MATHS

BOOKS - CENGAGE MATHS (ENGLISH)

CONTINUITY AND DIFFERENTIABILITY

Single Correct Answer Type

1. If $f(x)=\left\{\left(\sin\left(\frac{2x^2}{a}\right)+\cos\left(\frac{3x}{b}\right)\right)^{ab/x^2}, x\neq 0 \& e^3 atx=0\right\}$ is continuous at $x=0 \forall b\in R$ then minimum value of $a$is $-1/8$ b. $-1/4$ c. $-1/2$ d. 0

A. $-1/8$

B. $-1/4$

C. $-1/2$

D. 0

Answer: B

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2. Let $f: R \to R$ be any function. Also $g: R \to R$ is defined by $g(x) = |f(x)|$ for all x. Then g is

a. Onto if f is onto b. One-one if f is one-one c. Continuous if f is continuous d. None of these

A. onto if f is onto

B. one-one if f is one-one

C. continuous if f is continuous

D. None of these

Answer: C

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3. Let

$$f(x) = \left[\frac{1-\sin \pi x}{1+\cos 2\pi x}, x < \frac{1}{2} \text{ and } p, x = \frac{1}{2} \text{ and } \frac{\sqrt{2x-1}}{\sqrt{4+\sqrt{2x-1}}-2}\right.$$

.Determine the value of p, if possible, so that the function is continuous at $x = \frac{1}{2}$.

A. 1

B. $1/4$

C. 4

D. none of these

Answer: D

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4. For which of the following functions $f(0)$ exists such that $f(x)$ is continuous at $f(x) = \frac{1}{(\log)_e|x|}$ b. $f(x) = \frac{1}{(\log)_e|x|}$ c. f(x)=x sinpi/x d. $f(x) = \frac{1}{1+2^{\cot x}}$

A. $f(x) = \dfrac{1}{\log_e |x|}$

B. $f(x) = \cos\left(\dfrac{|\sin x|}{x}\right)$

C. $f(x) = x\dfrac{\sin(\pi)}{x}$

D. $f(x) = (1) = \dfrac{1}{1 + 2^{\cot x}}$

Answer: C

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5. Let $f(x) = x^3 - x^2 - 3x - 1, g(x) = (x+1)a$ and $h(x) = \dfrac{f(x)}{g(x)}$

where h is a rational function such that

(i) It is continuous everywhere except when $x = -1$,

(ii) $\lim_{x \to -1} h(x) = \dfrac{1}{2}$.

The value of h(1) is

A. $1/2$

B. $1/4$

C. $-1/2$

D. 1

Answer: C

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6. If the function $f(x) = \frac{3x^2 + ax + a + 3}{x^2 + x - 2}$ is continuous at $x = -2$, then the value of $f(-2)$ is

A. 0

B. -1

C. 1

D. 2

Answer: B

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7. Let $f(x) = \begin{cases} 8^{\frac{1}{x}}, & x < 0 \\ a[x], & a \in R - \{0\}, \quad x \geq 0 \end{cases}$ (where [.] denotes the greatest integer function).

Then f(x) is

A. continuous only at a finite number of points.

B. discontinuous at a finite number of points.

C. discontinuous at an infinite number of points.

D. discontinuous at x = 0.

Answer: C

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8. Let $f(x) = (1-x)^2 \sin^2 x + x^2$ for all x $\in$ R, and let $g(x) = \int \left(\frac{2(t-1)}{t+1} - \ln t \right) f(t) dt$ for t $\in$ [1, x] for all x $\in$ (1, ∞).Which of the following is true ?

A. f is continuous at $x = \pi/2$

B. f has an irremovable discontinuity at $x = \pi/2$

C. f has a removable discontinuity at $x = \pi/2$

D. none of these

Answer: B

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9. If

$$f(x) = \begin{cases} \sin\left(\frac{\pi}{2}\right)(x - [x]), & x < 55(b-1) \\ x = 5 \\ \frac{ab^2|x^2 - 11x + 24|}{x - 3}, & x > \end{cases}$$

is continuous at $x = 5, a, b \in R$ then ([.] denotes the greatest integer function) $a = \frac{25}{108}, b = \frac{6}{5}$ b. $a = \frac{6}{13}, b = \frac{17}{29}$ c. $a = \frac{1}{2}, b = \frac{25}{36}$ d. $a = \frac{23}{100}, b = \frac{6}{5}$

A. $a = \frac{25}{108}, b = \frac{6}{5}$

B. $a = \frac{6}{13}, b = \frac{17}{29}$

C. $a = \frac{1}{2}, b = \frac{25}{36}$

D. $a = (23), (100), b = \frac{6}{5}$

Answer: A

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10. The function $f(x)$ is discontinuous only at $x = 0$ such that $f^2(x) = 1 \forall x \in R$. The total number of such functions is 2 b. 3 c. 6 d. none of these

A. 2

B. 3

C. 6

D. none of these

Answer: C

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11. $f(x) = \left\{\left(x^2 + e^{\frac{1}{2-x}}\right)^{-1} k, x = 2, x \neq 2\right.$ is continuous from right at the point $x = 2,$ then k equals

a. 0

b. $1/4$

c. $-1/4$

d. none of these

A. 0

B. 1\4

C. $-1/4$

D. none of these

Answer: B

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12. Let $g(x) = f(f(x))$ where $f(x) = \{1 + x; 0 \leq x \leq 2\}$ and $f(x) = \{3 - x; 2 < x \leq 3\}$ then the

number of points of discontinuity of $g(x)$ in [0,3] is :

A. 0

B. 1

C. 2

D. 3

Answer: C

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13. If the function $f(x) = \frac{(128a + ax)^{1/8} - 2}{(32 + bx)^{1/5} - 2}$ is continuous at $x = 0$, then the value of a/b is $\frac{3}{5}f(0)$ b. $2^{8/5}f(0)$ c. $\frac{64}{5}f(0)$ d. none of these

A. $\frac{3}{5}f(0)$

B. $2^{8/5}f(0)$

C. $\frac{64}{5}f(0)$

D. none of these

Answer: C

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14. If $f(x) = \begin{cases} \dfrac{1-\cos\left(1-\frac{\cos x}{2}\right)}{2^m x^n} 1x = 0, x \neq 0 \end{cases}$ is continous at $x = 0$ then the value of $m + n$ is a. 2 b. 3 c. -3 d. 7

A. 2

B. 3

C. -3

D. 7

Answer: C

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15. Let $f(x) = \begin{cases} \frac{\alpha \cot x}{x} + \frac{\beta}{x^2} & 0 < |x| \leq 1 \\ \frac{1}{3} & x = 0 \end{cases}$. If $f(x)$ is continuous at $x = 0$ then the value of $\alpha^2 + \beta^2$ is

A. 1

B. 2

C. 5

D. 9

Answer: B

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16. Let $f(x) = \left\{ \frac{2}{1+x^2}, \xi srationalb, \xi srational \right.$ has exactly two points of continuity then the value of b are $(0, 3]$ b. $[0, 1]$ c. $(0, 2]$ d.φ

A. (0,3]

B. [0,1]

C. (0,2]

D. ϕ

Answer: C

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17. If $f(x)'\begin{cases}\sin\left(\frac{a-x}{2}\right)\tan\left[\frac{\pi x}{2a}\right] & \text{for} \quad x > a \\ \frac{\left[\cos\left(\frac{\pi x}{2a}\right)\right]}{a-x} & \text{for} \quad x < a\end{cases}$

(where [x] is the greatest integer function of x) and a gt 0, then

A. $f\left(a^{-}\right) < 0$

B. f has a removable discontinuity at x = a

C. f has an irremovable discontinuity at x = a

D. $f\left(a^{+}\right) < 0$

Answer: B

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18. Let $f(x) = [tanx[\cot x]], x\left[\frac{\pi}{12}, \frac{\pi}{12}\right]$, (where [.] denotes the greatest integer less than or equal to$x$). Then the number of points, where $f(x)$ is discontinuous is a. one b. zero c. three d. infinite

A. one

B. zero

C. three

D. infinite

Answer: C

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19. Let $f: [a, b] \to R$ be any function which is such that $f(x)$ is rational for irrational x and that $f(x)$ is iirrational for rational x, then in [a,b]

A. f is discontinuous everywhere

B. f is discontinuous only at x = 0 and discontinuous everywhere

C. f is continuous for all irrational x and discontinuous for rational x

D. f is continuous for rational x and discontinuous for irrational x

Answer: A

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20. If $f(x) = [x](\sin kx)^p$ is continuous for real x, then (where [.] represents the greatest integer function)

A. $k \in [n\pi, n \in I], p > 0$

B. $k \in \{2n\pi, n \in I\}, p > 0$

C. $k \in \{n\pi, n \in I\}, p \in R - \{0\}$

D. $k \in \{n\pi, nI, n \neq 0\}, p \in R - \{0\}$

Answer: A

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21. Statement 1: Minimum number of points of discontinuity of the function $f(x) = (g(x)[2x - 1] \forall x \in (-3, -1)$, where [.] denotes the greatest integer function and $g(x) = ax^3 = x^2 + 1$ is zero. Statement 2: $f(x)$ can be continuous at a point of discontinuity, say $x = c_1 of[2x - 1]$ if $g(c_1) = 0$. Statement 1 is True, Statement 2 is True, Statement 2 isa correct explanation for Statement 1. Statement 1 is True, Statement 2 is True, Statement 2 is NOT a correct explanation for statement 1. Statement 1 is True, Statement 2 is False Statement 1 is False, Statement 2 is True.

A. Statement 1 is True, Statement 2 is True, Statement 2 is a correct explaination for Statement 1.

B. Statement 1 is True, Statement 2 is True, Statement 2 is NOT a correct explanation for Statement 1/

C. Statement 1 is True, Statement 2 is False.

D. Statement 1 is False, Statement 2 is True.

Answer: D

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22. Number of points of discontinuity of $f(x) = \left[\sin^{-1} x\right] - [x]$ in its domain is equal to (where [.] denotes the greatest integer function) a. 0 b. 1 c. 2 d. 3

A. 0

B. 1

C. 2

D. 3

Answer: D

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23. If $g(x) = \left(\lim\right)_{m \to \infty} \frac{x^m f(x) + h(x) + 3}{2x^m + 4x + 1}$ when $x \neq 1 and g(1) = e^3$ such that $f(x), g(x) and h(x)$ are continuous functions at $x = 1$ then the value of $5f(1) - 2h(1)$ is 7 b. 6 c. 9 d. 8

A. 7

B. 6

C. 9

D. 8

Answer: B

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24. The number of points of discontinuity of $fx\Big) = \left[2x^2\right] - \{2x2\}^2$ (where [] denotes the greatest integer function and {} is fractional part of x) in the interval $(-2, 2),$ is 1 b. 6 c. 2 d. 4

A. 1

B. 6

C. 2

D. 5

Answer: B

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25. If $f(x) = \{(|x| - 3 \text{ when } x < 1), \text{and } (|x-2| + a, \text{ when } x \geq 1)$ &

$g(x) = \{2 - |x| \text{when } x < 2 \text{ and } sgn(x) - b \text{ , when } x \geq 2.$

if $h(x) = f(x) + g(x)$ is discontinuous at exactly one point, then -

(a). $a = -3, b = 0$

(b). $a = -3, b = -1$

(c) $a = 2, b = 1$

(d) $a = 0, b = 1$

A. $a = -3, b = 0$

B. $a = 0, b = 1$

C. $a = 2, b = 1$

D. $a = -3, b = 1$

Answer: D

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26. The function $f(x) = \frac{x^3}{8} - s \in \pi x + 4 \in [-4, 4]$ does not take the value -4 b. 10 c. 18 d. 12

A. -4

B. 10

C. 18

D. 12

Answer: C

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27. Let $f(x)$ be continuous functions $f: R\overrightarrow{R}$ satisfying $f(0) = 1 and f(2x) - f(x) = x \cdot$ Then the value of $f(3)$ is 2 b. 3 c. 4 d. 5

A. 2

B. 3

C. 4

D. 5

Answer: C

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28. about to only mathematics

A. $a = b = 4$

B. $a = b = -4$

C. $a = 4$ and $b = -4$

D. $a = -4$ and $b = 4$

Answer: C

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29. If $f(x) = \begin{cases} [x] + \sqrt{\{x\}}, & x < 1 \\ \frac{1}{[x] + \{x\}^2}, & x \geq 1 \end{cases}$, then

[where [.] and {.} represent the greatest integer and fractional part functions respectively]

A. f(x) is continuous at x = 1 but not differentiable

B. f(x) is not continuous at x = 1

C. f(x) is differentiable at x = 1

D. $\lim_{x \to 1} f(x)$ does not exist

Answer: A

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30. If is an even function such that $\lim_{h \to 0} \frac{f(h) - f(0)}{h}$ has some fininte non-zero value, then

A. f is continuous and derivable at x =0

B. f is continuous but not differentiable at x = 0

C. f may be discontinuous at x = 0

D. None of these

Answer: B

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31. Let $f(x)$ be differentiable for real x such that $f'(x) > 0 on (-\infty, -4)$, $f'(x) < 0 on (-4, 6)$, $f'(x) > 0 on (6, \infty)$, If $g(x) = f(10 - 2x)$, then the value of $g'(2)$ is a. 1 b. 2 c. 0 d. 4

A. 1

B. 2

C. 0

D. 4

Answer: C

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32. Number of points where $f(x) = x^2 - \left|x^2 - 1\right| + 2||x| - 1| + 2|x| - 7$ is non-differentiable is a. 0 b. 1 c. 2 d. 3

A. 0

B. 1

C. 2

D. 3

Answer: A

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33. If $f(x) = |x - 1|. ([x] = [-x]),$ then (where [.] represents greatest integer function)

A. f(a) is continuous and differentiable at x = 1

B. f(x) is discontinuous at x = 1

C. f(x) is continuous at x = 2

D. f(x) is continuous but non-differentiable at x= 1

Answer: D

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34. Number of point where function f(x) defined as $f:[0,2\pi]\to R, f(x)=\begin{cases}3-\left|\cos x-\frac{1}{\sqrt{2}}\right|, & |\sin x|<\frac{1}{\sqrt{2}}\\ 2+\left|\cos x+\frac{1}{\sqrt{2}}\right|, & |\sin x|\geq\frac{1}{\sqrt{2}}\end{cases}$ is non differentiable is

A. 2

B. 4

C. 6

D. 0

Answer: B

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35. Let $f(x) = \begin{cases} [x] & x \notin I \\ x - 1 & x \in I \end{cases}$ (where, [.] denotes the greatest integer function) and $g(x) = \begin{cases} \sin x + \cos x, & x < 0 \\ 1, & x \geq 0 \end{cases}$ Then for f(g(x)) at x = 0

A. $\lim_{x \to 0} g(g(x))$ exists but not continuous

B. continuous but not differentiable at x = 0

C. differentiable at x = 0

D. $\lim_{x \to 0} f(g(x))$ does not exist

Answer: C

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36. If

$f(x) = \{s \in (\cos^{-1} x) + \cos(\sin^{-1} x), x \leq 0 s \in (\cos^{-1} x) - \cos(\sin^{-1} x$

. Then at $x = 0$ $f(x)$ is continuous and differentiable $f(x)$ is continuous but not differentiable $f(x)$ not continuous but differentiable $f(x)$ is neither continuous nor differentiable

A. f(x) is continuous and differentiable

B. f(x) is continuous but not differentiable

C. f(x) not continuous but differentiable

D. f(x) is neither continuous nor differentiable

Answer: D

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37. If $f(x) = \max\{\tan x, \sin x, \cos x\}$ where $x \in \left[-\frac{\pi}{2}, \frac{3\pi}{2}\right)$ then the number of points, where f(x) is non -differentiable, is

A. 2

B. 3

C. 4

D. 5

Answer: B

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38. The number of points at which $g(x) = \frac{1}{1 + \frac{2}{f(x)}}$ is not differentiable, where $f(x) = \frac{1}{1 + \frac{1}{x}}$, is a. 1 b. 2 c. 3 d. 4

A. 1

B. 2

C. 3

D. 4

Answer: C

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39. Let $f(x) = (\lim)_{n \to \infty} \sum_{r=0}^{n-1} \frac{x}{(rx+1)\{(r+1)x+1\}}$. Then (A) $f(x)$ is continuous but not differentiable at $x = 0$ (B) $f(x)$ is both continuous but not differentiable at $x = 0$ (C) $f(x)$ is neither continuous not differentiable at $x = 0$ (D) $f(x)$ is a periodic function.

A. f(x) is continuous but not differentiable at x = 0

B. f(x) is both continuous and differentiable at x = 0

C. f(x) is neither continuous not differentiable at x = 0

D. f(x) is a periodic function

Answer: C

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40. Let the given function is differentiable at x = 1.

$$f(x) = \begin{cases} \lim\limits_{n \to \infty} \dfrac{ax(x-1)\left(\cot \cdot \frac{\pi x}{4}\right)^n + (px^2+2)}{\left(\cot, \frac{\pi x}{4}\right)^n + 1}, & x \in (0,1) \cup (1,2) \\ 0, & x = 1 \end{cases}$$

Then the value of $|a+p|$ is

A. 4

B. 6

C. 8

D. 10

Answer: B

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41. The value of p and q for which the function $f(x) = \left\{ \frac{\sin(p+1)x + \sin x}{x}, x < 0q, x = 0 \frac{\sqrt{x+x^2} - \sqrt{x}}{x^{3/2}}, x > 0 \right\}$ is continuous for all x in R, are: (1) $p = \frac{1}{2}, q = -\frac{3}{2}$ (2) $p = \frac{5}{2}, q = -\frac{1}{2}$ (3) $p = -\frac{3}{2}, q = \frac{1}{2}$ (4) $p = \frac{1}{2}, q = \frac{3}{2}$

A. $p = \frac{1}{2}, q = \frac{3}{2}$

B. $p = \frac{1}{2}, q = -\frac{3}{2}$

C. $p = \frac{5}{2}, q = \frac{1}{2}$

D. $p = -\frac{3}{2}, q = \frac{1}{2}$

Answer: D

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42. If $f: R \to R$ is a function defined by $f(x) = [x]\cos\left(\frac{2x-1}{2}\right)\pi$, where [x] denotes the greatest integer function, then f is

A. continuous for every real x.

B. discontinuous only at x=0.

C. discontinuous only at non-zero integral values of x.

D. continuous only at x=0.

Answer: A

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43. If the function

$$g(x) = \begin{cases} k\sqrt{x+1} & 0 \leq x \leq 3 \\ mx + 2 & 3 < x \leq 5 \end{cases}$$ is differentiable, then the value of k+m is

A. 2

B. $\frac{16}{5}$

C. $\frac{10}{3}$

D. 4

Answer: A

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44. For $x \in R$, $f(x) = |\log 2 - \sin x|$ and $g(x) = f(f(x))$, then

A. g'(0)= cos $(\log_e 2)$

B. $g'(0) = -\cos(\log_e 2)$

C. g is differentiable at x=0 and g'(0) =- sin $(\log_e 2)$

D. g is not differentiable at x=0

Answer: A

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45. Let $S = \left\{t \in R : f(x) = |x - \pi|\left(e^{|x|} - 1\right)\sin|x| \right.$ is not differentiable at t} Then the set S is equal to: (1) ϕ (2) {0} (3) $\{\pi\}$ (4) $\{0, \pi\}$

A. $\{0, \pi\}$

B. ϕ (an empty set)

C. $\{0\}$

D. $\{\pi\}$

Answer: B

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Multiple Correct Answer Type

1. Which of the following functions is/are discontinuous at $x = 1$?

$$f(x) = \frac{1}{1 + 2^{tanx}} \qquad g(x) = (\lim)_{x \overrightarrow{\infty}} \frac{1}{1 + n \in s^2(\pi x)}$$

$$h(x) = 2^{-2\hat{}}\left(\left(\left(\frac{1}{1-x}\right)\right)\right), x \neq 1 and h(1) = 1$$

$$\varphi(x) = \frac{x-1}{|x-1| + 2(x-1)^2}, x = 1 and \varphi(1) = 1$$

A. $f(x) = \dfrac{1}{1 + 2^{\tan x}}$

B. $g(x) = \lim\limits_{n \to \infty} \dfrac{1}{1 + n \sin^2(\pi x)}$

C. $h(x) = 2^{-2^{\left(\frac{1}{1-X}\right)}}, x \neq 1$ and $h(1) = 1$

D. $\phi(x) = \dfrac{x-1}{|x-1| + 2(x-1)^2}, x \neq 1$ and $\phi(1) = 1$

Answer: A

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2. f is a continous function in $[a, b]$; g is a continuous function in [b,c]. A function h(x) is defined as

$h(x) = f(x) f$ or $x \in [a, b), g(x) f$ or $x \in (b, c]$ if f(b) =g(b) then

A. h(x) may or may not be continuous in [a, c]

B. $h\left(b^{+}\right) = g\left(b^{-}\right)$ and $h\left(b^{-}\right) = f\left(b^{+}\right)$

C. $h\left(b^{-}\right) = g\left(b^{+}\right)$ and $h\left(b^{+}\right) = f\left(b^{-}\right)$

D. h(x) has a removable discontinuity at x = b

Answer: C::D

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3. If the function $f(x)$ defined as $f(x)$ defined as $f(x) = \left\{3, x = 0\left(1 + \frac{ax + bx^3}{x^2}\right), x > 0\right.$ is continuous at $x = 0$, then $a = 0$ b. $b = e^3$ c. $a = 1$ d. $b = (\log)_e 3$

A. a = 0

B. $b = e^3$

C. $a = 1$

D. $b = \log_e 3$

Answer: A::D

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4. Given $f(x) = \left\{3 - \left[\cot^{-1}\left(\frac{2x^3 - 3}{x^2}\right)\right] f \text{ or } x > 0 \text{ and } \{x^2\}\cos\left(e^{\frac{1}{x}}\right) f \text{ or } x <$ (where {} and [] denotes the fractional part and the integral part functions respectively). Then which of the following statements do/does not hold good?

A. $f\left(0^-\right) = 0$

B. $f\left(0^+\right) = 3$

C. If $f(0) = 0,$ then f(x) is continuous at x = 0

D. Irremovable discontinuity of f at x = 0

Answer: B::D

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5. Let $f(x) = \begin{cases} x\left[\frac{1}{x}\right] + x[x] & \text{if} \quad x \neq 0 \\ 0 & \text{if} \quad x = 0 \end{cases}$ (where [x] denotes the greatest integer function). Then the correct statement is/are

A. Limit exists for $x = -1$.

B. f(x) has a removable discontinuity at x = 1.

C. f(x) has a non removable discontinuity at x = 2.

D. f(x) is discontinuous at all positive integers.

Answer: A::B::C::D

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6. A function $f: R\overrightarrow{R}$ is defined as $f(x) = (\lim)_{n\overrightarrow{\infty}} \frac{ax^2 + bx + c + e^{nx}}{1 + \cdot e^{nx}}$ is continuous on then Point lies on the space Point represents the 2-dimensional Cartesian plane Locus of

point $(a, c) and (c, b)$ intersect at one point Point (a, b, c) lies on the plane in space

A. point (a, b, c) lies on line in space

B. point (a, b) represents the 2-dimensional Cartesian plane

C. Locus of point (a, c) and (c, b) intersect at one point

D. point (a, b, c) lies on the plane in space

Answer: A::B::C

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7. Let f be a function with continuous second derivative and $f(0) = f'(0) = 0.$ Determine a function g by $g(x) = \begin{cases} \frac{f(x)}{x}, x \neq 00, x = 0 \end{cases}$ Then which of the following statements is correct? g has a continuous first derivative g has a first derivative g is continuous but g fails to have a derivative g has a first derivative but the first derivative is not continuous

A. g has a continuous first derivative

B. g has a first derivative

C. g is continuous but g fails to have a derivative

D. g has a first derivative but the first derivative is not continuous

Answer: A::B

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8. Let $f(x)$ be a function defined on $(-a, a)$ with $a > 0$. Assume that $f(x)$ is continuous at $x = 0 and (\lim)_{x \to 0} \frac{f(x) - f(kx)}{x} = \alpha, where k \in (0, 1)$ then $f'(0^+) = 0$ b. $f'(0^-) = \frac{\alpha}{1-k}$ c. $f(x)$ is differentiable at $x = 0$ d. $f(x)$ is non-differentiable at $x = 0$

A. $f'(0^+) = 0$

B. $f'(0^-) = \frac{\alpha}{1-k}$

C. f(x) is defferentiable at x = 0

D. f(x) is non-differentiable at x = 0

Answer: B::C::D

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9. If $f(x) = x^{1/3}(x-2)^{2/3}$ for all x, then the domain of f' is a. $x \in R - \{0\}$ b. $\{x \mid x > 0\}$ c. $x \in R - \{0, 2\}$ d. $x \in R$

A. $x \in R - \{0\}$

B. $\{x \mid x > 0\}$

C. $x \in R - \{0, 2\}$

D. $x \in R$

Answer: C

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Comprehension Type

1. Consider two function $y = f(x)$ and $y = g(x)$ defined as

$$f(x) = \begin{cases} ax^2 + b & 0 \le x \le 1 \\ bx + 2b & 1 < x \le 3 \\ (a-1)x + 2c - 3 & 3 < x \le 4 \end{cases}$$

and $$g(x) = \begin{cases} cx + d & 0 \le x \le 2 \\ ax + 3 - c & 2 < x < 3 \\ x^2 + b + 1 & 3 \ge x \le 4 \end{cases}$$

$\lim_{x \to 2} \frac{f(x)}{|g(x)| + 1}$ exists and f is differentiable at x = 1. The value of limit will be

A. $k \in (-1, 0)$

B. $k \in (\infty, 0)$

C. $k \in (1, 5)$

D. $k \in (-1, 1)$

Answer: A

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2. Consider two function $y = f(x)$ and $y = g(x)$ defined as

$$f(x) = \begin{cases} ax^2 + b & 0 \le x \le 1 \\ bx + 2b & 1 < x \le 3 \\ (a-1)x + 2c - 3 & 3 < x \le 4 \end{cases}$$

and $$g(x) = \begin{cases} cx + d & 0 \le x \le 2 \\ ax + 3 - c & 2 < x < 3 \\ x^2 + b + 1 & 3 \ge x \le 4 \end{cases}$$

$\lim_{x \to 2} \dfrac{f(x)}{|g(x)| + 1}$ exists and f is differentiable at x = 1. The value of limit will be

A. -2

B. -1

C. 0

D. 2

Answer: C

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Exercises Single Correct Answer Type

1. Which of the following functions have finite number of points of discontinuity in R (where, $[\cdot]$ represents greatest integer function) ?

A. tan x

B. x[x]

C. $\frac{|x|}{x}$

D. $\sin[\pi x]$

Answer: C

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Single Correct Answer Type

1. The function $f(x) = \frac{4 - x^2}{4x - x^3}$ is

A. discontinuious at only point

B. discontinuous exaclty at two points

C. discontinous exactly at three points

D. none of these

Answer: C

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2. If $f(x) = \frac{\tan\left(\frac{\pi}{4} - x\right)}{\cot 2x}$ for $x \neq \frac{\pi}{4}$, find the value which can be assigned to $f(x)$ at $x = \frac{\pi}{4}$ so that the function $f(x)$ becomes continuous every where in $\left[0, \frac{\pi}{2}\right]$.

A. 1

B. 43467

C. 43468

D. -1

Answer: B

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3. If the function $f(x) = \dfrac{2x - \sin^{-1} x}{2x + \tan^{-1} x}$ is continuous at each point of its domain, then the value of $f(0)$ (a) 2 (b) $\dfrac{1}{3}$ (c) $-\dfrac{1}{3}$ (d) $\dfrac{2}{3}$

A. 2

B. 43468

C. 43499

D. $-\dfrac{1}{3}$

Answer: B

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4. The function $f(x) = \dfrac{\left(3^x - 1\right)^2}{\sin x \cdot \ln(1 + x)}, x \neq 0,$ is continuous at $x = 0,$ Then the value of $f(0)$ is

A. $2 \log_e 3$

B. $\left(\log_e 3\right)^2$

C. $\log_e 6$

D. none of these

Answer: B

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5. If $f(x) = \frac{x - e^x + \cos 2x}{x^2}, x \neq 0$ is continuous at x = 0, then

A. $f(0) = 5/2$

B. $[f(0)] = -2$

C. $\{f(0) = 0$

D. $[f(0)]\{f(0)\} = -1.5$

Answer: D

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6. if $f(x) = \begin{cases} \frac{8^x - 4^x - 2^x + 1}{x^2} & x > 0 \\ x^2 & x \leq 0 \end{cases}$

is continuous at x=0 , then the value of λ is

A. $4 \log_e 2$

B. $2 \log_e 2$

C. $\log_e 2$

D. none of these

Answer: C

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7. If $f(x) = \frac{a \cos x - \cos bx}{x^2}, x \neq 0 and f(0) = 4$ is continous at $x = 0$, then the ordered pair (a, b) is $(\pm 1, 3)$ b. $(1, \pm 3)$ c. $(-1, -3)$ d. $(-1, 3)$

A. $\pm 1, 3)$

B. $(1, \pm 3)$

C. $(-1, -3)$

D. $(1, 3)$

Answer: B

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8. Let f be a continuous function on R. If $f(1/4^n) = (\sin e^n)e^{-n^2} + \frac{n^2}{n^2+1}$, then f(0) is

A. 1

B. $1/2$

C. 0

D. none of these

Answer: A

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9. If $f(x) = \frac{x^2 - bx + 25}{x^2 - 7x + 10}$ for $x \neq 5$ and f is continuous at $x = 5$ then $f(5) =$

A. 0

B. 5

C. 10

D. 25

Answer: A

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10. The function $f(x) = \frac{\tan|\pi[x - \pi]|}{1 + [x]^2}$, where [x] denotes the greatest integer less than or equal to x, is

A. f(x) is discontinuous at some x

B. f(x) is continuous at all x, but the derivative f'(x) does not exist for some x

C. $f'(x)$ exists for all x, but f''(x) does not exist for some x

D. f'(x) exists for all x

Answer: D

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11. if $f(x) = \begin{cases} \frac{1-|x|}{1+x} & x \neq -1 \\ 1 & x = -1 \end{cases}$ then f([2x]) , where [.] represents the greatest integer function , is

A. discontinuous at x=-1

B. continuous at x=0

C. continuous at x=1/2

D. continuous at x=1

Answer: B

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12. Let $f(x) = \left\{ \frac{x-4}{|x-4|} + a, x < 4a + b, \frac{x-4}{|x-4|} + b, x > 4 \right.$ Then $f(x)$ is continous at $x = 4$ when $a = 0, b = 0$ b. $a = 1, b = 1$ c. $a = -1, b = 1$ d. $a = -1, b = -1$

A. a=0,b=0

B. a=1,b=1

C. a=-1,b=1

D. a=1,b=-1

Answer: D

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13. Which of the following is true about

$$Let f(x) = \begin{cases} \frac{x-2}{|x-2|}\left(\frac{x^2-1}{x^2+1}\right) & x \neq 2 \\ \frac{3}{5} & x = 2 \end{cases} ?$$

A. f(X) is continuous at x=2

B. f(x) has removable discontinuity ata x=2.

C. f(x) has non-removable discontinuity at x=2 .

D. Discontiuity at x=2 can be removed by redefining the function at x=2.

Answer: C

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14. if $f(x) = \begin{cases} x+2 & \text{if } x < 0 \\ -x^2 - 2 & \text{if } 0 \leq x < 1 \\ x & \text{if } x \geq 1 \end{cases}$

then the number of points of discontinuity of |f(x)|is (a) 1 (b) 2 (c) 3 (d) none of these

A. 1

B. 2

C. 3

D. none of these

Answer: A

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15. Which of the following statements is always true? ([.] represents the greatest integer function. a) If f(x) is discontinuous then $|f(x)|$ is discontinuous b) If $f(x)$ is discontinuous then f(|x|) is discontinuous c) f(x)=[g(x)] is discontinous when `g(x) is an integer d) none of these

A. if f(x) is discontinuous then |f(x)| is discontinuous

B. if f(x) is discontinuous , then f(|x|) is discontinuous .

C. f(x) =[g(x)] is discontinuous , when g(x) is an integer

D. none of these

Answer: D

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16. The number of point $f(x) = \begin{cases} [\cos \pi x] & 0 \leq x < 1 \\ |2x-3|[x-2] & 1 < x \leq 2 \end{cases}$ is discontinuous at Is ([.] denotes the greatest intgreal function)

A. two

B. three

C. four

D. zero

Answer: B

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17. A point where function $f(x) = [\sin[x]]$ is not continuous in $(0, 2\pi)$ [.] denotes the greatest integer $\leq x,$ is

A. $(3, 0)$

B. $(2, 0)$

C. $(1, 0)$

D. none of these

Answer: D

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18. The function $f(x) = \{x\}\sin(\pi[x])$, where $[.]$ denotes the greatest integer function and $\{.\}$ is the fraction part function , is discontinuous at

A. all x

B. all integer points

C. no x

D. x which is not an integer

Answer: C

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19. The function f(x) is defined by $f(x) = \{\log_{4x-3}(x^2 - 2x + 5)$ if 3/4 < x< 1 & x>1

4 when x=1}

A. is continuous at x=1

B. is discontinous at x==1 since $f(1^+)$ does not exist though $f(1^-)$ exsits

C. is disccontinous at x=1 since $f(1^-)$ odes not exsits though $f(1^+)$ exsits

D. is discontinuous at x=1 since neither $f(1^+)n$ or f^{1^-} exists.

Answer: D

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20. $f(x) = [x^2] - \{x\}^2$, where [.] and {.} denote the greatest integer function and the fractional part function , respectively , is

A. continuous at x=1,-1

B. continuous at x=-1 but not at x=1

C. continuous at x=1 but not at x=1

D. discontinuous at x=1 and x=-1

Answer: D

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21. if $f(x) = \{x^2\}$, where {x} denotes the fractional part of x , then

A. f(X) is continuous at x=-2 but not at x=2

B. f(x) is continuous at x=2 but not at x=-2

C. f(x) is continuous at x=2 and x=-2

D. f(x) is discontinuous at x=-2 and at x=2

Answer: B

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22. Given

$f(x) = \left\{3 - \left[\cot^{-1}\left(\frac{2x^3 - 3}{x^2}\right)\right] f \text{ or } x > 0 \text{ and } \{x^2\}\cos\left(e^{\frac{1}{x}}\right) f \text{ or } x <$

(where {} and [] denotes the fractional part and the integral part functions respectively). Then which of the following statements do/does not hold good?

A. 0

B. 1

C. -1

D. none of these

Answer: A

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23. Let f(x) be defined in the interval [0,4] such that

$$f(x) = \begin{cases} 1-x & 0 \le x \le 1 \\ x+2 & 1 < x < 2 \\ 4-x & 2 \le x \le 4 \end{cases}$$, then the number of points where f(x) is discontinuous is (a) 1 (b) 2 (c) 3 (d) none of these

A. 1

B. 2

C. 3

D. none of these

Answer: B

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24. The function defined by f(x) $= (-1)^{[x^3]}$ ([.] denotes the greatest integer function) satidfies

A. discontinuous for $x = n^{1/3}$ wheren is any integer

B. f(3/2)=1

C. f'(x)=1 for $-1 < x < 1$

D. none of these

Answer: A

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25. $f(x) = \lim_{n \to \infty} \frac{(x-1)^{2n} - 1}{(x-1)^{2n} + 1}$ is discontinuous at (A) x=0 only (B) x=2 only (C) x=0 and 2 (D) none of these

A. x= 0 only

B. x=2 only

C. x= 0 and 2

D. none of these

Answer: C

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26. Let $f: R \to R$ be given by $f(x) = \{5x \text{ ,if x in } Q \text{ , x^ } 2 + 6, \text{ if } x \in R - Q \text{ then}$

A. f is continuous at x=2 and x=3

B. f is not continuous at x=2 and x=3

C. f is continuous at x=2 but not at x=3

D. f is continuous at x=3 but not at x=2

Answer: A

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27. Let $f(x) = \lim_{n \to \infty} (\sin x)^{2n}$

A. Discontinuous at infinite number of points

B. discontinuous at $x = \frac{\pi}{2}$

C. Discontinuous at $x = -\frac{\pi}{2}$

D. none of these

Answer: D

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28. If $f(x) = \{(\sin x; x \text{ rational}) (\cos x; x \text{ is irrational})$ then the function is

A. $x = n\pi + \pi/4, n \in I$

B. $x = n\pi + \pi/8, n \in I$

C. $x = n\pi + \pi/6, n \in I$

D. $x = n\pi + \pi/3, n \in I$

Answer: A

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29. Let $f(x)\lim_{x\to\infty}\frac{\log,(2+x)-x^{2x}\sin x}{1+x^{2n}}$ then:

A. f is continuous at x=1

B. $\lim_{x\to 1^+} f(x)=\log 3$

C. $\lim_{x\to 1^+} f(x)=-\sin 1$

D. $\lim_{x\to 1^-} f(x)$ does not exist

Answer: C

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30. $f(x)=\lim_{n\to\infty}\sin^{2n}(\pi x)+\left[x+\frac{1}{2}\right]$, where [.] denotes the greatest integer function, is

A. continuous ar x=1 but discontinuous at x=3/2

B. cotinuous at x=1 but x=3//2

C. discontinuous at x= 1 and x= 3/2

D. c=discontinous at x=1 but continuous at =3/2

Answer: A

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31. If $f(x) = sgn\left(\sin^2 x - \sin x - 1\right)$ has exactly four points of discontinuity for $x \in (0, n\pi), n \in N$, then

A. minimum value of n is 5

B. maximum value of n is 6

C. there are exaclty two posoible values of n

D. none of these

Answer: C

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32. if $f(x) = x^2 - ax + 3, x$ is rational and $f(x) = 2 - x, x$ is irrational is continuous at exactly two points, then the possible values of a

A. $(2, \infty)$

B. $(- \infty, 3)$

C. $(- \infty, \ - 1), \ \cup (3, \infty)$

D. none of these

Answer: C

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33. If $f(x) = [x]\sin\left(\frac{\pi}{[x+1]}\right)$, where [.] denotes the greatest integer function, then the set of point of discontiuity of f in its domain is

A. all $x \in I - \{ - 1\}$

B. $allx \in I - \{0\}$

C. $allx \in I$

D. $allx \in I - \{-1, 0\}$

Answer: A

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34. $f(x) = \begin{cases} \frac{x}{2x^2 + |x|} & x \neq 0 \\ 1 & x = 0 \end{cases}$ then f(x) is

A. continuous but non- differentiable at x=0

B. differentiable at x=0

C. discontinuous at x=0

D. none of these

Answer: C

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35. Let a function f(x) be definded by $f(x) = \frac{x - |x-1|}{x}$

which of the following is not true ?

A. Discontinuous at x=0

B. Discontinuous at x=1

C. Not differentiable at x=0

D. Not differentiable at x=1

Answer: B

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36. Let $f(x) = \begin{cases} \min(x, x^2) & x \geq 0 \\ \max(2x, x-1) & x < 0 \end{cases}$, then which of the following is not true ?

A. f(x) is continuous at x=0

B. f(x) is not differentiable at x=1

C. f(x) is not differentiable at exactly three points

D. none of these

Answer: D

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37. The function $f(x) = \sin^{-1}(\cos x)$is

A. not different at $x = \frac{\pi}{2}$

B. differentiable at $\frac{3\pi}{2}$

C. differentiable at x=0

D. differentiable at $x = 2\pi$

Answer: B

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38. Let [x] denotes the greatest integer less than or equal to x and $f(x) = \left[\tan^2 x\right]$. Then

A. $\lim_{x \to 0}$ f(x) does not exist

B. f(x) is continuous at x=0

C. f(x) is not differentiable at x=0

D. $f'(0) = 1$

Answer: B

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39. if $f(x) = \begin{cases} 2x - [x] + x\sin(x - [x]) & x \neq 0 \\ 0 & x = 0 \end{cases}$ where [.] denotes the greatest integer function then

A. f(X) is differentiable at x=0

B. f(x) is differentiable at x=2

C. f(x) is continuous but not differentiable at x=0

D. none of these

Answer: D

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40. Which of the following function is non- differentiable ?

A. $f(x) = (e^x - 1)\left|e^{2x} - 1\right| \text{in} R$

B. $f(x) = \dfrac{x-1}{x^2+1} inR$

C. $f(x) = \begin{cases} ||x-3|-1| & x < 3 \\ \frac{x}{3}[x] - 2 & x \geq 3 \end{cases}$

where [.] represents the greatest integer function

D. $f(x) = 3(x-2)^{\frac{3}{4}} + 3inR$

Answer: D

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41. The number of value of $x \in [0, 2]$ at which $f(x) = \left|x - \frac{1}{2}\right| + |x - 1| + \tan x$ is not differentiable at (a) 0 (b) 1 (c) 3 (d) none of these

A. 0

B. 1

C. 3

D. none of these

Answer: C

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42. Which of the following fiunction is not differentiable at x=1?

A. $f(x) = \left(x^2 - 1\right)|(x - 1)(x - 2)|$

B. $f(x) = \sin(|x - 1|) - |x - 1|$

C. $f(x) = \tan(|x - 1|) + |x - 1|$

D. none of these

Answer: C

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43. $f(x) = \begin{cases} xe^{-\left(\frac{1}{x}+\frac{1}{|x|}\right)} & x \neq 0 \\ a & x = 0 \end{cases}$ the value of a , such that f(x) is differentiable at x=0, is equal to

A. 1

B. -1

C. 0

D. none of these

Answer: D

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44. about to only mathematics

A. $a = b = c = 0$

B. $a = 0, b = 0, c \in R$

C. $b = c = 0, a \in R$

D. $c = 0, a = 0, b \in R$

Answer: B

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45. Let $f(x) = \begin{cases} \sin 2x & \text{if } 0 \leq x \leq \frac{\pi}{6} \\ ax + b & \text{if } \frac{\pi}{6} < x < 1 \end{cases}$ If $f(x)$ and $f'(x)$ are continuous then a & b are (A) $a = 1, b = \frac{1}{\sqrt{2}} + \frac{\pi}{6}$ (B) $a = \frac{1}{\sqrt{2}}, b = \frac{1}{\sqrt{2}}$ (C) $a = 1, b = \frac{\sqrt{3}}{2} - \frac{\pi}{6}$ (D) None of these

A. $a = 1, b = \frac{1}{\sqrt{2}} + \frac{\pi}{6}$

B. $a = \frac{1}{\sqrt{2}}, b = \frac{1}{\sqrt{2}}$

C. $a = 1, b = \frac{\sqrt{3}}{2} - \frac{\pi}{6}$

D. none of these

Answer: C

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46. If $f(x) = \{x^3, \text{ if } x^2 < 1 \text{ and } x, \text{ if } x^2 \geq 1$ then $f(x)$ is differentiable at (a) $(-\infty, \infty) - \{1\}$ (b) $(-\infty, \infty) - \{1, -1\}$ (c) $(-\infty, \infty) - \{1, -1, 0\}$ (d) $(-\infty, \infty) - \{-1\}$

A. $(-\infty, \infty) - \{1\}$

B. $(-\infty, \infty) - \{1 - 1\}$

C. $(-\infty, \infty) - \{1 - 1, 0\}$

D. $(-\infty, \infty) - \{-1\}$

Answer: B

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47. if $f(x) = \left(x^2 - 4\right)\left|\left(x^3 - 6x^2 + 11x - 6\right)\right| + \frac{x}{1 + |x|}$ then set of points at which the function if non differentiable is

A. {-2,2,1,3}

B. {-2,0,3}

C. {-2,2,0}

D. {1,3}

Answer: D

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48. if $f(x) = \cos \pi(|x| + [x])$, where [.] denotes the greatest integer , function then which is not true ?

A. continuous at x=1/2

B. continuous at x=0

C. Differentiable in (-1,0)

D. Differentiable in (0,1)

Answer: B

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49. If $f(x) = \begin{cases} e^{x^2+x} & x > 0 \\ ax + b & x \leq 0 \end{cases}$ is differentiable at $x = 0$, then (a) $a = 1, b = -1$ (b) $a = -1, b = 1$ (c) $a = 1, b = 1$ (d) $a = -1, b = -1$

A. a=1,b=-1

B. a=-1,b=1

C. a=1,b=1

D. a=-1,b=-1

Answer: C

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50. if $f(x) = \begin{cases} x-1 & x<0 \\ x^2-2x & x \geq 0 \end{cases}$, then

A. f(|x|) is discontinuous at x=0

B. f(x) | is differerntiavble at x=0

C. |f(x)| is non - differentiable at =0,2

D. |f(x)| is comtinuous at x=0

Answer: C

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51. If $f(x) = \{|1-4x^2|, 0 \leq x < 1 \text{ and } [x^2-2x], 1 \leq x < 2$ where [.] denotes the greatest integer function, then

A. differentiable for all x

B. continnous at x=1

C. non- differntiable at x=1

D. none of these

Answer: C

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52. Show that the function $f(x) = \left\{ x^m \sin\left(\frac{1}{x}\right), \quad x \neq 0 \text{ , } 0 \ x = 0 \right.$ is continuous but not differentiable at $x = 0$, if (0 < m<1)

A. $a \in (-1, 0)$

B. $a \in (0, 2]$

C. $a \in (0, 1]$

D. $a \in [1, 2\}$

Answer: C

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53. If $x + 4|y| = 6y$, then y as a function of x is

A. continuous at x=0

B. derivable at x=0

C. $\frac{dy}{dx} = \frac{1}{2}$ for all x

D. none of these

Answer: A

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54. Let g(x) be a polynomial of degree one and f(x) be defined by $f(x) = -g(x), x \leq 0$ and $|x|^{\sin x}, x > 0$ If f(x) is continuous satisfying $f'(1) = f(-1)$, then g(x) is

A. (1+ sin1)x+1

B. (1-sin 1)x + 1

C. (sin 1 - 1)x-1

D. none of these

Answer: B

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55. If $f(x) = |1 - x|$, then the points where $\sin^{-1}(f|x|)$ is non-differentiable are

A. {0,1}

B. {0,-1}

C. {0,1,-1}

D. none of these

Answer: C

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56. Given that $f(x) = xg(x)/|x| g(0) = g'(0) = 0$ and $f(x)$ is continuous at x=0 then the value of f'(0)

A. does not exist

B. is-1

C. is 1

D. is 0

Answer: D

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57. If $f(x) = \{\sin x, x < 0$ and $\cos x - |x-1|, x \leq 0$ then $g(x) = f(|x|)$ is non-differentiable for

A. no value of x

B. exctly one value of x

C. exactly three values of x

D. none of these

Answer: C

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58. $f(x) = \max\left\{\frac{x}{n}, |\sin \pi x|\right\}, n \in N.$ has maximum points of non-differentiability for $x \in (0, 4)$, Then n cannot be (A) 4 (B) 2 (C) 5 (D) 6

A. 4

B. 2

C. 5

D. 6

Answer: B

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59. $f(x)=\begin{cases}1-\sqrt{1-x^2} & \text{if} \quad -1\leq x\leq 1\\ 1+\log\frac{1}{x} & \text{if} \quad x>1\end{cases}$ (a) continuous and differentiable at $x=1$ (b) continuous but not differentiable at $x=1$ (c) neither continuous nor differentiable at $x=1$ (d) none of these

A. continuous and differentibable at x=1

B. continuous but not differentiable at x=1

C. neither continuous nor differentiable at =1

D. none of these

Answer: B

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60. The set of all points where $f(x)=\sqrt[3]{x^2|x|}-|x|-1$ is not differentiable is

A. {0}

B. (-1,0,1}

C. {0,1}

D. none of these

Answer: D

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61. Let f(x) be a continuous function for all $x \in R$ and $f'(0) = 1$ then $g(x) = f(|x|) - \sqrt{\frac{1 - \cos 2x}{2}}, atx = 0,$

A. is differentiable at x=0 and its value is 1

B. is disfferentiable at x=0 and its value is 0

C. is non-differentaible at x=0 as its graph has sharp turn at x=o

D. is non - differentiable at x=0 as its graph has veritical tanent at x=0

Answer: B

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62. If $f(x) = \begin{cases} x\left(\frac{e^{1/x}-e^{-1/x}}{e^{1/x}+e^{1/x}}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$, then at x = 0 f(x) is

A. f(x) is discontinuous at x=0

B. f(x) is continuous but non- differentiable at x=0

C. f(x) is differtiable at x=0

D. f'(0)=2

Answer: C

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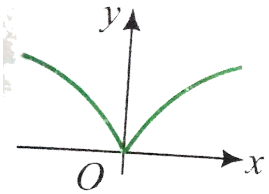
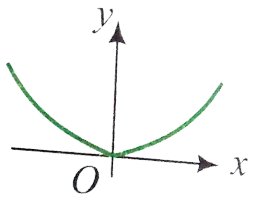
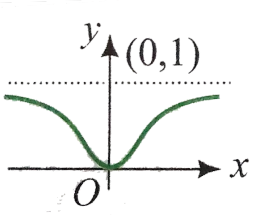
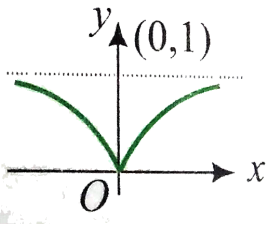
63. Let $y = f(x) = \begin{cases} e^{\frac{1}{x^2}} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ then which of the following can best represent the graph of $y = f(x)$?

A.



B. 

C. 

D. 

Answer: C

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64. If $f(2+x)=f(-x)$ for all $x \in R$ then differentiability at x=4 implies differentiability at (a) x=1 (b) x=-1 (c) x=-2 (d) cannot say anything

A. x=1

B. x=-1

C. x=-2

D. cannot say anything

Answer: C

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65. Number of points where the function

$$f(x)=\begin{cases}1+\left[\cos\frac{\pi x}{2}\right] & 1<x\leq 2\\ 1-\{x\} & 0\leq x<1\\ |\sin\pi x| & -1\leq x<0\end{cases}$$

and $f(1)=0$ is continuous but non differentiable :

(where [.] denotes greatest integer function and {.} denotes fractional part function)

A. 0

B. 1

C. 2

D. none of these

Answer: B

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66. Let $\lim_{n \to \infty} \frac{\left(x^2 + 2x + 3 + \sin \pi x\right)^n - 1}{\left(x^2 + 2x + 3 + \sin \pi x\right)^n + 1}$. then

A. f(x) is continuous and differentiable for all $x \in R$

B. f(X) is continuous but not differentiable for all $x \in R$

C. f(x) is discontinuous at infinite number of points

D. f(x) is discontinuous at finite number of points

Answer: A

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Multiple Correct Answers Type

1. Which of the statement(s) is / are incorrect ?

A. if f+g is continuous at x=a , then f and g are continuous at x=a .

B. if $\lim_{x \to a} (fg)$ exists , then both $\lim_{x \to a} f$ and $\lim_{x \to a}$ g exist

C. Discontinuity at $x = a \Rightarrow$ non - existence of limit.

D. All function definfed on a closed interval attain a maximum or minimum value in that interval.

Answer: A::B::C::D

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2. A function f is defined on an interval [a, b]. Which of the following statement(s) is are incorrect? (A) If f(a) and f(b) have opposite signs,then there must be a point $c \in (a, b)$ such that f(c)=0.

A. if f(a) and f(b) have opposite sings then there must be a point $c \in (a, b)$ such that f(c) =0

B. if f is continuous on $[a, b], f(a) < 0, \text{ and } f(b) > 0,$ then there must be point $c \in (a, b)$ such that f(c) =0.

C. if f is continuous on [a,b] , and there is point c in (a,b) such that f(c) -0 , then f(a) and f(b) have opposite signs

D. if f has no zeros on [a,b] then f(a) and f(b) have the same sign.

Answer: A::C::D

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3. Which of the following function has / have a removable discontinuity at the indicated point ?

A. $f(x) = \frac{x^2 - 2x - 8}{x + 2}$ at x=-2

B. $f(x) = \frac{x - 7}{|x - 7|}$ at x=7

C. $f(x) = \frac{x^3 + 64}{|x + 4|}$ at x=-4

D. $f(x) = \frac{3 - \sqrt{x}}{9 - x}$ at x=9

Answer: A::C::D

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4. Which of the following function (s) has/have removable discontinuity at $x = 1$ (A) $f(x) = \dfrac{1}{\ln(|x|)}$ (B) $f(x) = \dfrac{x^2 - 1}{x^3 - 1}$ (c) $f(x) = 2^{-2^{\frac{1}{1-x}}}$ (D) $f(x) = \dfrac{\sqrt{x+1} - \sqrt{2x}}{x^2 - x}$

A. $f(x) = \dfrac{1}{\text{in}|x|}$

B. $f(x) = \dfrac{x^2 - 1}{x^3 - 1}$

C. $f(x) = 2^{-2^{\frac{1}{1-x}}}$

D. $f(x) = \dfrac{\sqrt{x+1} - \sqrt{2x}}{x^2 - x}$

Answer: A::B::C::D

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5. If $f(x) = [|x|]$ where $[.]$ denotes the greatest integer function , then which of the following is not true ?

A. f(x) is continuous $\forall x \in R$

B. f(x) is continuous from right and discontinuous from left $\forall s \in N$

C. f(x) is conttinuous from left and discontinuous from right $\forall x \in I$

D. f(x) is continuous at x=0

Answer: B::D

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6. $f(x) = sgn(x^2 - ax + 1)$ has maximum number of points of discontinuity then

A. $a \in (2, \infty)$

B. $a \in (-\infty, -2)$

C. $a \in (-2, 2)$

D. none of these

Answer: A::B

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7. A function is defined as $f(x) = \lim_{x \to \infty} \left[\cos^{2n} x \text{ , if } x < 0\right)$, $n\sqrt{\sqrt{1+x^n},}$ if $0 \leq x, +1)$, $\frac{1}{1+x^n},$ if $x > 1$ which of the following does not hold good?

A. continuous at x=0 but discontinuous at x=1

B. continuous at x=1 but discontinuous at x=0

C. continuous both as x=1 and x=0

D. discontinuous both at x=1 and x=0

Answer: A::B::C

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8. (a) Draw the graph of

$$f(x) = = \begin{cases} 1, & |x| \geq 1 \\ \frac{1}{n^2}, & \frac{1}{n} < |x| < \frac{1}{n-1}, n = 2,3,... \\ 0, & x = 0 \end{cases}$$

(b) Sketch the region $y \leq -1$.

(c) Sketch the region $|x| < 3$.

A. is discontinuous at infinte points

B. is continuous everywhere

C. is discontinuous only at $x = \frac{1}{n}, n \in Z - \{0\}$

D. none of these

Answer: A::C

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9. Let $f(x) = [x]$ and $g(x) = \begin{cases} 0, & x \in Z \\ x^2, & x \in R - Z \end{cases}$, then (where $[\cdot]$ denotes greatest integer function)

A. $\lim_{x \to 1} g(x)$exists but g(x) oin not continuous at x=1

B. f(x) is not continuous at x=1

C. gof is continuous for all x

D. fog is continuous for all x

Answer: A::B::C

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10. If $f(x) = \sum_{r=1}^{n} a_r |x|^r$, where a_i s are real constants, then f(x) is

A. contionuous at x=1 for all a_1

B. differentiable at x=0 for all $a_{2k+1} = 0$

C. differentaiable at x=0 for all $a_i \in R$

D. none of these

Answer: A::B

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11. If $f(x) = \{(|x| - 3 \text{ when } x < 1), \text{and } (|x - 2| + a, \text{ when } x \geq 1)$ &

$g(x) = \{2 - |x| \text{when } x < 2 \text{ and } sgn(x) - b \text{ , when } x \geq 2.$

if $h(x) = f(x) + g(x)$ is discontinuous at exactly one point, then -

(a). $a = -3, b = 0$

(b). $a = -3, b = -1$

(c) $a = 2, b = 1$

(d) $a = 0, b = 1$

A. a=-3, b=0

B. a=2,b=1

C. a=2,b=0

D. a=-3,b=1

Answer: A::B

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12. $f(x)=\begin{cases}\left(\frac{3}{2}\right)^{\frac{\cot 3x}{\cot 2x}} & 0\leq x<\frac{\pi}{2}\\ b+3 & x=\frac{\pi}{2}\\ (1+|\cot x|)^{\frac{a\tan x|}{b}} & \frac{\pi}{2}<x<\pi\end{cases}$ is continuous at $x=\frac{\pi}{2}$, then

A. a=0

B. a=2

C. b=-2

D. b=2

Answer: A::C

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13. If $f(x)=sgn(\cos 2x-2\sin x+3)$, where sgn () is the signum function, then f(x)

A. is continuous over its domin

B. has a missing point discontinuity

C. has isolated point discontiuity

D. removable discontinuity

Answer: C::D

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14. Let

$$f(x) = \begin{cases} \dfrac{e^x - 1 + ax}{x^2}, & x > 0 \text{ and } b, x = 0 \text{ and } \dfrac{\sin\left(\frac{x}{2}\right)}{x}, x < 0 \end{cases},$$

then

A. f(X) is continuous at x=0 if a=- 1, $b = \frac{1}{2}$

B. f(x) is discontinuous at x=0 if b$b \neq \frac{1}{2}$

C. f(x) has irremovable diisontinuity at x-0, if $a \neq -1$

D. f(x) has removable discontinuity at $x = 0$ if $a \equiv -1, b \neq \frac{1}{2}$

Answer: A::B::C::D

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15. Find the value of a, b if $f(x) = \begin{cases} \frac{ae^{1/|x+2|}-1}{2-e^{1/|x+2|}} & ; -3 < x < -2 \\ b & ; x = -2 \\ \sin\left(\frac{x^4-16}{x^5+32}\right) & ; -2 < x < 0 \end{cases}$ is continuous at $x = -2$.

A. $a = \sin\frac{2}{5}$

B. $b = -\sin\frac{2}{5}$

C. $a = -\sin\frac{1}{5}$

D. $b = \sin\frac{1}{5}$

Answer: A::B

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16. The function $f(x) = \begin{cases} 5x - 4 & \text{for } 0 < x \leq 1 \\ 4x^2 - 3x & \text{for} 1 < x < 2 \\ 3x + 4 & \text{for} x \geq 2 \end{cases}$ *is*

A. continuous at x-1 and x=2

B. continuous at x=1 but not derivable at x=2

C. continuous at x=1 and 2 but not derivable at x=1

D. continuous at x=1 and 2 but not derivable derivable at x=1 and x=2

Answer: A::B

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17. Which of the following is true for $f(x) = sgn(x) \times \sin x$

A. (a) Discontinuous no where

B. (b) An even function

C. (c) Discontinuous everywhere

D. (d) Continuous everywhere

Answer: A::C

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18. $f(x)$ is differentiable function and $(f(x).\, g(x))$ is differentiable at $x = a$. Then (a) $g(x)$ must be differentiable at $x = a$ (b.) if $g(x)$ is discontinuous, then $f(a) = 0$ (c.) if $f(a) \neq 0$, then $g(x)$ must be differentiable (d.) none of these

A. g(X) must be differentiable at x=a

B. if g(X) is discontinuous , then f(a)=0

C. if $f(a) \neq 0,$ then g(x) must be differentiable

D. none of these

Answer: B::C

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19. Let $f(x) = \begin{cases} xe^x & x \leq 0 \\ x + x^2 - x^3 & x > 0 \end{cases}$ then the correct statement is (a) f is continuous and differentiable for all x (b) f is continuous but not differentiable at $x = 0$ (c) f is continuous and differentiable for all x. (d) f ' is continuous but not differentiable at $x = 0$

A. f is continuous and differentiable for all x,

B. f is continuous but not differentiable ata x=0

C. f is continuous and differentiable for all x.

D. f ' is continuous but not differentiable at x=0.

Answer: A::C

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20. Let $f(x) = \frac{[x]+1}{\{x\}+1}$ for $f: \left[0, \frac{5}{2}\right) \to \left(\frac{1}{2}, 3\right]$, where $[\cdot]$ represents the greatest integer function and $\{\cdot\}$ represents the fractional part of x. Draw the graph of $y = f(x)$. Prove that $y = f(x)$ is bijective. Also find the range of the function.

A. f(x) is injective discontinuous funtion

B. f(x) f(x) is surjective non - differntiable function .

C. $\min\left(\lim_{x\to 1^-} f(x),\ \lim_{x\to 1^+} f(x)\right) = f(1).$

D. max (x values of point of discontinuity function .

Answer: A::B::D

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21. If f(x)=$\begin{cases} \frac{x\log\cos x}{\log(1+x^2)} & x\neq 0 \\ 0 & x=0 \end{cases}$ then

A. f(X) is not continuous at x=0

B. f(x) is continuous at x=0

C. f(x) is continuous at x=0 but not differentiable at x=0

D. f(x) is discontinuous at x=0

Answer: B::D

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22. If $f(x) = x + |x| + \cos\left(\left[\pi^2\right]x\right)$ and $g(x) = \sin x,$ where [.] denotes the greatest integer function, then

A. f(x) + g(x) is continuous everywhere ,

B. f(x) + g(x) is differentiable everywhere

C. $f(x) \times g(x)$ is differentiable everywhere

D. $f(x) \times g(x)$ is continuous but not differentiable at x=0

Answer: A::C

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23. Let f : $R \to R$be any function and $g(x) = \frac{1}{f(x)}$ then which of the following is / are not true ? (a) g is onto of f is onto (b) g is one - one if f is onto (c) g is continuous if is continuous (d) g is differentiable if f is differentiable

A. g is onto of f is onto

B. g is one - one if f is onto

C. g is continuous if is continuous

D. g is differentiable if f is differentiable

Answer: A::C::D

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24. If

$f(x) = \{'x^2(sgn[x]) + \{x\}, 0 \leq x \leq 2''\sin x + |x-3|, 2 < x < 4,$

(where[.] & {.} greatest integer function & fractional part functiopn respectively), then -

Option 1. f(x) is differentiable at x = 1

Option 2. f(x) is continuous but non-differentiable at x

Option 3. f(x) is non-differentiable at x = 2

Option 4. f(x) is discontinuous at x = 2

A. f(x) is differentiable at x=1

B. f(x) is continuous but non - differentiable at x=1

C. f(x) is non-differnentiable at x=2

D. f(x) is discontinuous at x=2

Answer: B::C::D

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25. Which of the following function is thrice differentiable at x=0?

A. $f(x) = |x^3|$

B. $f(x) = x^3|x|$

C. $f(x) = |x|\sin^3 x$

D. $f(x) = x|\tan^3 x|$

Answer: B::C::D

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26. Let f(x) $= \left[\sin^4 x\right]$ then (where [.] represents the greatest integer function).

A. (a) f(x) is continuous at x=0

B. (b) f(x) is differentiable at x=0

C. (c) f(x) is non-differnentiable at x=0

D. (d) $f'(0) = 1$

Answer: A::B

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27. $f: [0, 1] \to R$is defined as

$f(x)\begin{cases} x^3(1-x)\sin\left(\frac{1}{x^2}\right) & 0 < x \leq 1 \\ 0 & x = 0 \end{cases}$ (a) f is continuous but not derivable in [0,1] (b) f is ontinuous in [0,1] (c) f is bounded in [0,1] (d) f' is bounded in [0,1]

A. f is continuous but not derivable in [0,1]

B. f is ontinuous in [0,1]

C. f is bounded in [0,1]

D. f' is bounded in [0,1]

Answer: B::C::D

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28. Let $f(x) = \begin{bmatrix} x^2 + a & 0 \leq x < 1 \\ 2x + b & 1 \leq x \leq 2 \end{bmatrix}$ and $g(x) = \begin{bmatrix} 3x + b & 0 \leq x < 1 \\ x^3 & 1 \leq x \leq 2 \end{bmatrix}$

If derivative of $f(x)$ w.r.t. $g(x) atx = 1$ exists and is equal to λ, then which of the followig is/are correct?

A. a=-1

B. b=-2

C. $\left(\frac{df}{dg}\right)_{x=1} = 2/3$

D. $a = \pm 1, b = \pm 2$

Answer: A::B::C

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29. If $f(x)=\begin{cases}\frac{a\cos x+bx\sin x+ce^{x}-2x}{x^{2}} & x\neq 0\\ 0 & x=0\end{cases}$ is differentiable at $x=0$, then (a) $a+b+c=2$ (b) $a+b=-4$ (c) $f'(0)=\frac{1}{3}$ (d) $a-c=4$

A. $a+b+c=2$

B. $a+b=-4$

C. $f'(0)=1/3$

D. $a-c=4$

Answer: B::C

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Linked Comprehension Type

1. Let $f(x) = \begin{cases} \frac{a(1-x\sin x)+b\cos x+5}{x^2}, & x<0 \\ 3, & x=0 \\ \left[1+\left(\frac{cx+dx^3}{x^2}\right)\right]^{1/x}, & x>0 \end{cases}$ If f is continuous at x = 0, then (a + b + c + d) is

A. [-10,10]

B. [-5,5]

C. [-12,12]

D. none of these

Answer: B

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2. Let $f(x) = \begin{cases} \frac{a(1-x\sin x)+b\cos x+5}{x^2}, & x<0 \\ 3, & x=0 \\ \left[1+\left(\frac{cx+dx^3}{x^2}\right)\right]^{1/x}, & x>0 \end{cases}$ If f is continuous at x = 0, then (a + b + c + d) is

A. $\log_e 9$

B. $\log_e 2$

C. 2

D. 1

Answer: A

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3. Let $f(x) = \begin{cases} \frac{a(1-x\sin x)+b\cos x+5}{x^2}, & x<0 \\ 3, & x=0 \\ \left[1+\left(\frac{cx+dx^3}{x^2}\right)\right]^{1/x}, & x>0 \end{cases}$ If f is continuous at x = 0, then (a + b + c + d) is

A. only one real , positive root

B. only one real , negative root

C. three real roots

D. none of these

Answer: B

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4. Consider

$f(x) = x^2 + ax + 3$ and $g(x) = x + b$ and $F(x) = \lim_{n \to \infty} \frac{f(x) + x^{2n} g(x}{1 + x^{2n}}$

If F(x) is continuous at x=-1, then

A. b=a+3

B. b=a-1

C. a=b-2

D. none of these

Answer: A

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5. Consider

$f(x) = x^2 + ax + 3$ and $g(x) = x + b$ and $F(x) = \lim_{n \to \infty} \frac{f(x) + x^{2n} g(x}{1 + x^{2n}}$

If F(x) is continuous at x=-1, then

A. a+b=-2

B. a-b=3

C. a+b=5

D. none of these

Answer: C

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6. Consider

$f(x) = x^2 + ax + 3$ and $g(x) = x + b$ and $F(x) = \lim_{n \to \infty} \frac{f(x) + x^{2n} g(x}{1 + x^{2n}}$

If F(x) is continuous at x=-1, then

A. imaginary roots

B. both the roots positive

C. both the roots negative

D. roots of oppostie signs

Answer: D

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7. $Let f(x) = \begin{cases} x+2 & 0 \le x < 2 \\ 6-x & x \ge 2 \end{cases}, g(x) = \begin{cases} 1+\tan x & 0 \le x < \frac{\pi}{4} \\ 3-\cot x & \frac{\pi}{4} \le x < \pi \end{cases}$

$f(g(x))$ is

A. discontinuous at $x = \pi/4$

B. differentiable at $x = \pi/4$

C. continuous but non - differentiable $x = \pi/4$

D. differentiable at$x = \pi/4$, but derivative is not continuous

Answer: C

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8. $Let f(x) = \begin{cases} x+2 & 0 \le x < 2 \\ 6-x & x \ge 2 \end{cases}, g(x) = \begin{cases} 1+\tan x & 0 \le x < \frac{\pi}{4} \\ 3-\cot x & \frac{\pi}{4} \le x < \pi \end{cases}$

$f(g(x))$ is

A. 1

B. 2

C. 3

D. 4

Answer: B

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9. $Let f(x) = \begin{cases} x+2 & 0 \le x < 2 \\ 6-x & x \ge 2 \end{cases}, g(x) = \begin{cases} 1+\tan x & 0 \le x < \frac{\pi}{4} \\ 3-\cot x & \frac{\pi}{4} \le x < \pi \end{cases}$

$f(g(x))$ is

A. $(-\infty, \infty)$

B. $(4, \infty)$

C. $(-\infty, 4]$

D. none of these

Answer: C

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10.

$$Let f(x) = \begin{cases} [x] & -2 \leq x \leq -\frac{1}{2} \\ 2x^2 - 1 & -\frac{1}{2} < x \leq 2 \end{cases} \text{ and } g(x) = f(|x|) + |f(x)|,$$

where [.] represents the greatest integer function .

the number of point where $|f(x)|$ is non - differentiable is

A. 3

B. 4

C. 2

D. 5

Answer: A

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11.

$$Let f(x) = \begin{cases} [x] & -2 \leq x \leq -\frac{1}{2} \\ 2x^2 - 1 & -\frac{1}{2} < x \leq 2 \end{cases} \text{ and } g(x) = f(|x|) + |f(x)|,$$

where [.] represents the greatest integer function .

the number of point where g(x) is non - differentiable is

A. 4

B. 5

C. 2

D. 3

Answer: D

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12.

$$Let f(x) = \begin{cases} [x] & -2 \leq x \leq -\frac{1}{2} \\ 2x^2 - 1 & -\frac{1}{2} < x \leq 2 \end{cases} \text{ and } g(x) = f(|x|) + |f(x)|,$$

where [.] represents the greatest integer function .

the number of point where g(x) is discontinuous is

A. 1

B. 2

C. 3

D. none of these

Answer: B

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13. Given the continuous fuunction

$$y = f(x) = \begin{cases} x^2 + 10x + 8 & x \leq -2 \\ ax^2 + bx + c & -2 < x < 0, a \neq 0 \\ x^2 + 2x & x \geq 0 \end{cases}$$

if a line L touches the graph of y=f(x) at three points , then

if y= f(x) is differentiable at x=0, then the value of B

A. is -1

B. is 2

C. is 4

D. connot be determined

Answer: B

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14. $f(x) = \begin{cases} b\sin^{-1}\left(\frac{x+c}{2}\right) & -\frac{1}{2} < x < 0 \\ \frac{1}{2} & x = 0 \\ \frac{e^{ax/2}-1}{x} & 0 < x < \frac{1}{2} \end{cases}$ If f(x) is differentiable at x=0 and $|c| < \frac{1}{2}$, then find the values of a and prove that $64b^2 - \left(4 - c^2\right)$

A. $1/2$

B. 1

C. $3/2$

D. 2

Answer: A

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15. $f(x) = \begin{cases} b\sin^{-1}\left(\frac{x+c}{2}\right) & -\frac{1}{2} < x < 0 \\ \frac{1}{2} & x = 0 \\ \frac{e^{ax/2}-1}{x} & 0 < x < \frac{1}{2} \end{cases}$ If f(x) is differentiable at x=0 and $|c| < \frac{1}{2}$, then find the values of a and prove that $64b^2 - \left(4 - c^2\right)$

A. $16b^2 = 4 - c^2$

B. $16b^2 = 1 - 4c^2$

C. $64b^2 = 4 - c^2$

D. none of these

Answer: C

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Numerical Value Type

1. The number of points of discontinuity for f(x) = sgn(sin x), $x \in [0, 4\pi]$ is _____.

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2. The number of points where $f(x) = sgn\left(x^2 - 3x + 2\right) + [x - 3], x \in [0, 4],$ is discontinuous is (where [.] denotes the greatest integer function)____.

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3. Number of points where $f(x) = \sqrt{x^2} + [x]^2, x \in [-2, 2]$ is discontinuous is (where [.] re[presents the greatest interger function)_____.

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4. Let $f(x) \lim_{x \to \infty} \frac{\log,(2+x) - x^{2x} \sin x}{1 + x^{2n}}$ then:

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5. Let

$f(x) = \left\{ \frac{x}{2} - 1, 0 \leq x \leq 1\frac{1}{2}, 1 \leq x \leq 2 \right\} g(x) = (2x+1)(x-k) + 3, 0$

then $g(f(x))$ is continuous at x=1 if k equal to:

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6. A differentiable function f Is satifying the relation $f(x+y) = f(x) + f(y) + 2xy(x+y) - \frac{1}{3} \forall x, y \in R$ and $\lim_{h \to 0} \frac{3f(h) -}{6h}$. Then the value of [f(2)]is (where [x] represents the greatest integer function)_____.

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7. The least integral value of p for which f"(x) is everywhere continuous where $f(x) = \left\{ x^p \sin\left(\frac{1}{x}\right) + x|x|, x \neq 0 \text{ and } 0, x = 0 \right.$ is _____'

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8. The number of points where $f(x) = [x + 1/3] + [x + 2/3]$, [.] denotes the greatest integer function , is discontinuous for $x \in (0, 3)$ is _____.

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9. $Let f(x)$ and $g(x)$ be two continuous function and $h(x) = \lim_{n \to \infty} \frac{x^{2n}.\, f(x) + x^{2m}.\, g(x)}{(x^{2n} + 1)}$. if the limit of h(x) exists at x=1, then one root of f(x)-g(x) =0 is _____.

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10. Number of points of discontinuity of $f(x) = \left[x^3 + 3x^2 + 7x + 2\right]$, where [.] represents the greatest integer function in $[0, 1]$ is _____.

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11. If $f(x) = \dfrac{x}{1 + (\log x)(\log x)\ldots. \infty}$, $\forall x \in [1, 3]$ is non-differentiable at x = k. Then, the value of $\left[k^2\right]$, is (where $[\cdot]$ denotes greatest integer function).

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12. If the function $f(x) = \dfrac{\tan(\tan x) - \sin(\sin x)}{\tan x - \sin x}(x \neq 0)$: is continuous at $x = 0$,then find the value of $f(0)$

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13. The number of points of non- differentiability of function f(x) = max $\{\sin^{-1}|\sin x|, \cos^{-1}|\sin x|\}, 0 < x < 2\pi,$ is _____.

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14. The function f(x) is discontinuous only at x = 0 such that $f^2(x) = 1 \forall x \in R$. The total number of such functions is

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Jee Advanced Previous Year

1. Let $f(x) = \begin{cases} x^2\left|\cos\frac{\pi}{x}\right|, & x \neq 0, x \in R \\ 0, & x = 0 \end{cases}$, then f is

A. differentiable both at x=0 at x=2

B. differentiable at x=0 but not differentiable at x=2

C. not differentiable at x=0 but differentiable at x=2

D. differentiable neither at x=0 nor at x=2

Answer: B

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2. If $f(x) = \begin{cases} -x - \frac{\pi}{2} & x \leq -\frac{\pi}{2} \\ -\cos x & -\frac{\pi}{2} < x \leq 0 \\ x - 1 & 0 < x \leq 1 \\ \text{In } x & x > 1 \end{cases}$ then which one of the following is not correct?

A. f(x) is continuous at $x = -\pi/2$

B. f(x) is not differentiable at x=0

C. f(x) is differtiable at x=1

D. f(x) is differentiable at x=-3/2

Answer: A::B::C::D

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3. Let $f: R \to R$ be a function such that $f(x+y) = f(x) + f(y),\ \forall x, y \in R$. If f(x) is differentiable at x = 0, then

A. f(x) is differentiable only in a finite interval containing zero

B. f(x) is continuous $\forall x \in R$

C. f'(x) is constant $\forall x \in R$

D. f(x) is differentiable except at finitely many points

Answer: B::C

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4. Let $a, b \in \mathbb{R}$ and $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = a\cos\left(\left|x^3 - x\right|\right) + b|x|\sin\left(\left|x^3 + x\right|\right)$. Then f is

A. differentiable at x=0 if a=0 and b=1

B. differentiable at x=1 if a=1 and b=0

C. NOT differentiable at x=0 if a=1 and b=0

D. Not differentiable at x=1 if a=1 and b=1

Answer: A::B

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5. Let $f: \left[-\frac{1}{2}, 2\right] \to R$ and $g: \left[-\frac{1}{2}, 2\right] \to R$ be function defined by $f(x) = [x^2 - 3]$ and $g(x) = |x|f(x) + |4x - 7|f(x)$ where [y] deonotes the greatest integer less than or equal to y for $y \varepsilon r$. Then

A. f is discontinuous exactly at three points in $\left[-\frac{1}{2}, 2\right]$

B. f si discontinuous exactly at four points in $\left[-\frac{1}{2}, 2\right]$

C. g is NOT differentiable exactly at four points in $\left(-\frac{1}{2}, 2\right)$

D. g is NOT differentiable exactly at five points in $\left(-\frac{1}{2}, 2\right)$

Answer: B::C

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6. Let $[x]$ be the greatest integer less than or equal to x. Then, at which of the following point (s) function $f(x) = x\cos(\pi(x + [x]))$ is discontinuous? $x = 1$ (b) $x = -1$ (c) $x = 0$ (d) $x = 2$

A. x=-1

B. x=0

C. x=2

D. x=1

Answer: B::C::D

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7. Let $f: R \to R$ and $g: R \to R$ be respectively given by $f(x) = |x| + 1$ and $g(x) = x^2 + 1$. Define $h: R \to R$ by $h(x) = \{ \max \{f(x), g(x)\}, \text{ if } x \leq 0 \text{ and } \min \{f(x), g(x)\}, \text{ if } x >$ .The number of points at which $h(x)$ is not differentiable is

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Illustration

1. A function $f(x)$ satisfies the following property: $f(x\dot{y}) = f(x)f(y)$. Show that the function $f(x)$ is continuous for all values of x if it is continuous at $x = 1$.

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2. Statement-1: If $|f(x)| \leq |x|$ for all $x \in R$ then $|f(x)|$ is continuous at 0. Statement-2: If $f(x)$ is continuous then $|f(x)|$ is also continuous.

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3. Let $f(x) = \left\{\frac{\log(1+x)^{1+x} - x}{x^2}\right\}$. Then find the value of $f(0)$ so that the function f is continuous at $x = 0$.

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4. $x^2 + (f(x) - 2)x - \sqrt{3}.\ f(x) + 2\sqrt{3} - 3 = 0,$ then the value of $f(\sqrt{3})$

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5. Let : $f(x) = \begin{cases} \frac{a+3\cos x}{x^2} & x < 0 \\ b\tan\left(\frac{\pi}{[x+3]}\right) & x \geq 0 \end{cases}$ If $f(x)$ is continuous at $x = 0,$ then find $a\ and\ b$, where $[.]$ denotes the greatest integer function.

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6. If the function f as defined below is continuous at x=0find the values of a,b and c

$f(x) = \left\{ \frac{\sin(a+1)x + \sin x}{x}, x < 0 \text{ and } c, x = 0, \text{ and } \frac{\sqrt{x + bx^2} - \sqrt{x}}{bx^{\frac{3}{2}}} \right.$

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7. If the function $g(x)\begin{cases}\frac{e^{px}+\log_e(1+4x)+q}{x^3}, x\neq 0\\ r, x=0\end{cases}$

continuous at x=0 , then find the values of p,q and r.

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8. Find the points of discontinuity of the following fiunctions over R.

$(i) f(x)=\frac{1}{2\sin x-1}$ $(ii) f(x)=\frac{1}{x^2-3|x|+2}$

$(iii) f(x)=\frac{1}{x^4+x^2+1}$

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9. What kind of discpntinuity following functions have ? If possible , redefine the function to make it continuous .

(a) $f(x)=[x] at x=0,$ where [x] repesents greates interger function

(b) $f(x)=[x]+[-x] at x=1,$ where [x] represents greatest integer fuction .

(c) $f(x)=\frac{e^1}{2} at x=0.$

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10. What type of discontinuity does $f(x) = \frac{1}{\log_e|x^2-4|}$ have?

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11. What type of discontinuity does $f(x) = \cos^{-1}\left(\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)$ have ?

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12. Let $f(x)$ be a function defined as $f(x) = \begin{cases} \frac{x^2-1}{x^2-2|x-1|-1}, & x \neq 1 \\ \frac{1}{2}, & x = 1 \end{cases}$ Discuss the continuity of the function at $x = 1$.

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13. if $f(x)=\begin{cases}\cos^{-1}\{\cos x\} & x<\frac{\pi}{2}\\ \pi[x]-1 & x\geq\frac{\pi}{2}\end{cases}$. where [.] repesents greatest funcrtion and {.} represents fractional part function , then discuss the continuity of f(x) at $x=\frac{\pi}{2}$.

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14. Test the continnuity of f(x) at x=0 if

$$f(x)=\begin{cases}(x+1)^{2-\left(\frac{1}{|x|}+\frac{1}{x}\right)} & x\neq 0\\ 1 & x=0\end{cases}.$$

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15. Discuss continuity of $f(x)=[\sin x]-[\cos x]$ at $x=\pi/2$, where [.] represent the greatest integer function .

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16. Let $f(x) = \begin{cases} \frac{(\log)_e \cos x}{1 + x^2 4 - 1}, x > 0 \frac{e^{\sin 4x} - 1}{(\log)_e (1 + \tan 2x), x < 0} \end{cases}$ Find the value of $f(0)$ which makes the function continuous at $x = 0$

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17. A function f(x) is defined as followings : $f(x) = \{ax - b, x \leq 1, 3x, 1 < x < 2, bx^2 - a, x \geq 2$ Prove that if f(x) is continuous at x=1 but discontinuous at x =2. then the locus of the point (a,b) is a straight excluding the point where it cuts the line $y = 3$.

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18. Let F and G be two real-valued function defined on R (the set of all real numbers)

F(x)=$\begin{cases} |x + 1| \\ x \end{cases}$ {:(,xle0,),(,xgt0,):}$G(x) =$ {:{(,|x|+1,),(,-|x-2,):} $\begin{matrix} x \leq 1 \\ x > 1 \end{matrix}$

and $H(x) = F(x) + G(x)$.

Point of discontinuity of H(x) are

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19. If $f: R \to R$ is a function defined by $f(x) = [x]\cos\left(\frac{2x-1}{2}\right)\pi$, where [x] denotes the greatest integer function, then f is

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20. Find the points of discontinuity of following functions (where ,[.] repesents greatest integer fuction)

(i) $f(x) = [x/3], x \in [0, 30]$

$(ii) f(x) = [\log_e x]$

$(iii) f(x) = \left[\sin^{-1} x\right]$

$f(x) = \left[\frac{2}{1+x^2}\right], x > 0$

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21. Draw the graph and find the points of discontinuity $f(x) = [2\cos x]$, $x \in [0, 2\pi]$. ([.] represents the greatest integer function.)

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22. Draw the graph and discuss the continuity of $f(x) = [\sin x + \cos x], x \in [0, 2\pi]$, where [.] represents the greatest integer function.

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23. If the function $f(x) = \left[\frac{(x-2)^3}{a}\right] \sin(x-2) + a\cos(x-2)$, $[.]$ denotes the greatest integer function, is continuous in $[4, 6]$, then find the values of a.

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24. Discuss continuity of

(i) $f(x) = sgn\left(x^3 - x\right)(ii) f(x) = sgn(2\cos x - 1)$

(iii) $f(x) = sgn\left(x^2 - 2x + 3\right)$

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25. If $f(x) = sgn(2\sin x + a)$ is continuous for all x, then find the possible values of a.

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26. Discuss the continuity of $f(x) = |x| sgn\left(x^3 - x\right)$

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27. if $f(x)\begin{cases} sgn(x-2) \times [\log_e x] & 1 \le x \le 3 \\ \{x^2\} & 3 < x \le 3.5 \end{cases}$

where [.] dentes inteatest function ans {.} repesents fractional part function find the points where the continuity of f(x) shoukd be checked , Hence find he points of discontinuity .

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28. Discuss the continuity of $f(x) = (\lim)_{n \to \infty} \frac{x^{2n} - 1}{x^{2n} + 1}$

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29. Discuss continuity of function $f(x) = 1 + \lim_{n \to \infty} \cos^{2n} x$. Draw the graph of the function and find the period of the function .

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30. Find the values of a if $f(x) = (\lim)_{n \to \infty} \frac{ax^{2n} + 2}{x^{2n} + a + 1}$ is continuous at $x = 1$.

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31. Discuss the continuity of function

$$f(x) = \begin{cases} 1 & \text{if x is rational} \\ 0 & \text{if x is irrational} \end{cases}$$

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32. Let f(x) be given that $f(x) = \begin{cases} x & \text{if x is rational} \\ 1-x & \text{if x is irrational} \end{cases}$

The number of points at which f(x) is continuous, is

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33. For $x > 0, leth(x) = \begin{cases} \frac{1}{q}, & \text{if } x = \frac{p}{q} \text{ and } 0, \text{ if } xisirrational \end{cases}$ where $p, q > 0$ are relatively prime integers. Then prove that $f(x)$ is continuous for all irrational values of x.

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34. If $f(x) = \frac{x+1}{x-1} and g(x) = \frac{1}{x-2}, then$ discuss the continuity of $f(x), g(x), and fog(x)$.

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35. If $f(x) = \begin{cases} x-2 & x \leq 0 \\ 4-x^2 & x > 0 \end{cases}$, $discusscont \in uityofy$=f(f(x))

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36. Show that the function $f(x) = (x-a)^2(x-b)^2 + x$ takes the value $\frac{a+b}{2}$ for some value of $x \in [a, b]$.

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37. Using intermediate value theorem, prove that there exists a number x such that $x^{2005} + \frac{1}{1+\sin^2 x} = 2005.$

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38. Let $f(x)$ be a continuous function defined for $1 \leq x \leq 3$. If $f(x)$ takes rational values for all x and $f(2) = 10$ then the value of $f(1.5)$ is :

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39. Let $f: [0, 1] \overrightarrow{0, 1}$ be a continuous function. Then prove that $f(x) = x$ for at least one $0 \leq x \leq 1$.

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40. Let $f(x)$ be a function satisfying the condition $f(-x) = f(x)$ for all real x. If $f'(0)$ exists, then its value is equal to

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41. Discuss the differentiability of $f(x) = (2x - 5)^{3/5}$ at $x = \frac{5}{2}$.

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42. Discuss the differentiability of f(x)=$\begin{cases} \frac{\sin x^2}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$ at x=0

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43. Discuss the differentiability of $f(x) = \{x \sin(1nx^2), x \neq 00, x = 0atx = 0$

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44. Discuss the differentiability of $f(x) = \begin{cases} (x-e)2^{-2^{\left(\frac{1}{(e-x)}\right)}}, x \neq e \end{cases}$ and 0 at x=e . at the point x=e

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45. $f(x) = \sqrt{1-\sqrt{1-x^2}}$ then at $x = 0$,value of f(x) is

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46. Prove the each of the following function is differentiable at x=0.

$(i) f(x) = \cos|x| (ii) f(x) = x|x|$

$(iii) f(x) = |x^3| (iv) f(x) = \frac{x}{1+|x|}$

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47. If

$f(x) = \{x, x \leq 1, x^2 + bx + c, x > 1'' f \in db$ and c if $functioniscont$

x=1`

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48. Find the values of a and b if $f(x) = \{a + \sin^{-1}(x+b), x \geq 1$ and $x, x < 1$ is differentiable at $x = 1.$

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49.

$f(x) = \{ax(x-1) + b, x < 1x - 1, 1 \leq x \leq 3. px^2 + qx + 2, x > 3$

Find the values of the constants $a, b, pandq$ so that all the following conditions are satisfied $f(x)$ is continuous for all x. $f(1)$ does not exist. $f'(x)$ is continuous at $x = 3$

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50. $Let f(x) = \begin{bmatrix} a\sqrt{x+1} & 0 < x < 3 \\ bx + 2 & 3 \leq x < 5 \end{bmatrix}$ if f(x) is differentiable at x=3 then find the values of a and b.

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51. Discuss the differentiability of $f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

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52. Discuss the differentiability of $f(x) = \sin|x|$

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53. Drew the graph of the function and find the points of non-differentiability .(i) f(x) = min ,{x, sin x}.`

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54. If $f(x) = \max\left\{x^2 + 2ax + 1, b\right\}$ has two points of non-differentiability, then prove that $a^2 > 1 - b$

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55. Test the continuity and differentiability of the function $f(x) = \left|\left(x + \frac{1}{2}\right)[x]\right|$ by drawing the graph of the function when $-2 \leq x < 2$, where $[.]$ represents the greatest integer function.

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56. Discuss the continuity and differentiability of the function $f(x) = |x| + |x - 1|$ in the interval (-1, 2).

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57. Discuss the differentiability of $f(x) = [x] + |1 - x|, x \in (-1, 3), where [.]$ represents greatest integer function.

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58. The set of points, where f(x)= $\frac{x}{1 + |x|}$ is differentiable, is

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59. Discuss the differentiability of $f(x) = (x^2 - 1)|x^2 - x - 2| + \sin(|x|)$.

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60. Discuss the differentiability of $f(x) = |x|\sin x + |x| - 2sgn(x-2) + |x-3|$.

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61. Discuss differentiability of $f(x) = \left(e^{|x|-1} - 1\right)|x^4 - x^2|$.

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62. Let $f(x) = e^{x-1} - ax^2 + b$ and $g(x) = \begin{cases} e^{x-1} & x \leq 1 \\ x^2 + 1 & x > 1 \end{cases}$, then find the values of a and b such that $f(x) \times g(x)$ is differentiable at $x = 1$.

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63. Prove that function $f(x) = (\sin \pi x)(x-1)^{1/5}$ is continuous and differentiable at x=1

Also show that f'(x) is differentiable at x=1.

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64. The set of points where $x^2|x|$ is thrice differentiable, is

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Solved Examples

1. Examine the continuity at $x = 0$ of the sum function of the infinite series:

$$\frac{x}{1+x} + \frac{x}{(x+1)(2x+1)} + \frac{x}{(2x+1)(3x+1)} + \ldots\ldots\ldots\ldots\ldots\infty$$

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2. A function f(x) is defind as

$$f(x) = \begin{cases} x^2 + ax + 1 & \text{x is rational} \\ ax^2 + 2x + b & \text{x is irrational} \end{cases}$$

is continuous at x=1and 3 , them find the values of a and b.

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3. Let [x] denote the greatest integer less than or equal to x and g (x) be given by $g(x) = \begin{cases} [f(x)] & x \in (0,\pi/2) \cup (\pi/2,\pi) \\ 3 & x = \frac{\pi}{2} \end{cases}$ where, $f(x) = \frac{2(\sin x - \sin^n x) + |\sin x - \sin^n x|}{2(\sin x - \sin^n x) - |\sin x - \sin^n x|}, n \in R^+$ then at $x = \frac{\pi}{2}, g(x)$, is

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4. The function $f(x) = [x] + \sqrt{\{x\}}$, where [.] denotes the greatest Integer function and {.} denotes the fractional part function respectively, is discontinuous at

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5. The function $y = f(x)$ is defined by $x = 2t - |t|, y = t^2 + |t|, t \in R$ in the interval $x \in [-1, 1]$, then

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6. If $f(x) = \begin{cases} x - 3, & x < 0 \\ x^2 - 3x + 2, & x \geq 0 \end{cases}$ and let $g(x) = f(|x|) + |f(x)|$. Discuss the differentiability of g(x).

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7. Let $f: R \to R$ satisfying $|f(x)| \leq x^2$, $\forall x \in R$, then show that f(x) is differentiable at x = 0.

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8. If a function $f: [-2a, 2a] \to R$ is an odd function such that, $f(x) = f(2a - x)$ for $x \in [a, 2a]$ and the left-hand derivative at $x = a$

is 0, then find the left-hand derivative at $x = -a$.

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9. If f(x) is continuous and differentiable function $f\left(\frac{1}{n}\right) = 0 \forall n \leq 1$ and $n \in Z$. then prove that $f(0) = 0$ and $f'(0) = 0$

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10. If f(x) is continuous and differerntiable function such that $f\left(\frac{1}{n}\right) = 0$ for all $n \in N$, then

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11. Let $f(x) = x^3 - x^2 - x + 1$ and $g(x) = \{\max\{f(t); 0 \leq t \leq x\}, 0 \leq x \leq 1, 3 - x, 1 \leq x \leq 2$ Discuss

the continuity and differentiability of the function g (x) in the interval (0, 2).

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12. Let $\alpha \in R$. Prove that a function $f: R \to R$ is differentiable at α if and only if there is a function $g: R \to R$ which is continuous at α and satisfies $f(x) - f(\alpha) = g(x)(x - \alpha)$, $\forall x \in R$.

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13. $f'(0) = \lim_{n \to \infty} nf\left(\frac{1}{n}\right)$ and $f(0) = 0$ Using this, find `lim_(n->oo) ((n+1)(2/pi)cos^(- 1)(1/n)-n)),|cos^(-1)1/n|

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14. about to only mathematics

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Concept Application Exercise 4 1

1. A function $f(x)$ satisfies the following property: $f(x\dot{y}) = f(x)f(y)$. Show that the function $f(x)$ is continuous for all values of x if it is continuous at $x = 1$.

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2. Find the value of $f(0)$ so that the function. $f(x) = \frac{\sqrt{1+x} - 1 + x3}{x} becomescont \in uousatx = 0$

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3. The function $f: R\sim\{0\}\overrightarrow{R}$ given by $f(x) = \frac{1}{x} - \frac{2}{e^{2x} - 1}$ can be made continuous at x = 0 by defining f(0) as (1) 2 (2) -1 (3) 0 (4) 1

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4. *Let* $f(x) = \dfrac{1-\tan x}{4x-\pi}, x \neq \dfrac{\pi}{4}, x \in \left[0, \dfrac{\pi}{2}\right]$, If $f(x) is$ continuous in $\left[0, \dfrac{\pi}{4}\right]$, then find the value of $f\left(\dfrac{\pi}{4}\right)$.

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5. If $f(x) = \left(\tan\left(\dfrac{\pi}{4} + (\log)_e x\right)\right)^{(\log)_x e}$ is to be made continuous at $x = 1$, then what is the value of $f(1)$?

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6. If the function $f(x) = \dfrac{x^2 - (A+2)x + A}{x-2}, f$ or $x \neq 2 and f(2) = 2$, is continuous at $x = 2$, then find the value of A.

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7. The values of a and b so that the function

$$f(x)=\begin{cases} x+a\sqrt{2}\sin x, & 0\le x<\pi/4 \\ 2x\cot x+b, & \pi/4\le x\le\pi/2 \\ a\cos 2x-b\sin x, & \pi/2<x\le\pi \end{cases}$$ is continuous for $x\in[0,\pi]$, are

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8. Let

$$f(x)=\begin{cases} \dfrac{1-\cos 4x}{x^2}, & \text{if } x<0 \\ a, & \text{if } x=0 \\ \dfrac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}, & \text{if} \end{cases}$$

Determine the value of a so that $f(x)$ is continuous at $x=0$.

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9. Let $f(x)=\begin{cases} \{1+|\sin x|\}^{a/|\sin x|}, & -\pi/6<x<0 \\ b, & x=0 \\ e^{\tan 2x/\tan 3x}, & 0<x<\pi/6 \end{cases}$ Determine a and b such that f(x) is continuous at x = 0

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10. Let f (x) be a continous function in $[-1, 1]$ such that

$$f(x) = \begin{cases} \frac{\ln\left(ax^2+bx+c\right)}{x^2} & -1 \leq x < 0 \\ 1 & x = 0 \\ \frac{\sin\left(e^{x^2}-1\right)}{x^2} & 0 < x \leq 1 \end{cases}$$

Then which of the following is/are corrent

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11. Which of the following functions is not continuous $\forall x \in R$?

A. $\sqrt{2\sin x + 3}$

B. $\frac{e^x + 1}{e^x + 3}$

C. $\left(\frac{2^{2x} + 1}{2^3 + 5}\right)^{5:7}$

D. $\sqrt{sgnx + 1}$

Answer:

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12. Discuss the continuity of $f(x) = \begin{cases} \frac{x^2}{|x|}, x \neq 00, x = 0 \end{cases}$

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13. Let $f(x) = \begin{cases} (1+3x)^{\frac{1}{x}}, x \neq 0 \text{ and } e^3, x = 0. \end{cases}$ Discuss the continuity of $f(x)$ at $(a) x = 0$, (b) $x = 1$.

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14. Discuss the conttinuity of $f(x) = \begin{cases} \frac{x-1}{e^{\frac{1}{x-1}}} & x = 0 \\ 0 & x = 0 \end{cases}$ at x=1.

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15. Discuss the continuity of $f(x) = \begin{cases} \frac{x^4 - 5x^2 + 4}{|(x-1)(x-2)}, x \neq 1, 26, x = 112, x = 2 \end{cases}$

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16. Discuss the contiuity of the funtion f(x) =[x] +|1-x|, $-1 \leq x \leq 3$, where [.] represents the greatest integer function .

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Concept Application Exercise 4 2

1. Find the value of x in $[1, 3]$ where the function $\left[x^2+1\right]$([.] represents the greatest integer function) is discontinuous.

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2. Find the number of points of discontinuity for $f(x) = [6 \sin x], 0 \leq \pi$([.] represents the greatest integer function).

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3. Discuss the continuity of $f(x) = \left[\tan^{-1} x\right]$([.] represents the greatest integer function).

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4. Discuss the continuity of $f(x) = \left\{\cot^{-1} x\right]$({.} represents the fractional part function).

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5. Discuss the continuity of $f(x) = (\log|x|)sgn\left(x^2 - 1\right), x \neq 0.$

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6. $Let f(x) = \begin{cases} [\sin \pi x] & 0 \leq x \leq 1 \\ sgn\left(x - \frac{5}{4}\right) \times \left\{x - \frac{2}{3}\right\} & 1 \leq x \leq 2 \end{cases}$

when [.] denotes the greatest integer function and {.} represents the fractional part function ,At what points should the continuity be checked ? Hence , find the points of discontinuity .

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7. Consider $f(x) = \lim_{x-\infty} \frac{x^n - \sin x^n}{x^n + \sin x^n}$ for $x > 0, x \neq 1, f(1) = 0$ then

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8. Discuss the continuity of $f(x) \in [0, 2], where f(x) = (\lim)_{n \to \infty} \left(\sin\left(\pi \frac{x}{2}\right)\right)^{2n}$

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9. Discuss the continuity of $f(x) = \{x^2, x is rational - x^2, x is irrational.$

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10. Find the value of a for which $f(x) = \{x^2, x \in Qx + a, x \notin Q$ is not continuous at any x.

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11. If $y = \frac{1}{t^2 + t - 2}, wheret = \frac{1}{x - 1},$ then find the number of points where $f(x)$ is discontinuous.

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12. Find the points of discontinuity of the function: $f(x) = \frac{1}{1 - e^{\frac{x-1}{x-2}}}$

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13. Let $f(x) = \begin{cases} 1 + x & 0 \leq x \leq 2 \\ 3 - x & 2 < x \leq 3 \end{cases}$ then discuss the continuity of g(x) =f(f(x)).

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Concept Application Exercise 4 3

1. Prove that $f(x) = \frac{x^3}{4} - \sin \pi x + 3$ takes the value of $\frac{7}{3}$ for $x \in [-2, 2]$.

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2. Leg f be continuous on the interval [0,1] to R such that $f(0) = f(1)$. Prove that there exists a point c in $\left[\frac{0, 1}{2}\right]$ such that $f(c) = f\left(c + \frac{1}{2}\right)$.

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3. Suppose f is a continuous map from R to R and $f(f(a)) = a$ for some a. Show that there is some b such that $f(b) = b$.

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Concept Application Exercise 4 4

1. If f is an even function such that $\left(\lim\right)_{h \to 0} \frac{f(h) - f(0)}{h}$ has some finite non-zero value, then prove that $f(x)$ is not differentiable at $x = 0$.

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2. Let f(x) be a function satisfying $f(x + y) = f(x) + f(y)$ and $f(x) = xg(x) \text{For all } x, y \in R$, where g(x) is continuous. Then,

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3. $f(x) = [2x] \sin 3\pi x and f'\left(k'\right) = \lambda k\pi(- 1)^k$ (where [.] denotes the greatest integer function and $k \in N)$, then find the value of λ .

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4. let $f(x) = \begin{cases} g(x).\cos\left(\frac{1}{x}\right) & \text{if } x \neq 0 \text{ and } 0 \text{ if } x = 0 \end{cases}$, where g(x) is an even function differentiable at x= 0 passing through the origin then f'(0)

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5. If $f(x) = \left(\sin^{-1} x\right)^2 \cdot \cos\left(\frac{1}{x}\right) \text{if} x \neq 0; f(0) = 0, f(x)$ is continuous at x= 0 or not ?

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6. $Let f(x) = \begin{cases} \sqrt{x}(1 + x\sin(1/x)) & x > 0 \\ -\sqrt{(-x)}(1 + \sin(1/x)) & x < 0 \\ 0 & x = 0 \end{cases}$

Discuss differentiability at x=0.

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Concept Application Exercise 4 5

1. Discuss the continuity and differentiability of $f(x) = |x+1| + |x| + |x-1| \, \forall x \in R$; also draw the graph of $f(x)$

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2. Find x where $f(x) = \max \left\{ \sqrt{x(2-x)}, 2-x \right\}$ is non-differentiable.

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3. Discuss the differentiability of $f(x) = mim. \{|x|, |x-2|, 2-|x-1|\}$.

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4. Discuss the differentiability of function $f(x) = x - \left|x - x^2\right|$

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5. Discuss the differentiability of $f(x)|[x]|x \mid \text{in} -1 < x \leq 2,$ where [.] repesents the greatest intger function .

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6. Discuss the differentiabilty of $f(x) = \cos(\cos x)$

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7. Discuss the differentiability of $f(x) = \max\{\tan^{-1} x, \cot^{-1} x\}.$

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8. Find the values of $aandb$ if $f(x) = \{ax^2 + 1, x \leq 1x^2 + ax + b, x > 1isd \Leftrightarrow erentiableatx = 1$

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9. Discuss the differentiability of $f\left(x=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right.$

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10. Discuss the differentiability of $f(x)=\left|\left|x^2-4\right|-12\right|$.

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11. Discuss the differentiability of $f(x)=x[x]\{x\}$in interval [-1,2] , where [.] and {.} denotes the greatest integer function and fractional part fntion , respectively .

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12. Discuss the continuity and differentiability in $[0,2]$ of $f(x)=\left\{|2x-3|[x], x\geq 1 \sin\left(\frac{\pi x}{2}\right), x<1\right.$ to where [.] denotes the greatest integer function.

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13. For which values of x function f(x)=$(\sin \pi x)|x-1||x-2||x-3|$ is non - differentiable ?

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14. Number of points where $f(x)=|xsgn\left(1-x^2\right)|$ is non-differentiable is a. 0 b. 1 c. 2 d. 3

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15. $Let f(x)=\begin{cases}\left(\cos\frac{1}{x}\right)(\log(1+x))^2 & x\geq 0\\ 0 & x\leq 0\end{cases}$. Prove that f(x) if differentiable but derivative is not continuous at x=0.

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Matrix Match Type

1. Match the following lists:

List I	List II
a. $f(x) = \lvert x^3 \rvert$ is	**p.** continuous in $(-1, 1)$
b. $f(x) = \sqrt{\lvert x \rvert}$ is	**q.** differentiable in $(-1, 1)$
c. $f(x) = \lvert \sin^{-1} x \rvert$ is	**r.** differentiable in $(0, 1)$
d. $f(x) = \cos^{-1} \lvert x \rvert$ is	**s.** not differentiable at least at one point in $(-1, 1)$

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2. In the following ,[x] denotes the greatest integer less than or equal to x. Now, match th following lists:

List I	List II
a. $f(x) = x\lvert x \rvert$ is	**p.** Continuous in $(-1, 1)$
b. $f(x) = \sqrt{\lvert x \rvert}$ is	**q.** Differentiable in $(-1, 1)$
c. $f(x) = x + [x]$ is	**r.** Strictly increasing in $(-1, 1)$
d. $f(x) = \lvert x - 1 \rvert$ is	**s.** Not differentiable at least at one point in $(-1, 1)$

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