MATHS

BOOKS - SAI MATHS (TELUGU ENGLISH)

FUNCTIONS AND MATHEMATICAL INDUCTION

Problems

1. If f : $N \to R$ is defined by $f(1) = -1$ and $f(n+1) = 3f(n) + 2$ for $n > 1$, then f is

A. one - one

B. onto

C. a constant function

D. $f(n) > 0$ for $n > 1$

Answer: C

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2. The remainder of $n^4 - 2n^3 - n^2 + 2n - 26$ when divided by 24 is

A. 20

B. 21

C. 22

D. 23

Answer: C

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3. If $f: R \to R, g: R \to R$ are defined by $f(x) = 5x - 3, g(x) = x^2 + 3$, then $\left(gof^{-1}\right)(3) =$

A. $\frac{25}{9}$

B. $\frac{111}{25}$

C. $\frac{9}{25}$

D. $\frac{25}{111}$

Answer: B

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4. If $A = \left\{x \in R / \frac{\pi}{4} \leq x \leq \frac{\pi}{3}\right\}$ and f(x)=sinx-x, then f(A) is equal to

A. $\left[\frac{\sqrt{3}}{2} - \frac{\pi}{3}, \frac{1}{\sqrt{2}} - \frac{\pi}{4}\right]$

B. $\left[-\frac{1}{\sqrt{2}} - \frac{\pi}{4}, \frac{\sqrt{3}}{2} - \frac{\pi}{3}\right]$

C. $\left[-\frac{\pi}{3}, \frac{-\pi}{4}\right]$

D. $\left[\frac{\pi}{4}, \frac{\pi}{3}\right]$

Answer: A

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5. The value of the sum 1.2.3+2.3.4+3.4.5+... upto n terms is equal to

A. $\frac{1}{6}n^2\left(2n^2+1\right)$

B. $\frac{1}{6}\left(n^2-1\right)(2n-1)(2n+3)$

C. $\frac{1}{8}\left(n^2+1\right)\left(n^2+5\right)$

D. $\frac{1}{4}n(n+1)(n+2)(n+3)$

Answer: D

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6. If $x=\frac{1}{5}+\frac{1.3}{5.10}+\frac{1.3.5}{5.10.15}+\ldots.\infty$ then find $3x^2+6x.$

A. 1

B. 2

C. 3

D. 4

Answer: B

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7. If R is the set of all real numbers and f : R $-\{2\} \to$ R is defined by $f(x) = \frac{2+x}{2-x}$ for $x \in R - \{2\}$

A. $R - \{-2\}$

B. R

C. $R - \{1\}$

D. $R - \{-1\}$

Answer: D

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8. Let Q be the set of all rational number in $[0, 1]$ and $f: [0, 1] \to [0, 1]$ be defined by $f(x) = \begin{cases} x & \text{for} \quad x \in Q \\ 1-x & \text{for} \quad x \notin Q \end{cases}$ Then the set

$S = \{x \in [0, 1](fof)(x) = x\}$ is equal to

A. $[0, 1]$

B. $-Q$

C. $[0, 1] - Q$

D.

Answer: A

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9. $\sum_{k=1}^{2n+1} (-1)^{k-1} k^2$ equals to

A. $(n-1)(2n-1)$

B. $(n+1)(2n+1)$

C. $(n+1)(2n-1)$

D. $(n-1)(2n+1)$

Answer: B

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10. If $f(x) = (p - x^n)^{1/n}$, $p > 0$ and n is a positive integer, then $f[f(x)]$ is equal to

A. x

B. x^n

C. $p^{1/n}$

D. $p - x^n$

Answer: A

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11. The value of $\left\{x \in R\left[\log(1.6)^{1-x^2} - (0.625)^{6(1+8)}\right] \in R\right\}$

A. $(-\infty, -1) \cup (7, \infty)$

B. $(-1, 5)$

C. $(1, 7)$

D. $(-1, 7)$

Answer: D

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12. If $f: R \to R^2$ and $R^+ \to R$ are such that $g\{f(x)\} = |\sin x|$ and $f\{(gx)\} = \left(\sin\sqrt{x}\right)^2$, then a possible choice for f and g is

A. $f(x) = x^2, g(x) = \sin\sqrt{x}$

B. $f(x) = \sin x, g(x) = |x|$

C. $f(x) = \sin^2 x, g(x) = \sqrt{x}$

D. $f(x) = x^2, g(x) = \sqrt{x}$

Answer: C

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13. Define $f: Z \to Z$ by $f(x) = \begin{cases} x/2 & (\text{x is even}) \\ 0 & (\text{x is odd}) \end{cases}$ then f is

A. onto but not one-to-one

B. one-to-one but not onto

C. one-to-one and onto

D. neither one-to-one nor onto

Answer: A

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14. If $\frac{1}{2 \times 4} + \frac{1}{4 \times 6} + \frac{1}{6 \times 8} + \ldots.$ (n terms) $= \frac{kn}{n+1}$, then k is equal to

A. $\frac{1}{4}$

B. $\frac{1}{2}$

C. 1

D. $\frac{1}{8}$

Answer: A

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15. If $f: [2, \infty) \to B$ defined by $f(x) = x^2 - 4x + 5$ is a bijection, then $B =$

A. $[0, \infty)$

B. $[1, \infty)$

C. $[4, \infty)$

D. $[5, \infty)$

Answer: B

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16. If $f: R \to R$ is defined by $f(x) = \left[\frac{x}{5}\right]$ for $x \in R$, where [y] denotes the greatest integer not exceeding y, then $\{f(x) : |x| < 71\}$ is equal to

A. $\{-14, -13, \ldots 0, \ldots., 13, 14\}$

B. $\{-14, -13, \ldots, 0, \ldots, 14, 15\}$

C. $\{-15, -14, \ldots., 0, \ldots., 14, 15\}$

D. $\{-15, -14, \ldots 0, \ldots, 13, 14\}$

Answer: D

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17. If a,b and n are natural numbers, then $a^{2n-1} + b^{2a-1}$ is divisible by

A. $a + b$

B. $a - b$

C. $a^3 + b^3$

D. $a^2 + b^2$

Answer: A

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18. If

$f(0) = 0, f(1) = 1, f(2) = 2$ and $f(x) = f(x-2) + f(x-3)$ for $x =$

then $f(9) =$

A. 12

B. 13

C. 14

D. 10

Answer: D

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19. Let R denote the set of all real numbers and R^+ denote the set of all positive real numbers. For the subsets A and B of R define $f: A \to B$ by $f(x) = x^2$ for $x \in A$. Observe the two lists given below

	Column I		Column II
$A.$	f is one-one and onto, if	1.	$A = R^+$,B=R
$B.$	f is one-one but not onto, if	2.	$A = B = R$
$C.$	f is onto but not one-one, if	3.	A=R, $B = R^+$
$D.$	f is neither one-one nor onto, if	4.	$A = B = R^+$

A. A-1, B-2, C-3, D-4

B. A-4, B-2, C-1, D-3

C. A-4, B-1, C-3, D-2

D. A-4, B-2, C-3, D-1

Answer: C

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20. The numbers $a_n = 6^n - 5n$ for $n = 1, 2, 3, \ldots$ whrn divided by 25 leave the remainder

A. 9

B. 7

C. 3

D. 1

Answer: D

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21. If $f: [2, 3] \to \mathbb{R}$ is defined by $f(x) = x^3 + 3x - 2$, then the range $f(x)$ is contained in the interval :

A. [1, 12]

B. [12, 34]

C. [35, 50]

D. [-12, 12]

Answer: B

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22. $\left\{x \in \mathbb{R}: \frac{2x-1}{x^3+4x^2+3x} \in \mathbb{R}\right\} =$

A. $R - \{0\}$

B. $R - \{0, 1, 3\}$

C. $R - \{0, -1, -3\}$

D. $R - \left\{0, -1, -3, +\frac{1}{2}\right\}$

Answer: C

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23. Using mathematical induction, the numbers a_n's are defined by,

$a_0 = 1, a_{n+1} = 3n^2 + n + a_n, (n \geq 0).$

Then, a_n is equal to

A. $n^3 + n^2 + 1$

B. $n^3 + n^2 + 1$

C. $n^3 - n^2 + 1$

D. $n^3 - n^2$

Answer: B

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24. If $\mathbb{R} \to C$ is defined by $f(x) = e^{2ix} f$ or $x \in \mathbb{R}$ then, f is (where C denotes the set of all complex numbers)

A. one-one

B. onto

C. one-to-one and onto

D. neither one-one nor onto

Answer: D

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25. If $f:[-6,6] \to \mathbb{R}$ defined by $f(x) = x^2 - 3$ for $x \in R$ then $(fofof)(-1) + (fofof)(0) + (fofof)(1) =$

A. $f\left(4\sqrt{2}\right)$

B. $f\left(3\sqrt{2}\right)$

C. $f\left(2\left(\sqrt{2}\right)\right.$

D. $f\left(\sqrt{2}\right)$

Answer: A

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26. If $f:Rr \to \mathbb{R}$ and $g:\mathbb{R} \to \mathbb{R}$ are defined by $f(x) = |x|$ and $g(x) = [x-3]$ for $x \in \mathbb{R}$, then $\{g(f(x)):-8/5 < x < 8/5\} =$

A. $\{0,1\}$

B. $\{1, 2\}$

C. $\{-3, -2\}$

D. $\{2, 3\}$

Answer: C

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27. For any integer $n \geq 1$, the sum $\sum_{k=1}^{n} k(k+2)$ is equal to

A. $\frac{n(n+1)(n+2)}{6}$

B. $\frac{n(n+1)(2n+1)}{6}$

C. $\frac{n(n+1)(2n+7)}{6}$

D. $\frac{n(n+1)(2n+9)}{6}$

Answer: C

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28. If $f: R \to R$ is defined by $f(x) = \frac{1}{2 - \cos 3x}$ for each $x \in R$ then the range of f is

A. $\left(\frac{1}{3}, 1\right)$

B. $\left[\frac{1}{3}, 1\right]$

C. (1, 2)

D. [1, 2]

Answer: B

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29. If $f: R \to R$ and $g: R \to R$ are defined by $f(x) = x - \{x\}$ and $g(x) = [x]$ for $x \in R$, where [x] is greatest integer not exceeding x, then for every $x \in R$, $f(g(x))$ is equal to

A. x

B. 0

C. f(x)

D. g(x)

Answer: B

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30. If $S_n = 1^3 + 2^3 + \ldots + n^3$ and $T_n = 1 + 2 + \ldots. + n$, then

A. $S_n = T_{n^3}$

B. $S_n = T_{n^2}$

C. $S_n = T_n^2$

D. $S_n = T_n^3$

Answer: C

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31. If $f: \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = x - [x] - \frac{1}{2}$ for $x \in \mathbb{R}$, where $[x]$ is the greatest integer not exceeding x, then $\left\{x \in \mathbb{R}: f(x) = \frac{1}{2}\right\} =$

A. z, the set of all integers

B. N, the set of all natural numbers

C. ϕ, an empty set

D. R

Answer: C

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32. If $f: R \to R$ is defined by f(x)=[2x]-2[x] for $x \in R$, then the range of f is (Here [x] denotes the greatest integer not exceding x)

A. $\{x \in R: 0 \leq x \leq 1\}$

B. {0, 1}

C. $\{x \in R: x > 0\}$

D. $\{x \in R : x \leq 0\}$

Answer: B

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33. If $f : R \to R$ is defined by

$$f(x) = \begin{cases} x + 4 & \text{for } x < -4 \\ 3x + 2 & \text{for } -4 \leq x < 4 \\ x - 4 & \text{for } x \geq 4 \end{cases}$$

then match the following columns and choose the correct answer.

	column I		column II
(A)	$f(-5) + f(-4)$	(1)	14
(B)	$f(\|f(-8)\|)$	(2)	4
(C)	$f(f(-7)) + f(3)$	(3)	-11
(D)	$f(f(f(0))) + 1$	(4)	-1
		(5)	5
		(6)	0

A. A-3, B-6, C-2, D-5

B. A-3, B-4, C-2, D-5

C. A-4, B-3, C-2, D-1

D. A-3, B-6, C-5, D-2

Answer: A

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34. For all integers, $n \geq 1$ which of the following is divisible by 9.

A. $8^n + 1$

B. $4^n - 3n - 1$

C. $3^{2n} + 3n + 1$

D. $10^n + 1$

Answer: B

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35. $\{x \in R : [x - |x| = 5\}$ is equal to

A. R, the set of all real numbers

B. ϕ, an empty set

C. $\{x \in R: < 0\}$

D. $\{x \in R: \geq 0\}$

Answer: B

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36. The function $f: C \to C$ defined by $f(x) = \frac{ax + b}{cx + d}$ for $x \in C$, where $bd \neq 0$ reduces to a constant function, if

A. a = c

B. b = d

C. ad = bc

D. ab = cd

Answer: C

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37. If N denotes the set of all positive integers and if $f: N \to N$ is defined by f(n) = the sum of positive divisors of n then, $f\left(2^k 3\right)$, where k is a positive integer, is

A. $2^{k+1} - 1$

B. $2\left(2^{k+1} - 1\right)$

C. $3\left(2^{k+1} - 1\right)$

D. $4\left(2^{k+1} - 1\right)$

Answer: D

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38. $\{n(n+1)(2n+1) : n \in 1\} \subset$

A. $\{6k : k \in I\}$

B. $\{12k : k \in I\}$

C. $\{18k : k \in I\}$

D. $\{24k : k \in I\}$

Answer: A

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39. If $f : N \to Z$ is defined by

$$f(x) = \begin{cases} 2 \text{ if } n = 3k & , k \in Z \\ 10 \text{ if } n = 3k+1 & , k \in Z \\ 0 \text{ if } n = 3k+2 & , k \in Z \end{cases}$$

then $\{n \in N : f(n) > 2)$ is equal to

A. {3, 6, 4}

B. {1, 4, 7}

C. {4, 7}

D. {7}

Answer: B

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40. The function $f: R \to R$ is defined by $f(x) = 3^{-x}$. Observe the following statements

I. f is one - one

II. f is onto

III. F is a decreasing function

Out of these, true statements are :

A. only, I, II

B. only II, III

C. only I, III

D. I, II, III

Answer: C

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41. If $n \in N$, and the period of $\frac{\cos nx}{\sin(x/n)}$ is 4π, then n=

A. 4

B. 3

C. 2

D. 1

Answer: C

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42. If $f(x) = \begin{cases} [x], & \text{if } -3 < x \leq -1 \\ |x|, & \text{if } -1 < x < 1 \\ |[x]|, & \text{if } 1 \leq x < 3 \end{cases}$

then the set $\{x : f(x) \geq 0\}$ is equal to

A. $(-1, 3)$

B. $[-1, 3)$

C. $(-1, 3]$

D. $[-1, 3]$

Answer: A

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43. $\sum_{k=1}^{5} \frac{1^3 + 2^3 + \ldots + k^3}{1 + 3 + 5 + \ldots. + (2k-1)}$ is equal to

A. 22.5

B. 24.5

C. 28.5

D. 32.5

Answer: A

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44. If $f: \mathbb{R} \to \mathbb{R}$ and $g: \mathbb{R} \to \mathbb{R}$ are defined by $f(x) = 2x + 3$ and $g(x) = x^2 + 7$, then the values of x such that $g(f(x)) = 8$ are

A. 1, 2

B. $-1, 2$

C. $-1, \ -2$

D. $1, \ -2$

Answer: C

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45. Suppose $f:[-2, 2] \to \mathbb{R}$ is defined by $f(x) = \begin{cases} -1 & \text{for} \quad -2 \leq x \leq 0 \\ x-1 & \text{for} \quad 0 \leq x \leq 2 \end{cases}$, then $\{x \in [-2, 2] : x \leq 0 \text{ and } f(|x|) = x\} =$

A. $\{-1\}$

B. $\{0\}$

C. $\left\{-\frac{1}{2}\right\}$

D. ϕ

Answer: C

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46. If $f: R \to R$ and $g: R \to R$ are given by $f(x) = |X|$ and $g(x) = [x]$ for each $x \in R$ then $\{x \in R: g(f(x)) \leq f(g(x))\}$ is equal to

A. $z \cup (-\infty, 0)$

B. $(-\infty, 0)$

C. z

D. R

Answer: D

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47. The period of the function $f(\theta) = \sin\frac{\theta}{3} + \cos\frac{\theta}{2}$ is

A. 3π

B. 6π

C. 9π

D. 12π

Answer: D

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48. If $t_n = \frac{1}{4}(n+2)(n+3)$ for n = 1, 2, 3,... then $\frac{1}{t_1} + \frac{1}{t_2} + \ldots + \frac{1}{t_{2003}}$ is equal to

A. $\frac{4006}{3006}$

B. $\frac{4003}{3007}$

C. $\frac{4006}{3008}$

D. $\frac{4006}{3009}$

Answer: D

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49. Let $A = \{x \in R : x \neq 0, \ -4 \leq x \leq 4\}$ and $f : A \to R$ is defined by $f(x) = \frac{|x|}{x}$ for $x \in A$. Then the range of f is

A. $\{1, \ -1\}$

B. $\{x : 0 \leq x \leq 1\}$

C. $\{1\}$

D. $\{x : \ -4 \leq x \leq 0\}$

Answer: A

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50. If $f(x) = \frac{\cos^2 x + \sin^4 x}{\sin^2 x + \cos^4 x}$ for $x \in R$ then $f(2002) =$

A. 1

B. 2

C. 3

D. 4

Answer: A

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51. The function $f: R \to R$ defined by $f(x) = \cos^2 x + \sin^4 x$ for $x \in R$. Then $f(R) =$

A. $\left(\frac{3}{4}, 1\right]$

B. $\left[\frac{3}{4}, 1\right)$

C. $\left[\frac{3}{4}, 1\right]$

D. $\left(\frac{3}{4}, 1\right)$

Answer: C

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52. If $f: R \to$ and $g: R \to R$ are defined by $f(x) = 3x - 4$ and $g(x) = 2 + 3x$ then $\left(g^{-1} \text{ of}^{-1}\right)(5) =$

A. 1

B. $\frac{1}{2}$

C. $\frac{1}{3}$

D. $\frac{1}{4}$

Answer: C

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53. If $f(x) = \sin^2\left(\frac{\pi}{8} + \frac{x}{2}\right) - \sin^2\left(\frac{\pi}{8} - \frac{x}{2}\right)$, then the period of f is

A. $\frac{\pi}{3}$

B. $\frac{\pi}{2}$

C. π

D. 2π

Answer: D

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54. If $f(x) = \left(25 - x^4\right)^{1/4}$ for $0 < x < \sqrt{5}$ then $f\left(f\left(\frac{1}{2}\right)\right) =$

A. 2^{-4}

B. 2^{-3}

C. 2^{-2}

D. 2^{-1}

Answer: D

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55. Define $f: Z \to Z$ by $f(x) = \begin{cases} x/2 & (\text{x is even}) \\ 0 & (\text{x is odd}) \end{cases}$ then f is

A. onto but not one-one

B. one-one but onto

C. one-one and onto

D. neither one-one nor onto

Answer: A

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56. Let $f: R \to R$ be defined by $f(x) = \{(x+2, (x \leq -1)), (x^2, (-1 \leq x \leq 1)), (2-x), (x \geq 1)\}$. Then the value of $f(-1.75) + f(0.5) + f(1.5)$ is

A. 0

B. 1

C. 2

D. -1

Answer: B

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57. Two functions $f: R \to R, g: R \to R$ are defined as follows :

$f(x) = \begin{cases} 0 & (\text{x rational}) \\ 1 & (\text{x irrational}) \end{cases}, \quad g(x) = \begin{cases} -1 & (\text{x rational}) \\ 0 & (\text{x irrational}) \end{cases}$ then

$(fog)(\pi) + (gof)(e) =$

A. 0

B. -1

C. 2

D. 1

Answer: B

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58. If $2^3 + 4^3 + 6^3 + \ldots + (2n)^3 = hn^2(n+1)^2$ then h is equal to

A. $\frac{1}{2}$

B. 1

C. $\frac{3}{2}$

D. 2

Answer: D

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