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## MATHS

## BOOKS - CENGAGE MATHS (ENGLISH)

## GRAPHS OF ELEMENTARY FUNCTIONS

## Illustrations

1. The equation of $a$ tangent to the parabola $y^{2}=8 x i s y=x+2$. The point on this line from which the other tangent to the parabola is perpendicular to the given tangent is (1) $(-1,1)(2)(0,2)(3)(2,4)(4)(-2,0)$
2. Draw the graph of $f(x)=\frac{x^{3}-x}{x^{2}-1}$.

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3. Graph of $y=f(x)$ and $y=g(x)$ is given in the following figure. If $h(x)=f(g(x))$, then find the value of $h^{\prime}(2)$.

4. Let $f(x)=(1-x)^{2} \sin ^{2} x+x^{2}$ for all $\mathrm{x} \in \mathrm{R}$, and let $g(x)=\int\left(\frac{2(t-1)}{t+1}-\ln t\right) f(t) d t$ for $\mathrm{t} \in[1, \mathrm{x}]$ for all $\mathrm{x} \in(1$, $\infty)$.Which of the following is true ?

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5. Sketch the regions satisfying the following inequalities:
(a) $x>2$
(b) $|y| \geq 1$

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6. Shade the regions where points satisfy $|x-y|<1$.
7. Plot the region satisfying $|x|+|y| \leq 2$ and $|x|+|y|>2$. Watch Video Solution
8. If $x<2$, then find the values of $x^{2}$ graphically.

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9. If $x<-1$, then find the values of $x^{2}$ graphically.

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10. Draw the graph of $f(x)=\left\{\begin{array}{l}x^{3}, x^{2}<1 \\ x, x^{2} \geq 1\end{array}\right.$
11. If $x>2$, then find the values of $1 / x$ graphically.

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12. If $x<-1$, then find the values of $1 / x$ graphically.

## D Watch Video Solution

13. When $x>-2$, find the values of $1 / x$.

## - Watch Video Solution

14. When $x<3$, find the values of $1 / x$.
15. Draw the graph of $\frac{1}{x}+\frac{1}{y}=1$.

## - Watch Video Solution

16. Draw the graph of $y=\frac{1}{x^{2}}$.

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17. Draw the graphs of following quadratic functions.
(i) $y=x^{2}+x+1$
(ii) $y=x^{2}-2 x-3$
(iii) $y=2+x-x^{2}$
(iv) $y=x-1-x^{2}$
18. The following figure shows the graph of $f(x)=a x^{2}+b x+c$, find the signs of $a, b$ and $c$.


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19. Let $f(x)=2 x(2-x), 0 \leq x \leq 2$. Then find the number of solutions of $f(f(f(x)))=\frac{x}{2}$.

## (D) Watch Video Solution

20. 

$f: R \rightarrow R$
is
defined
as
$f(x)=\left\{\begin{array}{ll}x^{2}+k x+3, & \text { for } x \geq 0 \\ 2 k x+3, & \text { for } x<0\end{array}\right.$. If $f(x)$ is injective, then find the values of $k$.

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21. If $f(x)=x^{3}+4 x^{2}+\lambda x+1$ is a monotonically decreasing function of $x$ in the largest possible interval $\left(-2,-\frac{2}{3}\right)$.

Then (a) $\lambda=4$ (b) $\lambda=2 \lambda=-1$ (d) $\lambda$ has no real value
(D) Watch Video Solution
22. For what real values of $a$ do the roots of the equation
$x^{2}-2 x-\left(a^{2}-1\right)=0$ lie between the roots of the equation $x^{2}-2(a+1) x+a(a-1)=0$.

## (D) Watch Video Solution

23. Find the value of $a$ for which $a x^{2}+(a-3) x+1<0$ for at least one positive real $x$.

## D Watch Video Solution

24. Consider the inequation $9^{x}-a 3^{x}-a+3 \leq 0$, where a is real parameter.

The given inequality has at least one negative solution for $a \in$
(a) $(-\infty, 2)$
(b) $(3, \infty)$
(c) $(-2, \infty)$
(d) $(2,3)$

## (D) Watch Video Solution

25. Let $a, b, c$ be real. If $a x^{2}+b x+c=0$ has two real roots aand $\beta$, where $\alpha<-1$ and $\beta>1$, then show that $1+\frac{c}{a}+\left|\frac{b}{a}\right|<0$

## (D) Watch Video Solution

26. If $b>a$, then the equation $(x-a)(x-b)-1=0$ has
(a) Both roots in $(a, b)$
(b) Both roots in $(-\infty, a)$
(c) Both roots in $(b,+\infty)$
(d) One root in $(-\infty, a)$ and the other in $(b,+\infty)$
27. When $x>-2$, find the values of $|x|$ graphically.

## (D) Watch Video Solution

28. When $x<3$, find the values of $|x|$ graphically.

## (D) Watch Video Solution

29. If $2 \leq|x| \leq 5$, then find the values of x from the graph of
$y=|x|$.

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30. Draw the graph of $f(x)=\frac{|x-1|}{x-1}$.
31. Draw the graph of $x+|y|=2 y$ and check the differentiability.

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32. Draw the graph of $f(x)=(x+2)|x-1|$.

## D Watch Video Solution

33. 

Draw
the
graph
of the
function
$f(x)=x-\left|x-x^{2}\right|,-1 \leq x \leq 1$ and find the points of non-differentiability.
34. Solve : $x^{2}-|x+2|+x>0$

## (D) Watch Video Solution

35. Draw the graph of $f(x)=|2 x-1|+|2 x-3|$. Find the range of the function.

## (D) Watch Video Solution

36. Draw the graph of $f(x)=|x|-|2 x-3|$. Find the range of the function.

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37. Let $f(x)=x+2|x+1|+2|x-1|$. Find the values of $k$ if $f(x)=k$
(i) has exactly one real solution,
(ii) has two negative solutions,
(iii) has two solutions of opposite sign.

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38. $f(x)=|a x-b|+c|x| \forall x \in(-\infty, \infty)$, where $a>0, b>0, c>0$. Find the condition if $f(x)$ attains the minimum value only at one point.

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39. about to only mathematics
A. On the left of $x=c$
B. On the right of $x=c$
C. At no point
D. At all points

## Answer:

## (D) Watch Video Solution

40. If a continuous function $f$ defined on the real line R assume positive and negative values in R , then the equation $f(x)=0$ has a root in R. For example, if it is known that a continuous function $f$ on R is positive at some point and its minimum value is negative, then the equation $f(x)=0$ has a root in R .

Consider $f(x)=k e^{x}-x$, for all real x where k is a real constant.

The positive value of k for which $k e^{x}-x=0$ has only one root is
A. No point
B. One point
C. Two points
D. More than two points

## Answer:

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41. If a continuous function $f$ defined on the real line R assume positive and negative values in R , then the equation $f(x)=0$ has a root in R. For example, if it is known that a continuous function $f$ on R is positive at some point and its minimum
value is negative, then the equation $f(x)=0$ has a root in R .
Consider $f(x)=k e^{x}-x$, for all real x where k is a real constant.

For $\mathrm{k}>0$, the set of all values of k for which $y=k e^{x}-x=0$
has two distinct roots is
A. a. $\frac{1}{e}$
B. b. 1
C. c. $e$
D. d. $\log _{e} 2$

## Answer:

## (D) Watch Video Solution

42. If $x$ and $y$ are arbitrary extensive variables, then
A. $\left(0, \frac{1}{e}\right)$
B. $\left(\frac{1}{e}, 1\right)$
C. $\left(\frac{1}{e}, \infty\right)$
D. $(0,1)$

## Answer:

- Watch Video Solution

43. Find the number of solution of $2^{x}+3^{x}+4^{x}-5^{x}=0$
(D) Watch Video Solution
44. Draw the graph of $y=\log _{x} \sqrt{x}$
45. Find the number of roots of the equation $x \log _{e} x=1$.

## D Watch Video Solution

46. If the graphs of the functions $y=\log _{e} x$ and $y=a x$ intersect at exactly two points, then find the value of $a$.

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47. draw the graph of $f(x)=x+[x]$, [.] denotes greatest integer function.
48. Draw the graph of the function
$f(x)=x-|x 2-x|-1 \leq x \leq 1$, where [ $\cdot$ ] denotes the greatest integer function. Find the points of discontinuity and non-differentiability.

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49. Draw the graph of $f(x)=\left[x^{2}\right], x \in[0,2)$, where $[\cdot]$ denotes the greatest integer function.

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50. Draw the graph of $f(x)=[\sqrt{x}], x \in[0,16)$, where $[\cdot]$ denotes the greatest ineger function.
51. Draw the graph of $y=[x]+\sqrt{x-[x]}$, where $[\cdot]$ denotes the greatest ineger function.

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52. Draw the graph of $f(x)=\left[\log _{e} x\right], e^{-2}<x<10$, where [.] represents the greatest integer function.

## D Watch Video Solution

53. Solve $x^{2}-4 x-[x]=0$ (where [] denotes the greatest integer function).
54. Sketch the region of relation $[x]+[y]=5, x, y \geq 0$, where [ $\cdot$ ] denots the greatest integer function.

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55. Draw the graph of $f(x)=\{2 x\}$, where $\{\cdot\}$ represents the fractional part function.

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56. Find the domain of $f(x)=\sqrt{|x|-\{x\}}$ (where $\{\cdot\}$ denots the fractional part of x .
57. Solve : $x^{2}=\{x\}$, where $\{x\}$ represents the fractional part function.

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58. Draw the graph of $y^{2}=\{x\}$, where $\{\cdot\}$ represents the fractional part function.

## - Watch Video Solution

59. Draw the graph of $y=\frac{1}{\{x\}}$, where $\{\cdot\}$ denotes the fractional part function.
60. Solve : $4\{x\}=x+[x]$ (where $[\cdot]$ denotes the greatest integer function and $\{\cdot\}$ denotes the fractional part function.

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61. Given the graph of the function $y=f(x)$, draw the graph of $y=\operatorname{sgn}(x)$.

62. Draw the graph of $f(x)=\operatorname{sgn}\left(x^{3}-x\right)$.

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63. Draw the graph of $f(x)=\operatorname{sgn}\left(\log _{e} x\right)$.

## D Watch Video Solution

64. Let a function $f(x)$ be defined in $[-2,2]$ as
$f(x)=\left\{\begin{array}{ll}\{x\}, & -2 \leq x<-1 \\ |\operatorname{sgn} x|, & -1 \leq x \leq 1 \\ \{-x\}, & 1<x \leq 2\end{array}\right.$ where $\{x\}$ and $\operatorname{sgn} x$
denote fractional part and signum functions, respectively. Then
find the area bounded by the graph of $f(x)$ an the $x$-axis.
65. Let $f: R \rightarrow R$ be defined as $f(x)=e^{\operatorname{sgn} x}+e^{x^{2}}$. Then find the range of the function, and also indentify the type of the function : one-one or many-one.

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66. Draw the graph of the function $f(x)=\max \left\{x, x^{2}\right\}$ and write its equivalent definition.

## (D) Watch Video Solution

67. Let $f: R \rightarrow R$ be a function defined by
$f(x)=\max \cdot\left\{x, x^{3}\right\}$. The set of all points where $f(x)$ is NOT differenctiable is
(a) $\{-1,1\}$
(b) $\{-1,0\}$
(c) $\{0,1\}$
(d) $\{-1,0,1\}$

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68. Find the equivalent definition of
$f(x)=\max ^{2},(-x)^{2}, 2 x(1-x) w h r e 0 \leq x \leq 1$

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69. Let $\mathrm{f}: R \rightarrow R$ and $g: R \rightarrow R$ be respectively given by
$f(x)=|x|+1$ and $\left.g(x)=x^{2}+1\right)$. Define $h: R \rightarrow R$ by
$h(x)= \begin{cases}\max \{f(x), g(x)\} & \text { if } x \leq 0 \\ \min \{f(x), g(x)\} & \text { if } x>0\end{cases}$
then number of point at which $h(x)$ is not differentiable is
70. Sketch the region of the points satisfying $\max .\{|x|,|y|\} \leq 4$.

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71. 

Consider
the
regions
$A=\left\{(x, y) \mid x^{2}+y^{2} \leq 100\right\}$ and $\left.B=|x \quad y| \sin (x+y)>0\right\}$
in the plane. Then the area of the region $A \cap B$ is

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72. Draw the graphs of the following parabolas:
(i) $x=y^{2}-2 y-3$
(ii) $x=6+y-y^{2}$

## (D) Watch Video Solution

73. Find the number of roots of the equation $e^{x}=\sqrt{-x}$.

## (D) Watch Video Solution

74. Let $g(x)=\sqrt{x-2 k}, \forall 2 k \leq x<2(k+1)$, where $k \in$ integer. Check whether $g(x)$ is periodic or not.

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75. Plot the region in the first quadrant in which points are nearer to the origin than to the line $x=3$.
76. Draw the graph of $y=\sqrt{x^{2}-1}$

## - Watch Video Solution

77. Draw the graph of $y=-\sqrt{6-3 x^{2}}$

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78. The eccentricity of the ellipse

$$
9 x^{2}+25 y^{2}-18 x-100 y-116=0 \text { is a. } \frac{25}{16} \text { b. } \frac{4}{5} \text { c. } \frac{16}{25} \text { d. } \frac{5}{4}
$$

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79. Find the area enclosed by the curves
$y=\sqrt{x}$ and $x=-\sqrt{y}$ and the circle $x^{2}+y^{2}=2$ above the

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80. Consider a square with vertices at $(1,1),(-1,1),(-1,-1), \operatorname{and}(1,-1)$. Set $S$ be the region consisting of all points inside the square which are nearer to the origin than to any edge. Sketch the region $S$ and find its area.

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## Exercises

1. Draw the graph of $y=\frac{1}{(1 / x)}$.
2. 

(a)

Draw
the
graph
of
$f(x)= \begin{cases}1, & |x| \geq 1 \\ \frac{1}{n^{2}}, & \frac{1}{n}<|x|<\frac{1}{n-1}, n=2,3, \ldots \\ 0, & x=0\end{cases}$
(b) Sketch the region $y \leq-1$.
(c) Sketch the region $|x|<3$.

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3. Sketch the regions which points satisfy $|x+y| \geq 2$.

## - Watch Video Solution

4. Sketch the region satisfying $|x|<|y|$.
5. For a point $P$ in the plane, let $d_{1}(P) a n d d_{2}(P)$ be the distances of the point $P$ from the lines $x-y=0$ and $x+y=0$ respectively. The area of the region $R$ consisting of all points $P$ lying in the first quadrant of the plane and satisfying $2 \leq d_{1}(P)+d_{2}(P) \leq 4$, is

## (D) Watch Video Solution

6. Draw the graph of $y=\frac{x-1}{x-2}$.

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7. The following figure shows the graph of $f(x)=a x^{2}+b x+c$, then find the sign of values of
$a, b$ and $c$.


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8. The entire graph of the equation $y=x^{2}+k x-x+9$ in strictly above the $x$-axis if and only if (a) $k<7$ (b) $-5<k<7$
(c). $k>-5$ (d) none of these
A. $k<7$
B. $-5<k<7$
C. $k>-5$
D. None of these

## Answer:

## D Watch Video Solution

9. If $x^{2}+2 a x+a<0 \forall x \in[1,2]$, the find the values of $a$.

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10. Draw the graph of $f(x)=x|x|$.
11. Draw the graph of the function: Solve $\left|\frac{x^{2}}{x-1}\right| \leq 1$ using the graphical method.

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12. Draw the graph of $y=\left|x^{2}-2 x\right|-x$.

## - Watch Video Solution

13. Draw the graph of $y= \begin{cases}2^{x}, & x^{2}-2 x \leq 0 \\ 1+3.5 x-x^{2} & x^{2}-2 x>0\end{cases}$

## - Watch Video Solution

14. Draw the graph of $f(x)=|x-1|+|2 x-3|$. Find the range of the function.

## (D) Watch Video Solution

15. 

Draw
the
graph
of
$f(x)=y=|x-1|+3|x-2|-5|x-4|$ and find the values of $\lambda$ for which the equation $f(x)=\lambda$ has roots of opposite sign.

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16. Find the set of real value(s) of $a$ for which the equation
$|2 x+3|+|2 x-3|=a x+6$ has more than two solutions.

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17. Draw the graph of $y=2^{\frac{(|x|+x)}{x}}$.

## - Watch Video Solution

18. Draw the graph of $y=x^{\frac{1}{\log _{e} x}}$.

## (D) Watch Video Solution

19. Find the number of solutions to the equation $x+\log _{e} x=0$.

## (D) Watch Video Solution

20. Draw the graph of $f(x)=x+[x]$, [.] denotes greatest integer function.
21. Given $f(x)$ is a periodic function with period 2 and it is defined as

$$
f(x)= \begin{cases}{\left[\cos \frac{\pi x}{2}\right]+1,} & 0<x<1 \\ 2-x, & 1 \leq x<2\end{cases}
$$

Here [.] represents the greatest integer $\leq x$. If $f(0)=1$, then draw the graph of the function for $x \in[-2,2]$.

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22. Draw the region of relation $[x][y]=6, x, y \geq 0$. Here $[\cdot]$ denotes the greatest integer function.

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23. $\lim _{x \rightarrow c} f(x)$ does not exist for
wher [.] represent greatest integer function $\{$.$\} represent$
fractional part function

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24. Let $f(x)=\frac{[x]+1}{\{x\}+1}$ for $f:\left[0, \frac{5}{2}\right) \rightarrow\left(\frac{1}{2}, 3\right]$, where $[\cdot]$
represents the greatest integer function and $\{\cdot\}$ represents the fractional part of x .

Draw the graph of $y=f(x)$. Prove that $y=f(x)$ is bijective.
Also find the range of the function.

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25. Draw the graph of $y=2^{\{x\}}$, where $\{\cdot\}$ represents the fractional part function.
26. Find tha area of the region containing the points $(x, y)$ satisfying $4 \leq x^{2}+y^{2} \leq 2(|x|+|y|)$.

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27. Draw the graph of $y=-\sqrt{x^{2}+2}$

## (D) Watch Video Solution

28. Draw the graph of $y=|x|^{1 / 2}$ for $-1 \leq x \leq 1$.

## (D) Watch Video Solution

29. Draw the graph of $f(x)=\operatorname{sgn}\left(\log _{0.5} x\right)$.
30. The graph of $y=f(x)$ is as shown in the following figure. Draw the graph of $y=[f(x)]$.


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31. The function $y=f(x)=\lim _{n \rightarrow \infty} \frac{x^{2 n}-1}{x^{2 n}+1}$. Is this function same as the function $g(x)=\operatorname{sgn}(|x|)-1)$.
32. An even periodic function $f: R \rightarrow R$ with period 4 is such that
$f(x)=\left\{\begin{array}{cl}\max \cdot\left(|x|, x^{2}\right), & 0 \leq x<1 \\ x, & 1 \leq x \leq 2\end{array}\right.$. Then draw the graph of $y=f(x)$ for $x \in R$

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33. 

The
function
$f(x)=\max \{(1-x),(1+x), 2\}, x \in(-\infty, \infty)$ is
A. Continuous at all points
B. Differentiable at all points
C. Differentiable at all points except at

$$
x=1 \text { and } x=-1
$$

D. Continuous at all points except at $x=1$ and $x=-1$, where it is discontinuous

## Answer:

## (D) Watch Video Solution

34. Check the differentiability if $f(x)=\min \cdot\left\{1, x^{2}, x^{3}\right\}$.
