



MATHS

BOOKS - SAI MATHS (TELUGU ENGLISH)

FUNCTIONS AND MATHEMATICAL INDUCTION

Problems

1. If $f : N \rightarrow R$ is defined by $f(1) = -1$ and $f(n+1) = 3f(n) + 2$ for $n > 1$, then f is

- A. one - one
- B. onto
- C. a constant function
- D. $f(n) > 0$ for $n > 1$

Answer: C



[View Text Solution](#)

2. The remainder of $n^4 - 2n^3 - n^2 + 2n - 26$ when divided by 24 is

A. 20

B. 21

C. 22

D. 23

Answer: C



[View Text Solution](#)

3. If $f: R \rightarrow R$, $g: R \rightarrow R$ are defined by $f(x) = 5x - 3$, $g(x) = x^2 + 3$,

then $(g \circ f^{-1})(3) =$

A. $\frac{25}{9}$

B. $\frac{111}{25}$

C. $\frac{9}{25}$

D. $\frac{25}{111}$

Answer: B

 [Watch Video Solution](#)

4. If $A = \left\{ x \in R / \frac{\pi}{4} \leq x \leq \frac{\pi}{3} \right\}$ and $f(x) = \sin x - x$, then $f(A)$ is equal to

A. $\left[\frac{\sqrt{3}}{2} - \frac{\pi}{3}, \frac{1}{\sqrt{2}} - \frac{\pi}{4} \right]$

B. $\left[-\frac{1}{\sqrt{2}} - \frac{\pi}{4}, \frac{\sqrt{3}}{2} - \frac{\pi}{3} \right]$

C. $\left[-\frac{\pi}{3}, \frac{-\pi}{4} \right]$

D. $\left[\frac{\pi}{4}, \frac{\pi}{3} \right]$

Answer: A

 [Watch Video Solution](#)

5. The value of the sum $1.2.3+2.3.4+3.4.5+\dots$ upto n terms is equal to

A. $\frac{1}{6}n^2(2n^2 + 1)$

B. $\frac{1}{6}(n^2 - 1)(2n - 1)(2n + 3)$

C. $\frac{1}{8}(n^2 + 1)(n^2 + 5)$

D. $\frac{1}{4}n(n + 1)(n + 2)(n + 3)$

Answer: D



Watch Video Solution

6. If $x = \frac{1}{5} + \frac{1.3}{5.10} + \frac{1.3.5}{5.10.15} + \dots \infty$ then find $3x^2 + 6x$.

A. 1

B. 2

C. 3

D. 4

Answer: B



Watch Video Solution

7. If R is the set of all real numbers and $f : R - \{2\} \rightarrow R$ is defined by

$$f(x) = \frac{2+x}{2-x} \text{ for } x \in R - \{2\}$$

A. $R - \{-2\}$

B. R

C. $R - \{1\}$

D. $R - \{-1\}$

Answer: D



View Text Solution

8. Let Q be the set of all rational number in $[0, 1]$ and $f : [0, 1] \rightarrow [0, 1]$ be

defined by $f(x) = \begin{cases} x & \text{for } x \in Q \\ 1-x & \text{for } x \notin Q \end{cases}$ Then the set

$S = \{x \in [0, 1] \mid (f \circ f)(x) = x\}$ is equal to

A. $[0, 1]$

B. $-\mathcal{Q}$

C. $[0, 1] - \mathcal{Q}$

D.

Answer: A



Watch Video Solution

9. $\sum_{k=1}^{2n+1} (-1)^{k-1} k^2$ equals to

A. $(n - 1)(2n - 1)$

B. $(n + 1)(2n + 1)$

C. $(n + 1)(2n - 1)$

D. $(n - 1)(2n + 1)$

Answer: B



View Text Solution

10. If $f(x) = (p - x^n)^{1/n}$, $p > 0$ and n is a positive integer, then $f[f(x)]$ is equal to

A. x

B. x^n

C. $p^{1/n}$

D. $p - x^n$

Answer: A



View Text Solution

11. The value of $\left\{ x \in R \left[\log(1.6)^{1-x^2} - (0.625)^{6(1+8)} \right] \in R \right\}$

A. $(-\infty, -1) \cup (7, \infty)$

B. $(-1, 5)$

C. $(1, 7)$

D. $(-1, 7)$

Answer: D



View Text Solution

12. If $f: R \rightarrow R^2$ and $R^+ \rightarrow R$ are such that $g\{f(x)\} = |\sin x|$ and $f\{(gx)\} = (\sin \sqrt{x})^2$, then a possible choice for f and g is

A. $f(x) = x^2, g(x) = \sin \sqrt{x}$

B. $f(x) = \sin x, g(x) = |x|$

C. $f(x) = \sin^2 x, g(x) = \sqrt{x}$

D. $f(x) = x^2, g(x) = \sqrt{x}$

Answer: C



[View Text Solution](#)

13. Define $f: Z \rightarrow Z$ by $f(x) = \begin{cases} x/2 & (\text{x is even}) \\ 0 & (\text{x is odd}) \end{cases}$ then f is

- A. onto but not one-to-one
- B. one-to-one but not onto
- C. one-to-one and onto
- D. neither one-to-one nor onto

Answer: A



[Watch Video Solution](#)

14. If $\frac{1}{2 \times 4} + \frac{1}{4 \times 6} + \frac{1}{6 \times 8} + \dots$ (n terms) $= \frac{kn}{n+1}$, then k is equal to

- A. $\frac{1}{4}$
- B. $\frac{1}{2}$

C. 1

D. $\frac{1}{8}$

Answer: A



[View Text Solution](#)

15. If $f: [2, \infty) \rightarrow B$ defined by $f(x) = x^2 - 4x + 5$ is a bijection, then

$B =$

A. $[0, \infty)$

B. $[1, \infty)$

C. $[4, \infty)$

D. $[5, \infty)$

Answer: B



[Watch Video Solution](#)

16. If $f: R \rightarrow R$ is defined by $f(x) = \left[\frac{x}{5} \right]$ for $x \in R$, where $[y]$ denotes the greatest integer not exceeding y , then $\{f(x) : |x| < 71\}$ is equal to

- A. $\{-14, -13, \dots, 0, \dots, 13, 14\}$
- B. $\{-14, -13, \dots, 0, \dots, 14, 15\}$
- C. $\{-15, -14, \dots, 0, \dots, 14, 15\}$
- D. $\{-15, -14, \dots, 0, \dots, 13, 14\}$

Answer: D



[View Text Solution](#)

17. If a, b and n are natural numbers, then $a^{2n-1} + b^{2a-1}$ is divisible by

- A. $a + b$
- B. $a - b$
- C. $a^3 + b^3$
- D. $a^2 + b^2$

Answer: A



[View Text Solution](#)

18.

If

$f(0) = 0, f(1) = 1, f(2) = 2$ and $f(x) = f(x - 2) + f(x - 3)$ for $x = 3, 4, \dots$

then $f(9) =$

A. 12

B. 13

C. 14

D. 10

Answer: D



[Watch Video Solution](#)

19. Let R denote the set of all real numbers and R^+ denote the set of all positive real numbers. For the subsets A and B of R define $f: A \rightarrow B$ by

$f(x) = x^2$ for $x \in A$. Observe the two lists given below

Column I

Column II

- | | |
|--|---------------------|
| A. f is one-one and onto, if | 1. $A = R^+, B = R$ |
| B. f is one-one but not onto, if | 2. $A = B = R$ |
| C. f is onto but not one-one, if | 3. $A = R, B = R^+$ |
| D. f is neither one-one nor onto, if | 4. $A = B = R^+$ |

A. A-1, B-2, C-3, D-4

B. A-4, B-2, C-1, D-3

C. A-4, B-1, C-3, D-2

D. A-4, B-2, C-3, D-1

Answer: C



[View Text Solution](#)

20. The numbers $a_n = 6^n - 5n$ for $n = 1, 2, 3, \dots$ when divided by 25 leave the remainder

A. 9

B. 7

C. 3

D. 1

Answer: D



[View Text Solution](#)

21. If $f: [2, 3] \rightarrow \mathbb{R}$ is defined by $f(x) = x^3 + 3x - 2$, then the range $f(x)$ is contained in the interval :

A. $[1, 12]$

B. $[12, 34]$

C. $[35, 50]$

D. $[-12, 12]$

Answer: B

 [Watch Video Solution](#)

$$22. \left\{ x \in \mathbb{R} : \frac{2x - 1}{x^3 + 4x^2 + 3x} \in \mathbb{R} \right\} =$$

A. $\mathbb{R} - \{0\}$

B. $\mathbb{R} - \{0, 1, 3\}$

C. $\mathbb{R} - \{0, -1, -3\}$

D. $\mathbb{R} - \left\{ 0, -1, -3, +\frac{1}{2} \right\}$

Answer: C

 [Watch Video Solution](#)

23. Using mathematical induction, the numbers a_n 's are defined by,

$$a_0 = 1, a_{n+1} = 3n^2 + n + a_n, (n \geq 0).$$

Then, a_n is equal to

A. $n^3 + n^2 + 1$

B. $n^3 + n^2 + 1$

C. $n^3 - n^2 + 1$

D. $n^3 - n^2$

Answer: B



[View Text Solution](#)

24. If $\mathbb{R} \rightarrow C$ is defined by $f(x) = e^{2ix}$ or $x \in \mathbb{R}$ then, f is (where C denotes the set of all complex numbers)

A. one-one

B. onto

C. one-to-one and onto

D. neither one-one nor onto

Answer: D



[Watch Video Solution](#)

25. If $f: [-6, 6] \rightarrow \mathbb{R}$ defined by $f(x) = x^2 - 3$ for $x \in \mathbb{R}$ then $(f \circ f \circ f)(-1) + (f \circ f \circ f)(0) + (f \circ f \circ f)(1) =$

A. $f(4\sqrt{2})$

B. $f(3\sqrt{2})$

C. $f(2(\sqrt{2}))$

D. $f(\sqrt{2})$

Answer: A



Watch Video Solution

26. If $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ are defined by $f(x) = |x|$ and $g(x) = [x - 3]$ for $x \in \mathbb{R}$, then $\{g(f(x)) : -8/5 < x < 8/5\} =$

A. $\{0, 1\}$

B. $\{1, 2\}$

C. $\{-3, -2\}$

D. $\{2, 3\}$

Answer: C



Watch Video Solution

27. For any integer $n \geq 1$, the sum $\sum_{k=1}^n k(k+2)$ is equal to

A. $\frac{n(n+1)(n+2)}{6}$

B. $\frac{n(n+1)(2n+1)}{6}$

C. $\frac{n(n+1)(2n+7)}{6}$

D. $\frac{n(n+1)(2n+9)}{6}$

Answer: C



View Text Solution

28. If $f: R \rightarrow R$ is defined by $f(x) = \frac{1}{2 - \cos 3x}$ for each $x \in R$ then the range of f is

A. $\left(\frac{1}{3}, 1\right)$

B. $\left[\frac{1}{3}, 1\right]$

C. $(1, 2)$

D. $[1, 2]$

Answer: B

 [Watch Video Solution](#)

29. If $f: R \rightarrow R$ and $g: R \rightarrow R$ are defined by $f(x) = x - \{x\}$ and $g(x) = [x]$ for $x \in R$, where $[x]$ is greatest integer not exceeding x , then for every $x \in R$, $f(g(x))$ is equal to

A. x

B. 0

C. $f(x)$

D. $g(x)$

Answer: B

 [View Text Solution](#)

30. If $S_n = 1^3 + 2^3 + \dots + n^3$ and $T_n = 1 + 2 + \dots + n$, then

A. $S_n = T_{n^3}$

B. $S_n = T_{n^2}$

C. $S_n = T_n^2$

D. $S_n = T_n^3$

Answer: C

 [View Text Solution](#)

31. If $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = x - [x] - \frac{1}{2}$ for $x \in \mathbb{R}$, where $[x]$ is the greatest integer not exceeding x , then $\left\{ x \in \mathbb{R} : f(x) = \frac{1}{2} \right\} =$

- A. \mathbb{Z} , the set of all integers
- B. \mathbb{N} , the set of all natural numbers
- C. ϕ , an empty set
- D. \mathbb{R}

Answer: C



[Watch Video Solution](#)

32. If $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = [2x] - 2[x]$ for $x \in \mathbb{R}$, then the range of f is (Here $[x]$ denotes the greatest integer not exceeding x)

- A. $\{x \in \mathbb{R} : 0 \leq x \leq 1\}$
- B. $\{0, 1\}$
- C. $\{x \in \mathbb{R} : x > 0\}$

$$D. \{x \in R: x \leq 0\}$$

Answer: B



Watch Video Solution

33. If $f: R \rightarrow R$ is defined by

$$f(x) = \begin{cases} x + 4 & \text{for } x < -4 \\ 3x + 2 & \text{for } -4 \leq x < 4 \\ x - 4 & \text{for } x \geq 4 \end{cases}$$

then match the following columns and choose the correct answer.

column I

column II

- | | |
|-----------------------|---------|
| (A) $f(-5) + f(-4)$ | (1) 14 |
| (B) $f(f(-8))$ | (2) 4 |
| (C) $f(f(-7)) + f(3)$ | (3) -11 |
| (D) $f(f(f(0))) + 1$ | (4) -1 |
| | (5) 5 |
| | (6) 0 |

A. A-3, B-6, C-2, D-5

B. A-3, B-4, C-2, D-5

C. A-4, B-3, C-2, D-1

D. A-3, B-6, C-5, D-2

Answer: A



Watch Video Solution

34. For all integers, $n \geq 1$ which of the following is divisible by 9.

A. $8^n + 1$

B. $4^n - 3n - 1$

C. $3^{2n} + 3n + 1$

D. $10^n + 1$

Answer: B



View Text Solution

35. $\{x \in R : [x - |x| = 5]\}$ is equal to

A. R, the set of all real numbers

B. ϕ , an empty set

C. $\{x \in R: < 0\}$

D. $\{x \in R: \geq 0\}$

Answer: B



[View Text Solution](#)

36. The function $f: C \rightarrow C$ defined by $f(x) = \frac{ax + b}{cx + d}$ for $x \in C$, where $bd \neq 0$ reduces to a constant function, if

A. $a = c$

B. $b = d$

C. $ad = bc$

D. $ab = cd$

Answer: C



[View Text Solution](#)

37. If N denotes the set of all positive integers and if $f: N \rightarrow N$ is defined by $f(n) =$ the sum of positive divisors of n then, $f(2^k 3)$, where k is a positive integer, is

A. $2^{k+1} - 1$

B. $2(2^{k+1} - 1)$

C. $3(2^{k+1} - 1)$

D. $4(2^{k+1} - 1)$

Answer: D



[View Text Solution](#)

38. $\{n(n+1)(2n+1) : n \in I\} \subset$

A. $\{6k : k \in I\}$

B. $\{12k : k \in I\}$

C. $\{18k : k \in I\}$

D. $\{24k : k \in I\}$

Answer: A



[View Text Solution](#)

39. If $f: N \rightarrow Z$ is defined by

$$f(x) = \begin{cases} 2 & \text{if } n = 3k, \quad k \in Z \\ 10 & \text{if } n = 3k + 1, \quad k \in Z \\ 0 & \text{if } n = 3k + 2, \quad k \in Z \end{cases}$$

then $\{n \in N : f(n) > 2\}$ is equal to

A. $\{3, 6, 4\}$

B. $\{1, 4, 7\}$

C. $\{4, 7\}$

D. $\{7\}$

Answer: B



[View Text Solution](#)

40. The function $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = 3^{-x}$. Observe the following statements

I. f is one - one

II. f is onto

III. f is a decreasing function

Out of these, true statements are :

A. only I, II

B. only II, III

C. only I, III

D. I, II, III

Answer: C



[Watch Video Solution](#)

41. If $n \in \mathbb{N}$, and the period of $\frac{\cos nx}{\sin(x/n)}$ is 4π , then $n =$

A. 4

B. 3

C. 2

D. 1

Answer: C



Watch Video Solution

$$42. \text{ If } f(x) = \begin{cases} [x], & \text{if } -3 < x \leq -1 \\ |x|, & \text{if } -1 < x < 1 \\ |[x]|, & \text{if } 1 \leq x < 3 \end{cases}$$

then the set $\{x : f(x) \geq 0\}$ is equal to

A. $(-1, 3)$

B. $[-1, 3)$

C. $(-1, 3]$

D. $[-1, 3]$

Answer: A



[View Text Solution](#)

43. $\sum_{k=1}^5 \frac{1^3 + 2^3 + \dots + k^3}{1 + 3 + 5 + \dots + (2k - 1)}$ is equal to

A. 22.5

B. 24.5

C. 28.5

D. 32.5

Answer: A



[View Text Solution](#)

44. If $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ are defined by $f(x) = 2x + 3$ and $g(x) = x^2 + 7$, then the values of x such that $g(f(x)) = 8$ are

A. 1, 2

B. -1, 2

C. -1, -2

D. 1, -2

Answer: C

 [Watch Video Solution](#)

45. Suppose $f: [-2, 2] \rightarrow \mathbb{R}$ is defined by

$$f(x) = \begin{cases} -1 & \text{for } -2 \leq x \leq 0 \\ x - 1 & \text{for } 0 \leq x \leq 2 \end{cases},$$

then

$$\{x \in [-2, 2] : x \leq 0 \text{ and } f(|x|) = x\} =$$

A. $\{-1\}$

B. $\{0\}$

C. $\left\{-\frac{1}{2}\right\}$

D. ϕ

Answer: C



Watch Video Solution

46. If $f: R \rightarrow R$ and $g: R \rightarrow R$ are given by $f(x) = |X|$ and $g(x) = [x]$ for each $x \in R$ then $\{x \in R: g(f(x)) \leq f(g(x))\}$ is equal to

A. $z \cup (-\infty, 0)$

B. $(-\infty, 0)$

C. z

D. R

Answer: D



View Text Solution

47. The period of the function $f(\theta) = \sin \frac{\theta}{3} + \cos \frac{\theta}{2}$ is

A. 3π

B. 6π

C. 9π

D. 12π

Answer: D



Watch Video Solution

48. If $t_n = \frac{1}{4}(n+2)(n+3)$ for $n = 1, 2, 3, \dots$ then $\frac{1}{t_1} + \frac{1}{t_2} + \dots + \frac{1}{t_{2003}}$ is equal to

A. $\frac{4006}{3006}$

B. $\frac{4003}{3007}$

C. $\frac{4006}{3008}$

D. $\frac{4006}{3009}$

Answer: D

 [View Text Solution](#)

49. Let $A = \{x \in R : x \neq 0, -4 \leq x \leq 4\}$ and $f: A \rightarrow R$ is defined by

$f(x) = \frac{|x|}{x}$ for $x \in A$. Then the range of f is

A. $\{1, -1\}$

B. $\{x : 0 \leq x \leq 1\}$

C. $\{1\}$

D. $\{x : -4 \leq x \leq 0\}$

Answer: A

 [Watch Video Solution](#)

50. If $f(x) = \frac{\cos^2 x + \sin^4 x}{\sin^2 x + \cos^4 x}$ for $x \in R$ then $f(2002) =$

A. 1

B. 2

C. 3

D. 4

Answer: A



Watch Video Solution

51. The function $f: R \rightarrow R$ defined by

$f(x) = \cos^2 x + \sin^4 x$ for $x \in R$. Then $f(R) =$

A. $\left(\frac{3}{4}, 1\right]$

B. $\left[\frac{3}{4}, 1\right)$

C. $\left[\frac{3}{4}, 1\right]$

D. $\left(\frac{3}{4}, 1\right)$

Answer: C



Watch Video Solution

52. If $f: R \rightarrow R$ and $g: R \rightarrow R$ are defined by $f(x) = 3x - 4$ and $g(x) = 2 + 3x$ then $(g^{-1} \circ f^{-1})(5) =$

A. 1

B. $\frac{1}{2}$

C. $\frac{1}{3}$

D. $\frac{1}{4}$

Answer: C



Watch Video Solution

53. If $f(x) = \sin^2\left(\frac{\pi}{8} + \frac{x}{2}\right) - \sin^2\left(\frac{\pi}{8} - \frac{x}{2}\right)$, then the period of f is

A. $\frac{\pi}{3}$

B. $\frac{\pi}{2}$

C. π

D. 2π

Answer: D



Watch Video Solution

54. If $f(x) = (25 - x^4)^{1/4}$ for $0 < x < \sqrt{5}$ then $f\left(f\left(\frac{1}{2}\right)\right) =$

A. 2^{-4}

B. 2^{-3}

C. 2^{-2}

D. 2^{-1}

Answer: D



Watch Video Solution

55. Define $f: Z \rightarrow Z$ by $f(x) = \begin{cases} x/2 & (\text{x is even}) \\ 0 & (\text{x is odd}) \end{cases}$ then f is

A. onto but not one-one

B. one-one but onto

C. one-one and onto

D. neither one-one nor onto

Answer: A



Watch Video Solution

56. Let $f: R \rightarrow R$ be defined by

$$f(x) = \{(x + 2, (x \leq -1)), (x^2, (-1 \leq x \leq 1)), (2 - x), (x \geq 1)\}.$$

Then the value of $f(-1.75) + f(0.5) + f(1.5)$ is

A. 0

B. 1

C. 2

D. -1

Answer: B



Watch Video Solution

57. Two functions $f: R \rightarrow R$, $g: R \rightarrow R$ are defined as follows :

$$f(x) = \begin{cases} 0 & (\text{x rational}) \\ 1 & (\text{x irrational}) \end{cases}, \quad g(x) = \begin{cases} -1 & (\text{x rational}) \\ 0 & (\text{x irrational}) \end{cases} \quad \text{then}$$

$$(f \circ g)(\pi) + (g \circ f)(e) =$$

A. 0

B. -1

C. 2

D. 1

Answer: B



Watch Video Solution

58. If $2^3 + 4^3 + 6^3 + \dots + (2n)^3 = hn^2(n+1)^2$ then h is equal to

A. $\frac{1}{2}$

B. 1

C. $\frac{3}{2}$

D. 2

Answer: D



View Text Solution