



## MATHS

### BOOKS - SAI MATHS (TELUGU ENGLISH)

### FUNCTIONS AND MATHEMATICAL INDUCTION

#### Problems

1. If  $f : N \rightarrow R$  is defined by  $f(1) = -1$  and  $f(n+1) = 3f(n) + 2$  for  $n > 1$ , then  $f$  is

- A. one - one
- B. onto
- C. a constant function
- D.  $f(n) > 0$  for  $n > 1$

**Answer: C**



[View Text Solution](#)

2. The remainder of  $n^4 - 2n^3 - n^2 + 2n - 26$  when divided by 24 is

A. 20

B. 21

C. 22

D. 23

**Answer: C**



[View Text Solution](#)

3. If  $f: R \rightarrow R$ ,  $g: R \rightarrow R$  are defined by  $f(x) = 5x - 3$ ,  $g(x) = x^2 + 3$ ,

then  $(g \circ f^{-1})(3) =$

A.  $\frac{25}{9}$

B.  $\frac{111}{25}$

C.  $\frac{9}{25}$

D.  $\frac{25}{111}$

**Answer: B**

 [Watch Video Solution](#)

4. If  $A = \left\{ x \in R / \frac{\pi}{4} \leq x \leq \frac{\pi}{3} \right\}$  and  $f(x) = \sin x - x$ , then  $f(A)$  is equal to

A.  $\left[ \frac{\sqrt{3}}{2} - \frac{\pi}{3}, \frac{1}{\sqrt{2}} - \frac{\pi}{4} \right]$

B.  $\left[ -\frac{1}{\sqrt{2}} - \frac{\pi}{4}, \frac{\sqrt{3}}{2} - \frac{\pi}{3} \right]$

C.  $\left[ -\frac{\pi}{3}, \frac{-\pi}{4} \right]$

D.  $\left[ \frac{\pi}{4}, \frac{\pi}{3} \right]$

**Answer: A**

 [Watch Video Solution](#)

5. The value of the sum  $1.2.3+2.3.4+3.4.5+\dots$  upto  $n$  terms is equal to

A.  $\frac{1}{6}n^2(2n^2 + 1)$

B.  $\frac{1}{6}(n^2 - 1)(2n - 1)(2n + 3)$

C.  $\frac{1}{8}(n^2 + 1)(n^2 + 5)$

D.  $\frac{1}{4}n(n + 1)(n + 2)(n + 3)$

**Answer: D**



**Watch Video Solution**

6. If  $x = \frac{1}{5} + \frac{1.3}{5.10} + \frac{1.3.5}{5.10.15} + \dots \infty$  then find  $3x^2 + 6x$ .

A. 1

B. 2

C. 3

D. 4

**Answer: B**



**Watch Video Solution**

7. If  $R$  is the set of all real numbers and  $f : R - \{2\} \rightarrow R$  is defined by

$$f(x) = \frac{2+x}{2-x} \text{ for } x \in R - \{2\}$$

A.  $R - \{-2\}$

B.  $R$

C.  $R - \{1\}$

D.  $R - \{-1\}$

**Answer: D**



**View Text Solution**

8. Let  $Q$  be the set of all rational number in  $[0, 1]$  and  $f : [0, 1] \rightarrow [0, 1]$  be

defined by  $f(x) = \begin{cases} x & \text{for } x \in Q \\ 1-x & \text{for } x \notin Q \end{cases}$  Then the set

$S = \{x \in [0, 1] \mid (f \circ f)(x) = x\}$  is equal to

A.  $[0, 1]$

B.  $-\mathcal{Q}$

C.  $[0, 1] - \mathcal{Q}$

D.

**Answer: A**



**Watch Video Solution**

9.  $\sum_{k=1}^{2n+1} (-1)^{k-1} k^2$  equals to

A.  $(n - 1)(2n - 1)$

B.  $(n + 1)(2n + 1)$

C.  $(n + 1)(2n - 1)$

D.  $(n - 1)(2n + 1)$

**Answer: B**



**View Text Solution**

10. If  $f(x) = (p - x^n)^{1/n}$ ,  $p > 0$  and  $n$  is a positive integer, then  $f[f(x)]$  is equal to

A.  $x$

B.  $x^n$

C.  $p^{1/n}$

D.  $p - x^n$

**Answer: A**



**View Text Solution**

11. The value of  $\left\{ x \in R \left[ \log(1.6)^{1-x^2} - (0.625)^{6(1+8)} \right] \in R \right\}$

A.  $(-\infty, -1) \cup (7, \infty)$

B.  $(-1, 5)$

C.  $(1, 7)$

D.  $(-1, 7)$

**Answer: D**



**View Text Solution**

12. If  $f: R \rightarrow R^2$  and  $R^+ \rightarrow R$  are such that  $g\{f(x)\} = |\sin x|$  and  $f\{(gx)\} = (\sin \sqrt{x})^2$ , then a possible choice for  $f$  and  $g$  is

A.  $f(x) = x^2, g(x) = \sin \sqrt{x}$

B.  $f(x) = \sin x, g(x) = |x|$

C.  $f(x) = \sin^2 x, g(x) = \sqrt{x}$

D.  $f(x) = x^2, g(x) = \sqrt{x}$

**Answer: C**





[View Text Solution](#)

13. Define  $f: Z \rightarrow Z$  by  $f(x) = \begin{cases} x/2 & (\text{x is even}) \\ 0 & (\text{x is odd}) \end{cases}$  then  $f$  is

- A. onto but not one-to-one
- B. one-to-one but not onto
- C. one-to-one and onto
- D. neither one-to-one nor onto

**Answer: A**



[Watch Video Solution](#)

14. If  $\frac{1}{2 \times 4} + \frac{1}{4 \times 6} + \frac{1}{6 \times 8} + \dots$  (n terms)  $= \frac{kn}{n+1}$ , then  $k$  is equal to

- A.  $\frac{1}{4}$
- B.  $\frac{1}{2}$

C. 1

D.  $\frac{1}{8}$

**Answer: A**



[View Text Solution](#)

15. If  $f: [2, \infty) \rightarrow B$  defined by  $f(x) = x^2 - 4x + 5$  is a bijection, then

$B =$

A.  $[0, \infty)$

B.  $[1, \infty)$

C.  $[4, \infty)$

D.  $[5, \infty)$

**Answer: B**



[Watch Video Solution](#)

16. If  $f: R \rightarrow R$  is defined by  $f(x) = \left[ \frac{x}{5} \right]$  for  $x \in R$ , where  $[y]$  denotes the greatest integer not exceeding  $y$ , then  $\{f(x) : |x| < 71\}$  is equal to

- A.  $\{-14, -13, \dots, 0, \dots, 13, 14\}$
- B.  $\{-14, -13, \dots, 0, \dots, 14, 15\}$
- C.  $\{-15, -14, \dots, 0, \dots, 14, 15\}$
- D.  $\{-15, -14, \dots, 0, \dots, 13, 14\}$

**Answer: D**



[View Text Solution](#)

17. If  $a, b$  and  $n$  are natural numbers, then  $a^{2n-1} + b^{2n-1}$  is divisible by

- A.  $a + b$
- B.  $a - b$
- C.  $a^3 + b^3$
- D.  $a^2 + b^2$

**Answer: A**



[View Text Solution](#)

**18.**

If

$f(0) = 0, f(1) = 1, f(2) = 2$  and  $f(x) = f(x - 2) + f(x - 3)$  for  $x = 3, 4, \dots$

then  $f(9) =$

A. 12

B. 13

C. 14

D. 10

**Answer: D**



[Watch Video Solution](#)

19. Let  $R$  denote the set of all real numbers and  $R^+$  denote the set of all positive real numbers. For the subsets  $A$  and  $B$  of  $R$  define  $f: A \rightarrow B$  by

$f(x) = x^2$  for  $x \in A$ . Observe the two lists given below

Column I

Column II

- |  |                     |
|--|---------------------|
| A. $f$ is one-one and onto, if         | 1. $A = R^+, B = R$ |
| B. $f$ is one-one but not onto, if     | 2. $A = B = R$      |
| C. $f$ is onto but not one-one, if     | 3. $A = R, B = R^+$ |
| D. $f$ is neither one-one nor onto, if | 4. $A = B = R^+$    |

A. A-1, B-2, C-3, D-4

B. A-4, B-2, C-1, D-3

C. A-4, B-1, C-3, D-2

D. A-4, B-2, C-3, D-1

**Answer: C**



[View Text Solution](#)

20. The numbers  $a_n = 6^n - 5n$  for  $n = 1, 2, 3, \dots$  when divided by 25 leave the remainder

A. 9

B. 7

C. 3

D. 1

**Answer: D**



[View Text Solution](#)

21. If  $f: [2, 3] \rightarrow \mathbb{R}$  is defined by  $f(x) = x^3 + 3x - 2$ , then the range  $f(x)$  is contained in the interval :

A.  $[1, 12]$

B.  $[12, 34]$

C.  $[35, 50]$

D.  $[-12, 12]$

**Answer: B**

 [Watch Video Solution](#)

$$22. \left\{ x \in \mathbb{R} : \frac{2x - 1}{x^3 + 4x^2 + 3x} \in \mathbb{R} \right\} =$$

A.  $\mathbb{R} - \{0\}$

B.  $\mathbb{R} - \{0, 1, 3\}$

C.  $\mathbb{R} - \{0, -1, -3\}$

D.  $\mathbb{R} - \left\{ 0, -1, -3, +\frac{1}{2} \right\}$

**Answer: C**

 [Watch Video Solution](#)

23. Using mathematical induction, the numbers  $a_n$ 's are defined by,

$$a_0 = 1, a_{n+1} = 3n^2 + n + a_n, (n \geq 0).$$

Then,  $a_n$  is equal to

A.  $n^3 + n^2 + 1$

B.  $n^3 + n^2 + 1$

C.  $n^3 - n^2 + 1$

D.  $n^3 - n^2$

**Answer: B**



[View Text Solution](#)

24. If  $\mathbb{R} \rightarrow C$  is defined by  $f(x) = e^{2ix}$  or  $x \in \mathbb{R}$  then,  $f$  is (where  $C$  denotes the set of all complex numbers)

A. one-one

B. onto

C. one-to-one and onto

D. neither one-one nor onto

**Answer: D**



[Watch Video Solution](#)



25. If  $f: [-6, 6] \rightarrow \mathbb{R}$  defined by  $f(x) = x^2 - 3$  for  $x \in \mathbb{R}$  then  $(f \circ f \circ f)(-1) + (f \circ f \circ f)(0) + (f \circ f \circ f)(1) =$

A.  $f(4\sqrt{2})$

B.  $f(3\sqrt{2})$

C.  $f(2(\sqrt{2}))$

D.  $f(\sqrt{2})$

**Answer: A**



**Watch Video Solution**

26. If  $f: \mathbb{R} \rightarrow \mathbb{R}$  and  $g: \mathbb{R} \rightarrow \mathbb{R}$  are defined by  $f(x) = |x|$  and  $g(x) = [x - 3]$  for  $x \in \mathbb{R}$ , then  $\{g(f(x)) : -8/5 < x < 8/5\} =$

A.  $\{0, 1\}$

B.  $\{1, 2\}$

C.  $\{-3, -2\}$

D.  $\{2, 3\}$

**Answer: C**



**Watch Video Solution**

27. For any integer  $n \geq 1$ , the sum  $\sum_{k=1}^n k(k+2)$  is equal to

A.  $\frac{n(n+1)(n+2)}{6}$

B.  $\frac{n(n+1)(2n+1)}{6}$

C.  $\frac{n(n+1)(2n+7)}{6}$

D.  $\frac{n(n+1)(2n+9)}{6}$

**Answer: C**



**View Text Solution**

28. If  $f: R \rightarrow R$  is defined by  $f(x) = \frac{1}{2 - \cos 3x}$  for each  $x \in R$  then the range of  $f$  is

A.  $\left(\frac{1}{3}, 1\right)$

B.  $\left[\frac{1}{3}, 1\right]$

C.  $(1, 2)$

D.  $[1, 2]$

**Answer: B**

 [Watch Video Solution](#)

29. If  $f: R \rightarrow R$  and  $g: R \rightarrow R$  are defined by  $f(x) = x - \{x\}$  and  $g(x) = [x]$  for  $x \in R$ , where  $[x]$  is greatest integer not exceeding  $x$ , then for every  $x \in R$ ,  $f(g(x))$  is equal to

A.  $x$

B.  $0$

C.  $f(x)$

D.  $g(x)$

**Answer: B**

 [View Text Solution](#)

30. If  $S_n = 1^3 + 2^3 + \dots + n^3$  and  $T_n = 1 + 2 + \dots + n$ , then

A.  $S_n = T_{n^3}$

B.  $S_n = T_{n^2}$

C.  $S_n = T_n^2$

D.  $S_n = T_n^3$

**Answer: C**

 [View Text Solution](#)

31. If  $f: \mathbb{R} \rightarrow \mathbb{R}$  is defined by  $f(x) = x - [x] - \frac{1}{2}$  for  $x \in \mathbb{R}$ , where  $[x]$  is the greatest integer not exceeding  $x$ , then  $\left\{ x \in \mathbb{R} : f(x) = \frac{1}{2} \right\} =$

- A.  $\mathbb{Z}$ , the set of all integers
- B.  $\mathbb{N}$ , the set of all natural numbers
- C.  $\phi$ , an empty set
- D.  $\mathbb{R}$

**Answer: C**



[Watch Video Solution](#)

32. If  $f: \mathbb{R} \rightarrow \mathbb{R}$  is defined by  $f(x) = [2x] - 2[x]$  for  $x \in \mathbb{R}$ , then the range of  $f$  is (Here  $[x]$  denotes the greatest integer not exceeding  $x$ )

- A.  $\{x \in \mathbb{R} : 0 \leq x \leq 1\}$
- B.  $\{0, 1\}$
- C.  $\{x \in \mathbb{R} : x > 0\}$

$$D. \{x \in R: x \leq 0\}$$

**Answer: B**



**Watch Video Solution**

**33.** If  $f: R \rightarrow R$  is defined by

$$f(x) = \begin{cases} x + 4 & \text{for } x < -4 \\ 3x + 2 & \text{for } -4 \leq x < 4 \\ x - 4 & \text{for } x \geq 4 \end{cases}$$

then match the following columns and choose the correct answer.

column I

column II

- |                       |         |
|-----------------------|---------|
| (A) $f(-5) + f(-4)$   | (1) 14  |
| (B) $f( f(-8) )$      | (2) 4   |
| (C) $f(f(-7)) + f(3)$ | (3) -11 |
| (D) $f(f(f(0))) + 1$  | (4) -1  |
|                       | (5) 5   |
|                       | (6) 0   |

A. A-3, B-6, C-2, D-5

B. A-3, B-4, C-2, D-5

C. A-4, B-3, C-2, D-1

D. A-3, B-6, C-5, D-2

**Answer: A**



**Watch Video Solution**

**34.** For all integers,  $n \geq 1$  which of the following is divisible by 9.

A.  $8^n + 1$

B.  $4^n - 3n - 1$

C.  $3^{2n} + 3n + 1$

D.  $10^n + 1$

**Answer: B**



**View Text Solution**

**35.**  $\{x \in R : [x - |x| = 5]\}$  is equal to

A. R, the set of all real numbers

B.  $\phi$ , an empty set

C.  $\{x \in R: < 0\}$

D.  $\{x \in R: \geq 0\}$

**Answer: B**



[View Text Solution](#)

36. The function  $f: C \rightarrow C$  defined by  $f(x) = \frac{ax + b}{cx + d}$  for  $x \in C$ , where  $bd \neq 0$  reduces to a constant function, if

A.  $a = c$

B.  $b = d$

C.  $ad = bc$

D.  $ab = cd$

**Answer: C**



[View Text Solution](#)



37. If  $N$  denotes the set of all positive integers and if  $f: N \rightarrow N$  is defined by  $f(n) =$  the sum of positive divisors of  $n$  then,  $f(2^k 3)$ , where  $k$  is a positive integer, is

A.  $2^{k+1} - 1$

B.  $2(2^{k+1} - 1)$

C.  $3(2^{k+1} - 1)$

D.  $4(2^{k+1} - 1)$

**Answer: D**



**View Text Solution**

38.  $\{n(n+1)(2n+1) : n \in I\} \subset$

A.  $\{6k : k \in I\}$

B.  $\{12k : k \in I\}$

C.  $\{18k : k \in I\}$

D.  $\{24k : k \in I\}$

**Answer: A**



[View Text Solution](#)

39. If  $f: N \rightarrow Z$  is defined by

$$f(x) = \begin{cases} 2 & \text{if } n = 3k, \quad k \in Z \\ 10 & \text{if } n = 3k + 1, \quad k \in Z \\ 0 & \text{if } n = 3k + 2, \quad k \in Z \end{cases}$$

then  $\{n \in N : f(n) > 2\}$  is equal to

A.  $\{3, 6, 4\}$

B.  $\{1, 4, 7\}$

C.  $\{4, 7\}$

D.  $\{7\}$

**Answer: B**



[View Text Solution](#)

40. The function  $f: \mathbb{R} \rightarrow \mathbb{R}$  is defined by  $f(x) = 3^{-x}$ . Observe the following statements

I.  $f$  is one - one

II.  $f$  is onto

III.  $f$  is a decreasing function

Out of these, true statements are :

A. only I, II

B. only II, III

C. only I, III

D. I, II, III

**Answer: C**



[Watch Video Solution](#)

41. If  $n \in \mathbb{N}$ , and the period of  $\frac{\cos nx}{\sin(x/n)}$  is  $4\pi$ , then  $n =$

A. 4

B. 3

C. 2

D. 1

**Answer: C**



**Watch Video Solution**

$$42. \text{ If } f(x) = \begin{cases} [x], & \text{if } -3 < x \leq -1 \\ |x|, & \text{if } -1 < x < 1 \\ |[x]|, & \text{if } 1 \leq x < 3 \end{cases}$$

then the set  $\{x : f(x) \geq 0\}$  is equal to

A.  $(-1, 3)$

B.  $[-1, 3)$

C.  $(-1, 3]$

D.  $[-1, 3]$

**Answer: A**



[View Text Solution](#)

43.  $\sum_{k=1}^5 \frac{1^3 + 2^3 + \dots + k^3}{1 + 3 + 5 + \dots + (2k - 1)}$  is equal to

A. 22.5

B. 24.5

C. 28.5

D. 32.5

**Answer: A**



[View Text Solution](#)

44. If  $f: \mathbb{R} \rightarrow \mathbb{R}$  and  $g: \mathbb{R} \rightarrow \mathbb{R}$  are defined by  $f(x) = 2x + 3$  and  $g(x) = x^2 + 7$ , then the values of  $x$  such that  $g(f(x)) = 8$  are

A. 1, 2

B. -1, 2

C. -1, -2

D. 1, -2

**Answer: C**

 [Watch Video Solution](#)

**45.** Suppose  $f: [-2, 2] \rightarrow \mathbb{R}$  is defined by

$$f(x) = \begin{cases} -1 & \text{for } -2 \leq x \leq 0 \\ x - 1 & \text{for } 0 \leq x \leq 2 \end{cases},$$

then

$$\{x \in [-2, 2] : x \leq 0 \text{ and } f(|x|) = x\} =$$

A.  $\{-1\}$

B.  $\{0\}$

C.  $\left\{-\frac{1}{2}\right\}$

D.  $\phi$

**Answer: C**



**Watch Video Solution**

46. If  $f: R \rightarrow R$  and  $g: R \rightarrow R$  are given by  $f(x) = |X|$  and  $g(x) = [x]$  for each  $x \in R$  then  $\{x \in R: g(f(x)) \leq f(g(x))\}$  is equal to

A.  $z \cup (-\infty, 0)$

B.  $(-\infty, 0)$

C.  $z$

D.  $R$

**Answer: D**



**View Text Solution**

47. The period of the function  $f(\theta) = \sin \frac{\theta}{3} + \cos \frac{\theta}{2}$  is

A.  $3\pi$

B.  $6\pi$

C.  $9\pi$

D.  $12\pi$

**Answer: D**



**Watch Video Solution**

**48.** If  $t_n = \frac{1}{4}(n+2)(n+3)$  for  $n = 1, 2, 3, \dots$  then  $\frac{1}{t_1} + \frac{1}{t_2} + \dots + \frac{1}{t_{2003}}$  is equal to

A.  $\frac{4006}{3006}$

B.  $\frac{4003}{3007}$

C.  $\frac{4006}{3008}$

D.  $\frac{4006}{3009}$

**Answer: D**



 [View Text Solution](#)

49. Let  $A = \{x \in R : x \neq 0, -4 \leq x \leq 4\}$  and  $f: A \rightarrow R$  is defined by

$f(x) = \frac{|x|}{x}$  for  $x \in A$ . Then the range of  $f$  is

A.  $\{1, -1\}$

B.  $\{x : 0 \leq x \leq 1\}$

C.  $\{1\}$

D.  $\{x : -4 \leq x \leq 0\}$

**Answer: A**

 [Watch Video Solution](#)

50. If  $f(x) = \frac{\cos^2 x + \sin^4 x}{\sin^2 x + \cos^4 x}$  for  $x \in R$  then  $f(2002) =$

A. 1

B. 2

C. 3

D. 4

**Answer: A**



**Watch Video Solution**

51. The function  $f: R \rightarrow R$  defined by

$f(x) = \cos^2 x + \sin^4 x$  for  $x \in R$ . Then  $f(R) =$

A.  $\left(\frac{3}{4}, 1\right]$

B.  $\left[\frac{3}{4}, 1\right)$

C.  $\left[\frac{3}{4}, 1\right]$

D.  $\left(\frac{3}{4}, 1\right)$

**Answer: C**



**Watch Video Solution**

52. If  $f: R \rightarrow R$  and  $g: R \rightarrow R$  are defined by  $f(x) = 3x - 4$  and  $g(x) = 2 + 3x$  then  $(g^{-1} \circ f^{-1})(5) =$

A. 1

B.  $\frac{1}{2}$

C.  $\frac{1}{3}$

D.  $\frac{1}{4}$

**Answer: C**



**Watch Video Solution**

53. If  $f(x) = \sin^2\left(\frac{\pi}{8} + \frac{x}{2}\right) - \sin^2\left(\frac{\pi}{8} - \frac{x}{2}\right)$ , then the period of  $f$  is

A.  $\frac{\pi}{3}$

B.  $\frac{\pi}{2}$

C.  $\pi$

D.  $2\pi$

**Answer: D**



**Watch Video Solution**

54. If  $f(x) = (25 - x^4)^{1/4}$  for  $0 < x < \sqrt{5}$  then  $f\left(f\left(\frac{1}{2}\right)\right) =$

A.  $2^{-4}$

B.  $2^{-3}$

C.  $2^{-2}$

D.  $2^{-1}$

**Answer: D**



**Watch Video Solution**

55. Define  $f: Z \rightarrow Z$  by  $f(x) = \begin{cases} x/2 & (\text{x is even}) \\ 0 & (\text{x is odd}) \end{cases}$  then  $f$  is

A. onto but not one-one

B. one-one but onto

C. one-one and onto

D. neither one-one nor onto

**Answer: A**



**Watch Video Solution**

56. Let  $f: R \rightarrow R$  be defined by

$$f(x) = \{(x + 2, (x \leq -1)), (x^2, (-1 \leq x \leq 1)), (2 - x), (x \geq 1)\}.$$

Then the value of  $f(-1.75) + f(0.5) + f(1.5)$  is

A. 0

B. 1

C. 2

D. -1

**Answer: B**



Watch Video Solution

57. Two functions  $f: R \rightarrow R$ ,  $g: R \rightarrow R$  are defined as follows :

$$f(x) = \begin{cases} 0 & (\text{x rational}) \\ 1 & (\text{x irrational}) \end{cases}, \quad g(x) = \begin{cases} -1 & (\text{x rational}) \\ 0 & (\text{x irrational}) \end{cases} \quad \text{then}$$

$$(f \circ g)(\pi) + (g \circ f)(e) =$$

A. 0

B. -1

C. 2

D. 1

Answer: B



Watch Video Solution

58. If  $2^3 + 4^3 + 6^3 + \dots + (2n)^3 = hn^2(n+1)^2$  then h is equal to

A.  $\frac{1}{2}$

B. 1

C.  $\frac{3}{2}$

D. 2

**Answer: D**



**View Text Solution**