MATHS

BOOKS - CENGAGE MATHS (ENGLISH)

ELLIPSE

Single Correct Answer Type

1. The second degree equation $x^2 + 4y^2 - 2x - 4y + 2 = 0$ represents

A. a parabola

B. a pair of straight line

C. an ellipse

D. a hyperbola

Answer: C

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2. In the standard ellipse, the lines joining the ends of the minor axis to one focus are at right angles. The distance between the focus and the nearer vertex is $\sqrt{10} - \sqrt{5}$. The equation of the ellipse

A. $\frac{x^2}{36} + \frac{y^2}{18} = 1$

B. $\frac{x^2}{40} + \frac{y^2}{20} = 1$

C. $\frac{x^2}{20} + \frac{y^2}{10} = 1$

D. $\frac{x^2}{10} + \frac{y^2}{5} = 1$

Answer: D

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3. The foci of an ellipse are $(-2, 4)$ and (2,1). The point $\left(1, \frac{23}{6}\right)$ is an extremity of the minor axis. What is the value of the eccentricity?

A. $\frac{9}{13}$

B. $\frac{3}{\sqrt{13}}$

C. $\frac{2}{\sqrt{13}}$

D. $\frac{4}{13}$

Answer: B

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4. Let $Q = \left(3, \sqrt{5}\right), R = \left(7, 3\sqrt{5}\right)$. A point P in the XY-plane varies in such a way that perimeter of ΔPQR is 16. Then the maximum area of ΔPQR is

A. 6

B. 12

C. 18

D. 9

Answer: B

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5. The eccentricity of the ellipse $(x-3)^2+(y-4)^2=\frac{y^2}{9}$

A. $\frac{\sqrt{3}}{2}$

B. $\frac{1}{3}$

C. $\frac{1}{3\sqrt{2}}$

D. $\frac{1}{\sqrt{3}}$

Answer: B

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6. Area bounded by the circle which is concentric with the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$ and which passes through $\left(4, -\frac{9}{5}\right)$, the vertical chord common to both circle and ellipse on the positive side of x-axis is

A. $\frac{481}{25}\tan^{-1}\left(\frac{9}{20}\right)-\frac{36}{5}$

B. $2\tan^{-1}\left(\frac{9}{20}\right)$

C. $\frac{481}{25}\tan^{-1}\left(\frac{9}{20}\right)$

D. none of these

Answer: A

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7. If A and B are foci of ellipse $(x-2y+3)^2+(8x+4y+4)^2=20$ andP is any point on it, then $PA+PB=$

A. 2

B. 4

C. $\sqrt{2}$

D. $2\sqrt{2}$

Answer: B

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8. The distance between directrix of the ellipse $(4x-8)^2+16y^2=\left(x+\sqrt{3}y+10\right)^2$ is

A. 12

B. 16

C. 20

D. 24

Answer: B

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9. A chord is drawn passing through $P(2, 2)$ on the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ such that it intersects the ellipse at A and B. Then maximum value of $PA.\ PB$ is

A. $\frac{61}{4}$

B. $\frac{59}{4}$

C. $\frac{71}{4}$

D. $\frac{63}{4}$

Answer: B

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10. If (x, y) lies on the ellipse $x^2 + 2y^3 = 2$, then maximum value of $x^2 + y^2 + \sqrt{2}xy - 1$ is

A. $\frac{\sqrt{5}+1}{2}$

B. $\frac{\sqrt{5}-1}{2}$

C. $\frac{\sqrt{5}+1}{4}$

D. $\frac{\sqrt{5}-1}{4}$

Answer: A

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11. If the eccentric angles of two points P and Q on the ellipse $\frac{x^2}{28}+\frac{y^2}{7}=1$ whose centre is C differ by a right angle then the area of ΔCPQ is

A. 5

B. 6

C. 7

D. 8

Answer: C

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12. P and Q are points on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ whose center is C. The eccentric angles of P and Q differ by a right angle. If $\angle PCQ$ minimum, the eccentric angle of P can be (A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{12}$

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{12}$

Answer: B

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13. If eccentric angle of a point lying in the first quadrant on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ be θ and the line joining the centre to that point makes an

angle ϕ with the x-axis, then $\theta - \phi$ will be maximum when θ is equal to

A. $\tan^{-1}\sqrt{\frac{a}{b}}$

B. $\tan^{-1}\sqrt{\frac{b}{a}}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{3}$

Answer: A

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14. Let P and Q be points of the ellipse $16x^2 + 25y^2 = 400$ so that $PQ = 96/25$ and P and Q lie above major axis. Circle drawn with PQ as diameter touch major axis at positive focus, then the value of slope m of PQ is

A. -1

B. $1/2$

C. 2

D. $1/3$

Answer: A

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15. If the reflection of the ellipse $\frac{(x-4)^2}{16}+\frac{(y-3)^2}{9}=1$ in the mirror line $x-y-2=0$ is $k_1x^2+k_2y^2-160x-36y+292=0$, then $\frac{k_1+k_2}{5}$ is equal to

A. 4

B. 5

C. 6

D. 7

Answer: B

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16. A point P moves on x-y plane such that $PS + PS' = 4$ where $S(K, 0)$ and $S'(-K, 0)$, then which of the following is not true about the locus of P?

A. ellipse if $K \in (-2, 2)$

B. pair of coincidence lines if $K = \pm 2$

C. empty if $K \in (-\infty, -2) \cup (2, \infty)$

D. none of these

Answer: D

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17. The ratio of the area enclosed by the locus of the midpoint of PS and area of the ellipse is (P-be any point on the ellipse and S, its focus)

A. $\frac{1}{2}$

B. $\frac{1}{3}$

C. $\frac{1}{5}$

D. $\frac{1}{4}$

Answer: D

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18. Find the set of those value(s) of α for which $\left(7-\frac{5\alpha}{4}, \alpha\right)$ lies inside the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$

A. 0

B. 1

C. 2

D. 3

Answer: B

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19. The coordinates of the vertices $BandC$ of a triangle ABC are (2, 0) and (8, 0), respectively. Vertex A is moving in such a way that $4\frac{\tan B}{2}\frac{\tan C}{2}=1$. Then find the locus of A

A. $\frac{(x-5)^2}{25}+\frac{y^2}{16}=1$

B. $\frac{(x-5)^2}{16}+\frac{y^2}{25}=1$

C. $\frac{(x-5)^2}{25}+\frac{y^2}{9}=1$

D. $\frac{(x-5)^2}{9}+\frac{y^2}{25}=1$

Answer: A

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20. PQ and QR are two focal chords of an ellipse and the eccentric angles of P,Q,R are $2\alpha, 2\beta, 2\gamma$, respectively then $\tan \beta\gamma$ is equal to

A. $\cot \alpha$

B. $\cot^2 \alpha$

C. $2 \cot \alpha$

D. None of these

Answer: B

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21. the value of λ for which the line $2x - \frac{8}{3}\lambda y = -3$ is a normal to the conic $x^2 + \frac{y^2}{4} = 1$ is:

A. $\frac{3}{8}$

B. $\frac{1}{2}$

C. $-\frac{\sqrt{3}}{2}$

D. $\frac{\sqrt{3}}{2}$

Answer: D

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22. If the length of the major axis intercepted between the tangent and normal at a point $P(a\cos\theta, b\sin\theta)$ on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is equal to the length of semi-major axis, then eccentricity of the ellipse is

A. $\frac{\cos\theta}{\sqrt{1-\cos\theta}}$

B. $\frac{\sqrt{1-\cos\theta}}{\cos\theta}$

C. $\frac{\sqrt{1-\cos\theta}}{\sin\theta}$

D. $\frac{\sin\theta}{\sqrt{1-\sin\theta}}$

Answer: B

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23. How many tangents to the circle $x^2+y^2=3$ are normal tothe ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$?

A. 3

B. 2

C. 1

D. 0

Answer: D

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24. An ellipse passes through the point (2,3) and its axes along the coordinate axes, $3x + 2y - 1 = 0$ is a tangent to the ellipse, then the equation of the ellipse is

A. $\frac{x^2}{4} + 4y^2 = 1$

B. $\frac{x^2}{8} + \frac{y^2}{1} = 1$

C. $4x^2 + \frac{y^2}{4} = 1$

D. No such ellipse exists

Answer: D

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25. If $x\cos\alpha + y\sin\alpha = 4$ is tangent to $\frac{x^2}{25} + \frac{y^2}{9} = 1$, then the value of α is

A. $\tan^{-1}(3/\sqrt{7})$

B. $\tan^{-1}(7/3)$

C. $\tan^{-1}(\sqrt{3}/7)$

D. $\tan^{-1}(3/7)$

Answer: A

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26. If the normal at any point P of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ meets the coordinate axes at M and N respectively, then $|PM| : |PN|$ equals

A. $4:3$

B. $16:9$

C. $9:16$

D. $3:4$

Answer: C

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27. If the normal at any point P on the ellipse cuts the major and minor axes in G and g respectively and C be the centre of the ellipse, then

A. $a^2(CG)^2 + b^2(Cg)^2 = \left(a^2 - b^2\right)^2$

B. $a^2(CG)^2 - b^2(Cg)^2 = \left(a^2 - b^2\right)^2$

C. $a^2(CG)^2 - b^2(Cg)^2 = \left(a^2 + b^2\right)^2$

D. None of these

Answer: A

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28. The area of the parallelogram formed by the tangents at the points whose eccentric angles are $\theta, \theta + \frac{\pi}{2}, \theta + \pi, \theta + \frac{3\pi}{2}$ on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

A. ab

B. 4ab

C. 3ab

D. 2ab

Answer: D

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29. The straight line $\frac{x}{4}+\frac{y}{3}=1$ intersects the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ at two points A and B, there is a point P on this ellipse such that the area of ΔPAB is equal to $6\left(\sqrt{2}-1\right)$. Then the number of such points (P) is/are

A. 0

B. 1

C. 2

D. 3

Answer: D

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30. The tangent at any point on the ellipse $16x^2 + 25y^2 = 400$ meets the tangents at the ends of the major axis at T_1 and T_2. The circle on T_1T_2 as diameter passes through

A. $(3, 0)$

B. $(0, 0)$

C. $(0, 3)$

D. $(4, 0)$

Answer: A

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31. The minimum value of $\left\{(r+5-4|\cos\theta|)^2+(r-3|\sin\theta|)^2\right\}\forall r,\theta\in R$ is

A. 0

B. 2

C. 3

D. None of these

Answer: A

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32. Let S_1 and S_2 denote the circles $x^2 + y^2 + 10x - 24y - 87 = 0$ and $x^2 + y^2 - 10x - 24y + 153 = 0$ respectively. The value of a for which the line $y = ax$ contains the centre of a circle which touches S_2 externally and S_1 internally is

A. $\pm \frac{3}{10}$

B. $\pm \frac{1}{5}$

C. $\pm \frac{\sqrt{13}}{10}$

D. $\pm \frac{10}{13}$

Answer: C

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33. If ω is one of the angles between the normals to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the point whose eccentric angles are θ and $\frac{\pi}{2}+\theta$, then prove that $\frac{2\cot\omega}{\sin 2\theta}=\frac{e^2}{\sqrt{1-e^2}}$

A. $\frac{e^2}{\sqrt{1-e^2}}$

B. $\frac{e^2}{\sqrt{1+e^2}}$

C. $\frac{e^2}{1-e^2}$

D. $\frac{e^2}{1+e^2}$

Answer: A

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34. From any point on the line $(t+2)(x+y)=1, t \neq -2$, tangents are drawn to the ellipse $4x^2+16y^2=1$. It is given that chord of contact passes through a fixed point. Then the number of integral values of 't' for which the fixed point always lies inside the ellipse is

A. 0

B. 1

C. 2

D. 3

Answer: C

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35. At a point P on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ tangents PQ is drawn. If the point Q be at a distance $\frac{1}{p}$ from the point P, where 'p' is distance of the tangent from the origin, then the locus of the point Q is

A. (a) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1+\frac{1}{a^2b^2}$

B. (b) $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1-\frac{1}{a^2b^2}$

C. (c) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{1}{a^2b^2}$

D. (d) $\frac{x^2}{a^2}-\frac{y^2}{b^2}=\frac{1}{a^2b^2}$

Answer: A

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36. From a point P perpendicular tangents PQ and PR are drawn to ellipse $x^2+4y^2=4$, then locus of circumcentre of triangle PQR is

A. $x^2 + y^2 = \frac{16}{5}\left(x^2 + 4y^2\right)^2$

B. $x^2 + y^2 = \frac{5}{16}\left(x^2 + 4y^2\right)^2$

C. $x^2 + 4y^2 = \frac{16}{5}\left(x^2 + y^2\right)^2$

D. $x^2 + 4y^2 = \frac{16}{5}\left(x^2 + y^2\right)^2$

Answer: B

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37. If the normal at any point P on ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ meets the auxiliary circle at Q and R such that $\angle QOR = 90^\circ$ where O is centre of ellipse, then

A. (a) $a^4 + 2b^3 \geq 3a^2b^2$

B. (b) $a^4 + 2b^4 \geq 5a^2b^2 + 2a^3b$

C. (c) $a^4 + 2b^4 \geq 3a^2b^2 + ab$

D. (d) None of these

Answer: B

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38. Tangents are drawn from any point on the circle $x^2 + y^2 = 41$ to the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ then the angle between the two tangents is

A. $\frac{\pi}{4}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{2}$

Answer: D

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39. If radius of the director circle of the ellipse $\frac{(3x+4y-2)^2}{100}+\frac{(4x-3y+5)^2}{625}=1$ is

A. 6

B. $\sqrt{34}$

C. $\sqrt{29}$

D. $\sqrt{26}$

Answer: C

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40. If the curve $x^2 + 3y^2 = 9$ subtends an obtuse angle at the point $(2\alpha, \alpha)$ then a possible value of α^2 is

A. 1

B. 2

C. 3

D. 4

Answer: B

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41. An ellipse has the points $(1, -1)$ and $(2, -1)$ as its foci and $x + y = 5$ as one of its tangent then the value of $a^2 + b^2$ where a, b are the lenghta of semi major and minor axes of ellipse respectively is :

A. $\frac{41}{2}$

B. 10

C. 19

D. $\frac{81}{4}$

Answer: D

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42. An ellipse has foci at $F_1(9, 20)$ and $F_2(49, 55)$ in the xy-plane and is tangent to the x-axis. Find the length of its major axis.

A. 85

B. 75

C. 65

D. 55

Answer: A

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43. The maximum distance of the centre of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ from the chord of contact of mutually perpendicular tangents of the ellipse is

A. $\frac{144}{5}$

B. $\frac{9}{5}$

C. $\frac{16}{5}$

D. $\frac{8}{5}$

Answer: C

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44. P_1 and P_2 are the lengths of the perpendicular from the foci on the tangent of the ellipse and P_3 and P_4 are perpendiculars from extermities of major axis and P from the centre of the ellipse on

the same tangent, then $\frac{P_1P_2 - P^2}{P_3P_4 - P^2}$ equals (where e is the eccentricity of the ellipse)

A. e

B. $\sqrt{e}$

C. e^2

D. none of these

Answer: C

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45. From the focus $(-5, 0)$ of the ellipse $\frac{x^2}{45} + \frac{y^2}{20} = 1$, a ray of light is sent which makes angle $\cos^{-1}\left(\frac{-1}{\sqrt{5}}\right)$ with the positive direction of X-axis upon reacting the ellipse the ray is reflected from it. Slope of the reflected ray is

A. $-\frac{3}{2}$

B. $-\frac{7}{3}$

C. $-\frac{5}{4}$

D. $-\frac{2}{11}$

Answer: D

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46. Let $5x - 3y = 8\sqrt{2}$ be normal at $P\left(\frac{5}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right)$ to an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b.$ If m, m' are feet of perpendiculars from foci s, s' respectively. or tangents at p, then point of intersection of sm' and $s'm$ is

A. $\left(\frac{5}{2}, 0\right)$

B. $\left(0, \frac{5}{2}\right)$

C. $\left(\frac{41}{10\sqrt{2}}, \frac{3}{2\sqrt{2}}\right)$

D. $\left(\frac{3}{2\sqrt{2}}, \frac{41}{10\sqrt{2}}\right)$

Answer: C

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47. If the normals at α, β, γ and δ on an ellipse are concurrent then the value of $(\sigma \cos \alpha)(\sigma \sec \alpha)$ I

A. 2

B. 4

C. 6

D. none of these

Answer: B

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48. Prove that the chords of contact of pairs of perpendicular tangents to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ touch another fixed ellipse.

A. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{1}{(2a^2+b^2)}$

B. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{2}{(a^2-b^2)}$

C. $\frac{x^2}{a^4}+\frac{y^2}{b^4}=\frac{1}{(a^2+b^2)}$

D. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=\frac{2}{(3a^2-b^2)}$

Answer: C

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49. Consider an ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$ with centre c and a point P on it with eccentric angle $\frac{\pi}{4}$. Nomal drawn at P intersects the major and minor axes in A and B respectively. N_1 and N_2 are the feet of the perpendiculars from the foci S_1 and S_2 respectively on the tangent at P and N is the foot of the perpendicular from the centre of the ellipse on the normal at P. Tangent at P intersects the axis of x at T.

A.
P	Q	R	S
2	3	4	1

B.
P	Q	R	S
3	1	4	2

C.
P	Q	R	S
2	4	1	3

D.
P	Q	R	S
4	1	2	3

Answer: C

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Multiple Correct Answers Type

1. In triangle $ABC, a = 4$ and $b = c = 2\sqrt{2}$. A point P moves within the triangle such that the

square of its distance from BC is half the area of rectangle contained by its distance from the other two sides. If D be the centre of locus of P, then

A. locus of P is an ellipse with eccentricity $\sqrt{\frac{2}{3}}$

B. locus of P is a hyperbola with eccentricity $\sqrt{\frac{3}{2}}$

C. area of the quadr5ilateral $ABCD = \frac{16}{3}$ sq. units

D. area of the quadrilateral $ABCD = \frac{32}{3}$ sq. units

Answer: A::C

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2. Extremities of the latera recta of the ellipses $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1(a > b)$ having a given major axis 2a lies on

A. $x^2 = a(a - y)$

B. $x^2 = a(a + y)$

C. $y^2 = a(a + x)$

D. $y^2 = a(a - x)$

Answer: A::B

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3. Identify correct statement(s) about conic $\sqrt{(x-5)^2+(y-7)^2}+\sqrt{(x+1)^2+(y+1)^2}=12$

A. centre of conic is (2,3)

B. conic is hyperbola with foci (5,7) and $(-1,-1)$

C. conic is ellipse with major axis $4x-3y+1=0$

D. eccentricity of conic is $\frac{5}{7}$

Answer: A::C

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4. P and Q are two points on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ whose eccentric angles are differ by 90°, then

A. Locus of point of intersection of tangents at P and Q is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2$

B. Locus of mid-point (P, Q) is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{1}{2}$

C. Product of slopes of OP and OQ ehere O is the centre is $\frac{-b^2}{a^2}$

D. Max. area of ΔOPQ is $\frac{1}{2}ab$

Answer: A::B::C::D

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5. For the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$

A. (a) The foci of each ellipse always lie within the other ellipse

B. (b) Their auxiliary circles are the same

C. (c) Their director circles are the same

D. (d) The ellipses encloses the same area

Answer: B::C::D

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6. AB and CD are two equal and parallel chords of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. Tangents to the ellipse at A and B intersect at P and tangents at C and D at Q. The line PQ

A. passes through the origin

B. is bisected at the origin

C. cannot pass through the origin

D. is not bisected at the origin

Answer: A::B

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