



## MATHS

### BOOKS - CENGAGE MATHS (ENGLISH)

#### DEFINITE INTEGRATION

##### Illustration Type

1. Evaluate the following definite integrals as limit of sum  $\int_1^2 x^2 dx$ .

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2. Evaluate:  $\int_a^b e^x dx$  using limit of sum

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3. Evaluate:  $\int_a^b \sin x dx$  using limit of sum

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4. Evaluate  $\int_a^b \frac{dx}{\sqrt{x}}$ , where  $a, b > 0$ .

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5. Let L be the line of intersection of the planes  $2x + 3y + z = 1$  and  $x + 3y + 2z = 2$ . If L makes an angle  $\alpha$  with the positive x-axis, then  $\cos \alpha$  equals

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6. Evaluate:  $\int_{-\frac{\pi}{2}}^{2\pi} \sin^{-1}(\sin x) dx$

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7. Evaluate  $\int_0^1 \frac{1}{\sqrt{1-x^2} \sin^{-1}(2x\sqrt{1-x^2})} dx$ .

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8. Evaluate:  $\int_0^{2\pi} [\sin x] dx$ , where  $[.]$  denotes the greatest integer function.

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9. Prove that  $\frac{1 + \sqrt{2}}{2} < \int_0^{\pi/2} \frac{\sin x}{x} dx < \frac{\pi + 2\sqrt{2}}{4}$

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10. Evaluate:  $\int_{-1}^0 \frac{dx}{x^2 + 2x + 2}$

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11. Let  $P(x)$  be a polynomial of least degree whose graph has three points of inflection  $(-1, -1)$ ,  $(1, 1)$  and a point with abscissa 0 at which the curve is inclined to the axis of abscissa at an angle of  $60^\circ$ . Then find the value of  $\int_0^1 P(x) dx$

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12. Let  $f$  be a continuous function on  $[a, b]$ . Prove that there exists a number  $x \in [a, b]$  such that  $\int_a^x f(t) dx = \int_x^b f(t) dt$

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13.  $\int_0^1 \frac{dx}{e^x + e^{-x}}$

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14. Evaluate  $\left(\frac{\int_0^\pi}{2}\right) \frac{\tan x dx}{1 + m^2 \tan^2 x}$

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15. Find the mistake in the following evaluation of the integral

$$I = \int_0^\pi \frac{dx}{1 + 2\sin^2 x}, \quad \text{then} \quad : \quad I = \int_0^\pi \frac{dx}{\cos^2 x + 3\sin^2 x}$$
$$= \int_0^\pi \frac{\sec^2 x dx}{1 + 3\tan^2 x} = \frac{1}{\sqrt{3}} \left[ \tan^{-1}(\sqrt{3}\tan x) \right]_0^\pi = 0$$

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16. Let  $\frac{d}{dx}(F(x)) = \frac{e^{\sin x}}{x}, x > 0$ . If  $\int_1^4 2 \frac{e^{\sin(x^2)}}{x} dx = F(k) - F(1)$ , then possible value of k is:

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17. If  $\int_a^b (f(x) - 3x)dx = a^2 - b^2$  then the value of  $f\left(\frac{\pi}{6}\right)$  is \_\_\_



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18. If  $f(0) = 1, f(2) = 3, f'(2) = 5$ , then find the value of  $I_1 = \int_0^1 xf'(2x)dx$



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19. Evaluate:  $\int_0^1 \log x dx$



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20. Evaluate:  $\int_0^{\frac{1}{2}} \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx$



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21. If  $\lambda = \int_0^1 \frac{e^t}{1+t}$ , then  $\int_0^1 e^t \log_e(1+t) dt$  is equal to

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22. If  $\int_0^1 e^{-x} \wedge 2 dx = a$ , then find the value of  $\int_0^1 x^2 e^{-x} \wedge 2 dx$  in terms of  $a$ .

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23. If  $f(x) = x + \sin x$ , then find the value of  $\int_{\pi}^{2\pi} f^{-1}(x) dx$ .

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24. Find the value of  $\int_0^{\pi/2} \cos^5 x \sin^7 x dx$

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25. Evaluate  $\lim_{n \rightarrow \infty} \left[ \frac{1}{(n+1)(n+2)} + \frac{1}{(n+2)(n+4)} + \dots + \frac{1}{6n^2} \right]$

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26. Evaluate:  $(\lim)_{n \rightarrow \infty} n \left[ \frac{1}{na} + \frac{1}{na+1} + \frac{1}{na+2} + \dots + \frac{1}{nb} \right]$

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27. Evaluate:  $(\lim)_{n \rightarrow \infty} \left( \frac{(n+1)(n+2)(n+n)^{\frac{1}{n}}}{n} \right)$

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28. Evaluate:  $(\lim)_{n \rightarrow \infty} \frac{(1^2 + 2^2 + 3^2 + \dots + n^2)(1^3 + 2^3 + 3^3 + \dots + n^3)}{1^6 + 2^6 + 3^6 + \dots + n^6}$

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29. Prove that  $0 < \int_0^1 \frac{x^7 dx}{(1+x^8)^{\frac{1}{3}}} < \frac{1}{8}$

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30. Evaluate:  $\int_0^1 x \frac{dx}{\sqrt{1-x^2}}$

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31. Let  $I_1 = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x}{x} dx$ ,  $I_2 = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin(\sin x)}{\sin x} dx$ ,  $I_3 = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin(\tan x)}{\tan x} dx$ . Then arrange in the decreasing order in which values  $I_1, I_2, I_3$  lie.

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32. Prove that  $1 < \int_0^2 \left( \frac{5-x}{9-x^2} \right) dx < \frac{6}{5}$



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33. Estimate the absolute value of the integral  $\int_{10}^{19} \frac{\sin x}{1+x^8} dx$



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34. Prove that  $\int_0^1 \sqrt{(1+x)(1+x^3)} dx$  cannot exceed  $\sqrt{\frac{15}{8}}$ .



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35. Prove that  $\int_a^b f(x) dx = (b-a) \int_0^1 f((b-a)x+a) dx$



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36. Evaluate  $\int_{-1}^2 |x^3 - x| dx$



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37. Evaluate:  $\int_{-1}^{3/2} |x \sin \pi x| dx$

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38. Show that  $\int_a^b \frac{|x|}{x} dx = |b| - |a|$ .

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39. If  $f(n) = \int_0^{2015} \frac{e^x}{1+x^n} dx$ , then find the value of  $\lim_{n \rightarrow \infty} f(n)$

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40. Let:  $f(x) = \int_0^x |2t - 3| dt$ . Then discuss continuity and differentiability of

$$f(x) \text{ at } x = \frac{3}{2}$$

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41. A continuous real function  $f$  satisfies

$$f(2x) = 3 \left( f(x) \forall x \in \mathbb{R} \text{ if } \int_0^1 f(x) dx = 1, \text{ then find the value of } \int_1^2 f(x) dx \right)$$

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42. Let  $g(x) = \int_0^x f(t) dt$ , where  $f$  is such that  $\frac{1}{2} \leq f(t) \leq 1$ , for  $t \in [0, 1]$  and  $0 \leq f(t) \leq \frac{1}{2}$ , for  $t \in [1, 2]$ . Then prove that  $\frac{1}{2} \leq g(2) \leq \frac{3}{2}$ .

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43. If  $[x]$  denotes the greatest integer less than or equal to  $x$ , then find

the value of the integral  $\int_0^2 x^2 [x] dx$ .

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44. Evaluate:  $\int_0^{\frac{5\pi}{2}} [\tan x] dx$ , where  $[.]$  denotes the greatest integer function.

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45. Evaluate:  $\int_0^{10\pi} [\tan^{-1}x] dx$ , where  $[x]$  represents greatest integer function.

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46. Evaluate:  $\int_0^2 [x^2 - x + 1] dx$ , where  $[.]$  denotes the greatest integer function.

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47. Prove that  $\int_0^\infty [ne^{-x}] dx = 1n \left( \frac{n^n}{n!} \right)$ , where  $n$  is a natural number greater than 1 and  $[.]$  denotes the greatest integer function..

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48. Evaluate:  $\int_0^{\sqrt{3}} \frac{1}{1+x^2} \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx$

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49. Evaluate  $\int_{\pi/6}^{\pi/3} \frac{\sqrt{(\sin x)} dx}{\sqrt{(\sin x)} + \sqrt{(\cos x)}}$

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50. Evaluate:  $\int_0^a \frac{dx}{x + \sqrt{a^2 - x^2}}$  or  $\int_0^{\frac{\pi}{2}} \frac{d\theta}{1 + \tan \theta}$

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51. Evaluate  $\int_0^{\pi} \frac{\sin 6x}{\sin x} dx$ .

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52. Evaluate:  $\int_0^{\frac{\pi}{2}} \log\left(\frac{4 + 3\sin x}{4 + 3\cos x}\right) dx$

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53. Evaluate:  $\int_{-\pi}^{3\pi} \log(\sec\theta - \tan\theta) d\theta$

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54. Prove that  $\int_0^{2a} f(x) dx = \int_0^a [f(a-x) + f(a+x)] dx$

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55. Evaluate  $\int_0^{\pi/4} \ln(1 + \tan x) dx$

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56. Evaluate:  $\int_{-5}^5 x^2 \left[ x + \frac{1}{2} \right] dx$  (where  $[.]$  denotes the greatest integer function).

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57. Evaluate:  $\int_{-\pi}^{\pi} \frac{x \sin x dx}{e^x + 1}$

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58. Evaluate:  $\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x + \cos x} dx$

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59. Prove that  $\int_0^1 \tan^{-1} \left( \frac{1}{1-x+x^2} \right) dx = 2 \int_0^1 \tan^{-1} x dx$ . Hence or otherwise, evaluate the integral  $\int_0^1 \tan^{-1} (1-x+x^2) dx$

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60. Show that  $\int_0^{\frac{\pi}{2}} \sqrt{(\sin 2\theta)} \sin \theta d\theta = \frac{\pi}{4}$



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61. For  $\theta \in \left(0, \frac{\pi}{2}\right)$ , prove that  $\int_0^{\theta} \log(1 + \tan \theta \tan x) dx = \theta \log(\sec \theta)$



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62. Evaluate the definite integral:  $\int_{-\frac{1}{\sqrt{3}}}^{\frac{1}{\sqrt{3}}} \left( \frac{x^4}{1-x^4} \right) \cos^{01} \left( \frac{2x}{1+x^2} \right) dx$ .



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64. Evaluate  $\int_0^{2\pi} \frac{dx}{1 + 3\cos^2 x}$

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65. Evaluate  $\int_0^{2\pi} \frac{x \cos^{2n} x}{\cos^{2n} x + \sin^{2n} x} dx$

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66. Evaluate:  $\int_0^{\pi} e^{|\cos x|} \left( 2s \in \left( \frac{1}{2} \cos x \right) + 3 \cos \left( \frac{1}{2} \cos x \right) \right) \sin x dx$

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67.  $\int_0^{\pi} x \log \sin x dx$

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68. Evaluate:  $\int_{-\pi/4}^{\pi/4} \log(\sin x + \cos x) dx$

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69. Evaluate:  $\int_0^{\pi} x \cot x dx$

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70. Evaluate:  $\int_0^{\infty} \log\left(x + \frac{1}{x}\right) \frac{dx}{1+x^2}$

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71. Evaluate:  $\int_{-\pi/2}^{\pi/2} \sqrt{\cos x - \cos^3 x} dx$

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72. Evaluate:  $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \pi \log \left( \frac{a - \sin \theta}{a + \sin \theta} \right) d\theta, a > 0$

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73. Evaluate:  $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \pi \log \left\{ \frac{ax^2 + bx + c}{ax^2 - bx + c} (a + b)|\sin x| \right\} dx$

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74. Evaluate:  $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x^9 - 3x^5 + 7x^3 - x + 1}{\cos^2 x} dx$

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75. If  $f$  is an odd function, then evaluate  $I = \int_{-a}^a \frac{f(\sin x) dx}{f(\cos x) + f(\sin^2 x)}$

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76. Evaluate:  $\int_{-\frac{1}{2}}^{\frac{1}{2}} \left[ \left( \frac{x+1}{x-1} \right)^2 + \left( \frac{x-1}{x+1} \right)^2 - 2 \right]^{\frac{1}{2}} dx$

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77. Find the value of  $\int_{-2}^2 \frac{\sin^{-1}(\sin x) + \cos^{-1}(\cos x)}{(1+x^2) \left( 1 + \left[ \frac{x^2}{5} \right] \right)} dx$ , where  $[\cdot]$  represents

the greatest integer function.

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78. Determine the value of  $\int_{-\pi}^{\pi} \frac{2x(1 + \sin x)}{1 + \cos^2 x} dx$ .

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79. Evaluate  $\int_0^{16\pi/3} |\sin x| dx$ .

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80. Evaluate  $\frac{\int_0^n [x] dx}{\int_0^n \{x\} dx}$  (where  $[x]$  and  $\{x\}$  are integral and fractional parts of  $x$  respectively and  $n \in \mathbb{N}$ ).

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81. Let  $f(x)$  be a continuous and periodic function such that  $f(x) = f(x + T)$  for all  $x \in \mathbb{R}$ ,  $T > 0$ . If  $\int_{-2T}^{a+5T} f(x) dx = 19(aT)$  and  $\int_0^T f(x) dx = 2$ , then find the value of  $\int_0^a f(x) dx$ .

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82. If  $g(x) = \int_0^x \cos^4 t dt$ , then prove that  $g(x + \pi) = g(x) + g(\pi)$ .

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83. Evaluate:  $\int_{-\frac{\pi}{4}}^{n\pi - \frac{\pi}{4}} |\sin x + \cos x| dx$

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84. Evaluate:  $\int_0^x [\text{cost}] dt$  where  $x \in \left(2n\pi, \left(4n + 1\frac{\pi}{2}\right)\right)$ ,  $n \in N$ , and  $[\cdot]$  denotes the greatest integer function.

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85. Let  $f$  be a real-valued function satisfying  $f(x) + f(x+4) = f(x+2) + f(x+6)$ . Prove that  $\int_x^{x+8} f(t) dt$  is constant function.

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86. A periodic function with period 1 is integrable over any finite interval. Also, for two real numbers  $a, b$  and two unequal non-zero positive integers  $m$  and  $n$   $\int_a^{a+n} f(x) dx = \int_b^{b+m} f(x) dx$  calculate the value of  $\int_m^n f(x) dx$



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87. If  $y = \int_{x^2}^{x^3} \frac{1}{\log t} dt$  ( $x > 0$ ), then find  $\frac{dy}{dx}$



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88. If  $\int_0^y \cos t^2 dt = \int_0^{x^2} \frac{\sin t}{t} dt$ , then  $\frac{dy}{dx}$  is equal to



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89. If  $x = \int_0^y \frac{dt}{\sqrt{1+9t^2}}$  and  $\frac{d^2y}{dx^2} = ay$ , then  $f \in da$



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90. If  $\int_{\sin x}^1 t^2 f(t) dt = 1 - \sin x$ , where  $x \in \left(0, \frac{\pi}{2}\right)$ , then find the value of

$$f\left(\frac{1}{\sqrt{3}}\right).$$



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91. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a differentiable function having  $f(2) = 6$ ,  $f'(2) = \frac{1}{48}$ . Then

evaluate  $\lim_{x \rightarrow 2} \int_6^{f(x)} \frac{4t^3}{x-2} dt$



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92. Evaluate:  $(\lim)_{x \rightarrow \infty} \frac{\left(\int_0^x e^x \wedge 2 dx\right)^2}{\int_0^x e^{2x} \wedge 2 dx}$



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93. Prove that:  $y = \int_{\frac{1}{8}}^{\sin^2 x} \sin^{-1} \sqrt{t} dt + \int_{\frac{1}{8}}^{\cos^2 x} \cos^{-1} \sqrt{t}$ , where  $0 \leq x \leq \frac{\pi}{2}$ , is the equation of a straight line parallel to the x-axis. Find the equation.

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94. If  $f(x) = \int_{x^2}^{x^2+1} e^{-t^2} dt$ , then find the interval in which  $f(x)$  increases.

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95. Let  $f: (0, \infty) \rightarrow \mathbb{R}$  be a differentiable function satisfying,  $\int_0^x (1-t)f(t) dt = \int_0^x tf(t) dx \in \mathbb{R}^+$  and  $f(1) = 1$ . Determine  $f(x)$ .

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96. Let  $f: \mathbb{R} \rightarrow (0, \infty)$  be a real valued function satisfying  $\int_0^x tf(x-t) dt = e^{2x} - 1$  then find  $f(x)$  ?

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97. Let  $f: R \rightarrow R$  be a differentiable function satisfying

$$f(x) = x^2 + 3 \int_0^x e^{-t^3} \cdot f(x - t^3) dt. \text{ Then find } f(x).$$

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98. If  $y = \int_0^x f(t) \sin\{k(x - t)\} dt$ , then prove that  $\frac{d^2 y}{dx^2} + k^2 y = kf(x)$

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99. Prove that  $\int_0^x e^{xt} e^{-t^2} dt = e^{x^2/4} \int_0^x e^{-t^2/4} dt$ .

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100. show that the sum of the two integrals

$$\int_{-4}^{-5} e^{(x+5)^2} dx + 3 \int_{\frac{1}{3}}^{\frac{2}{3}} e^9 \left(x - \frac{2}{3}\right)^2 dx \text{ is zero}$$



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101. Compute the integrals:  $\int_0^{\infty} f(x^n + x^{-n}) \log x \frac{dx}{x}$



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102. Compute the integrals:  $\int_0^{\infty} f(x^n + x^{-n}) \log x \frac{dx}{1+x^2}$



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103. Compute the integrals:  $\int_{\frac{1}{e}}^e \frac{1}{x} \sin\left(x - \frac{1}{x}\right) dx$



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104. Let  $A = \int_0^{\infty} \frac{\log x}{1+x^3} dx$ . Then find the value of  $\int_0^{\infty} \frac{x \log x}{1+x^3} dx$  in terms of  $A$ .



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105. If  $\int_0^1 \frac{\sin t}{1+t} dt = \alpha$ , then the value of the integral  $\int_{4\pi-2}^{4\pi} \frac{\sin\left(\frac{t}{2}\right)}{4\pi+2-t} dt$  is (1)  
 2 $\alpha$  (2) -2 $\alpha$  (3)  $\alpha$  (4) - $\alpha$

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106. Prove that  $\int_0^{\tan^{-1}x} \frac{1}{x} dx = \frac{1}{2} \int_0^{\frac{\pi}{2}} \frac{x}{\sin x} dx$ .

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107. For  $x > 0$ , let  $f(x) = \int_1^x \frac{\log t}{1+t} dt$ . Find the function  $f(x) + f\left(\frac{1}{x}\right)$  and find the value of  $f(e) + f\left(\frac{1}{e}\right)$ .

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108. Determine a positive integer  $n$  such that  $\int_0^{\frac{\pi}{2}} x^n \sin x dx = \frac{3}{4}(\pi^2 - 8)$

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109. Determine a positive integer  $n \leq 5$  such that  $\int_0^1 e^x (x - 1)^n = 16 - 6e$

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110. Prove that:  $I_n = \int_0^{\infty} x^{2n+1} e^{-x^2} dx = \frac{n!}{2}, n \in N.$

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111. If  $I_n = \int_0^1 \frac{dx}{(1+x^2)^n}; n \in N,$  then prove that  $2nI_{n+1} = 2^{-n} + (2n-1)I_n$

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112. If  $I_n = \int_0^{\frac{\pi}{2}} \sin^n x dx$ , then show that  $I_n = ((n-1)n)I_{n-2}$ .

Hence prove that

$$I_n = \begin{cases} \left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\left(\frac{n-5}{n-4}\right)\cdots\left(\frac{1}{2}\right)\frac{\pi}{2} & \text{if } n \text{ is even} \\ \left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\left(\frac{n-5}{n-4}\right)\cdots\left(\frac{2}{3}\right)1 & \text{if } n \text{ is odd} \end{cases}$$



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### Solved Example Type

1.  $f, g, h,$  are continuous in

$[0, a], f(a-x) = f(x), g(a-x) = -g(x), 3h(x) - 4h(a-x) = 5$ . Then prove that

$$\int_0^a f(x)g(x)h(x)dx = 0$$



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2. Evaluate  $\int_0^{\frac{\pi}{2}} \frac{\sin 3x}{\sin x + \cos x} dx$ .



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3. Given a function  $f: [0, 4] \rightarrow \mathbb{R}$  is differentiable, then prove that for some

$$\alpha, \beta \in (0, 2), \int_0^4 f(t) dt = 2\alpha f(\alpha^2) + 2\beta f(\beta^2).$$



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4. Prove that  $\int_0^{\infty} \frac{\sin^2 x}{x^2} dx = \int_0^{\infty} (\sin x) x dx$



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5. If  $\int_0^{\frac{\pi}{2}} \log \sin \theta d\theta = k$ , then find the value of  $\int_0^{\frac{\pi}{2}} \left( \frac{\theta}{\sin \theta} \right)^2 d\theta$  in terms of  $k$



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6. Evaluate :  $\int_0^{\pi} \frac{x^2 \sin 2x \cdot \sin\left(\frac{\pi}{2} \cdot \cos x\right)}{2x - \pi} dx$

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7. Find the value of  $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{\pi + 4x^3}{2 - \cos\left(|x|\frac{\pi}{3}\right)} dx$

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8. It is known that  $f(x)$  is an odd function and has a period  $p$ . Prove that

$\int_a^x f(t) dt$  is also periodic function with the same period.

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9. Evaluate:  $\int_0^{\frac{\pi}{4}} \left( \tan^{-1} \left( \frac{2\cos^2\theta}{2 - \sin 2\theta} \right) \right) \sec^2\theta d\theta$



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10. If  $f(x) = \frac{\sin x}{x} \forall x \in (0, \pi]$ , prove that  $\frac{\pi}{2} \int_0^{\frac{\pi}{2}} f(x)f\left(\frac{\pi}{2} - x\right) dx = \int_0^{\pi} f(x) dx$

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11. Let  $f(x)$  be a continuous function  $\forall x \in R$ , except at  $x = 0$ , such that

$\int_x^a \frac{f(t)}{t} dt$ , provethat  $\int_0^a f(x) dx = \int_0^a g(x) dx$

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12. If  $\int_0^x \sin(f(t)) dt = (x + 2) \int_0^x t \sin(f(t)) dt$ , where  $x > 0$ , then show that

$$f'(x) \cot f(x) + \frac{3}{1+x} = 0.$$

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13. Show that:  $\int_0^{\pi/2} f(\sin 2x) \sin x dx = \sqrt{2} \int_0^{\pi/4} f(\cos 2x) \cos x dx$



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14. Let  $a + b = 4$ , where  $a < 2$ , and let  $g(x)$  be a differentiable function. If

$\frac{dg}{dx} > 0$  for all  $x$ , prove that

$$\int_0^a g(x) dx + \int_0^b g(x) dx \in \text{crerasesas}(b - a) \in \text{crerases}$$



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15. If  $f$  is a continuous function with  $\int_0^x f(t) dt \rightarrow \infty$  as  $|x| \rightarrow \infty$  then show

that every line  $y = mx$  intersects the curve  $y^2 + \int_0^x f(t) dt = 2$



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16. If  $f(x + f(y)) = f(x) + y \forall x, y \in \mathbb{R}$  and  $f(0) = 1$ , then prove that

$$\int_0^2 f(2 - x) dx = 2 \int_0^1 f(x) dx.$$



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17. Suppose  $f$  is a real-valued differentiable function defined on  $[1, \infty]$  with

$f(1) = 1$ . Moreover, suppose that  $f$  satisfies

$$f'(x) = \frac{1}{x^2 + f^2(x)} \text{ Show that } f(x) < 1 + \frac{\pi}{4} \forall x \geq 1.$$

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18. Let  $f$  be a continuous function on  $[a, b]$ . If

$$F(x) = \left( \int_a^x f(t) dt - \int_x^b f(t) dt \right) (2x - (a + b)),$$
 then prove that there exist some

$$c \in (a, b) \text{ such that } \int_a^c f(t) dt - \int_c^b f(t) dt = f(c)(a + b - 2c)$$

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19.  $f(x)$  is a continuous and bijective function on  $\mathbb{R}$ . If  $\forall t \in \mathbb{R}$ , then the

area bounded by  $y = f(x)$ ,  $x = a - t$ ,  $x = a$ , and the  $x$ -axis is equal to the

area bounded by  $y = f(x)$ ,  $x = a + t$ ,  $x = a$ , and the  $x$ -axis. Then prove that

$$\int_{-\lambda}^{\lambda} f^{-1}(x) dx = 2a\lambda \text{ (given that } f(a) = 0)$$

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20. If  $f(x) = x + \int_0^1 t(x+t)f(t)dt$ , then find the value of the definite integral  $\int_0^1 f(x)dx$ .

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Cae Type

1. Evaluate the following integrals using limit of sum.

$$\int_a^b \cos x dx$$

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2. Evaluate the following integrals .

$$\int_a^b x^3 dx$$

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3. Find the value of  $\int_0^4 [x] dx$ , where  $[.]$  represents the greatest integer function.

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4. If  $f(x) = \begin{cases} 1 - |x| & |x| \leq 1 \\ 0 & |x| > 1 \end{cases}$  and  $g(x) = f(x - 1) + f(x + 1)$ , then find the value of  $\int_{-3}^5 g(x) dx$ .

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5. Consider the integral  $I = \int_0^{2\pi} \frac{dx}{5 - 2\cos x}$

Making the substitution  $\tan \frac{x}{2} = t$ , we have

$$I = \int_0^{2\pi} \frac{dx}{5 - 2\cos x} = \int_0^0 \frac{2dt}{(1+t^2)[5 - 2(1-t^2)/(1+t^2)]} = 0$$

The result is obviously wrong, since the integrand is positive and consequently the integral of this function cannot be equal to zero. Find the mistake.

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6. Evaluate the following :  $\int_0^{\pi} \frac{dx}{1 + \sin x}$

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7. Evaluate:  $\int_1^{\infty} (e^{x+1} + e^{3-x})^{-1} dx$

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8. Evaluate:  $\int_0^{\frac{1}{\sqrt{2}}} \frac{\sin^{-1} x}{(1-x^2)\sqrt{1-x^2}} dx$

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9. Evaluate:  $\int_0^1 \frac{2-x^2}{(1+x)\sqrt{1-x^2}} dx$

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10. Evaluate the following :  $\int_0^{\pi/2} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$

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11. Evaluate:  $\int_{\pi/6}^{\pi/4} \frac{1 + \cot x}{e^x \sin x} dx$

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12. Evaluate  $\int_0^1 \frac{e^{-x} dx}{1 + e^x}$

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13. Prove that  $\int_0^{102} (x-1)(x-2)(x-100)$

$$x \left( \frac{1}{(x-1) + \frac{1}{(x-2)} + \frac{1}{(x-100)}} dx = 101! - 100! \right)$$

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14. Show that :  $\int_0^1 \frac{\log x}{(1+x)} dx = - \int_0^1 \frac{\log(1+x)}{x} dx$

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15. If  $\int_0^1 \frac{e^t}{1+t} dt = a$ , then find the value of  $\int_0^1 \frac{e^t}{(1+t)^2} dt$  in terms of  $a$ .

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16. Let  $f$  be a one to one continuous function such that  $f(2) = 3$  and  $f(5) = 6$ . Given  $\int_2^5 f(x) dx = 17$ , then find the value of  $\int_3^7 f^{-1}(x) dx$ .

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17. Evaluate:  $(\lim)_{n \rightarrow \infty} \left( \frac{1}{\sqrt{4n^2 - 1}} + \frac{1}{\sqrt{4n^2 - 2^2}} + \dots + \frac{1}{\sqrt{3n^2}} \right)$



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18.  $\lim_{n \rightarrow \infty} \left[ \frac{1}{n^2} \cdot \sec^2\left(\frac{1}{n^2}\right) + \frac{2}{n^2} \cdot \sec^2\left(\frac{4}{n^2}\right) + \dots + \frac{1}{n} \cdot \sec^2 1 \right]$



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19. Evaluate  $(\lim)_{n \rightarrow \infty} \sum_{k=1}^n \frac{k}{n^2 + k^2}$



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20. Evaluate the following limit:

$$\lim_{n \rightarrow \infty} \frac{\sum_{r=1}^n \sqrt{r} \sum_{r=1}^h \frac{1}{\sqrt{r}}}{\sum_{r=1}^n r}$$



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21. Evaluate the following limit:

$$\lim_{n \rightarrow \infty} \left[ \frac{n!}{n^n} \right]^{1/n}$$



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22. Prove that  $4 \leq \int_1^3 \sqrt{3+x^2} \leq 4\sqrt{3}$



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23. If  $I_1 = \int_0^1 2^{x^2} dx$ ,  $I_2 = \int_0^1 2^{x^3} dx$ ,  $I_3 = \int_1^2 x^2 dx$ ,  $I_4 = \int_1^2 2^{x^3} dx$ , then which of the following is/are true?

a)  $I_1 > I_2$

(b)  $I_2 > I_2$

c)  $I_3 > I_4$

(d)  $I_3 < I_4$

A.  $I_1 > I_2$

B.  $I_2 > I_1$

C.  $I_3 > I_4$

D.  $I_3 < I_4$

**Answer: A::D**

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24. If  $I_1 = \int_0^{\pi/2} \cos(\sin x) dx$ ,  $I_2 = \int_0^{\pi/2} \sin(\cos x) dx$ , and  $I_3 = \int_0^{\pi/2} \cos x dx$ , then find the order in which the values  $I_1, I_2, I_3$ , exist.

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25. Prove that  $\pi/6$

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26. Evaluate  $\int_0^{\pi/2} |\sin x - \cos x| dx$ .

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27. Evaluate:  $\int_{-1}^4 f(x) dx = 4$  and  $\int_2^4 (3 - f(x)) dx = 7$ , then find the value of  $\int_2^{-1} f(x) dx$ .

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28. Evaluate  $\int_1^5 \sqrt{x-2} \sqrt{x-1} dx$ .

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29. Evaluate:  $\int_{-1}^3 \left( \tan^{-1} \left( \frac{x}{x^2+1} \right) + \tan^{-1} \left( \frac{x^2+1}{x} \right) \right) dx$

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30. Evaluate  $\int_1^a x \cdot a^{-[\log_e x]} dx$ , ( $a > 1$ ). Here  $[.]$  represents the greatest integer function.

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31. Evaluate  $\int_1^{e^6} \left[ \frac{\log x}{3} \right] dx$ , where  $[.]$  denotes the greatest integer function.

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32. Find the value of  $\int_{-1}^1 [x^2 + \{x\}] dx$ , where  $[.]$  and  $\{.\}$  denote the greatest function and fractional parts of  $x$ , respectively.

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33. Evaluate:-  $\int_0^\pi [\cot x] dx$ , where  $[.]$  denotes the greatest integer function.

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34. Prove that  $\int_0^x [t] dt = \frac{[x]([x] - 1)}{2} + [x](x - [x])$ , where  $[.]$  denotes the greatest integer function.

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35. Evaluate:  $\int_0^{\infty} [2e^{-x}] dx$ , where  $[x]$  represents greatest integer function.

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36. If  $f(a + b - x) = f(x)$ , then prove that  $\int_a^b x f(x) dx = \frac{a + b}{2} \int_a^b f(x) dx$ .

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37. The value of the integral  $\int_3^6 \frac{\sqrt{x}}{\sqrt{9-x} + \sqrt{x}} dx$  is

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38. Find the value of  $\int_0^1 \sqrt[3]{2x^3 - 3x^2 - x + 1} dx$ .

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39. Show that  $\int_0^\pi f(x(\sin x)) dx = \frac{\pi}{2} \int_0^\pi f(\sin x) dx$ .

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40. Find the value of  $\int_0^1 x(1-x)^n dx$

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41. If a continuous function  $f$  on  $[0, a]$  satisfies  $f(x)f(a-x) = 1$ ,  $a > 0$ , then

find the value of  $\int_0^a \frac{dx}{1+f(x)}$ .

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42. If  $f$  and  $g$  are continuous functions on  $[0, a]$  satisfying

$$f(x) = f(a - x) \text{ and } g(x)(a - x) = 2, \text{ then show that } \int_0^a f(x)g(x)dx = \int_0^a f(x)dx.$$

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43. Find the value of  $\int_0^{\pi/2} \sin 2x \log \tan x dx$ .

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44. The value of  $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1 + a^x} dx, a > 0$  is

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45. Evaluate:  $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$

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46. Evaluate  $\int_0^{\pi} \frac{x dx}{1 + \cos \alpha \sin x}$ , where  $0 < \alpha < \pi$ .

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47. Find the value of  $\int_0^{2\pi} \frac{1}{1 + \tan^4 x} dx$

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48.  $\int_0^{2\pi} \sin^{100} x \cos^{99} x dx$

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49. For  $U_n = \int_0^1 x^n (2-x)^n dx$ ;  $V_n = \int_0^1 x^n (1-x)^n dx$   $n \in \mathbb{N}$ , which of the following statement(s) is/are true? (a)  $U_n = 2^n V_n$  (b)  $U_n = 2^{-n} V_n$  (c)  $U_n = 2^{2n} V_n$  (d)  $V_n = 2^{-2n} U_n$

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50. Evaluate:  $\int_0^{\pi} \log(1 + \cos x) dx$

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51. Find the value of  $\int_0^1 \left\{ (\sin^{-1} x) / x \right\} dx$

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52. Evaluate  $\int_{-\infty}^0 \frac{te^t}{\sqrt{1 - e^{2t}}} dt$

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53. If  $I_1 = \int_0^{\pi} x f(\sin^3 x + \cos^2 x) dx$  and  $I_2 = \int_0^{\frac{\pi}{2}} f(\sin^3 x + \cos^2 x) dx$ , then relate  $I_1$  and  $I_2$

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54. Evaluate:  $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x \cos^2 x (\sin x + \cos x) dx$

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55. Evaluate:  $\int_{-1}^1 \frac{x^3 + |x| + 1}{x^2 + 2|x| + 1} dx$

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56. Evaluate the following:  $\int_{-\pi}^{\pi} (1 - x^2) \sin x \cos^2 x dx$

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57. Evaluate the following:  $\int_{-1}^1 \frac{\sin x - x^2}{3 - |x|} dx$

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58. Evaluate the following:  $\int_{-1/2}^{1/2} \cos x \log \frac{1-x}{1+x} dx$

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59.  $\int_{-\frac{\pi}{2}}^{-\frac{2}{3\pi}} \{(\pi+x)^3 + \cos^2(x+3\pi)\} dx$  is equal to (A)  $\frac{\pi}{4} - 1$  (B)  $\frac{\pi^4}{32}$  (C)  $\frac{\pi^4}{32} + \frac{\pi}{2}$   
(D)  $\frac{\pi}{2}$

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60. Evaluate:  $\int_0^{100} (x - [x]) dx$  (where  $[.]$  represents the greatest integer function).

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61. Evaluate:  $\int_0^{100\pi} \sqrt{(1 - \cos 2x)} dx$

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62. If  $\int_0^{n\pi} f(\cos^2 x) dx = k \int_0^\pi f(\cos^2 x) dx$ , then  $f \in \mathcal{D}$  the value of  $k$

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63. Evaluate  $\int_0^{n\pi+t} (|\cos x| + |\sin x|) dx$ , where  $n \in \mathbb{N}$  and  $t \in [0, \pi/2]$ .

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64. Find the value of  $\int_0^{10} e^{2x - [2x]} d(x - [x])$  where  $[.]$  denotes the greatest integer function).

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65. If  $f(x)$  is a function satisfying  $f(x+a) + f(x) = 0$  for all  $x \in \mathbb{R}$  and positive constant  $a$  such that  $\int_b^{c+b} f(x) dx$  is independent of  $b$ , then find the least positive value of  $\cdot$

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66. Show that  $\int_0^{n\pi+v} |\sin x| dx = 2n + 1 - \cos v$ , where  $n$  is a positive integer and  $0 \leq v < \pi$

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67. If  $\int_{\frac{\pi}{3}}^x \sqrt{3 - \sin^2 t} dt + \int_0^y \cos t dt = 0$ , the  $\neq$  value  $\frac{dy}{dx}$

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68. If  $f(x) = e^{g(x)}$  and  $g(x) = \int_2^x \frac{tdt}{1+t^4}$ , then find the value of  $f'(2)$

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69. Evaluate  $(\lim)_{x \rightarrow 4} \int_4^x \frac{4t - f(t)}{(x-4)} dt$

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70. Evaluate:  $(\lim)_{x \rightarrow 2} \frac{\int_0^x \cos t^2 dt}{x}$

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71. Find the points of minima for  $f(x) = \int_0^x t(t-1)(t-2)dt$

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72. Find the equation of tangent to  $y = \int_{x^2}^{x^3} \frac{dt}{\sqrt{1+t^2}}$  at  $x = 1$ .

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73. If  $f(x) = \int_{\frac{x^2}{16}}^{x^2} \frac{\sin x \sin \sqrt{\theta}}{1 + \cos^2 \sqrt{\theta}} d\theta$ , then find the value of  $f' \left( \frac{\pi}{2} \right)$ .

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74. Let  $f(x)$  be a continuous and differentiable function such that

$$f(x) = \int_0^x \sin(t^2 - t + x) dt \text{ Then prove that } f'(x) + f(x) = \cos x^2 + 2x \sin x^2$$



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75. Let  $f(x)$  be a differentiable function satisfying

$$f(x) = \int_0^x e^{2tx - t^2} \cos(x - t) dt, \text{ then find the value of } f'(0).$$



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76. If  $\int_0^1 \frac{e^t dt}{t+1} = a$ , the  $\neq$  valuate  $\int_{b-1}^b \frac{e^{-t} dt}{t-b-1}$



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77. If  $f(x) = \int_1^x (\log t) (1 + t + t^2) dt \forall x \geq 1$ , then prove that  $f(x) = f\left(\frac{1}{x}\right)$ .



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78.  $f(x) = \int_1^x \frac{\tan^{-1}(t)}{t} dt \forall x \in \mathbb{R}^+$ , then  $f \in \mathbb{R}$  the value of  $f(e^2) - f\left(\frac{1}{e^2}\right)$

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79. Evaluate:  $\int_{\sqrt{2}-1}^{\sqrt{2}+1} \frac{(x^2 - 1)}{(x^2 + 1)^2} dx$

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80. Evaluate:  $\int_0^{e-1} \frac{x^2+2x-1}{x+1} dx + \int_1^e x \log x e^{\frac{x^2-2}{2}} dx$

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81. Find the value of  $\int_{\frac{1}{2}}^2 e^{\left|x - \frac{1}{x}\right|} dx$ .

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82. If  $I_1 = \int_0^1 \frac{dx}{e^x(1+x)}$  and  $I_2 = \int_0^{\pi/4} \frac{e^{\tan^7 \theta} \sin \theta}{(2 - \tan^2 \theta) \cos^3 \theta d\theta}$ , then find the value of  $\frac{I_1}{I_2}$ .

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83. If  $I_k = \int_1^e (1/x)^k dx$  ( $k \in I^+$ ), then find the value of  $I_4$ .

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84. Given  $I_m = \int_1^e (\log x)^m dx$ , then prove that  $\frac{I_m}{1-m} + mI_{m-2} = e$

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85. If  $I_n = \int_0^\pi x^n \sin x dx$ , then find the value of  $I_5 + 20I_3$ .

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86. If  $L(m, n) = \int_0^1 t^m (1+t)^n dt$ , then prove that

$$L(m, n) = \frac{2^n}{m+1} - \frac{n}{m+1} L(m+1, n-1)$$

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87. If  $I_n = \int_0^1 x^n (\tan^{-1} x) dx$ , then prove that  $(n+1)I_n + (n-1)I_{n-2} = -\frac{1}{n} + \frac{\pi}{2}$

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88. If  $I_{m,n} = \int_0^{\frac{\pi}{2}} \sin^m x \cos^n x dx$ , Then show that

$$I_{m,n} = \frac{m-1}{m+n} I_{m-2,n} \quad (m, n \in \mathbb{N}) \quad \text{Hence, prove that}$$

$$I_{m,n} = f(x) = \left\{ \frac{(n-1)(n-3)(m-5)(n-1)(n-3)(n-5)}{(m+n)(m+n-2)(m+n-4)} \right\} \frac{\pi}{4} \text{ when both } m \text{ and } n$$

are even

$$\left. \frac{(m-1)(m-3)(m-5)(n-1)(n-3)(n-5)}{(m+n)(m+n-2)(m+n-4)} \right\}$$

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## Scq Type

1. Let  $f(x) = \lim_{n \rightarrow \infty} \frac{1}{n} \left( \left(x + \frac{1}{n}\right)^2 + \left(x + \frac{2}{n}\right)^2 + \dots + \left(x + \frac{n-1}{n}\right)^2 \right)$

Then the minimum value of  $f(x)$  is

- A. 1/4
- B. 1/6
- C. 1/9
- D. 1/12

**Answer: D**



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2. If  $S_n = \left[ \frac{1}{1 + \sqrt{n}} + \frac{1}{2 + \sqrt{2n}} + \dots + \frac{1}{n + \sqrt{n^2}} \right]$ , then  $(\lim)_{n \rightarrow \infty} S_n$  is equal to

(a) log 2 (b) log 4 (c) log 8 (d) none of these

A. log 2

B. log 4

C. log 8

D. none of these

**Answer: B**

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3. The value of  $(\lim)_{n \rightarrow \infty} \sum_{r=1}^{4n} \frac{\sqrt{n}}{\sqrt{r}(3\sqrt{r} + \sqrt{n})^2}$  is equal to  $\frac{1}{35}$  (b)  $\frac{1}{4}$  (c)  $\frac{1}{10}$  (d)

$\frac{1}{5}$

A.  $\frac{1}{35}$

B.  $\frac{1}{14}$

C.  $\frac{1}{10}$

D.  $\frac{1}{5}$

**Answer: C**



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4. The value of

$$\lim_{n \rightarrow \infty} \frac{(1^2 + 2^2 + \dots + n^2)(1^3 + 2^3 + \dots + n^3)(1^4 + 2^4 + \dots)}{(1^5 + 2^5 + \dots + n^5)^2}$$

is equal to

A.  $\frac{3}{5}$

B.  $\frac{4}{5}$

C.  $\frac{2}{5}$

D.  $\frac{1}{5}$

**Answer: A**



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5. The value of  $\lim_{n \rightarrow \infty} \left[ \tan \frac{\pi}{2n} \tan \frac{2\pi}{2n} \tan \frac{n\pi}{2n} \right]^{1/n}$  is (a)  $e$  (b)  $e^2$  (c)  $1$  (d)  $e^3$

A.  $e$

B.  $e^2$

C.  $1$

D.  $e^3$

**Answer: C**



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6.  $\int_{2-a}^{2+a} f(x) dx$  is equal to [where  $f(2 - \alpha) = f(2 + \alpha) \forall \alpha \in R$

(a)  $2 \int_2^{2+a} f(x) dx$  (b)  $2 \int_0^a f(x) dx$  (c)  $2 \int_2^2 f(x) dx$  (d) none of these

A.  $2 \int_2^{2+a} f(x) dx$

B.  $2 \int_0^a f(x) dx$

C.  $2 \int_2^2 f(x) dx$

D. none of these

**Answer: A**

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7. If  $f(x) = \min(\{x\}, \{-x\})$ ,  $x \in R$ , where  $\{x\}$  denotes the fractional part of  $x$ , then  $\int_{-100}^{100} f(x) dx$  is

A. (a) 50

B. (b) 100

C. (c) 200

D. (d) none of these

**Answer: A**

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8. Which of the following is incorrect ?

A. (a)  $\int_{a+c}^{b+c} f(x) dx = \int_a^b f(x+c) dx$

B. (b)  $\int_{ac}^{bc} f(x) dx = c \int_a^b f(cx) dx$

C. (c)  $\int_{-a}^a f(x) dx = \frac{1}{2} \int_{-a}^a f(x) + f(-x) dx$

D. (d) none of these

**Answer: D**

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9.  $\int \frac{1}{2 - 1} \int (e^x(2-x^2) dx) / ((1-x)\sqrt{1-x^2})$

A.  $\frac{\sqrt{e}}{2} (\sqrt{3} + 1)$

B.  $\frac{\sqrt{3e}}{2}$

C.  $\sqrt{3e}$

D.  $\sqrt{\frac{e}{3}}$

**Answer: C**



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10. If  $\int_{\log 2}^x \frac{dy}{\sqrt{e^y - 1}} = \frac{\pi}{6}$ , then  $x$  is equal to

(a) 4 (b)  $\ln 8$  (c)  $\ln 4$  (d) none of these

A. 4

B.  $\ln 8$

C.  $\ln 4$

D. none of these

**Answer: C**



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11. evaluate  $\int_{\frac{5}{2}}^5 \frac{\sqrt{(25-x^2)^3}}{x^4} dx$  (A)  $\frac{\pi}{6}$  (B)  $\frac{2\pi}{3}$  (C)  $\frac{5\pi}{6}$  (D)  $\frac{\pi}{3}$

A.  $\frac{\pi}{6}$

B.  $\frac{2\pi}{3}$

C.  $\frac{5\pi}{6}$

D.  $\frac{\pi}{3}$

**Answer: D**



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12. If  $f(x)$  satisfies the condition of Rolle's theorem in  $[1, 2]$ , then  $\int_1^2 f'(x) dx$  is equal to (a) 1 (b) 3 (c) 0 (d) none of these

A. 1

B. 3

C. 0

D. none of these

**Answer: C**



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13. The value of the integral  $\int_0^{\log 5} \frac{e^x \sqrt{e^x - 1}}{e^x + 3} dx$

A.  $3 + 2\pi$

B.  $4 - \pi$

C.  $2 + \pi$

D. none of these

**Answer: B**



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14. The value of the integral  $\int_0^1 \frac{dx}{x^2 + 2x \cos \alpha + 1}$ ,  $0 < \alpha < \pi$  is

A.  $\sin\alpha$

B.  $\alpha\sin\alpha$

C.  $\frac{\alpha}{\sin\alpha}$

D.  $\frac{\alpha}{2}\sin\alpha$

**Answer: C**



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15.  $\int_0^{\infty} \frac{dx}{[x + \sqrt{x^2 + 1}]^3}$  is equal to (a)  $\frac{3}{8}$  (b)  $\frac{1}{8}$  (c)  $-\frac{3}{8}$  (d) none of these

A.  $\frac{3}{8}$

B.  $\frac{1}{8}$

C.  $-\frac{3}{8}$

D. none of these

**Answer: A**



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16. If  $f(y) = e^y$ ,  $g(y) = y$ ,  $y > 0$ , and  $F(t) = \int_0^t f(t-y)g(y)dy$ , then

A.  $F(t) = e^t - (1 + t)$

B.  $F(t) = te^t$

C.  $F(t) = te^{-t}$

D.  $F(t) = 1 - e^t(1 + t)$

Answer: A



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17. If  $P(x)$  is a polynomial of the least degree that has a maximum equal to 6 at  $x = 1$ , and a minimum equal to 2 at  $x = 3$ , then  $\int_0^1 P(x)dx$  equals:

A.  $\frac{17}{4}$

B.  $\frac{13}{4}$

C.  $\frac{19}{4}$

D.  $\frac{5}{4}$

**Answer: C**

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18. The numbers of possible continuous  $f(x)$  defined in  $[0, 1]$  for which

$$I_1 = \int_0^1 f(x) dx = 1, I_2 = \int_0^1 x f(x) dx = 2, I_3 = \int_0^1 x^2 f(x) dx = 4 \text{ is/are } 1 \text{ (b) } \infty \text{ (c) } 2$$

(d) 0

A. 1

B.  $\infty$

C. 2

D. 0

**Answer: D**

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19. Suppose that  $F(x)$  is an antiderivative of  $f(x) = \frac{\sin x}{x}$ ,  $x > 0$ , then

$\int_1^3 \frac{\sin 2x}{x} dx$  can be expressed as

A.  $F(6) - F(2)$

B.  $\frac{1}{2}(F(6) - F(2))$

C.  $\frac{1}{2}(F(3) - F(1))$

D.  $2(F(6) - F(2))$

**Answer: A**



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20.  $\int_{-\frac{\pi}{3}}^0 \left[ \cot^{-1} \left( \frac{2}{2\cos x - 1} \right) + \cot^{-1} \left( \cos x - \frac{1}{2} \right) \right] dx$  is equal to  $\frac{\pi^2}{6}$  (b)  $\frac{\pi^2}{3}$  (c)

$\frac{\pi^2}{8}$  (d)  $\frac{3\pi^2}{8}$

A.  $\frac{\pi^2}{6}$

B.  $\frac{\pi^2}{3}$

C.  $\frac{\pi^2}{8}$

D.  $\frac{3\pi^2}{8}$

**Answer: A**

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21. Evaluate the definite integrals  $\int_0^{\pi/4} \frac{\sin x + \cos x}{25 - 16(\sin x - \cos x)^2} dx$

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22.  $\int_{-1}^1 \frac{e^{-\frac{1}{x}}}{x^2 \left(1 + e^{-\frac{2}{x}}\right)} dx$  is equal to :

A.  $\frac{\pi}{2} - 2\tan^{-1}e$

B.  $\frac{\pi}{2} - 2\cot^{-1}e$

C.  $2\tan^{-1}e$

D.  $\pi - 2\tan^{-1}e$

**Answer: D**



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23. If  $\int_0^{\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}$ , then  $\int_0^{\infty} \frac{\sin^3 x}{x} dx$  is equal to

A.  $\pi/2$

B.  $\pi/4$

C.  $\pi/6$

D.  $3\pi/2$

**Answer: B**



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24. The range of the function  $f(x) = \int_{-1}^1 \frac{\sin x dt}{1 + 2t \cos x + t^2}$  is

A.  $\left[ -\frac{\pi}{2}, \frac{\pi}{2} \right]$

B.  $[0, \pi]$

C.  $\{0, \pi\}$

D.  $\left\{ -\frac{\pi}{2}, \frac{\pi}{2} \right\}$

**Answer: D**



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25. If the function  $f: [0, 8] \rightarrow R$  is differentiable, then for  $0 < \alpha < 1$  and

$0 < \beta < 2$ ,  $\int_0^8 f(t) dt$  is equal to

A.  $3 \left[ \alpha^3 f(\alpha^2) + \beta^2 f(\beta^2) \right]$

B.  $3 \left[ \alpha^3 f(\alpha) + \beta^3 f(\beta) \right]$

C.  $3 \left[ \alpha^2 f(\alpha^3) + \beta^2 f(\beta^3) \right]$

$$D. 3[\alpha^2 f(\alpha^2) + \beta^2 f(\beta^2)]$$

**Answer: C**



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26. If  $f(x) = x^5 + 5x - 1$  then  $\int_5^{41} \frac{dx}{(f^{-1}(x))^5 + 5f^{-1}(x)}$  equals

A. 0

B.  $\log_e 3$

C.  $\log_e 4$

D.  $\log_e 6$

**Answer: D**



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27. Let  $f(0) = 0$  and  $\int_0^2 f'(2t)e^{f(2t)} dt = 5$ . then value of  $f(4)$  is  $\log 2$  (b)  $\log 7$  (c)  $\log 11$  (d)  $\log 13$

A.  $\log 2$

B.  $\log 7$

C.  $\log 11$

D.  $\log 13$

**Answer: C**



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28. If  $f(x) = \cos(\tan^{-1}x)$ , then the value of the integral  $\int_0^1 xf'(x) dx$  is

A.  $\frac{3 - \sqrt{2}}{2}$

B.  $\frac{3 + \sqrt{2}}{2}$

C. 1

$$D. 1 - \frac{3}{2\sqrt{2}}$$

**Answer: D**



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29. The equation of the curve is  $y = f(x)$ . The tangents at  $[1, f(1)]$ ,  $[2, f(2)]$ , and  $[3, f(3)]$  make angles  $\frac{\pi}{6}$ ,  $\frac{\pi}{3}$ , and  $\frac{\pi}{4}$ , respectively, with the positive direction of x-axis. Then the value of  $\int_2^3 f'(x)f'' dx + \int_1^3 f'' dx$  is equal to -  $\frac{1}{\sqrt{3}}$  (b)  $\frac{1}{\sqrt{3}}$  (e) 0 (d) none of these

A.  $-1/\sqrt{3}$

B.  $1/\sqrt{3}$

C. 0

D. none of these

**Answer: A**



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30. The value of  $\int_1^e \left( \frac{\tan^{-1}x}{x} + \frac{\log x}{1+x^2} \right) dx$  is (a)  $\tan e$  (b)  $\tan^{-1}e$  (c)  $\tan^{-1}\left(\frac{1}{e}\right)$  (d)

none of these

A.  $\tan e$

B.  $\tan^{-1}e$

C.  $\tan^{-1}(1/e)$

D. none of these

**Answer: B**



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31. If  $f(\pi) = 2$  and  $\int_0^\pi (f(x) + f'(x)) \sin x dx = 5$ , then  $f(0)$  is equal to ( it is given that  $f(x)$  is continuous in  $[0, \pi]$ )

A. 7

B. 3

C. 5

D. 1

**Answer: B**



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32. If  $\int_1^2 e^{x^2} dx = a$ , then  $\int_e^{e^4} \sqrt{\ln x} dx$  is equal to (a)  $2e^4 - 2e - a$  (b)  $2e^4 - e - a$  (c)  $2e^4 - e - 2a$  (d)  $e^4 - e - a$

A.  $2e^4 - 2e - a$

B.  $2e^4 - e - a$

C.  $2e^4 - e - 2a$

D.  $e^4 - e - a$

**Answer: B**



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33. If  $f(x)$  is continuous for all real values of  $x$ , then  $\sum_{r=1}^n \int_0^1 f(r-1+x) dx$  is equal to (a)  $\int_0^n f(x) dx$  (b)  $\int_0^1 f(x) dx$  (c)  $\int_0^1 f(x) dx$  (d)  $(n-1) \int_0^1 f(x) dx$

A.  $\int_0^n f(x) dx$

B.  $\int_0^1 f(x) dx$

C.  $n \int_0^1 f(x) dx$

D.  $(n-1) \int_0^1 f(x) dx$

**Answer: A**



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34. The value of  $\int_0^2 (3x^2 - 1) dx$

A.  $1 - \cos \alpha$

B.  $1 + \cos \alpha$

C. 1

D.  $\cos\alpha$

**Answer: C**



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35.  $f(x)$  is a continuous function for all real values of  $x$  and satisfies

$$\int_n^{n+1} f(x)dx = \frac{n^2}{2} \quad \forall n \in I$$

Then  $\int_{-3}^5 f(|x|)dx$  is equal to  $\frac{19}{2}$  (b)  $\frac{35}{2}$  (c)  $\frac{17}{2}$  (d)

none of these

A.  $19/2$

B.  $35/2$

C.  $17/2$

D. none of these

**Answer: B**



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36. If  $f(x) = \int_{-1}^x |t| dt$ , then for any  $x \geq 0$ ,  $f(x)$  equals

A.  $\frac{1}{2}(1 - x^2)$

B.  $\frac{1}{2}x^2$

C.  $\frac{1}{2}(1 + x^2)$

D. none of these

**Answer: C**



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37. If  $a > 0$  and  $A = \int_0^a \cos^{-1} x dx$ , and

$$\int_{-a}^a \left( \cos^{-1} x - \sin^{-1} \sqrt{1 - x^2} \right) dx = \pi a - \lambda A. \text{ Then } \lambda \text{ is}$$

A. 0

B. 2

C. 3

D. none of these

Answer: B



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38. The value of  $\int_1^a [x]f'(x)dx$ , where  $a > 1$ , and  $[x]$  denotes the greatest integer not exceeding  $x$ , is

$$af(a) - \{f(1)f(2) + \dots + f([a])\}$$

$$[a]f(a) - \{f(1) + f(2) + \dots + f([a])\}$$

$$[a]f(a) - \{f(1) + f(2) + \dots + fA\}$$

$$af([a]) - \{f(1) + f(2) + \dots + fA\}$$

A.  $af(a) - (f(1) + f(2) + \dots + f([a]))$

B.  $[a]f(a) - (f(1) + f(2) + \dots + f([a]))$

C.  $[a]f([a]) - (f(1) + f(2) + \dots + f(a))$

D.  $af([a]) - (f(1) + f(2) + \dots + f(a))$

Answer: B



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39.  $\int_3^{10} [\log[x]] dx$  is equal to (where  $[.]$  represents the greatest integer function)

A. 9

B.  $16 - e$

C. 10

D.  $10 + e$

**Answer: A**



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40.  $\int_{-1}^2 \left[ \frac{[x]}{1+x^2} \right] dx$ , where  $[.]$  denotes the greatest integer function, is equal to

A. -2

B. -1

C. zero

D. none of these

**Answer: B**



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41. The value of  $\int_{-2}^1 \left[ x \left[ 1 + \cos \left( \frac{\pi x}{2} \right) \right] + 1 \right] dx$ , where  $[.]$  denotes greatest integer function is

A. 1

B. 1/2

C. 2

D. none of these

**Answer: C**



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42. The value of  $\int_0^{2\pi} [2\sin x] dx$ , where  $[.]$  represents the greatest integral functions, is

A.  $\frac{-5\pi}{3}$

B.  $-\pi$

C.  $\frac{5\pi}{3}$

D.  $-2\pi$

**Answer: B**



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43.  $I_1 = \int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$ ,  $I_2 = \int_0^{2\pi} \cos^6 x dx$ ,

$I_3 = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^3 x dx$ ,  $I_4 = \int_0^1 \ln\left(\frac{1}{x} - 1\right) dx$ . Then

A.  $I_2 = I_3 = I_4 = 0$ ,  $I_1 \neq 0$

B.  $I_1 = I_2 = I_3 = 0, I_4 \neq 0$

C.  $I_1 = I_3 = I_4 = 0, I_2 \neq 0$

D.  $I_1 = I_2 = I_3 = 0, I_4 \neq 0$

**Answer: C**



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44. Given  $\int_0^{\pi/2} \frac{dx}{1 + \sin x + \cos x} = A$ . Then the value of the definite integral

$\int_0^{\pi/2} \frac{\sin x}{1 + \sin x + \cos x} dx$  is equal to

A.  $\frac{1}{2}A$

B.  $\frac{\pi}{2} - A$

C.  $\frac{\pi}{4} - \frac{1}{2}A$

D.  $\frac{\pi}{2} + A$

**Answer: C**



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45.

$$I_1 = \int_{-100}^{101} \frac{dx}{(5 + 2x - 2x^2)(1 + e^{2-4x})}$$

and  $I_2 = \int_{-100}^{101} \frac{dx}{5 + 2x - 2x^2}$ , then  $\frac{I_1}{I_2}$  is (a) 2 (b)  $\frac{1}{2}$  (c) 1 (d)  $-\frac{1}{2}$

A. 2

B.  $\frac{1}{2}$

C. 1

D.  $-\frac{1}{2}$

**Answer: B**



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46.  $\int_0^{\infty} \frac{xdx}{(1+x)(1+x^2)}$

A.  $\frac{\pi}{4}$

B.  $\frac{\pi}{2}$

C.  $\pi$

D. none of these

**Answer: A**

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47. Q.  $\int_0^{\pi} e^{\cos^2 x} (\cos^3(2n+1)x) dx, n \in I$

A.  $\pi$

B. 1

C. 0

D. none of these

**Answer: C**

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48. Let  $f$  be a positive function. If  $I_1 = \int_{1-k}^k x f[x(1-x)] dx$  and  $I_2 = \int_{1-k}^k f[x(1-x)] dx$ , where  $2k - 1 > 0$ . Then  $\frac{I_1}{I_2}$  is

A. 2

B.  $k$

C.  $\frac{1}{2}$

D. 1

Answer: C



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49. If  $f(x) = \frac{e^x}{1 + e^x}$ ,  $I_1 = \int_{f(-a)}^{f(a)} x g(x(1-x)) dx$  and  $I_2 = \int_{f(-a)}^{f(a)} g(x(1-x)) dx$ , then the value of  $\frac{I_2}{I_1}$  is (a) -1 (b) -2 (c) 2 (d) 1

A. -1

B. -2

C. 2

D. 1

**Answer: C**



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50. The value of  $\int_1^2 \frac{x^2 + 1}{x^4 - x^2 + 1} \log\left(1 + x - \frac{1}{x}\right) dx$  is

A.  $\frac{\pi}{8} \log_e 2$

B.  $\frac{\pi}{2} \log_e 2$

C.  $-\frac{\pi}{2} \log_e 2$

D. none of these

**Answer: A**



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51. The value of the definite integral  $\int_0^{\frac{\pi}{2}} \sqrt{\tan x} dx$  is  $\sqrt{2}\pi$  (b)  $\frac{\pi}{\sqrt{2}}$   $2\sqrt{2}\pi$  (d)

$$\frac{\pi}{2\sqrt{2}}$$

A.  $\sqrt{2}\pi$

B.  $\frac{\pi}{\sqrt{2}}$

C.  $2\sqrt{2}\pi$

D.  $\frac{\pi}{2\sqrt{2}}$

**Answer: B**



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52.  $f(x) > 0 \forall x \in R$  and is bounded. If

$$\lim_{n \rightarrow \infty} \left[ \int_0^a \frac{f(x)dx}{f(x) + f(a-x)} + a \int_a^{2a} \frac{f(x)dx}{f(x) + f(3a-x)} + a^2 \int_{2a}^{3a} \frac{f(x)dx}{f(x) + f(5a-x)} + \dots + a^{n-1} \int_{(n-1)a}^{na} \frac{f(x)dx}{f(x) + f[(2n-1)a-x]} \right] = 7/5$$

(where  $a < 1$ ), then  $a$  is equal to

A.  $\frac{2}{7}$

B.  $\frac{1}{7}$

C.  $\frac{14}{19}$

D.  $\frac{9}{14}$

**Answer: C**

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53. If  $\int_0^1 \cot^{-1}(1 - x + x^2) dx = \lambda \int_0^1 \tan^{-1} x dx$ , then  $\lambda$  is equal to (a) 1 (b) 2 (c) 3 (d) 4

A. 1

B. 2

C. 3

D. 4

**Answer: B**

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54. The value of the definite integral  $\int_{-1}^1 (1+x)^{1/2}(1-x)^{3/2} dx$  equals

A.  $\pi$

B.  $\frac{3\pi}{4}$

C.  $\frac{\pi}{4}$

D.  $\frac{\pi}{2}$

**Answer: D**



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55. The value of the integral  $\int_{-3\pi/4}^{5\pi/4} \frac{(\sin x + \cos x)}{e^{x-\pi/4} + 1} dx$  is

A. 0

B. 1

C. 2

D. none of these

**Answer: A**

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56.  $I_1 = \int_0^{\frac{\pi}{2}} \ln(\sin x) dx$ ,  $I_2 = \int_{-\pi/4}^{\pi/4} \ln(\sin x + \cos x) dx$ . Then

A.  $I_1 = 2I_2$

B.  $I_2 = 2I_1$

C.  $I_1 = 4I_2$

D.  $I_2 = 4I_1$

**Answer: A**

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57.

$$\text{If } I_1 = \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{1 + \cos^2 x} dx, I_2 = \int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{1 + \sin^2 x} dx$$

$$I_3 = \int_0^{\frac{\pi}{2}} \frac{1 + 2\cos^2 x \sin^2 x}{4 + 2\cos^2 x \sin^2 x} dx, \text{ then } I_1 = I_2 > I_3 \text{ (b) } I_3 > I_1 = I_2 \text{ (c) } I_1 = I_2 = I_3 \text{ (d)}$$

none of these

A.  $I_1 = I_2 > I_3$

B.  $I_3 > I_1 = I_2$

C.  $I_1 = I_2 = I_3$

D. none of these

**Answer: C**



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58. Evaluate :  $\int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$

A.  $\frac{\pi^2}{2}$

B.  $\frac{\pi^2}{4}$

C.  $\frac{\pi^2}{8}$

D.  $\frac{\pi^2}{16}$

**Answer: D**

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59. For  $x \in \mathbb{R}$ , and a continuous function  $f$  let  $I_1 = \int_{\sin^2 t}^{1 + \cos^2 t} x f\{x(2 - x)\} dx$  and

$$I_2 = \int_{\sin^2 t}^{1 + \cos^2 t} f\{x(2 - x)\} dx.$$

Then  $\frac{I_1}{I_2}$  is

A. -1

B. 1

C. 2

D. 3

**Answer: B**

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60. If  $\int_{-\pi}^{\frac{3\pi}{4}} \frac{e^{\frac{\pi}{4}} dx}{\left(e^x + e^{\frac{\pi}{4}}\right)(\sin x + \cos x)} = k \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sec x dx$ , then the value of  $k$  is  $\frac{1}{2}$  (b)  $\frac{1}{\sqrt{2}}$

(c)  $\frac{1}{2\sqrt{2}}$  (d)  $-\frac{1}{\sqrt{2}}$

A.  $\frac{1}{2}$

B.  $\frac{1}{\sqrt{2}}$

C.  $\frac{1}{2\sqrt{2}}$

D.  $-\frac{1}{\sqrt{2}}$

Answer: C



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61. The value of the definite integral

$\int_2^4 x(3-x)(4+x)(6-x)(10-x) + \sin x dx$  equals  $\cos 2 + \cos 4$  (b)  $\cos 2 - \cos 4$

$\sin 2 + \sin 4$  (d)  $\sin 2 - \sin 4$

A.  $\cos^2 + \cos^4$

B.  $\cos^2 - \cos^4$

C.  $\sin^2 + \sin^4$

D.  $\sin^2 - \sin^4$

**Answer: B**



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62. If  $I = \int_{-20\pi}^{20\pi} |\sin x| [\sin x] dx$  (where  $[\cdot]$  denotes the greatest integer function), then the value of  $I$  is -40 (b) 40 (c) 20 (d) -20

A. -40

B. 40

C. 20

D. -20

**Answer: A**

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63. The function  $f$  and  $g$  are positive and continuous. If  $f$  is increasing and  $g$  is decreasing, then  $\int_0^1 f(x)[g(x) - g(1-x)]dx$  is always non-positive is always non-negative can take positive and negative values none of these

- A. is always non-positive
- B. is always non-negative
- C. can take positive and negative values
- D. none of these

**Answer: A**

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64. Evaluate :  $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$

- A.  $\frac{\pi^2}{4}$

B.  $\frac{\pi^2}{2}$

C.  $\frac{3\pi^2}{2}$

D.  $\frac{\pi^2}{3}$

**Answer: A**

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65. If  $f(x) = \int_0^{\pi} \frac{t \sin t dt}{\sqrt{1 + \tan^2 x \sin^2 t}}$  for  $0 < x < \frac{\pi}{2}$  then

A.  $f(0^+) = -\pi$

B.  $f\left(\frac{\pi}{4}\right) = \frac{\pi^2}{8}$

C.  $f$  is continuous and differentiable in  $\left(0, \frac{\pi}{2}\right)$

D.  $f$  is continuous but not differentiable in  $\left(0, \frac{\pi}{2}\right)$

**Answer: C**

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66. about to only mathematics

A.  $3^8$

B.  $3^7$

C.  $3^9$

D. none of these

**Answer: B**



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67. The value of  $\int_0^{4\pi} \log_e |3\sin x + 3\sqrt{3}\cos x| dx$  then the value of I is equal to

A.  $\pi \log_e 3$

B.  $2\pi \log_e 3$

C.  $4\pi \log_e 3$

D.  $8\pi \log_e 3$

**Answer: C**



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68. The value of  $\int_0^\pi \frac{|x|\sin^2 x}{1 + 2|\cos x|\sin x} dx$  is equal to

A.  $\pi/4$

B.  $\pi/2$

C.  $\pi$

D.  $2\pi$

**Answer: B**



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69. The value of the integral  $\int_{-\pi}^{\pi} \sin mx \sin nx dx$ , for  $m \neq n (m, n \in I)$ , is 0 (b)  $\pi$   
 (c)  $\frac{\pi}{2}$  (d)  $2\pi$

A. 0

B.  $\pi$

C.  $\pi/2$

D.  $2\pi$

**Answer: A**



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70. If  $f(x)$  and  $g(x)$  are continuous functions, then

$$\int_{\ln \lambda}^{\ln(1/\lambda)} \frac{f(x^2/4)[f(x) - f(-x)]}{g(x^2/4)[g(x) + g(-x)]} dx$$

A. dependent on  $\lambda$

B. a non zero constant

C. zero

D. none of these

**Answer: C**



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71. The value of  $\int_0^1 \frac{\tan^{-1}\left(\frac{x}{x+1}\right)}{\tan^{-1}\left(\frac{1+2x-2x^2}{2}\right)} dx$  is

A.  $1/4$

B.  $1/2$

C. 1

D. 2

**Answer: B**



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72.  $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{e^{|\sin x|} \cos x}{(1 + e^{\tan x})} dx$  is equal to (a)  $e + 1$  (b)  $1 - e$  (c)  $e - 1$  (d) none of these

A.  $e + 1$

B.  $2e$

C.  $e - 1$

D.  $e - 2$

**Answer: C**



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73. Evaluate the following definite integral:  $\int_{-\pi}^{\pi} \frac{2x(1 + \sin x)}{1 + \cos^2 x} dx$

A.  $\pi$

B.  $\pi^2$

C.  $2\pi^2$

D.  $\pi^2/2$

**Answer: B**



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74. [ The value of  $\int_{-\pi}^{\pi} \sum_{r=0}^{999} \cos rx \left( 1 + \sum_{r=1}^{999} \sin rx \right) dx$ , is [ (1)  $2\pi$ , (2)  $999\pi$ , (3)  $0$  ]]

A.  $2\pi$

B.  $999\pi$

C.  $0$

D.  $\pi$

**Answer: A**



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75. Let  $T > 0$  be a fixed real number. Suppose  $f$  is continuous function such that for all  $x \in R$ ,  $f(x + T) = f(x)$ . If  $I = \int_0^T f(x) dx$ , then the value of

$$\int_3^{3+3T} f(2x) dx \text{ is } \frac{3}{2}I \text{ (b) } 2I \text{ (c) } 3I \text{ (d) } 6I$$

A.  $\frac{3}{2}I$

B.  $2I$

C.  $3I$

D.  $6I$

**Answer: C**



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76.  $\int_1^4 (x - 0.4) dx$  equals  $\left( \text{where } \{x\} \text{ is a } \frac{t}{i} \text{ onal part of } (x) \right)$  (a) 13 (b) 6.3 (c) 1.5 (d) 7.5

A. 13

B. 6.3

C. 1.5

**Answer: C****Watch Video Solution**

77. The value of  $\int_0^x [\text{cost}] dt, x \in \left[ (4n+1)\frac{\pi}{2}, (4n+3)\frac{\pi}{2} \right]$  and  $n \in N$ , is equal

to where  $[.]$  represents greatest integer function.  $\frac{\pi}{2}(2n-1) - 2x$

$\frac{\pi}{2}(2n-1) + x$   $\frac{\pi}{2}(2n+1) - x$  (d)  $\frac{\pi}{2}(2n+1) + x$

A.  $\frac{\pi}{2}(2n-1) - 2x$

B.  $\frac{\pi}{2}(2n-1) + x$

C.  $\frac{\pi}{2}(2n+1) - x$

D.  $\frac{\pi}{2}(2n+1) + x$

**Answer: C****Watch Video Solution**

78. Evaluate  $\int_0^{2\pi} [\sin x] dx$ , where  $[.]$  denotes the greatest integer function.

A.  $4n - \cos x$

B.  $4n - \sin x$

C.  $4n + 1 - \cos x$

D.  $4n - 1 - \cos x$

**Answer: C**



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79.  $\int_0^x \frac{2^t}{2^{[t]}} dt$ , where  $[.]$  denotes the greatest integer function and  $x \in \mathbb{R}^+$ ,

is equal to

A.  $\frac{1}{1n2} \left( [x] + 2^{\{x\}} - 1 \right)$

B.  $\frac{1}{1n2} \left( [x] + 2^{\{x\}} \right)$

C.  $\frac{1}{1n2} \left( [x] - 2^{\{x\}} \right)$

$$D. \frac{1}{1n2} \left( [x] + 2^{\{x\}} + 1 \right)$$

**Answer: A**



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**80.**  $f$  is an odd function, It is also known that  $f(x)$  is continuous for all values of  $x$  and is periodic with period 2. If  $g(x) = \int_0^x f(t)dt$ , then (a)  $g(x)$  is odd (b)  $g(n) = 0, n \in N$  (c)  $g(2n) = 0, n \in N$  (d)  $g(x)$  is non-periodic

A.  $g(x)$  is odd

B.  $g(n) = 0, n \in N$

C.  $g(2n) = 0, n \in N$

D.  $g(x)$  is non-periodic

**Answer: C**



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81. If  $g(x) = \int_0^x (|\sin t| + |\cos t|) dt$ , then  $g\left(x + \frac{\pi n}{2}\right)$  is equal to, where  $n \in N$ ,

a)  $g(x) + g(\pi)$

(b)  $g(x) + ng\left(\frac{\pi}{n2}\right)$

c)  $g(x) + g\left(\frac{\pi}{2}\right)$

(d) none of these

A.  $g(x) + g(\pi)$

B.  $g(x) + ng\left(\frac{\pi}{2}\right)$

C.  $g(x) + g\left(\frac{\pi}{2}\right)$

D. none of these

**Answer: B**



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82. If  $x = \int_c^{\sin t} \sin^{-1} z dz$ ,  $y = \int_k^{\sqrt{t} \sin z^2} \frac{dz}{z}$ , then  $\frac{dy}{dx}$  is equal to (a)  $\frac{\tan t}{2t}$  (b)  $\frac{\tan t}{t^2}$

(c)  $\frac{t}{2t^2}$  (d)  $\frac{\tan t}{2t^2}$

A.  $\frac{\tan t}{2t}$

B.  $\frac{\tan t}{t^2}$

C.  $\frac{\tan t}{2t^2}$

D.  $\frac{\tan t^2}{2t^2}$

**Answer: C**



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83. Let  $f(x) = \int_2^x \frac{dt}{\sqrt{1+t^4}}$  and  $g$  be the inverse of  $f$ . Then, the value of  $g'(0)$

is

A. 1

B. 17

C.  $\sqrt{17}$

D. none of these

**Answer: C**



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84. If  $f(x)$  is differentiable and  $\int_0^{t^2} xf(x)dx = \frac{2}{5}t^5$ , then  $f\left(\frac{4}{25}\right)$  equals (a)  $\frac{2}{5}$  (b)

$-\frac{5}{2}$  (c) 1 (d)  $\frac{5}{2}$

A. 2/5

B. -5/2

C. 1

D. 5/2

Answer: A



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85. If  $f(x) = \cos x - \int_0^x (x-t)f(t)dt$ , then  $f'(x) + f(x)$  is equal to

a)  $-\cos x$

(b)  $-\sin x$

c)  $\int_0^x (x-t)f(t)dt$

(d) 0

A.  $-\cos x$

B.  $-\sin x$

C.  $\int_0^x (x-t)f(t)dt$

D. 0

**Answer: A**



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**86.** A function  $f$  is continuous for all  $x$  (and not everywhere zero) such that

$f^2(x) = \int_0^x f(t) \frac{\cos t}{2 + \sin t} dt$ . Then  $f(x)$  is

A.  $\frac{1}{2} \ln \left( \frac{x + \cos x}{2} \right)$

B.  $\frac{1}{2} \ln \left( \frac{3}{2 + \cos x} \right)$

C.  $\frac{1}{2} \ln \left( \frac{2 + \sin x}{2} \right)$

D.  $\frac{\cos x + \sin x}{2 + \sin x}$

**Answer: C**



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87.  $\lim_{x \rightarrow 0} \frac{1}{x} \left[ \int_y^a e^{\sin^2 t} dt - \int_{x+y}^a e^{\sin^2 t} dt \right]$  is equal to (a)  $e^{\sin^2 y}$  (b)  $\sin 2y e^{\sin^2 y}$  (c) 0 (d) none of these

A. a.  $e^{\sin^2 y}$

B.  $\sin 2y e^{\sin^2 y}$

C. 0

D. none of these

**Answer: A**



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88. Let  $f(x) = \int_1^x \frac{e^t}{t} dt$ ,  $x \in \mathbb{R}^+$ . Then complete set of values of  $x$  for which  $f(x) \leq \ln x$  is

- A.  $(0, 1]$
- B.  $[1, \infty)$
- C.  $(0, \infty)$
- D. none of these

**Answer: A**



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89. If  $\int_0^x f(t) dt = x + \int_x^1 t f(t) dt$ , then the value of  $f(1)$

- A.  $1/2$
- B.  $0$
- C.  $1$

D.  $-1/2$

**Answer: A**



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90. If  $f(x) = 1 + \frac{1}{x} \int_1^x f(t) dt$ , then the value of  $f(e^{-1})$  is (a) 1 (b) 0 (c) -1 (d)

none of these

A. 1

B. 0

C. -1

D. none of these

**Answer: B**



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91. If  $\left[ f\left(\frac{\sqrt{3}}{2}\right) \right]$  is [.] denotes the greatest integer function) 4 (b) 5 (c) 6 (d)

-7

A. 4

B. 5

C. 6

D. -7

**Answer: B**



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92.  $f(x)$  is continuous function for all real values of  $x$  and satisfies

$$\int_0^x f(t) dt = \int_x^1 t^2 f(t) dt + \frac{x^{16}}{8} + \frac{x^6}{3} + a$$

Then the value of  $a$  is equal to: (a)  $-\frac{1}{24}$

(b)  $\frac{17}{168}$  (c)  $\frac{1}{7}$  (d)  $-\frac{167}{840}$

A. (a)  $-\frac{1}{24}$

B.  $\frac{17}{168}$

C.  $\frac{1}{7}$

D.  $-\frac{167}{840}$

**Answer: D**



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93. The value of  $\int_{\frac{1}{e} \rightarrow \tan x} \frac{tdt}{1+t^2} + \int_{\frac{1}{e} \rightarrow \cot x} \frac{dt}{t \cdot (1+t^2)} =$

A. (a) 0

B. (b) 2

C. (c) 1

D. (c) none of these

**Answer: C**



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94.  $\lim_{x \rightarrow \infty} \frac{\int_0^x \tan^{-1} t \, dt}{\sqrt{x^2 + 1}}$  is equal to

A.  $\frac{\pi}{2}$

B.  $\frac{\pi}{4}$

C. 1

D.  $\pi$

**Answer: A**



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95. A function  $f$  is defined by  $f(x) = \int_0^\pi \cos t \cos(x - t) dt$ ,  $0 \leq x \leq 2\pi$  then which of the following hold(s) good?

A.  $\frac{\pi}{4}$

B.  $\frac{\pi}{2}$

C.  $\frac{-\pi}{2}$

D.  $\frac{-\pi}{4}$

**Answer: C**



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96. If  $f'$  is a differentiable function satisfying  $f(x) = \int_0^x \sqrt{1 - f^2(t)} dt + \frac{1}{2}$  then the value of  $f(\pi)$  is equal to

A.  $-\frac{\sqrt{3}}{2}$

B.  $-\frac{1}{2}$

C.  $\frac{\sqrt{3}}{2}$

D.  $\frac{1}{2}$

**Answer: B**



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97. If  $\int_0^1 e^{x^2}(x - \alpha)dx = 0$ , then

A.  $1 < \alpha < 2$

B.  $\alpha < 0$

C.  $0 < \alpha < 1$

D.  $\alpha = 0$

**Answer: C**



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98. The value of the integral  $\int_0^1 e^{x^2} dx$  lies in the interval (a) (0, 1) (b) (-1, 0) (c) (1, e) (d) none of these

A. (0, 1)

B. (-1, 0)

C. (1, e)

D. none of these

**Answer: C**



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99. Given that  $f$  satisfies  $|f(u) - f(v)| \leq |u - v|f$  or  $u$  and  $v$  in  $[a, b]$  Then

$$\left| \int_a^b f(x) dx - (b - a)f(a) \right| \leq \text{(a) } \frac{(b - a)}{2} \text{ (b) } \frac{(b - a)^2}{2} \text{ (c) } (b - a)^2 \text{ (d) none of these}$$

A.  $\frac{(b - a)}{2}$

B.  $\frac{(b - a)^2}{2}$

C.  $(b - a)^2$

D. none of these

**Answer: B**



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100. The value of the integral  $\int_0^{\infty} \frac{x \log x}{(1+x^2)^2} dx$  (a) 0 (b)  $\log 7$  (c)  $5 \log 13$  (d)

none of these

A. 0

B.  $\log 7$

C.  $5 \log 13$

D. none of these

**Answer: A**



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101.  $\int_0^{\infty} \left( \frac{\pi}{1+\pi^2 x^2} - \frac{1}{1+x^2} \right) \log x dx$  is equal to (a)  $-\frac{\pi}{2} \log \pi$  (b) 0 (c)  $\frac{\pi}{2} \log 2$  (d)

none of these

A.  $-\frac{\pi}{2} \ln \pi$

B. 0

C.  $\frac{\pi}{2} \ln 2$

D. none of these

**Answer: A**

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102. If  $A = \int_0^{\pi} \frac{\cos x}{(x+2)^2} dx$ , then  $\int_0^{\pi/2} \frac{\sin 2x}{x+1} dx$  is equal to

A.  $\frac{1}{2} + \frac{1}{\pi+2} - A$

B.  $\frac{1}{\pi+2} - A$

C.  $1 + \frac{1}{\pi+2} - A$

D.  $A - \frac{1}{2} - \frac{1}{\pi+2}$

**Answer: A**

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103.  $\int_0^4 \frac{(y^2 - 4y + 5)\sin(y - 2)dy}{[2y^2 - 8y + 1]}$  is equal to

- A. 0
- B. 2
- C. -2
- D. none of these

**Answer: A**

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104.  $\int_{\sin\theta}^{\cos\theta} f(x\tan\theta)dx$  (where  $\theta \neq \frac{n\pi}{2}, n \in I$ ) is equal to

A.  $-\cos\theta \int_1^{\tan\theta} f(x\sin\theta)dx$

B.  $-\tan\theta \int_{\cos\theta}^{\sin\theta} f(x)dx$

C.  $\sin\theta \int_1^{\tan\theta} f(x\cos\theta)dx$

$$D. -\frac{1}{\tan\theta} \int_{\sin\theta}^{\sin\theta \tan\theta} f(x) dx$$

**Answer: A**



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105. If  $I_1 = \int_0^1 \frac{e^x}{1+x} dx$  and  $I_2 = \int_0^1 \frac{x^2}{e^{x^3}(2-x^3)} dx$  then  $\frac{I_1}{I_2}$  is

A.  $3/e$

B.  $e/3$

C.  $3e$

D.  $1/3e$

**Answer: C**



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106. Let  $I_1 = \int_{-2}^2 \frac{x^6 + 3x^5 + 7x^4}{x^4 + 2} dx$  and  $I_2 = \int_{-3}^1 \frac{2(x+1)^2 + 11(x+1) + 14}{(x+1)^4 + 2} dt$ .

Then the value of  $I_1 + I_2$  is (a) 8 (b)  $\frac{200}{3}$  (c)  $\frac{100}{3}$  (d) none of these

A. 8

B.  $200/3$

C.  $100/3$

D. noe

**Answer: C**



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107. Let  $f$  be integrable over  $[0, a]$  for any real value of  $a$ .

If  $I_1 = \int_0^{\pi/2} \cos\theta f(\sin\theta + \cos^2\theta) d\theta$  and  $I_2 = \int_0^{\pi/2} \sin 2\theta f(\sin\theta + \cos^2\theta) d\theta$ , then

A.  $I_1 = -2I_2$

B.  $I_1 = I_2$

C.  $2I_1 = I_2$

D.  $I_1 = -I_2$

**Answer: B**



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108. The value of  $\int_a^b (x - a)^3 (b - x)^4 dx$  is (a)  $\frac{(b - a)^4}{6^4}$  (b)  $\frac{(b - a)^8}{280}$  (c)  $\frac{(b - a)^7}{7^3}$

(d) none of these

A.  $\frac{(b - a)^4}{6^4}$

B.  $\frac{(b - a)^8}{280}$

C.  $\frac{(b - a)^7}{7^3}$

D. none of these

**Answer: B**



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109. If  $I(m, n) = \int_0^1 x^{m-1}(1-x)^{n-1} dx$ , then

A.  $I(m, n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m-n}} dx$

B.  $I(m, n) = \int_0^\infty \frac{x^m}{(1+x)^{m+n}} dx$

C.  $I(m, n) = \int_0^\infty \frac{x^{n-1}}{(1+x)^{m+n}} dx$

D.  $I(m, n) = \int_0^\infty \frac{x^n}{(1+x)^{m+n}} dx$

Answer: C



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110. The value of the definite integral  $\int_0^{\frac{\pi}{2}} \frac{\sin 5x}{\sin x} dx$  is 0 (b)  $\frac{\pi}{2}$  (c)  $\pi$  (d)  $2\pi$

A. 0

B.  $\frac{\pi}{2}$

C.  $\pi$

D.  $2\pi$

**Answer: A**



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111. If  $I_n = \int_0^\pi e^x (\sin x)^n dx$ , then  $\frac{I_3}{I_1}$  is equal to

A.  $3/5$

B.  $1/5$

C.  $1$

D.  $2/5$

**Answer: A**



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112. If  $f'(x) = f(x) + \int_0^1 f(x) dx$ , given  $f(0) = 1$ , then the value of  $f(\log_e 2)$  is

A.  $\frac{1}{3+e}$

B.  $\frac{5 - e}{3 - e}$

C.  $\frac{2 + e}{e - 2}$

D. none of these

**Answer: B**

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**113.** Let  $f(x)$  be positive, continuous, and differentiable on the interval

$(a, b)$  and  $(\lim)_{x \rightarrow a^+} f(x) = 1$ ,  $(\lim)_{x \rightarrow b^-} f(x) = 3^{\frac{1}{4}}$  If  $f'(x) \geq f^3(x) + \frac{1}{f(x)}$ , then the

greatest value of  $b - a$  is  $\frac{\pi}{48}$  (b)  $\frac{\pi}{36}$   $\frac{\pi}{24}$  (d)  $\frac{\pi}{12}$

A.  $\frac{\pi}{48}$

B.  $\frac{\pi}{36}$

C.  $\frac{\pi}{24}$

D.  $\frac{\pi}{12}$

**Answer: C**



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## Mcq Type

1. If  $f(x)$  is integrable over  $[1, 2]$  then  $\int_1^2 f(x) dx$  is equal to (a)

$$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^n f\left(\frac{r}{n}\right) \quad (\text{b}) \quad \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=n+1}^{2n} f\left(\frac{r}{n}\right) \quad (\text{c})$$

$$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^n f\left(\frac{r+n}{n}\right) \quad (\text{d}) \quad \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^{2n} f\left(\frac{r}{n}\right)$$

A. (a)  $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^n f\left(\frac{r}{n}\right)$

B.  $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=n+1}^{2n} f\left(\frac{r}{n}\right)$

C.  $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^n f\left(\frac{r+n}{n}\right)$

D.  $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^{2n} f\left(\frac{r}{n}\right)$

Answer: B::C



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2. If  $L = \lim_{n \rightarrow \infty} \frac{n^3(e^{1/n} + e^{2/n} + \dots + e)}{(n+1)^m(1^m + 4^m + \dots + n^{2m})}$  is non zero finite real,

then

A.  $L = 3(e - 1)$

B.  $L = 2(e - 1)$

C.  $m = 1/3$

D.  $m = 1/3$

**Answer: A:C**

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3. Let  $p = 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \dots + \frac{1}{\sqrt{120}}$  and  $q = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \dots + \frac{1}{\sqrt{121}}$  then

A.  $p > 20$

B.  $q < 20$

C.  $p + q < 40$

$$D. p + q > 40$$

**Answer: A::B::D**



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**4. about to only mathematics**

$$A. S_n < \frac{\pi}{3\sqrt{3}}$$

$$B. S_n > \frac{\pi}{3\sqrt{3}}$$

$$C. T_n < \frac{\pi}{3\sqrt{3}}$$

$$D. T_n > \frac{\pi}{3\sqrt{3}}$$

**Answer: A::D**



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5. The value of  $\int_0^1 \frac{2x^2 + 3x + 3}{(x+1)(x^2 + 2x + 2)} dx$  is

A.  $\frac{\pi}{4} + 2\log 2 - \tan^{-1} 2$

B.  $\frac{\pi}{4} + 2\log 2 - \tan^{-1} \frac{1}{3}$

C.  $2\log 2 - \cot^{-1} 3$

D.  $-\frac{\pi}{4} + \log 4 + \cot^{-1} 2$

**Answer: A::C::D**



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6. Let  $f(x) = \int_1^x \frac{3^t}{1+t^2} dt$ , where  $x > 0$ , Then

A. for  $0 < \alpha < \beta$ ,  $f(\alpha) < f(\beta)$

B. for  $0 < \alpha < \beta$ ,  $f(\alpha) > f(\beta)$

C.  $f(x) + \pi/4 < \tan^{-1} x \forall x \geq 1$

D.  $f(x) + \pi/4 > \tan^{-1} x \forall x \geq 1$

**Answer: A::D**



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7. If  $\int_a^b |\sin x| dx = 8$  and  $\int_0^{a+b} |\cos x| dx = 9$ , then find the value of  $\int_a^b x \sin x dx$ .

A.  $a + b = \frac{9\pi}{2}$

B.  $|a - b| = 4\pi$

C.  $\frac{a}{b} = 15$

D.  $\int_a^b \sec^2 x dx = 0$

**Answer: A::B**



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8. If  $g(x) = \int_0^x 2|t| dt$ , then

(a)  $g(x) = x|x|$  (b)  $g(x)$  is monotonic

(c)  $g(x)$  is differentiable at  $x = 0$  (d)  $g'(x)$  is differentiable at  $x = 0$

A.  $g(x) = x|x|$

B.  $g(x)$  is monotonic

C.  $g(x)$  is differentiable at  $x = 0$

D.  $g'(x)$  is differentiable at  $x = 0$

**Answer: A::B::C**

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9. If  $A_n = \int_0^{\frac{\pi}{2}} \frac{\sin(2n-1)x}{\sin x} dx$ ,  $b_n = \int_0^{\frac{\pi}{2}} \left( \frac{\sin nx}{\sin x} \right)^2 dx$  or  $n \in N$ , Then (a)

$A_{n+1} = A_n$  (b)  $B_{n+1} = B_n$  (c)  $A_{n+1} - A_n = B_{n+1}$  (d)  $B_{n+1} - B_n = A_{n+1}$

A.  $A_{n+1} = A_n$

B.  $B_{n+1} = B_n$

C.  $A_{n+1} - A_n = B_{n+1}$

D.  $B_{n+1} - B_n = A_{n+1}$

**Answer: A::D**



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10. The value of  $\int_0^{\infty} \frac{dx}{1+x^4}$  is (a) same as that of  $\int_0^{\infty} \frac{x^2+1}{1+x} dx$  (b)  $\frac{\pi}{2\sqrt{2}}$  (c)

same as that of  $\int_0^{\infty} \frac{x^2+1}{1+x^4} dx$  (d)  $\frac{\pi}{\sqrt{2}}$

A. same as that of  $\int_0^{\infty} \frac{x^2+1}{1+x^4} dx$

B.  $\frac{\pi}{2\sqrt{2}}$

C. same as that of  $\int_0^{\infty} \frac{x^2 dx}{1+x^4}$

D.  $\frac{\pi}{\sqrt{2}}$

Answer: B::C



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11. The value of  $\int_0^1 e^{x^2-x} dx$  is (a)  $< 1$  (b)  $> 1$  (c)  $> e^{-\frac{1}{4}}$  (d)  $< e^{-\frac{1}{4}}$

A.  $< 1$

B.  $> 1$

C.  $> e^{-\frac{1}{4}}$

D.  $< e^{-\frac{1}{4}}$

**Answer: A::C**



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12. If  $\int_a^b \frac{f(x)}{f(x) + f(a + b - x)} dx = 10$ , then

A.  $b = 22, a = 2$

B.  $b = 15, a = -5$

C.  $b = 10, a = -10$

D.  $b = 10, a = -2$

**Answer: A::B::C**



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13. The values of  $a$  for which the integral  $\int_0^2 |x - a| dx \geq 1$  is satisfied are  
(2,  $\infty$ ) (b) ( $-\infty$ , 0) (0, 2) (d) none of these

A.  $[2, \infty)$

B.  $(-\infty, 0]$

C. (0, 2)

D. none of these

**Answer: A::B::C**



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14. If  $f(x) = \int_0^x |t - 1| dt$ , where  $0 \leq x \leq 2$ , then

A. range of  $f(x)$  is  $[0, 1]$

B.  $f(x)$  is differentiable at  $x = 1$

C.  $f(x) = \cos^{-1}x$  has two real roots

D.  $f(1/2) = 1/2$

**Answer: B**



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15. If  $f(2 - x) = f(2 + x)$  and  $f(4 - x) = f(4 + x)$  for all  $x$  and  $f(x)$  is a function for which  $\int_0^2 f(x) dx = 5$ , then  $\int_0^{50} f(x) dx$  is equal to (a) 125 (b)  $\int_{-4}^{46} f(x) dx$  (c)  $\int_1^{51} f(x) dx$  (d)  $\int_2^{52} f(x) dx$

A. 125

B.  $\int_{-4}^{46} f(x) dt$

C.  $\int_1^{51} f(x) dx$

D.  $\int_2^{52} f(x) dx$

**Answer: A::B::D**



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16. If  $f(x) = \int_0^x (\cos(\sin t) + \cos(\cos t)) dt$ , then  $f(x + \pi)$  is (a)  $f(x) + f(\pi)$  (b)  $f(x) + 2(\pi)$  (c)  $f(x) + f\left(\frac{\pi}{2}\right)$  (d)  $f(x) + 2f\left(\frac{\pi}{2}\right)$

A.  $f(x) + f(\pi)$

B.  $f(x) + 2f(\pi)$

C.  $f(x) + f\left(\frac{\pi}{2}\right)$

D.  $f(x) + 2f\left(\frac{\pi}{2}\right)$

**Answer: A:D**

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17. If  $I_n = \int_0^{\pi/4} \tan^n x dx$ , ( $n > 1$  is an integer), then (a)  $I_n + I_{n-2} = \frac{1}{n+1}$  (b)  $I_n + I_{n-2} = \frac{1}{n-1}$  (c)  $I_2 + I_4, I_6, \dots$  are in H.P. (d)  $\frac{1}{2(n+1)} < I_n < \frac{1}{2(n-1)}$

A.  $I_n + I_{n-2} = \frac{1}{n+1}$

B.  $I_n + I_{n-2} = \frac{1}{n-1}$

C.  $I_2 + I_4, I_6, \dots$  are in H.P.

D.  $\frac{1}{2(n+1)} < I_n < \frac{1}{2(n-1)}$

**Answer: B::C::D**

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18. If  $I_n = \int_0^1 \frac{dx}{(1+x^2)^n}$ , where  $n \in \mathbb{N}$ , which of the following statements

hold good? (a)  $2nI_{n+1} = 2^{-n} + (2n-1)I_n$

(b)  $I_2 = \frac{\pi}{8} + \frac{1}{4}$  (c)  $I_2 = \frac{\pi}{8} - \frac{1}{4}$  (d)  $I_3 = \frac{3\pi}{32} + \frac{1}{4}$

A.  $2nI_{n+1} = 2^{-n} + (2n-1)I_n$

B.  $I_2 = \frac{\pi}{8} + \frac{1}{4}$

C.  $I_2 = \frac{\pi}{8} - \frac{1}{4}$

D.  $I_3 = \frac{3\pi}{32} + \frac{1}{4}$

**Answer: A::B::D**

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19. Let  $f: [1, \infty) \rightarrow \mathbb{R}$  and  $f(x) = \int_1^x \frac{e^t}{t} dt - e^x$ . Then  $f(x)$  is an increasing function  
 $\lim_{x \rightarrow \infty} f(x) \rightarrow \infty$   $f'(x)$  has a maxima at  $x = e$   $f(x)$  is a decreasing function

A.  $f(x)$  is an increasing function

B.  $\lim_{x \rightarrow \infty} f(x) \rightarrow \infty$

C.  $f'(x)$  has a maxima at  $x = e$

D.  $f(x)$  is a decreasing function

**Answer: A:B**



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20. If  $f(x) = \int_a^x [f(x)]^{-1} dx$  and  $\int_a^1 [f(x)]^{-1} dx = \sqrt{2}$ , then

A.  $f(2) = 2$

B.  $f(2) = 1/2$

C.  $f^{-1}(2) = 2$

D.  $\int_0^1 f(x) dx = \sqrt{2}$

**Answer: A::B::C**



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21. A continuous function  $f(x)$  satisfies the relation  $f(x) = e^x + \int_0^1 e^x f(t) dt$   
then  $f(1) =$

A.  $f(0) < 0$

B.  $f(x)$  is a decreasing function

C.  $f(x)$  is increasing function

D.  $\int_0^1 f(x) dx > 0$

**Answer: A::B**



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22.  $\int_0^x \left\{ \int_0^u f(t) dx \right\} du$  is equal to (a)  $\int_0^x (x-u)f(u)du$  (b)  $\int_0^x uf(x-u)du$  (c)  $x \int_0^x f(u)du$  (d)  $x \int_0^x uf(u-x)du$

A.  $\int_0^x (x-u)f(u)du$

B.  $\int_0^x uf(x-u)du$

C.  $x \int_0^x f(u)du$

D.  $x \int_0^x uf(u-x)du$

**Answer: A::B**



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23. Which of the following statement(s) is/are TRUE?

A. If function  $y = f(x)$  is continuous at  $x = c$  such that  $f(c) \neq 0$ , then

$f(x)f(c) > 0 \forall x \in (c-h, c+h)$ , where  $h$  is sufficiently small positive

quantity.

B.  $\lim_{n \rightarrow \infty} \frac{1}{n} \left( n \left( 1 + \frac{1}{n} \right) \left( 1 + \frac{2}{n} \right) \dots \left( 1 + \frac{n}{n} \right) \right) = 1 + 2 \ln 2.$

C. Let  $f$  be a continuous and non-negative function defined on  $[a, b]$  If

$$\int_a^b f(x) dx = 0, \text{ then } f(x) = \forall x \in [a, b]$$

D. Let  $f$  be continuous function defined on  $[a, b]$  such that  $\int_a^b f(x) dx = 0$

.Then there exists at least one  $c \in (a, b)$  for which  $f(c) = 0$

**Answer: A::C::D**

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24. If  $\int_0^x [x] dx = \int_0^{[x]} x dx, x \notin \text{integer}$  (where,  $[.]$  and  $\{.\}$  denotes the greatest integer and fractional parts respectively, then the value of  $4\{x\}$  is equal to ...

A.  $x \in [0, 1)$

B.  $\{x\} = 1/2$

C.  $\{x\} = 1/3$

D.  $x > 0$

**Answer: A::B**



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25. Consider the function  $f(\theta) = \int_0^1 \frac{|\sqrt{1-x^2} - \sin\theta|}{\sqrt{1-x^2}} dx$ , where  $0 \leq \theta \leq \frac{\pi}{2}$ ,

then

A.  $f_{\min} = \sqrt{2} - 1$

B.  $f_{\min} = \sqrt{2} + 1$

C.  $f_{\max} = 1$

D.  $f_{\max} = \frac{\pi}{2} - 1$

**Answer: A::B::C::D**



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26.  $f: [0, 1) \rightarrow R$  be a non increasing function then for  $\alpha \in (0, 1)$

$$A. \alpha \int_0^1 f(x) dx \leq \int_0^\alpha f(x) dx$$

$$B. \alpha \int_0^1 f(x) dx \geq \int_0^\alpha f(x) dx$$

$$C. \alpha^2 \int_0^1 f(x) dx \leq \int_0^\alpha f(x) dx$$

$$D. \sqrt{\alpha} \int_0^1 f(x) dx \geq \int_0^\alpha f(x) dx$$

**Answer: A::C**

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27. Let  $f(x)$  be a non-constant twice differentiable function defined on  $(\infty, \infty)$  such that  $f(x) = f(1 - x)$  and  $f''(1/4) = 0$ . Then

A.  $f'(x)$  vanishes at least twice on  $[0, 1]$

$$B. f'\left(\frac{1}{2}\right) = 0$$

$$C. \int_{-1/2}^{1/2} f\left(x + \frac{1}{2}\right) \sin x dx = 0$$

$$D. \int_0^{1/2} f(t) e^{\sin xt} dt = \int_{t/2}^1 f(1 - t) e^{\sin \pi t} dt$$

**Answer: A::B::C::D**



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Lc Type

1.  $y = f(x)$  satisfies the relation  $\int_2^x f(t)dt = \frac{x^2}{2} + \int_x^2 t^2 f(t)dt$

The range of  $y = f(x)$  is

A.  $[0, \infty)$

B.  $R$

C.  $(-\infty, 0]$

D.  $\left[-\frac{1}{2}, \frac{1}{2}\right]$

Answer: D



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2.  $y = f(x)$  satisfies the relation  $\int_2^x f(t) dt = \frac{x^2}{2} + \int_x^2 t^2 f(t) dt$

The value of  $\int_{-2}^2 f(x) dx$  is

- A. 0
- B. -2
- C.  $2\log_e 2$
- D. none of these

**Answer: A**



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3.  $y = f(x)$  satisfies the relation  $\int_2^x f(t) dt = \frac{x^2}{2} + \int_x^2 t^2 f(t) dt$

The value of  $x$  for which  $f(x)$  is increasing is

- A.  $(-\infty, 1]$
- B.  $[-1, \infty)$

C.  $[-1, 1]$

D. none of these

**Answer: C**



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4. Let  $f: R \rightarrow R$  be a differentiable function such that

$f(x) = x^2 + \int_0^x e^{-t} f(x-t) dt$ .  $f(x)$  increases for

A.  $x > 1$

B.  $x < -2$

C.  $x > 2$

D. none of these

**Answer: B**



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5. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a differentiable function such that

$$f(x) = x^2 + \int_0^x e^{-t} f(x-t) dt.$$

$y = f(x)$  is

- A. (a) injective but not surjective
- B. (b) surjective but not injective
- C. (c) bijective
- D. (d) neither injective nor surjective

**Answer: B**



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6. Let  $f(x)$  be a differentiable function such that  $f(x) = x^2 + \int_0^x e^{-t} f(x-t) dt$

then  $\int_0^1 f(x) dx =$

A.  $\frac{1}{4}$

B.  $-\frac{1}{12}$

C.  $\frac{5}{12}$

D.  $\frac{12}{7}$

**Answer: C**



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7.  $f(x)$  satisfies the relation  $f(x) - \lambda \int_0^{\pi/2} \sin x \cdot \cos t f(t) dt = \sin x$  If  $\lambda > 2$  then  $f(x)$  decreases in

A.  $(0, \pi)$

B.  $\left(\frac{\pi}{2}, 3\pi/2\right)$

C.  $(-\pi/2, \pi/2)$

D. none of these

**Answer: C**



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8.  $f(x)$  satisfies the relation  $f(x) - \lambda \int_0^{\pi/2} \sin x \cos t f(t) dt = \sin x$

If  $f(x) = 2$  has the least one real root, then

A.  $\lambda \in [1, 4]$

B.  $\lambda \in [-1, 2]$

C.  $\lambda \in [0, 1]$

D.  $\lambda \in [1, 3]$

**Answer: D**



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9.  $f(x)$  satisfies the relation  $f(x) - \lambda \int_0^{\pi/2} \sin x \cdot \cos t f(t) dt = \sin x$  If  $\lambda > 2$  then

$f(x)$  decreases in

A. 1

B.  $3/2$

C.  $4/3$

D. none of these

**Answer: C**



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10. Let  $f(x)$  and  $\phi(x)$  are two continuous function on  $R$  satisfying  $\phi(x) = \int_a^x f(t)dt, a \neq 0$  and another continuous function  $g(x)$  satisfying  $g(x + \alpha) + g(x) = 0 \forall x \in R, \alpha > 0$ , and  $\int_b^{2k} g(t)dt$  is independent of  $b$

If  $f(x)$  is an odd function, then

- A. (a)  $\phi(x)$  is also an odd function
- B. (b)  $\phi(x)$  is an even function
- C. (c)  $\phi(x)$  is neither an even nor an odd function
- D. (d) for  $\phi(x)$  to be an even function, it must satisfy  $\int_0^a f(x)dx = 0$

**Answer: B**



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11. Let  $f(x)$  and  $\phi(x)$  are two continuous function on  $R$  satisfying  $\phi(x) = \int_a^x f(t)dt, a \neq 0$  and another continuous function  $g(x)$  satisfying  $g(x + \alpha) + g(x) = 0 \forall x \in R, \alpha > 0$ , and  $\int_b^{2k} g(t)dt$  is independent of  $b$

If  $f(x)$  is an even function, then

- A.  $\phi(x)$  is also an even function
- B.  $\phi(x)$  is an odd function
- C.  $\phi(x)$  is an neither even nor odd function
- D. None of these

**Answer: D**

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12. Let  $f(x)$  and  $\phi(x)$  are two continuous function on  $R$  satisfying  $\phi(x) = \int_a^x f(t)dt, a \neq 0$  and another continuous function  $g(x)$  satisfying  $g(x + \alpha) + g(x) = 0 \forall x \in R, \alpha > 0$ , and  $\int_b^{2k} g(t)dt$  is independent of  $b$

Least positive value fo  $c$  if  $c, k, b$  are n A.P. is

A. 0

B. 1

C.  $\alpha$

D.  $2\alpha$

**Answer: D**



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**13.** Find the area of a parallelogram whose adjacent sides are given by the vectors  $\vec{a} = 3\hat{i} + \hat{j} + 4\hat{k}$  and  $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ .



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**14.** Evaluating integrals dependent on a parameter:

Differentiate I with respect to the parameter with in the sign an integrals taking variable of the integrand as constant. Now evaluate the integral so obtained as a function of the parameter then integrate then result of get

I. Constant of integration can be computed by giving some arbitrary values to the parameter and the corresponding value of I.

The value of  $\int_0^1 \frac{x^a - 1}{\log x} dx$  is

A.  $\log(a - 1)$

B.  $\log(a + 1)$

C.  $a \log(a + 1)$

D. none of these

**Answer: B**

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**15.** Evaluating integrals dependent on a parameter:

Differentiate I with respect to the parameter within the sign an integrals taking variable of the integrand as constant. Now evaluate the integral so obtained as a function of the parameter then integrate then result of get

I. Constant of integration can be computed by giving some arbitrary

values to the parameter and the corresponding value of  $I$ .

The value  $\int_0^{\pi/2} \log(\sin^2\theta + k^2\cos^2\theta) d\theta$ , where  $k \geq 0$ , is

A.  $\pi\log(1+k) + \pi\log 2$

B.  $\pi\log(1+k)$

C.  $\pi\log(1+k) - \pi\log 2$

D.  $\log(1+k) - \log 2$

**Answer: C**



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**16.**

The value of  $\frac{dI}{da}$  when  $I = \int_0^{\pi/2} \log\left(\frac{1+a\sin x}{1-a\sin x}\right) \frac{dx}{\sin x}$  (where  $|a| < 1$ ) is

A.  $\frac{\pi}{\sqrt{1-a^2}}$

B.  $-\pi\sqrt{1-a^2}$

C.  $\sqrt{1-a^2}$

$$D. \frac{\sqrt{1-a^2}}{\pi}$$

**Answer: A**

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**17. Evaluating integrals dependent on a parameter:**

Differentiate I with respect to the parameter within the sign an integrals taking variable of the integrand as constant. Now evaluate the integral so obtained as a function of the parameter then integrate then result of get I. Constant of integration can be computed by giving some arbitrary values to the parameter and the corresponding value of I.

If  $\int_0^{\pi} \frac{dx}{(a - \cos x)} = \frac{\pi}{\sqrt{a^2 - 1}}$ , then the value of  $\int_0^{\pi} \frac{dx}{(\sqrt{10} - \cos x)^3}$  is

A. (a)  $\frac{\pi}{81}$

B. (b)  $\frac{7\pi}{162}$

C. (c)  $\frac{7\pi}{81}$

D. (d) none of these

**Answer: C**

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18.  $f(x) = \sin x + \int_{-\pi/2}^{\pi/2} (\sin x + t \cos x) f(t) dt$

The range of  $f(x)$  is

A.  $\left[ -\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2} \right]$

B.  $\left[ -\frac{\sqrt{5}}{3}, \frac{\sqrt{5}}{3} \right]$

C.  $\left[ -\frac{\sqrt{5}}{2}, \frac{\sqrt{5}}{2} \right]$

D. none of these

**Answer: B**

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19.  $f(x) = \sin x + \int_{-\pi/2}^{\pi/2} (\sin x + t \cos x) f(t) dt$

$f(x)$  is not invertible for

A.  $x \in \left[ -\frac{\pi}{2} - \tan^{-1} 2, \frac{\pi}{2} - \tan^{-1} 2 \right]$

B.  $x \in \left[ \frac{\tan^{-1} 1}{2}, \pi + \tan^{-1} \frac{1}{2} \right]$

C.  $x \in \left[ \pi + \cot^{-1} 2, 2\pi + \cot^{-1} 2 \right]$

D. none of these

**Answer: D**



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20.  $f(x) = \sin x + \int_{-\pi/2}^{\pi/2} (\sin x + t \cos x) f(t) dt$

The value of  $\int_0^{\pi/2} f(x) dx$  is

A. 1

B. -2

C. -1

D. 2

**Answer: C**



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21. Let  $u = \int_0^{\infty} \frac{dx}{x^4 + 7x^2 + 1}$  and  $v = \int_0^{\infty} \frac{x^2 dx}{x^4 + 7x^2 + 1}$  then

A.  $\pi/3$

B.  $\pi/6$

C.  $\pi/12$

D.  $\pi/9$

**Answer: B**



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22. Let  $u = \int_0^{\infty} \frac{dx}{x^4 + 7x^2 + 1}$  and  $v = \int_0^x \frac{x^2 dx}{x^4 + 7x^2 + 1}$  then

A.  $\pi/3$

B.  $\pi/6$

C.  $\pi/12$

D.  $\pi/9$

**Answer: B**



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23. If  $f(x) = \int_0^1 \frac{dt}{1 + |x - t|}$ ,  $x \in R$ . The value of  $f'(1/2)$  is equal to

A.  $1/2$

B. 0

C. 1

D. 2

**Answer: B**



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24. If  $f(x) = \int_0^1 \frac{dt}{1 + |x - t|}$ ,  $x \in R$ . The value of  $f(1/2)$  is equal to

A.  $f(x)$  is decreasing for  $x > 1$

B.  $f(x)$  is increasing for  $x < 1$

C.  $f(1) = \log_e 2$

D.  $f(1/2) = \log_e(3/2)$

**Answer: D**



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25. Let  $f$  be a differentiable function satisfying

$$\int_0^{f(x)} f^{-1}(t) dt - \int_0^x (\cos t - f(t)) dt = 0 \text{ and } f\left(\frac{\pi}{2}\right) = \frac{2}{\pi}$$

The value of  $\int_0^{\pi/2} f(x)dx$  lies in the interval

A.  $\left(\frac{2}{\pi}, 1\right)$

B.  $\left(1, \frac{\pi}{2}\right)$

C.  $\left(\frac{3}{2}, \frac{\pi}{2}\right)$

D.  $\left(0, \frac{2}{\pi}\right)$

**Answer: B**



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26. Let  $f$  be a differentiable function satisfying

$$\int_0^{f(x)} f^{-1}(t)dt - \int_0^x (\cos t - f(t))dt = 0 \text{ and } f\left(\frac{\pi}{2}\right) = \frac{2}{\pi}$$

The value of  $\lim_{x \rightarrow 0} \frac{\cos x}{f(x)}$  is equal to where  $[.]$  denotes greatest integer function

A. 0

B. 1

C.  $1/2$

D. 2

**Answer: B**



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27. If  $U_n = \int_0^\pi \frac{1 - \cos nx}{1 - \cos x} dx$  where  $n$  is positive integer of zero, then

The value of  $U_n$  is a.  $\pi/2$  b.  $\pi$  c.  $n\pi/2$  d.  $n\pi$

A.  $\pi/2$

B.  $\pi$

C.  $n\pi/2$

D.  $n\pi$

**Answer: D**



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28. If  $U_n = \int_0^\pi \frac{1 - \cos nx}{1 - \cos x} dx$ , where  $n$  is positive integer or zero, then show that  $U_{n+2} + U_n = 2U_{n+1}$ . Hence, deduce that  $\int_0^{\frac{\pi}{2}} \frac{\sin^2 n\theta}{\sin^2 \theta} d\theta = \frac{1}{2} n\pi$ .

A.  $\pi/2$

B.  $\pi$

C.  $n\pi/2$

D.  $n\pi$

**Answer: C**

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29. Assertion : Millikan's experiment established that electric charge is quantised.

Reason : From this experiment mass of the electron could not be determined.

A.  $\frac{\pi}{8} (1 + \sqrt{2})$

B.  $\frac{\pi}{4}(1 + \sqrt{2})$

C.  $\frac{\pi}{8\sqrt{2}}$

D.  $\frac{\pi}{4\sqrt{2}}$

**Answer: A**



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**30.** Data could not be retrieved.

A. 4

B. 3

C. 2

D. 1

**Answer: D**



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31. Let the definite integral be defined by the formula

$\int_a^b f(x)dx = \frac{b-a}{2}(f(a) + f(b))$ . For more accurate result, for  $c \in (a, b)$ , we can

use  $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx = F(c)$  so that for  $c = \frac{a+b}{2}$  we get

$$\int_a^b f(x)dx = \frac{b-a}{4}(f(a) + f(b) + 2f(c)).$$

If  $f'(x) < 0 \forall x \in (a, b)$  and  $c$  is a point such that  $a < c < b$ , and  $(c, f(c))$  is the point lying on the curve for which  $F(c)$  is maximum then  $f(c)$  is equal to

- A.  $\frac{f(b) - f(a)}{b - a}$
- B.  $\frac{2(f(b) - f(a))}{b - a}$
- C.  $\frac{2f(b) - f(a)}{2b - a}$
- D. 0

**Answer: B**



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1. If  $[.]$  denotes the greatest integer function, then match the following lists:

List I	List II
a. $\int_{-1}^1 [x + [x + [x]]] dx$	p. 3
b. $\int_2^5 ([x] + [-x]) dx$	q. 5
c. $\int_{-1}^3 \operatorname{sgn}(x - [x]) dx$	r. 4
d. $25 \int_0^{\pi/4} (\tan^6(x - [x]) + \tan^4(x - [x])) dx$	s. -3



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2. Match the following lists:

List I	List II (which of the following functions appear in integration of functions in List I)
a. $\int \frac{x^2 - x + 1}{x^3 - 4x^2 + 4x} dx$	p. $\log  x $
b. $\int \frac{x^2 - 1}{x(x-2)^3} dx$	q. $\log  x-2 $
c. $\int \frac{x^3 + 1}{x(x-2)^2} dx$	r. $\frac{1}{(x-2)}$
d. $\int \frac{x^5 + 1}{x(x-2)^3} dx$	s. $x$



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3. Match the following lists:

List I	List II
<p>a. If <math>f(x)</math> is an integrable function for <math>x \in \left[ \frac{\pi}{6}, \frac{\pi}{3} \right]</math> and</p> $I_1 = \int_{\pi/6}^{\pi/3} \sec^2 \theta f(2 \sin 2\theta) d\theta, \text{ and}$ $I_2 = \int_{\pi/6}^{\pi/3} \operatorname{cosec}^2 \theta f(2 \sin 2\theta) d\theta, \text{ then } I_1/I_2 =$	<p>p. 3</p>
<p>b. If <math>f(x+1) = f(3+x) \forall x</math>, and the value of <math>\int_a^{a+b} f(x) dx</math> is independent of <math>a</math>, then the value of <math>b</math> can be</p>	<p>q. 1</p>
<p>c. The value of <math display="block">2 \int_1^4 \frac{\tan^{-1}[x^2]}{\tan^{-1}[x^2] + \tan^{-1}[25 + x^2 - 10x]} dx</math> (where <math>[\cdot]</math> denotes the greatest integer function) is</p>	<p>r. 2</p>
<p>d. If <math>I = \int_0^2 \sqrt{x + \sqrt{x + \sqrt{x + \dots \infty}}} dx</math> (where <math>x &gt; 0</math>), then <math>[I]</math> is equal to (where <math>[\cdot]</math> denotes the greatest integer function)</p>	<p>s. 4</p>



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4. Match the following lists:



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5. Let  $\int_0^{\infty} \frac{\sin x}{x} dx = \alpha$  Then match the following lists and choose the correct code. :



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6. If  $I(m) = \int_0^{\pi} \log_e(1 - 2m \cos x + m^2) dx$ , Then match the following lists and choose the correct code. :



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1. If the value of  $(\lim)_{n \rightarrow \infty} \left( n^{-\frac{3}{2}} \right) \sum_{j=1}^{6n} \sqrt{j}$  is equal to  $o\sqrt{N}$ , then the value of  $\frac{N}{12}$  is

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2.  $\lim_{n \rightarrow \infty} \frac{n}{2^n} \int_0^2 x^n dx$  equals

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3. A continuous real function  $f$  satisfies  $f(2x) = 3(f(x)) \forall x \in R$ . If  $\int_0^1 f(x) dx = 1$ , then find the value of  $\int_1^2 f(x) dx$ .

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4. Consider the polynomial  $f(x) = ax^2 + bx + c$ . If  $f(0), f(2) = 2$ , then the minimum value of  $\int_0^2 |f'(x)| dx$  is \_\_\_\_\_

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5. If  $I = \int_0^{3\pi/4} ((1+x)\sin x + (1-x)\cos x) dx$ , then the value of  $(\sqrt{2}-1)I$  is \_\_\_\_\_

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6. If  $\int_0^{100} f(x) dx = 7$ , then  $\sum_{r=1}^{100} \int_0^1 f(r-1+x) dx =$  \_\_\_\_\_.

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7. The value of  $\int_0^{3\pi} \frac{|\tan^{-1} \tan x| - |\sin^{-1} \sin x|}{|\tan^{-1} \tan x| + |\sin^{-1} \sin x|} dx$  is equal to

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8. Let  $f(x) = x^3 = \frac{3x^2}{2} + x + \frac{1}{4}$ . Then the value of  $\left(\int_{\frac{3}{4}}^{\frac{3}{4}} f(f(x)) dx\right)^{-1}$  is \_\_\_\_.

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9. The value of  $\int_0^1 \frac{\tan^{-1}x}{\cot^{-1}(1-x+x^2)} dx$  is \_\_\_\_.

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10. Let  $f(x)$  be a differentiable function symmetric about  $x = 2$ , then the value of  $\int_0^4 \cos(\pi x) f(x) dx$  is equal to \_\_\_\_.

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11. Let  $f: [0, \infty) \rightarrow \mathbb{R}$  be a continuous strictly increasing function, such that  $f^3(x) = \int_0^x t \cdot f^2(t) dt$  for every  $x \geq 0$ . Then value of  $f(6)$  is \_\_\_\_\_

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12. If  $f$  is continuous function and  $F(x) = \int_0^x \left( (2t + 3) \cdot \int_t^2 f(u) du \right) dt$ , then

$\left| \frac{F^2}{f(2)} \right|$  is equal to \_\_\_\_ .

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13. If the value of the definite integral  $\int_0^1 \frac{\sin^{-1} \sqrt{x}}{x^2 - x + 1} dx$  is  $\frac{\pi^2}{\sqrt{n}}$  (where  $n \in \mathbb{N}$ ), then the value of  $\frac{n}{27}$  is

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14. Let  $f(x) = \int_0^x \frac{dt}{\sqrt{1+t^3}}$  and  $g(x)$  be the inverse of  $f(x)$ . Then the value of  $4 \frac{g'(x)}{g(x)^2}$  is \_\_\_\_\_.



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15. Let  $g(x)$  be differentiable on  $R$  and  $\int_{\sin t}^1 x^2 g(x) dx = (1 - \sin t)$ , where  $t \in \left(0, \frac{\pi}{2}\right)$ . Then the value of  $g\left(\frac{1}{\sqrt{2}}\right)$  is \_\_\_\_\_



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16. If  $\int_0^\infty x^{2n+1} e^{-x^2} dx = 360$ , then the value of  $n$  is \_\_\_\_\_



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17. Let  $f(x)$  be a derivable function satisfying  $f(x) = \int_0^x e^t \sin(x-t) dt$  and  $g(x) = f'(x) - f(x)$ . Then the possible integers in the range of  $g(x)$  is \_\_\_\_\_



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18. Let  $f(x) = \frac{1}{x^2} \int_4^x (4t^2 - 2f(t)) dt$  then find  $9f(4)$



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19. If the value of the definite integral  $\int_0^1 (2007)C_7^{2000} x^{2000} (1-x)^7 dx$  is equal to  $\frac{1}{k}$ , where  $k \in N$ , then the value of  $\frac{k}{26}$  is \_\_\_\_



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20. If  $I_n = \int_0^1 (1-x^5)^n dx$ , then  $\frac{55 I_{10}}{7 I_{11}}$  is equal to



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21. Evaluate:  $5050 \frac{\int_0^1 (1 - x^{50})^{100} dx}{\int_0^1 (1 - x^{50})^{101} dx}$

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22. Let  $J = \int_{-5}^{-4} (3 - x^2) \tan(3 - x^2) dx$  and  $K = \int_{-2}^{-1} (6 - 6x + x^2) \tan(6x - x^2 - 6) dx$ . Then  $(J+K)$  equals \_\_\_\_\_

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23. The value of the definite integral  $\int_{2-1}^{\sqrt{2}+1} \frac{x^4 + x^2 + 2}{(x^2 + 1)^2} dx$  equals \_\_\_\_\_

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24. Consider a real valued continuous function  $f$  such that  $f(x) = \sin x + \int_{-\pi/2}^{\pi/2} (\sin x + t(f(t))) dt$ . If  $M$  and  $m$  are maximum and minimum

values of function  $f$ , then the value of  $M/m$  is \_\_\_\_\_.

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25. If  $f(x) = x + \int_0^1 t(x+t)f(t)dt$ , then the value of  $\frac{23}{2}f(0)$  is equal to \_\_\_\_\_

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26. Let  $y = f(x) = 4x^3 + 2x - 6$ , then the value of  $\int_0^2 f(x)dx + \int_0^{30} f^{-1}(y)dy$  is equal to \_\_\_\_\_.

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27. The value of  $\int_1^3 \left( \sqrt{1 + (x-1)^3} + (x^2 - 1)^{\frac{1}{3}} + 1 \right) dx$  is \_\_\_\_\_.

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28. The value of  $\int_0^1 \cos^{-1}\left(\left(x - x^2\right) - \sqrt{\left(1 - x^2\right)\left(2x - x^2\right)}\right) dx$  is equal to \_\_\_\_\_.

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Jee Main

1.  $\int_0^\pi [\cot x] dx$ , where  $[.]$  denotes the greatest integer function, is equal to

A.  $\frac{\pi}{2}$

B. 1

C. -1

D.  $-\frac{\pi}{2}$

Answer: D

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2. Let  $p(x)$  be a function defined on  $R$  such that  $p'(x) = p'(1-x)$  for all  $x \in [0, 1]$ ,  $p(0) = 1$  and  $p(1) = 41$ .

Then  $\int_0^1 p(x) dx$  is equals to (a)42 (b) $\sqrt{41}$  (c)21 (d)41

A. 42

B.  $\sqrt{41}$

C. 21

D. 41

**Answer: C**



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3. The value of  $\int_0^1 \frac{8 \log(1+x)}{1+x^2} dx$  is

A.  $\log 2$

B.  $\pi \log 2$

C.  $\frac{\pi}{8} \log 2$

D.  $\frac{\pi}{2} \log 2$

**Answer: B**



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4. For  $x \in \left(0, \frac{5\pi}{2}\right)$ , define  $f(x) = \int_0^x \sqrt{t} \sin t dt$ . Then  $f$  has

A. local maximum at  $\pi$  and local minima at  $2\pi$

B. local maximum at  $\pi$  and  $2\pi$

C. local minimum at  $\pi$  and  $2\pi$

D. local minimum at  $\pi$  and local maximum at  $2\pi$

**Answer: A**



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5. If  $g(x) = \int_0^x \cos^4 t dt$ , then  $g(x + \pi)$  equals to (a)  $\frac{g(x)}{g(\pi)}$  (b)  $g(x) + g(\pi)$  (c)  $g(x) - g(\pi)$  (d)  $g(x) \cdot g(\pi)$

A.  $\frac{g(x)}{g(\pi)}$

B.  $g(x) + g(\pi)$

C.  $g(x) - g(\pi)$

D.  $g(x) \cdot g(\pi)$

**Answer: B**



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6. Evaluate the integral  $\int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}}$

A. Statement I is true, statement II is true, statement II is a correct explanation for statement I

- B. Statement I is true, statement II is true, statement II is a not a correct explanation for statement I
- C. Statement I is true, statement II is false
- D. Statement I is false, statement II is true

**Answer: D**

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7. The intercepts on x-axis made by tangents to the curve,  $y = \int_0^x |t| dt, x \in R$ , which are parallel to the line  $y = 2x$ , are equal to (1)  $\pm 2$  (2)  $\pm 3$  (3)  $\pm 4$  (4)  $\pm 1$

- A.  $\pm 1$
- B.  $\pm 2$
- C.  $\pm 3$
- D.  $\pm 4$

**Answer: A**



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8. The integral  $\int_0^{\pi} \sqrt{1 + 4\sin^2 \frac{x}{2}} - 4\sin \frac{x}{2} dx$  is equals to (a)  $\pi - 4$  (b)  $\frac{2\pi}{3} - 4 - \sqrt{3}$   
(c)  $\frac{2\pi}{3} - 4 - \sqrt{3}$  (d)  $4\sqrt{3} - 4 - \frac{\pi}{3}$

A.  $\pi - 4$

B.  $\frac{2\pi}{3} - 4 - \sqrt{3}$

C.  $4\sqrt{3} - 4$

D.  $4\sqrt{3} - 4 - \frac{\pi}{3}$

**Answer: D**



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9. The integral  $\int_2^4 \frac{\log x^2}{\log x^2 + \log(36 - 12x + x^2)} dx$  is equal to

A. 2

B. 4

C. 1

D. 6

**Answer: C**



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10.  $\lim_{n \rightarrow \infty} \left( \frac{(n+1)(n+2)(n+3)\dots 2n}{n^{2n}} \right)^{\frac{1}{n}}$  is equal to

A.  $\frac{27}{e^{20}}$

B.  $\frac{9}{e^2}$

C.  $3\log 3 - 2$

D.  $\frac{18}{e^4}$

**Answer: A**



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11. The Integral  $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1 + \cos x}$  is equal to: (2) (3) (4)

A. -1

B. -2

C. 2

D. 4

Answer: C



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12. The value of  $\int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1 + 2^x} dx$  is

A.  $\pi/4$

B.  $\pi/8$

C.  $\pi/2$

D.  $4\pi$

**Answer: A**



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**Jee Advanced**

1. Let  $f$  be a non-negative function defined on the interval  $[0, 1]$ . If

$$\int_0^x \sqrt{1 - (f'(t))^2} dt = \int_0^x f(t) dt, \quad 0 \leq x \leq 1, \text{ and } f(0) = 0, \text{ then}$$

A.  $f\left(\frac{1}{2}\right) < \frac{1}{2}$  and  $f\left(\frac{1}{3}\right) > \frac{1}{3}$

B.  $f\left(\frac{1}{2}\right) > \frac{1}{2}$  and  $f\left(\frac{1}{3}\right) > \frac{1}{3}$

C.  $f\left(\frac{1}{2}\right) < \frac{1}{2}$  and  $f\left(\frac{1}{3}\right) < \frac{1}{3}$

D.  $f\left(\frac{1}{2}\right) > \frac{1}{2}$  and  $f\left(\frac{1}{3}\right) < \frac{1}{3}$

**Answer: C**



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2. The value of  $\int_0^1 \frac{x^4(1-x)^4}{1+x^2} dx$  is

A.  $\frac{22}{7} - \pi$

B.  $\frac{2}{105}$

C. 0

D.  $\frac{71}{15} - \frac{3\pi}{2}$

**Answer: A**



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3. Let  $f$  be a real-valued function defined on the interval  $(-1, 1)$  such that

$$e^{-x}f(x) = 2 + \int_0^x \sqrt{t^4 + 1} dt, \text{ for all, } x \in (-1, 1) \text{ and let } f^{-1} \text{ be the inverse}$$

function of  $f$ . Then  $(f^{-1})'(2)$  is equal to 1 (b)  $\frac{1}{3}$  (c)  $\frac{1}{2}$  (d)  $\frac{1}{e}$

A. 1

B. 1/3

C. 1/2

D. 1/e

**Answer: B**



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4. The value of  $\int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}} \frac{x \sin x^2}{\sin x^2 + \sin(\ln 6 - x^2)} dx$  is

A.  $\frac{1}{4} \ln \frac{3}{2}$

B.  $\frac{1}{2} \ln \frac{3}{2}$

C.  $\ln \frac{3}{2}$

D.  $\frac{1}{6} \ln \frac{3}{2}$

**Answer: A**

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5. Let  $f: [-1, 2] \rightarrow [0, \infty)$  be a continuous function such that  $f(x) = f(1-x)$  for all  $x \in [-1, 2]$ . Let  $R_1 = \int_{-1}^2 xf(x)dx$ , and  $R_2$  be the area of the region bounded by  $y = f(x)$ ,  $x = -1$ ,  $x = 2$ , and the  $x$ -axis. Then

$R_1 = 2R_2$  (b)  $R_1 = 3R_2$  (c)  $2R_1 = R_2$  (d)  $3R_1 = R_2$

A.  $R_1 = 2R_2$

B.  $R_1 = 3R_2$

C.  $2R_1 = R_2$

D.  $3R_1 = R_2$

**Answer: C**

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6. Let  $f: \left[\frac{1}{2}, 1\right] \rightarrow \mathbb{R}$  (the set of all real numbers) be a positive, non-constant, and differentiable function such that  $f'(x) < 2f(x)$  and  $f\left(\frac{1}{2}\right) = 1$ . Then the value of  $\int_{\frac{1}{2}}^1 f(x) dx$  lies in the interval (a)  $(2e - 1, 2e)$  (b)  $(3 - 1, 2e - 1)$  (c)  $\left(\frac{e - 1}{2}, e - 1\right)$  (d)  $\left(0, \frac{e - 1}{2}\right)$

A.  $(2e - 1, 2e)$

B.  $(e - 1, 2e - 1)$

C.  $\left(\frac{e - 1}{2}, e - 1\right)$

D.  $\left(0, \frac{e - 1}{2}\right)$

**Answer: D**



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7. Let  $f: [0, 2] \rightarrow \mathbb{R}$  be a function which is continuous on  $[0, 2]$  and is differentiable on  $(0, 2)$  with  $f(0) = 1$

Let:  $F(x) = \int_0^{x^2} f(\sqrt{t}) dt$  or  $x \in [0, 2]$  If  $F'(x) = f'(x)$  for all  $x \in (0, 2)$ , then

$F(2)$  equals  $e^2 - 1$  (b)  $e^4 - 1$   $e - 1$  (d)  $e^4$

A.  $e^2 - 1$

B.  $e^4 - 1$

C.  $e - 1$

D.  $e^4$

**Answer: B**

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8.  $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (2 \operatorname{cosec} x)^{17} dx$

A. (a)  $\int_0^{\log(1+\sqrt{2})} 2(e^u + e^{-u})^{16} du$

B. (b)  $\int_0^{\log(1+\sqrt{2})} 2(e^u + e^{-u})^{17} du$

C. (c)  $\int_0^{\log(1+\sqrt{2})} 2(e^u - e^{-u})^{17} du$

D. (d)  $\int_0^{\log(1+\sqrt{2})} 2(e^u - e^{-u})^{16} du$

**Answer: A**



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9. Let  $f(x) = \frac{192x^3}{2 + \sin^4 \pi x}$  for all  $x \in \mathbb{R}$  with  $f\left(\frac{1}{2}\right) = 0$ . If  $m \leq \int_{\frac{1}{2}}^1 f(x) dx \leq M$

then the possible values of  $m$  and  $M$  are (i)  $m = 13, M = 24$  (ii)

$m = \frac{1}{4}, M = \frac{1}{2}$  (iii)  $m = -11, M = 0$  (iv)  $m = 1, M = 12$

A.  $m = 13, M = 24$

B.  $m = \frac{1}{4}, M = \frac{1}{2}$

C.  $m = -11, M = 0$

D.  $m = 1, M = 12$

**Answer: D**



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10. Evaluate:  $\int_{-\pi/2}^{\pi/2} \frac{x^2 \cos x}{1 + e^x} dx$

A.  $\frac{\pi^2}{4} - 2$

B.  $\frac{\pi^2}{4} + 2$

C.  $\pi^2 - e^{\frac{\pi}{2}}$

D.  $\pi^2 + \frac{e^\pi}{2}$

**Answer: A**



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11. If  $I_n = \int_{-\pi}^{\pi} \frac{\sin nx}{(1 + \pi^x) \sin x} dx, n = 0, 1, 2, \dots$  then which one of the

following is not true ?

A.  $I_n = I_{n+2}$

B.  $\sum_{m=1}^{10} I_{2m+1} = 10\pi$

10

C.  $\sum_{m=1}^{10} I_{2m} = 0$

D.  $I_n = I_{n+1}$

**Answer: A::B::C**

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12. about to only mathematics

A.  $f'(x)$  exists for all  $x \in (0, \infty)$

B.  $f(x)$  exists for all  $x \in (0, \infty)$  and  $f$  is continuous on  $(0, \infty)$  but not differentiable on  $(0, \infty)$ .

C. There exists  $\alpha > 1$  such that  $|f(x)| < |f(x) + \alpha|$  for all  $x \in (\alpha, \infty)$

D. There exists  $\beta > 0$  such that  $|f(x)| + |f'(x)| \leq \beta$  for all  $x \in (0, \infty)$

**Answer: B::C**

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13. Let  $S$  be the area of the region enclosed by  $y = e^{-x^2}$ ,  $y = 0$ ,  $x = 0$ , and  $x = 1$ . Then  $S \geq \frac{1}{e}$  (b)  $S \geq 1 - \frac{1}{e}$  (c)

$$S \leq \frac{1}{4} \left( 1 + \frac{1}{\sqrt{e}} \right) \quad \text{(d) } S \leq \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{e}} \left( 1 - \frac{1}{\sqrt{2}} \right)$$

A.  $S \geq \frac{1}{e}$

B.  $S \geq 1 - \frac{1}{e}$

C.  $S \leq \frac{1}{4} \left( 1 + \frac{1}{\sqrt{e}} \right)$

D.  $S \leq \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{e}} \left( 1 - \frac{1}{\sqrt{2}} \right)$

**Answer: A::B::D**

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14. Find  $a$  for which

$$\lim_{n \rightarrow \infty} \frac{1^a + 2^a + 3^a + \dots + n^a}{(n+1)^{a-1} [(na+1) + (na+2) + \dots + (na+n)]} = \frac{1}{60}$$

A. 5

B. 7

C.  $\frac{-15}{2}$

D.  $\frac{-17}{2}$

**Answer: B::D**



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15. Let  $f: [a, b] \rightarrow [1, \infty)$  be a continuous function and let  $g: \mathbb{R} \rightarrow \mathbb{R}$  be defined as

$$g(x) = \begin{cases} 0 & \text{if } x < a \\ \int_a^x f(t) dt & \text{if } a \leq x \leq b \\ \int_a^b f(t) dt & \text{if } x > b \end{cases} \text{ Then}$$

A. (a)  $g(x)$  is continuous but not differentiable at  $a$

B. (b)  $g(x)$  is differentiable on  $\mathbb{R}$

C. (c)  $g(x)$  is continuous but not differentiable at  $b$

D. (d)  $g(x)$  is continuous and differentiable at either a or b but not

both

**Answer: A::C**



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16. Let  $f: (0, \infty) \in R$  be given

$$f(x) = \int_{1/x}^x e^{t + \frac{1}{t}} \frac{1}{t} dt, \text{ then}$$

A.  $f(x)$  is monotonically increasing on  $[1, \infty)$

B.  $f(x)$  is monotonically decreasing on  $(0, 1)$

C.  $f(x) + f\left(\frac{1}{x}\right) = 0$ , for all  $x \in (0, \infty)$

D.  $f(2^x)$  is an odd function of  $x$  on  $R$

**Answer: A::C::D**



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17. The option(s) with the values of  $a$  and  $L$  that satisfy the following

equation is (are) 
$$\frac{\int_0^{4\pi} te^{t(s \in^6 at + \cos^4 at)} dt}{\int_0^{\pi} te^{t(s \in^6 at + \cos^4 at)} dt} = L \quad a = 2, L = \frac{e^{4\pi} - 1}{e^{\pi} - 1} \quad (b)$$

$a = 2, L = \frac{e^{4\pi} + 1}{e^{\pi} + 1}$   $a = 4, L = \frac{e^{4\pi} - 1}{e^{\pi} - 1}$  (d)  $a = 4, L = \frac{e^{4\pi} + 1}{e^{\pi} + 1}$

A.  $a = 2, L = \frac{e^{4\pi} - 1}{e^{\pi} - 1}$

B.  $a = 2, L = \frac{e^{4\pi+1}}{e^{\pi} + 1}$

C.  $a = 4, L = \frac{e^{4\pi} - 1}{e^{\pi} - 1}$

D.  $a = 4, L = \frac{e^{4\pi} + 1}{e^{\pi} + 1}$

**Answer: A:C**



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18. Let  $f(x) = 7\tan^8 x + 7\tan^6 x - 3\tan^4 x - 3\tan^2 x$  for all  $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ . Then

the correct expression (s) is (are) (a)  $\int_0^{\frac{\pi}{4}} xf(x)dx = \frac{1}{12}$  (b)  $\int_0^{\frac{\pi}{4}} f(x)dx = 0$  (c)

$\int_0^{\frac{\pi}{4}} xf(x) = \frac{1}{6}$  (d)  $\int_0^{\frac{\pi}{4}} f(x)dx = \frac{1}{12}$

$$\text{A. } \int_0^{\pi/4} xf(x)dx = \frac{1}{12}$$

$$\text{B. } \int_0^{\pi/4} f(x)dx = 0$$

$$\text{C. } \int_0^{\pi/4} xf(x) = \frac{1}{6}$$

$$\text{D. } \int_0^{\pi/4} f(x)dx = 1$$

**Answer: A::B**

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19. find the period of  $\sin\left(\frac{x}{2}\right) - \cos\left(\frac{x}{3}\right)$  is

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→  
20. Let  $f: R_0, 1$  be a continuous function. Then, which of the following function (s) has (have) the value zero at some point in the interval (0,1)?

$$e^x - \int_0^x f(t)\sin t dt \quad (\text{b}) \quad f(x) + \int_0^{\frac{\pi}{2}} f(t)\sin t dt \quad x - \int_0^{\frac{\pi}{2}-x} f(t)\cos t dt \quad (\text{d}) \quad x^9 - f(x)$$

A.  $e^x - \int_0^x f(t) \sin t dt$

B.  $x^9 - f(x)$

C.  $f(x) + \int_0^{\frac{\pi}{2}} f(t) \sin t dt$

D.  $x - \int_0^{\frac{\pi}{2}-x} f(t) \cos t dt$

**Answer: B::D**



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21. If  $I \sum_{k=1}^{98} \int_k^{k+1} \frac{k+1}{x(x+1)} dx$ , then: (a)  $I < \frac{49}{50}$  (b)  $I > (\log)_e 99$  (c)  $I > \frac{49}{50}$  (d)

$I < (\log)_e 99$

A.  $I > \log_e 99$

B.  $I < \log_e 99$

C.  $I < \frac{49}{50}$

D.  $I > \frac{49}{50}$

**Answer: B::D**

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22. If  $g(x) = \int_{\sin x}^{\sin(2x)} \sin^{-1}(t) dt$ , then :

(a)  $g' \left( \frac{\pi}{2} \right) = -2\pi$  (b)  $g' \left( -\frac{\pi}{2} \right) = -2\pi$  (c)  $g' \left( -\frac{\pi}{2} \right) = 2\pi$  (d)  $g' \left( \frac{\pi}{2} \right) = 2\pi$

A.  $g' \left( \frac{\pi}{2} \right) = -2\pi$

B.  $g' \left( -\frac{\pi}{2} \right) = 2\pi$

C.  $g' \left( \frac{\pi}{2} \right) = 2\pi$

D.  $g' \left( -\frac{\pi}{2} \right) = -2\pi$

**Answer:**

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23. Given that for each  $a \in (0, 1)$ ,  $\lim_{(h \rightarrow 0^+)} \int_h^{1-h} t^{-a}(1-t)^{a-1} dt$  exists. Let this limit be  $g(a)$ . In addition it is given the function  $g(a)$  is differentiable on  $(0, 1)$ .

The value of  $g\left(\frac{1}{2}\right)$  is a.  $\frac{\pi}{2}$  b.  $\pi$  c.  $-\frac{\pi}{2}$  d. 0

A. a.  $\pi$

B. b.  $2\pi$

C. c.  $\frac{\pi}{2}$

D. d.  $\frac{\pi}{4}$

**Answer: A**



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24. Given that for each  $a \in (0, 1)$ ,  $\lim_{(h \rightarrow 0^+)} \int_h^{1-h} t^{-a}(1-t)^{a-1} dt$  exists. Let this limit be  $g(a)$ . In addition it is given the function  $g(a)$  is differentiable

on(0, 1).

The value of  $g\left(\frac{1}{2}\right)$  is a.  $\frac{\pi}{2}$  b.  $\pi$  c.  $-\frac{\pi}{2}$  d. 0

A.  $\frac{\pi}{2}$

B.  $\pi$

C.  $-\frac{\pi}{2}$

D. 0

**Answer: D**



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25. Let  $F: \mathbb{R} \rightarrow \mathbb{R}$  be a thrice differentiable function. Suppose that  $F(1) = 0$ ,  $F(3) = -4$  and  $F'(x) < 0$  for all  $x \in (1/2, 3)$ . Let  $f(x) = xF(x)$  for all  $x \in \mathbb{R}$ . Then the correct statement(s) is (are)

A. a.  $f'(1) < 0$

B. b.  $f(2) < 0$

C. c.  $f(x) \neq 0$  for an  $x \in (1, 3)$

D. d.  $f(x) = 0$  for some  $x \in (1, 3)$

**Answer: A::B::C**

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**26.** Let  $F: \mathbb{R} \rightarrow \mathbb{R}$  be a thrice differentiable function. Suppose that  $F(1) = 0$ ,  $F(3) = -4$  and  $F'(x) < 0$  for all  $x \in (1/2, 3)$ . Let  $f(x) = xF(x)$  for all  $x \in \mathbb{R}$ . Then the correct statement(s) is (are)

A.  $9f(3) + f(1) - 32 = 0$

B.  $\int_1^3 f(x) dx = 12$

C.  $9f(3) - f(1) + 32 = 0$

D.  $\int_1^3 f(x) dx = -12$

**Answer: C::D**

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27. Match the terms given in Column I with the compound given in Column II.

Column I	Column II
A. Acid rain	1. $\text{CHCl}_2\text{-CHF}_2$
B. Photochemical smog	2. CO
C. Combination with haemoglobin	3. $\text{CO}_2$
D. Depletion of ozone layer	4. $\text{SO}_2$
	5. Unsaturated hydrocarbons



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28. For any real number  $x$ , let  $[x]$  denote the largest integer less than or equal to  $x$ , Let  $f$  be a real-valued function defined on the interval  $[-10, 10]$  be  $f(x) = \{x - [x], \text{ if } [x] \text{ is odd, } 1 + [x] - x, \text{ if } [x] \text{ is even}$  Then the value of  $\frac{\pi^2}{10} \int_{-10}^{10} f(x) \cos \pi x dx$  is \_\_\_\_



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29. Let  $y'(x) + y(x)g'(x) = g(x)g'(x), y(0), x \in R$ , where  $f'(x)$  denotes  $\frac{dy(x)}{dx}$ , and  $g(x)$  is a given non-constant differentiable function on  $R$  with

$g(0) = g(2) = 0$ . Then the value of  $y(2)$  is \_\_\_\_\_

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30. The value of  $\int_0^1 4x^3 \left\{ \frac{d^2}{dx^2} (1 - x^2)^5 \right\} dx$  is

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31. Let  $f: R \rightarrow R$  be a continuous odd function, which vanishes exactly at one point and  $f(1) = \frac{1}{2}$ . Suppose that  $F(x) = \int_{-1}^x f(t) dt$  for all  $x \in [-1, 2]$  and  $G(x) = \int_{-1}^x t|f(f(t))| dt$  for all  $x \in [-1, 2]$ . If  $\lim_{x \rightarrow 1} \frac{F(x)}{G(x)} = \frac{1}{14}$ , Then the value of  $f\left(\frac{1}{2}\right)$  is

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32. If  $\alpha = \int_0^1 \left( e^{9x+3\tan^{-1}x} \right) \left( \frac{12+9x^2}{1+x^2} \right) dx$  where  $\tan^{-1}x$  takes only principal

values, then, find the value of  $(\log)_e |1 + \alpha| - \frac{3\pi}{4}$

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33. Let  $F(x) = \int_x^{x^2 + \frac{\pi}{6}} (2\cos^2 t) dt$  for all  $x \in \mathbb{R}$  and  $f: \left[0, \frac{1}{2}\right] \rightarrow [0, \infty)$  be a

continuous function. For  $a \in \left[0, \frac{1}{2}\right]$ , if  $F'(a)+2$  is the area of the region

bounded by  $x=0, y=0, y=f(x)$  and  $x=a$ , then  $f(0)$  is

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34. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a function defined by  $f(x) = \{[x], (x \leq 2) (0, x > 2)$  where

$[x]$  is the greatest integer less than or equal to  $x$ . If

$I = \int_{-1}^2 \frac{xf(x^2)}{2+f(x+1)} dx$ , then the value of  $(4I - 1)$  is

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35. The total number for distinct  $x \in [0, 1]$  for which  $\int_0^x \frac{t^2}{1+t^4} dt = 2x - 1$  is \_\_\_\_\_.

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36. Let  $f: R \rightarrow R$  be a differentiable function such that  $f(0) = 0, f\left(\frac{\pi}{2}\right) = 3$  and  $f'(0) = 1$ . If  $g(x) = \int_{\frac{\pi}{2}}^x [f'(t) \operatorname{cosec} t - \cot t \operatorname{cosec} t f(t)] dt$  for  $x \in \left(0, \frac{\pi}{2}\right]$ , then  $(\lim)_{x \rightarrow 0} g(x) =$

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37. For each positive integer  $n$ , let  $y_n = \frac{1}{n} \left( (n+1)(n+2)\dots(n+n) \right)^{\frac{1}{n}}$  For  $x \in R$  let  $[x]$  be the greatest integer less than or equal to  $x$ . If  $(\lim)_{n \rightarrow \infty} y_n = L$ , then the value of  $[L]$  is \_\_\_\_\_.

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38. The value of the integral  $\int_0^{\frac{1}{2}} \frac{1 + \sqrt{3}}{\left((x + 1)^2(1 - x)^6\right)^{\frac{1}{4}}} dx$  is \_\_\_\_\_.



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