). Let the values of *fandg* at the points -1, 0and2 be as given in the following table: , x = -1, x = 0, x = 2 f(x), 3, 6, 0 g(x), 0, 1, -1 In each of the intervals (-1, 0)and(0, 2) the function (f - 3g)'' never vanishes. Then the correct statement(s) is (are) f'(x) - 3g'(x) = 0 has exactly three solutions in $(-1, 0) \cup (0, 2)$. f'(x) - 3g'(x) = 0 has exactly one solutions in $(-1, 0) \cup f'(x) - 3g'(x) = 0$ has exactly one solutions in $(-1, 2) \cdot f'(x) - 3g'(x) = 0$ has exactly two solutions in (-1, 0) = 0.

A. f'(x) - 3g'(x) = 0 has exactly three solution in $(-1,0) \cup (0,2)$

B. f'(x) - 3g'(x) = 0 has exactly one solution in (-1,0)

C. f'(x) - 3g'(x) = 0 has exactly one solution in (0,2)

D. f'(x) - 3g'(x) = 0 has excatly two solutions in (-1,0) and exactly

two solution in (0,2)

Answer: B::C

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10. For every twice differentiable function $f: R \to [-2, 2]$ with $(f(0))^2 + (f'(0))^2 = 85$, which of the following statement(s) is (are) TRUE? There exist $r, s \in R$ where `roo)f(x)=1(d)There \exists alpha in (-4,\ 4) $sucht^{f}(alpha)+f''(alpha)=0$ and f^(prime)(alpha)!=0`

A. There exist r,s $\in R$, where r < s , such that f is one-one on the

open interval (r,s)

B. There exist $x_0 \in (\,-4,0)$ such that $|f'(x_0)| \leq 1$

C. $\lim_{x\,
ightarrow\,\infty}\,f(x)=1$

D. There exists $lpha \in (\,-\,4,\,4)$ such that $f(lpha)+f'\,{}'(lpha)=0$ and

f'(lpha)
eq 0

Answer: A::B::D



3. Find the equation of tangent to the curve
$$y = rac{\sin^{-1}(2x)}{1+x^2} atx = \sqrt{3}$$

4. The equation of the tangent tothe curve $y = \left\{x^2 \sin\left(\frac{1}{x}\right), x \neq 0 \text{ and } 0, x = 0 \text{ at the origin is} \right\}$ Watch Video Solution

5. Find the equation of normal line to the curve $y=x^3+2x+6$ which

is parallel to the line x + 14y + 4 = 0.



6. If the equation of the tangent to the curve $y^2 = ax^3 + b$ at point

(2,3)isy=4x-5 , then find the values of aandb .

7. For the curve $y = 4x^3 - 2x^5$, find all the points at which the tangents pass through the origin.

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8. Find the equation of tangent to the curve represented parametrically by the equations $x = t^2 + 3t + 1$ and $y = 2t^2 + 3t - 4$ at the point M (-1,10).

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9. For the curve xy = c, prove that the portion of the tangent intercepted between the coordinate axes is bisected at the point of contact.

10. If the tangent at any point $\left(4m^2, 8m^2
ight)$ of $x^3-y^2=0$ is a normal to

the curve $x^3-y^2=0$, then find the value of $m_{
m c}$

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11. Find all the tangents to the curve $y=\cos(x+y),\;-2\pi\leq x\leq 2\pi$

that are parallel to the line x + 2y = 0.

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12. Find the equation of all possible normals to the parabola $x^2 = 4y$

drawn from the point (1,2).



13. Find the equations of the tangents drawn to the curve $y^2 - 2x^3 - 4y + 8 = 0$ from the point (1, 2) .

14. From the point (1,1) tangents are drawn to the curve represented parametrically as $x = 2t - t^2$ and $y = t + t^2$. Find the possible points of contact.

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15. Show that the straight line $x \cos \alpha = p$ touches the curve $xy = a^2$, if

$$p^2 = 4a^2 \cos lpha \sin lpha$$

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16. Find the condition that the line Ax + By = 1 may be normal to the

curve
$$a^{n-1}y = x^n$$
.

17. Find the acute angle between the curves $y=ig|x\hat{2}-1ig|and$ $y=ig|x^2-3ig|$ at their points of intersection.



$$f(x)=2^x(\log)_exandg(x)=x^{2x}-1.$$

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20. Find the angle between the curves $y^2 = 4x$ and $y = e^{-x/2}$.



25. In the curve $x^{m+n} = a^{m-n}y^{2n}$, prove that the mth power of the

sub-tangent varies as the nth power of the sub-normal.



27. Find the shortest distance between the line y=x-2 and the

parabola $y=x^2+3x+2$

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28. about to only mathematics

29. Prove that points of the curve $y^2 = 4a \Big\{ x + a \sin \Big(rac{x}{a} \Big) \Big\}$ at which

tangents are parallel to x-axis lie on the parabola.



30. The tangent at any point on the curve $x = a \cos^3 \theta$, $y = a \sin^3 \theta$ meets the axes in PandQ. Prove that the locus of the midpoint of PQ is a circle.

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31. Displacement s of a particle at time t is expressed as $s = \frac{1}{2}t^3 - 6t$.

Find the acceleration at the time when the velocity vanishes (i.e., velocity tends to zero).



32. On the curve $x^3 = 12y$, find the interval of values of x for which the

abscissa changes at a faster rate than the ordinate?



33. एक आयत की लम्बाई x, 5 cm / min की दर से घट रही है और चौड़ाई y, 4 cm / min कि दर से बढ़ रही है जब x=8 cm और y= 6 cm है तब आयत के (a) परिमाप (b) क्षेत्रफल की परिवर्तन की दर ज्ञात कीजिए

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34. Let x be the length of one of the equal sides of an isosceles triangle, and let θ be the angle between them. If x is increasing at the rate (1/12) m/h, and θ is increasing at the rate of $\frac{\pi}{180}$ radius/h, then find the rate in m^3 / h at which the area of the triangle is increasing when $x = 12mandth\eta = \pi/4$. **35.** A lamp is 50ft above the ground. A ball is dropped from the same height from a point 30ft away from the light pole. If ball falls a distance $s = 16t^2ft$ in t second, then how fast is the shadow of the ball moving along the ground $\frac{1}{2}s$ later?

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36. If water is poured into an inverted hollow cone whose semi-vertical angel is 30^0 , show that its depth (measured along the axis) increases at the rate of 1 cm/s. Find the rate at which the volume of water increases when the depth is 24 cm.

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37. A horse runs along a circle with a speed of 20km/h. A lantern is at the centre of the circle. A fence is along the tangent to the circle at the point at which the horse starts. Find the speed with which the shadow of

the horse moves along the fence at the moment when it covers 1/8 of the circle in km/h.

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38. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall at the rate of 10 cm/s. How fast is the angle between the ladder and the ground decreasing when the foot of the ladder is 2 m away from the wall?

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39. The radius of the base of a cone is increasing at the rate of 3 cm/min and the altitude is decreasing at the rate of 4 cm/min. The rate of change of lateral surface when the radius is 7 cm and altitude is 24cm is $108\pi cm^2 / \min$ (b) $7\pi cm^2 / \min 27\pi cm^2 / \min$ (d) none of these



44. Find the approximate change in the volume \boldsymbol{V} of a cube of side \boldsymbol{x}

meters caused by increasing side by 1~% .



45. Discuss the applicability of Rolles theorem for the following functions

on the indicated intervals: $f(x) = |x| \in [-1, 1]$ $f(x) = 3 + (x-2)^{2/3}$ in [1,3] $f(x) = \tan \xi n[0, \pi]$ $f(x) = \log \left\{ \frac{x^2 + ab}{x(a+b)} \right\}$ in `[a , b],w h e r e-

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46. If the function $f(x) = x^3 - 6x^2 + ax + b$ defined on [1,3] satisfies Rolles theorem for $c = \frac{2\sqrt{3}+1}{\sqrt{3}}$ then find the value of aandb

47. Show that between any two roots of $e^{-x} - \cos x = 0$, there exists at least one root of $\sin x - e^{-x} = 0$

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48. How many roots of the equation (x-1)(x-2)(x-3) + (x-1)(x-2)(x-4) + (x-2)(x-3)(x-4) + (x-1)(x-3)(x-4) = 0` are positive?

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49. If 2a+3b+6c = 0, then show that the equation $ax^2 + bx + c = 0$ has

atleast one real root between 0 to 1.

50. Let f(x) be differentiable function and g(x) be twice differentiable function. Zeros of f(x), g'(x) be a,b respectively, `(a



51. Let f(x) be differentiable function and g(x) be twice differentiable function. Zeros of f(x), g'(x) be a,b respectively, `(a

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52. Let P(x) be a polynomial with real coefficients, Let $a, b \in R, a < b$,

be two consecutive roots of P(x). Show that there exists c such that

 $a \leq c \leq b$ and P'(c) + 100P(c) = 0.

53. The fucntion $f(x) = x^3 - 6ax^2 + 5x$ satisfies the conditions of Lagrange's mean value theorem over the interval [1, 2] and the value of c (that of LMVT) is $\frac{7}{4}$, then find the value of a.



54. If f: [5, 5]R is a differentiable function and if f'(x) does not vanish anywhere, then prove that f(5)f(5).

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55. Let f be differentiable for all x, If $f(1) = -2andf'(x) \geq 2$ for all

 $x\in [1,6], ext{ then find the range of values of } f(b) \cdot$
56. Let $f: [2,7] \overrightarrow{0,\infty}$ be a continuous and differentiable function. Then show that $(f(7) - f(2)) \frac{(f(7))^2 + (f(2))^2 + f(2)f(7)}{3} = 5f^2(c)f'(c)$, where $c \in [2,7]$.

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57. Let f(x)andg(x) be differentiable function in (a, b), continuous at

$$aandb, andg(x)
eq 0$$
 in $[a, b]$. Then prove that $rac{g(a)f(b) - f(a)g(b)}{g(c)f'(c) - f(c)g'(c)} = rac{(b-a)g(a)g(b)}{\left(g(c)
ight)^2}$

58. Using Lagranges mean value theorem, prove that
$$|\cos a - \cos b| < |a - b|$$
.
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59. Using lagrange's mean value theorem, show that

$$\frac{\beta - \alpha}{1 + \beta^2} < \tan^{-1}\beta - \tan^{-1}\alpha < \frac{\beta - \alpha}{1 + \alpha^2}, \beta > \alpha > 0.$$
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60. Prove that
$$rac{1}{28} < (28)^{1/3} - 3 < rac{1}{27}$$

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61. Let f(x)andg(x) be two differentiable functions in Randf(2)=8, g(2)=0, f(4)=10, andg(4)=8. Then prove that g'(x)=4f'(x) for at least one $x\in (2,4)$.

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62. Let f be continuous on [a, b], a > 0, and differentiable on (a, b).

Prove that there exists $c\in(a,b)$ such that

$$rac{bf(a)-af(b)}{b-a}=f(c)-cf^{\,\prime}(c)$$



Solved Examples

1. Prove that the equation of the normal to $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ is $y\cos\theta - x\sin\theta = a\cos 2\theta$, where θ is the angle which the normal makes with the axis of x.

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2. If the area of the triangle included between the axes and any tangent

to the curve $x^ny=a^n$ is constant, then find the value of n_{\cdot}

3. Show that the segment of the tangent to the curve $y=\frac{a}{2}In\left(\frac{a+\sqrt{a^2-x^2}}{a-\sqrt{a^2-x^2}}\right)-\sqrt{a^2-x^2} \text{ contained between the y=axis}$

and the point of tangency has a constant length.

Watch Video Solution 4. If the tangent at (x_1, y_1) to the curve $x^3 + y^3 = a^3$ meets the curve

again in $(x_2,y_2), ext{ then prove that } rac{x_2}{x_1}+rac{y_2}{y_1}= -1$

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5. Find the condition for the line y=mx to cut at right angles the conic

$$ax^2 + 2hxy + by^2 = 1.$$

6. If two curves $ax^2 + by^2 = 1$ and $a'x^2 + b'y^2 = 1$ intersect orthogonally, then show that $\frac{1}{a} - \frac{1}{b} = \frac{1}{a'} - \frac{1}{b'}$

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7. A man is moving away from a tower 41.6 m high at the rate of 2 m/sec. Find the rate at which the angle of elevation of the top of tower is changing, when he is at a distance of 30m from the foot of the tower. Assume that the eye level of the man is 1.6m from the ground.



8. If f is continuous and differentiable function and f(0)=1, f(1)=2,then prove that there exists at least one $c\in [0,1]f$ or $whichf'(c)(f(c))^{n-1}>\sqrt{2^{n-1}}$, where $n\in N$.

9. Let a, b, c be three real numbers such that a < b < c. f(x) is continuous in [a, c] and differentiable in (a, c) Also, f'(x) is strictly increasing in (a, c) Prove that (b-c)f(a) + (c-a)f(b) + (a-b)f(c) < 0.

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10. Use the mean value theorem to prove $e^x \geq 1 + x \, orall x \in R$

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11. Show that the square roots of two successive natural numbers

greater than N^2 differ by less than $rac{1}{2N}$.

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12. Using Rolles theorem, prove that there is at least one root in

$$\left(45^{\frac{1}{100}}, 46\right)$$

equation.

$$P(x) = 51x^{101} - 2323(x)^{100} - 45x + 1035 = 0.$$

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13. Let f(x0) be a non-constant thrice differentiable function defined on $(-\infty,\infty)$ such that $f(x) = f(6-x)andf'(0) = 0 = f'(x)^2 = f(5)$. If n is the minimum number of roots of $(f'(x)^2 + f'(x)f^x = 0$ in the interval [0,6], then the value of $\frac{n}{2}$ is____

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14. If f"(x) exists for all points in [a, b] and

 $rac{f(c)-f(a)}{c-a} = rac{f(b)-f(c)}{b-c}, ext{where} a < c < b, ext{ then show that there}$

exists a number 'k' such that f"(k)=0.

15. Let f defined on [0,1] be twice differentiable such that $|f(x)| \le 1$ for $x \in [0,1]$. if f(0)=f(1) then show that |f'(x)<1 for all $x \in [0,1]$

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Matrix Match Type

1. Match the following lists:

15.5	List I	List II
a.	A circular plate is expanded by heat from radius 6 cm to 6.06 cm. Approximate increase in the area is	p. 5
b.	If an edge of a cube increases by 2%, then the percentage increase in the volume is	q. 0.72 π

c. If the rate of decrease of $\frac{x^2}{2} - 2x + 5$ is thrice the rate of decrease of x, then x is equal to (rate of decrease is nonzero)	r. 6
d. The rate of increase in the area of an equailateral triangle of side 30 cm, when each side increases at the rate of 0.1 cm/s, is	s. $\frac{3\sqrt{3}}{2}$



2. Match the following lists:

List I: Curves	List II: Angle between the curves
a. $y^2 = 4x$ and $x^2 = 4y$	p. 90°
b. $2y^2 = x^3$ and $y^2 = 32x$	q. Any one of $\tan^{-1} \frac{3}{4}$ or $\tan^{-1}(16^{1/3})$
c. $xy = a^2$ and $x^2 + y^2 = 2a^2$	r. 0°
d. $y^2 = x$ and $x^3 + y^3 = 3xy$ at other than origin	s. $\tan^{-1} \frac{1}{2}$

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3. The curve $y=ax^2+bx^2+cx+5$ touches the x-axis at $P(\,-2,\,0)$

and cuts the y-axis at point Q, where its gradient is 3. Now, match the

following lists and then choose the correct code.

List I	List II
a. The value of <i>a</i> is	p. 3
b. The value of <i>b</i> is	q. 0
c. The value of c is	r. $-\frac{3}{4}$
d. The value of $y'(1)$ is	s. $-\frac{1}{2}$

٨	a	b	c	d
A.	\boldsymbol{s}	r	q	p
р	a	b	c	d
р.	q	s	r	p
c	a	b	c	d
C.	s	r	q	p
_	a	b	c	d
υ.	s	p	q	r

-

Answer: C