MATHS

BOOKS - SAI MATHS (TELUGU ENGLISH)

APPLICATIONS OF DERIVATIVES

Problems

1. Two particles P and Q located at the points with coordinates

$P\left(t^3-16t-3\right), Q\left(t+1, t^3-6t-6\right)$ are moving in a plane. The minimum distance between them in their motion is

A. 1

B. 5

C. 169

D. 49

Answer: D

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2. Define $f(x) = \begin{cases} x & (0 \leq x \leq 1) \\ 2 - x & (1 \leq x \leq 2) \end{cases}$

Then Rolle's theorem is not applicable to f(x) because

A. f(x) is not defined everywhere on [0, 2]

B. f(x) is not continuous on [0, 2]

C. f(x) is not differentiable on (1, 2)

D. f(x) is not differentiable on (0, 2)

Answer: D

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3. An equilateral triangle is of side 10 units. In measuring the side, an error of 0.05 units is made. Then the percentage error in the area of the triangle is

A. 5

B. 4

C. 1

D. 0.5

Answer: C

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4. If the lines $y = -4x + b$ are tangents to the curve $y = \frac{1}{x}$, then b =

A. ± 4

B. ± 2

C. ± 1

D. ± 8

Answer: A

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5. The volume of a sphere is increasing at the rate of 1200 c.cm/sec. The rate of increase in its surface area when the radius is 10 cm is

A. 120 sq.cm/sec

B. 240 sq.cm/sec

C. 200 sq.cm/sec

D. 100 sq.cm/sec

Answer: B

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6. The slope of the tangent to the curve $y = \int_0^x \frac{dt}{1+t^3}$ at the point where x = 1 is

A. $\frac{1}{4}$

B. $\frac{1}{3}$

C. $\frac{1}{2}$

D. 1

Answer: C

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7. If $x^2 + y^2 = 25$, then $\log_5[Max(3x + 4y)]$ is

A. 2

B. 3

C. 4

D. 5

Answer: A

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8. If f is defined in [1, 3] by $f(x) = x^3 + bx^2 + ax$ such that $f(1) - f(3) = 0$ and f'(c) =0 where $c = 2 + \frac{1}{\sqrt{3}}$, then (a, b) =

A. $(-6, 11)$

B. $\left(2 - \frac{1}{\sqrt{3}}, 2 + \frac{1}{\sqrt{3}}\right)$

C. $(11, -6)$

D. $(6, 11)$

Answer: C

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9. If the curves $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and $\frac{x^2}{25}+\frac{y^2}{16}=1$ cut each other orthogonally, then a^2-b^2 equals to

A. 9

B. 400

C. 75

D. 41

Answer: A

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10. The condition that $f(x) = ax^3 + bx^2 + cx + d$ has no extreme value, is

A. $b^> 3ac$

B. $b^> 4ac$

C. $b^= 3ac$

D. $b^< 3ac$

Answer: D

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11. If there is an error of ± 0.04 cm in the measurement of the diameter of a sphere, then the approximate percentage error in its volume, when the radius is 10 cm, is

A. ± 1.2

B. ± 0.06

C. ± 0.006

D. ± 0.6

Answer: D

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12. The value of c in the Langrange's mean value theorem for $f(x) = \sqrt{x-2}$ in the interval $[2, 6]$ is

A. $\frac{9}{12}$

B. $\frac{5}{2}$

C. 3

D. 4

Answer: C

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13. The relation between pressure p and volume V is given by $pV^{\frac{1}{4}} =$ constant. If the percentage decrease in volume is $\frac{1}{2}$, then the percentage increase in pressure is

A. $-\frac{1}{8}$

B. $\frac{1}{16}$

C. $\frac{1}{8}$

D. $\frac{1}{2}$

Answer: C

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14. If the curves $x^2 + py^2 = 1$ and $qx^2 + y^2 = 1$ are orthogonal to each other, then

A. $p - q = 2$

B. $\frac{1}{p} - \frac{1}{q} = 2$

C. $\frac{1}{p}+\frac{1}{q}=-2$

D. $\frac{1}{p}+\frac{1}{q}=2$

Answer: D

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15. The focal length of a mirror is given by $\frac{2}{f}=\frac{1}{v}-\frac{1}{u}$. In finding the values of u and v, the errors are equal to 'p'. Then, the relative error in f is

A. $\frac{p}{2}\left(\frac{1}{u}+\frac{1}{v}\right)$

B. $\frac{p}{2}\left(\frac{1}{u}+\frac{1}{v}\right)$

C. $\frac{p}{2}\left(\frac{1}{u}-\frac{1}{v}\right)$

D. $p\left(\frac{1}{u}-\frac{1}{v}\right)$

Answer: B

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16. The coordinates of the point P on the curve $x = a(\theta + \sin\theta), y = a(1 - \cos\theta)$, where the tangent is inclined at an angle $\frac{\pi}{4}$ to x-axis, are

A. $\left[a\left(\frac{\pi}{4}-1\right), a\right]$

B. $\left[a\left(\frac{\pi}{2}+1\right), a\right]$

C. $\left(a\frac{\pi}{2}, a\right)$

D. (a, a)

Answer: B

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17. If Δ is the area of the triangle formed by the positive x-axis and the normal and tangent

to the circle $x^2 + y^2 = 4$ $(1, \sqrt{3})$, then Δ is equal to

A. $\frac{\sqrt{3}}{2}$

B. $\sqrt{3}$

C. $2\sqrt{3}$

D. 6

Answer: C

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18. If the volume of a shpere increases at the rate of $2\pi\ cm^3/s$, then the rate of increase of its radius (in cm/s), when the volume is $288\pi\ cm^3$, is

A. $\frac{1}{36}$

B. $\frac{1}{72}$

C. $\frac{1}{18}$

D. $\frac{1}{9}$

Answer: B

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19. If $1^\circ = \alpha$ radians, then the approximate value of $\cos(60^\circ 1')$

A. $\frac{1}{2} + \frac{\alpha\sqrt{3}}{120}$

B. $\frac{1}{2} - \frac{\alpha}{120}$

C. $\frac{1}{2} - \frac{\alpha\sqrt{3}}{120}$

D. $\frac{1}{2} + \frac{\alpha}{120}$

Answer: C

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20. If the distance s travelled by a particle in time ti is given by $s = t^2 - 2t + 5$, then its acceleration is

A. 0

B. 1

C. 2

D. 3

Answer: C

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21. The length of the subtangent at any point x_1, y_1 on the curve $y = 5^x$

A. $5x_1$

B. $y_1 5x_1$

C. $\log_e 5$

D. $\dfrac{1}{\log_e 5}$

Answer: C

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22. The height of the cone of maximum volume inscribed in a sphere of radius R is

A. $\frac{R}{3}$

B. $2\frac{R}{3}$

C. $4\frac{R}{3}$

D. $\frac{4R}{\sqrt{3}}$

Answer: C

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23. The longest distance of the point (a, 0) from the curve $2x^2 + y^2 = 2x$ is

A. $1 + a$

B. $|1 - a|$

C. $\sqrt{1 - 2a + 2a^2}$

D. $\sqrt{1 - 2a + 3a^2}$

Answer: C

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24. If m_1 and m_2 are the roots of the equation

$$x^2 + (\sqrt{3}+2)x + (\sqrt{3}-1) = 0$$

then the area of the triangle formed by the lines $y = m_1x, y = m_2x$ and $y = c$

A. $\left(\frac{\sqrt{33}-\sqrt{11}}{4}\right).c^2$

B. $\left(\frac{\sqrt{33}+\sqrt{11}}{4}\right).c^2$

C. $\left(\frac{\sqrt{11}-\sqrt{33}}{2}\right).c^2$

D. $\frac{\sqrt{33}}{2}.c^2$

Answer: B

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25. There is an error of ± 0.04 cm in the measurement of the diameter of a sphere. When the radius is 10 cm, the percentage error in the volume of the sphere is

A. ± 1.2

B. ± 1.0

C. ± 0.8

D. ± 0.6

Answer: D

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26. The function $f(x) = x^3 + ax^2 + bx + c$, $a^2 \leq 3b$ has

A. one maximum value

B. one maximum value

C. no extreme value

D. one maximum and one minimum value

Answer: C

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27. The maximum value of $\frac{\log x}{x}$, `0

A. ∞

B. e

C. 1

D. e^{-1}

Answer: D

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28. The equation of the normal to the curve $y^4 = ax^3$ at (a, a) is

A. $x + 2y = 3a$

B. $3x - 4y + a = 0$

C. $4x + 3y = 7a$

D. $4x - 3y = 0$

Answer: C

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29. The angle between the curves $y^2 = 4x + 4$ and $y^2 = 36(9 - x)$ is

A. 30°

B. 45°

C. 60°

D. 90°

Answer: D

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30. If m and M respectively denote the minimum and maximum of $f(x) = (x-1)^2 + 3$ for $x \in [-3, 1]$, then the ordered pair (m, M) is equal to

A. $(-3, 19)$

B. $(3, 19)$

C. $(-19, 3)$

D. $(-19, 13)$

Answer: B

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31. The length of the subtangent at (2, 2) to the curve $x^5 = 2y^4$, is

A. $\frac{5}{2}$

B. $\frac{8}{5}$

C. $\frac{2}{5}$

D. $\frac{5}{8}$

Answer: B

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32. The lengths of tangent, subtangent, normal and subs normal for the curve $y = x^2 + x - 1$ at (1, 1) are A, B, C and D respectively, then their increasing order is

A. B, D, A, C

B. B, A, C, D

C. A, B, C, D

D. B, A, D, C

Answer: D

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33. The condition for $f(x) = x^3 + px^2 + qx + r(x \in R)$ to have no extreme value, is

A. $p^2 < 3q$

B. $2p^2 < q$

C. $p^2 < \frac{1}{4}q$

D. $p^2 < 3q$

Answer: A

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34. The circumference of a circle is measured as 56 cm with an error 0.02 cm. The percentage error in its area is

A. $\frac{1}{7}$

B. $\frac{1}{28}$

C. $\frac{1}{14}$

D. $\frac{1}{56}$

Answer: C

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35. Statement I $f(x) = xe^{-x}$ has the maximum at x = 1.

Statement II f'(1) = 0 and f"(1) < 0

A. Both I and II are true and II is the correct explanation for I

B. Both I and II are true and II is not the correct explanation for I

C. I is true but II is false

D. I is false but II is true

Answer: A

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36. If θ is the angle between the curves xy = 2 and $x^2 + 4y = 0$ then $\tan\theta$ is equal to

A. 1

B. -1

C. 2

D. 3

Answer: D

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37. In the interval $(-3, 3)$ the function $f(x) = \frac{x}{3} + \frac{3}{x}, x \neq 0$ is

A. Increasing

B. Decreasing

C. Neither increasing nor decreasing

D. Partly increasing and partly decreasing

Answer: B

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38. The perimeter of a asector is a constant. If its area is to be maximum, the sectorial angle is

A. $\frac{\pi^c}{6}$

B. $\frac{\pi^c}{4}$

C. 4^c

D. 2^c

Answer: D

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39. If x is real, then minimum value of $\frac{x^2 - x + 1}{x^2 + x + 1}$ is

A. $\frac{1}{3}$

B. 3

C. $\frac{1}{2}$

D. 1

Answer: A

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40. The extreme values of $4\cos\left(x^2\right)\cos\left(\frac{\pi}{3}+x^2\right)\cos\left(\frac{\pi}{3}-x^2\right)$ over R, are

A. $-1, 1$

B. $-2, 1$

C. $-3, 3$

D. $-4, 4$

Answer: A

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41. A stone thrown upwards has its equation of motion $S = 490t - 4.9t^2$. Then the maximum height reached by it is

A. 24500

B. 12500

C. 12250

D. 25400

Answer: C

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42. The radius of a circular plate is increases at the rate of 0.01 cm/s when the radius is 12 cm. Then the rate at which the area increases, is

A. 0.24π sq cm/s

B. 60π sq cm/s

C. 24π sq cm/s

D. 1.2π sq cm/s

Answer: A

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43. Statement I $f(x) = 2x^3 - 9x^2 + 12x - 3$ is increasing outside the interval (1, 2).

Statement II $f'(x) < 0$ for $x \in (1, 2)$.

A. Both I and II are true and II is the correct explanation for I

B. Both I and II are true and II is not the correct explanation for I

C. I is true but II is false

D. I is false but II is true

Answer: B

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44. A particle moves along the curve $y = x^2 + 2x$. Then the point on the curve such that x and y coordinates of the particle change with the same rate, is

A. (1, 3)

B. $\left(\frac{1}{2}, \frac{5}{2}\right)$

C. $\left(-\frac{1}{2}, -\frac{3}{4}\right)$

D. $(-1, -1)$

Answer: C

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45. A point is moving on $y = 4 - 2x^2$. The x-coordinate of the point is decreasing at the rate of 5 units per second. Then the rate at which y coordinate of the point is changing when the point is at (1, 2), is

A. 5 unit/s

B. 10 unit/s

C. 15 unit/s

D. 20 unit/s

Answer: D

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46. If $f: R \to R$ is an even function having derivatives of all orders, then an odd function among the following is

A. f''

B. f'''

C. $f' + f''$

D. $f'' + f'''$

Answer: B

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47. Match the column I (the curve $2y^2 = x + 1$) with column II (the slope of normals)

Column I	Column II
(A) $(7, 2)$	(1) $-4\sqrt{2}$
(B) $\left(0, \frac{1}{\sqrt{2}}\right)$	(2) -8
(C) $(1, -1)$	(3) 4
(D) $(3, \sqrt{2})$	(4) 0
	(5) $-2\sqrt{2}$

A. $(A) - 2, (B) - 4, (C) - 3, (D) - 1$

B. $(A) - 2, (B) - 5, (C) - 3, (D) - 1$

C. $(A) - 2, (B) - 3, (C) - 5, (D) - 1$

D. $(A) - 2, (B) - 5, (C) - 1, (D) - 3$

Answer: B

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48. The sum of two numbers is 20. If the product of the square of one number and cube of the other is maximum, then the numbers are

A. 12, 8

B. 3, 4

C. 9, 12

D. 15, 18

Answer: A

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49. A minimum value of $\int_0^x \left(te^{t^2}\right)dt$ IS

A. 0

B. 1

C. 2

D. 3

Answer: A

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50. Gas is being pumped into a spherical balloon at the rate of $30 ft^3 / \min$. The rate at which the radius increase when it reaches the value 15 ft. is

A. $\frac{1}{30\pi} ft / \text{ min}$

B. $\frac{1}{15\pi} ft / \text{ min}$

C. $\frac{1}{20} ft / \text{ min}$

D. $\frac{1}{25} ft / \text{ min}$

Answer: A

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51. The angle between the curves $y = \sin x$ and $y = \cos x$ is

A. $\tan^{-1}(2\sqrt{2})$

B. $\tan^{-1}(3\sqrt{2})$

C. $\tan^{-1}(2\sqrt{3})$

D. $\tan^{-1}(5\sqrt{2})$

Answer: A

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52. The minimum value of $2x^2 + x - 1$ is

A. $-\frac{1}{4}$

B. $\frac{3}{2}$

C. $-\frac{9}{8}$

D. $\frac{9}{8}$

Answer: C

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53. If $\log(1+x) - \frac{2x}{2+x}$ is increasing, then

A. $0 < x < \infty$

B. $-\infty < x < 0$

C. $-\infty < x < \infty$

D. $-1 < x < 2$

Answer: A

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54. The function $f(x) = xe^{-x}, \forall (x \in R)$ attains a maximum value at x equals to

A. 1

B. 2

C. $\frac{1}{e}$

D. 3

Answer: A

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55. The two curves $x = y^2, xy = a^3$ cut orthogonally at a point, then a^2 is equal to

A. $\frac{1}{3}$

B. $\frac{1}{2}$

C. 1

D. 1

Answer: B

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56. The approximate value of $(1.0002)^{3000}$ is

A. 1.2

B. 1.4

C. 1.6

D. 1.8

Answer: C

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57. The minimum value of $(x-\alpha)(x-\beta)$ is

A. 0

B. $\alpha\beta$

C. $\frac{1}{4}(\alpha-\beta)^2$

D. $-\frac{1}{4}(\alpha-\beta)^2$

Answer: D

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58. The equation of tangent to the curve $6y = 7 - x^3$ at (1, 1), is

A. $2x + y = 3$

B. $x + 2y = 3$

C. $x + y = -1$

D. $x + y + 2 = 0$

Answer: B

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59. The maximum value of xy subject to $x + y = 7$, is

A. 10

B. 12

C. $\frac{49}{4}$

D. $\frac{55}{4}$

Answer: C

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