# MATHS

# BOOKS - CENGAGE MATHS (ENGLISH)

# INTRODUCTION TO VECTORS

## Illustration 1

**1.** The vector $\vec{a} + \vec{b}$ bisects the angle between the vectors $\widehat{a}$ and $\hat{b}$ if (A) $\left|\vec{a}\right| + \left|\vec{b}\right| = 0$ (B) angle between $\vec{a}$ and $\vec{b}$ is zero (C) $\left|\vec{a}\right| = \left|\vec{b}\right| = 0$ (D) none of these

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## Illustration 2

**1.** if $\overrightarrow{A}o + \overrightarrow{O}B = \overrightarrow{B}O + \overrightarrow{O}C$ ,than prove that B is the midpoint of AC.

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## Illustration 3

**1.** $ABCDE$ is pentagon, prove that $\overrightarrow{A}B + \overrightarrow{B}C + \overrightarrow{C}D + \overrightarrow{D}E + \overrightarrow{E}A = \overrightarrow{0}$

$\overrightarrow{A}B + \overrightarrow{A}E + \overrightarrow{B}C + \overrightarrow{D}C + \overrightarrow{E}D + \overrightarrow{A}C = 3\overrightarrow{A}C$

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## Illustration 4

**1.** Prove that the resultant of two forces acting at point O and represented by $\overrightarrow{O}B$ and $\overrightarrow{O}C$ is given by $2\overrightarrow{O}D$ ,where D is the midpoint of BC.

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## Illustration 5

1. Prove that the sum of all vectors drawn from the centre of a regular octagon to its vertices is the zero vector.

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## Illustration 6

1. ABC is a triangle and P any point on BC. if $\overrightarrow{P}Q$ is the sum of $\overrightarrow{A}P + \overrightarrow{P}B + \overrightarrow{P}C$ , show that ABPQ is a parallelogram and Q , therefore , is a fixed point.

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## Illustration 7

**1.** Two forces $\vec{A}B$ and $\vec{A}D$ are acting at vertex A of a quadrilateral ABCD and two forces $\vec{C}B$ and $\vec{C}D$ at C prove that their resultant is given by 4 $\vec{E}F$, where E and F are the midpoints of AC and BD, respectively.

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## Illustration 8

**1.** If O($\vec{0}$) is the circumcentre and O' the orthocentre of a triangle ABC, then prove that

i. $\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} = \overrightarrow{OO'}$

ii. $\overrightarrow{O'A} + \overrightarrow{O'B} + \overrightarrow{O'C} = 2\overrightarrow{O'O}$

iii. $\overrightarrow{AO'} + \overrightarrow{O'B} + \overrightarrow{O'C} = 2\overrightarrow{AO} = \overrightarrow{AP}$

where AP is the diameter through A of the circumcircle.

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## Illustration 9

1. A unit vector of modulus 2 is equally inclined to $x$ - and $y$ -axes angle at an angle $\pi/3$. Find the length of projection of the vector on the $z$ -axis.

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## Illustration 10

1. If the projections of vector $\vec{a}$ on $x$ -, $y$ - and $z$ -axes are 2, 1 and 2 units ,respectively, find the angle at which vector $\vec{a}$ is inclined to the $z$ -axis.

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## Illustration 11

1. Find a vector of magnitude 8 units in the direction of the vector $\left(5\hat{i}-\hat{j}+2\hat{k}\right)$.

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## Illustration 12

**1.** Find the unit vector in the direction of vector $\overrightarrow{PQ}$, where $P$ and $Q$ are the points $(1, 2, 3)$ and $(4, 5, 6)$, respectively.

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## Illustration 13

**1.** If $\overrightarrow{a} = \left(-\hat{i} + \hat{j} - \hat{k}\right)$ and $\overrightarrow{b} = \left(2\hat{i} - 2\hat{j} + 2\hat{k}\right)$ then find the unit vector in the direction of $\left(\overrightarrow{a} + \overrightarrow{b}\right)$.

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## Illustration 14

1. Show that the points $A, B$ and C having position vectors $\left(3\hat{i}-4\hat{j}-4\hat{k}\right), \left(2\hat{i}-\hat{j}+\hat{k}\right)$and $\left(\hat{i}-3\hat{j}-5\hat{k}\right)$ respectively, from the vertices of a right-angled triangle.

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## Illustration 15

1. If 2$\overrightarrow{A}C$ = 3$\overrightarrow{C}B$ , then prove that 2$\overrightarrow{O}A$ =3$\overrightarrow{C}B$ then prove that 2$\overrightarrow{O}A$ + 3 $\overrightarrow{O}B$ =5$\overrightarrow{O}C$ where $O$ is the origin.

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## Illustration 16

1. Prove that points $\hat{i}+2\hat{j}-3\hat{k}, 2\hat{i}-\hat{j}+\hat{k}$ and $2\hat{i}+5\hat{j}-\hat{k}$ form a triangle in space.

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## Illustration 17

**1.** Find the position vector of a point R which divides the line joining the point $P\left(\hat{i}+2\hat{j}-\hat{k}\right)$ and $Q\left(-\hat{i}+\hat{j}+\hat{k}\right)$ in the ratio $2:1$, (i) internally and (ii) externally.

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## Illustration 18

**1.** If $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are the position vectors of points $A, B, C$ and $D$, respectively referred to the same origin O such that no three of these points are collinear and $\vec{a}+\vec{c}=\vec{b}+\vec{d}$, then prove that quadrilateral $ABCD$ is a parallelogram.

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## Illustration 19

**1.** Find the point of intersection of AB and $A($ 6,-7,0),B(16,-19,-4,) , C(0,3,-6) and D(2,-5,10).

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## Illustration 20

**1.** Find the angle of vector $\vec{a} = 6\hat{i} + 2\hat{j} - 3\hat{k}$ with $x$-axis.

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## Illustration 21

**1.** i. Show that the lines joining the vertices of a tetrahedron to the centroids of opposite faces are concurrent.

ii. Show that the joins of the midpoints of the opposite edges of a tetrahedron intersect and bisect each other.

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## Illustration 22

**1.** The midpoint of two opposite sides of a quadrilateral and the midpoint of the diagonals are the vertices of a parallelogram. Prove that using vectors.

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## Illustration 23

**1.** Check whether the three vectors $2\hat{i}+2\hat{j}+3\hat{k}, \vec{b}=-3\hat{i}+3\hat{j}+2\hat{k}$ and $\vec{c}=3\hat{i}+4\hat{k}$ form a triangle or not.

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## Illustration 24

**1.** Find the resultant of vectors $\overrightarrow{a} = \hat{i} - \hat{j} + 2\hat{k}$ and $\overrightarrow{b} = \hat{i} + 2\hat{j} - 4\hat{k}$.

Find the unit vector in the direction of the resultant vector.

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## Illustration 25

**1.** If in parallelogram ABCD, diagonal vectors are $\overrightarrow{AC} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\overrightarrow{BD} = -6\hat{i} + 7\hat{j} - 2\hat{k}$, then find the adjacent side vectors $\overrightarrow{AB}$ and $\overrightarrow{AD}$.

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## Illustration 26

**1.** If two sides of a triangle are $\hat{i}+2\hat{j}$ and $\hat{i}+\hat{k}$, then find the length of the third side.

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## Illustration 27

**1.** Three coinitial vectors of magnitudes a, 2a and 3a meet at a point and their directions are along the diagonals if three adjacent faces if a cube. Determined their resultant R. Also prove that the sum of the three vectors determinate by the diagonals of three adjacent faces of a cube passing through the same corner, the vectors being directed from the corner, is twice the vector determined by the diagonal of the cube.

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## Illustration 28

**1.** A stone is projectef from level ground such that its horizontal and vertical components of initial velocity are $u_x = 10\frac{m}{s}$ and $u_y = 20\frac{m}{s}$ respectively. Then the angle between velocity vector of stone one second before and one second after it attains maximum height is:

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## Illustration 29

**1.** If the resultant of two forces is equal in magnitude to one of the components and perpendicular to it direction, find the other components using the vector method.

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## Illustration 30

**1.** A man travelling towards east at 8km/h finds that the wind seems to blow directly from the north On doubling the speed, he finds that it appears to come from the north-east. Find the velocity of the wind.

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## Illustration 31

**1.** OABCDE is a regular hexagon of side 2 units in the XY-plane in the first quadrant. O being the origin and OA taken along the x-axis. A point P is taken on a line parallel to the z-axis through the centre of the hexagon at a distance of 3 unit from O in the positive Z direction. Then find vector AP.

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## Illustration 32

**1.** If $\vec{a} = 7\hat{i} - 4\hat{j} - 4\hat{k}$ and $\vec{b} = -2\hat{i} - \hat{j} + 2\hat{k}$, determine vector $\vec{c}$ along the internal bisector of the angle between vectors $\vec{a}$ and $\vec{b}$ such that $\left|\vec{c}\right| = 5\sqrt{6}$.

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## Illustration 33

**1.** Find a unit vector $\vec{c}$ if $-\hat{i} + \hat{j} - \hat{k}$ bisects the angle between vectors $\vec{c}$ and $3\hat{i} + 4\hat{j}$.

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## Illustration 34

**1.** The vectors $2\hat{i} + 3\hat{j}, 5\hat{i} + 6\hat{j}$ and $8\hat{j} + \lambda\hat{j}$ have their initial points at (1, 1). The value of $\lambda$ so that the vectors terminate on one straight line, is

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## Illustration 35

**1.** Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-zero vectors which are positive non-collinear. If $\vec{a} + 3\vec{b}$ is collinear with $\vec{c}$ and $\vec{b} + 2\vec{c}$ is collinear with $\vec{a}$ then $\vec{a}$ then $\vec{a} + 3\vec{b} + 6\vec{c}$ is:

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## Illustration 36

**1.** i. Prove that the points $\vec{a} - 2\vec{b} + 3\vec{c}, 2\vec{a} + 3\vec{b} - 4\vec{c}$ and $-7\vec{b} + 10\vec{c}$ are collinear, where $\vec{a}$, $\vec{b}$ and $\vec{c}$ are non-coplanar.

ii. Prove that the points $A(1, 2, 3), B(3, 4, 7)$ and $C(-3, -2, -5)$ are collinear. Find the ratio in which point C divides AB.

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## Illustration 37

**1.** Check whether the given three vectors are coplnar or non- coplanar :

$-2\hat{i} - 2\hat{j} + 4\hat{k}, \ -2\hat{i} + 4\hat{j} - 2\hat{k}, 4\hat{i} - 2\hat{j} - 2\hat{k}.$

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## Illustration 38

**1.** Prove that the four points $6\hat{i} - 7\hat{j}, 16\hat{i} - 19\hat{j} - 4\hat{k}, 3\hat{j} - 6\hat{k}$ and $2\hat{i} + 5\hat{j} + 10\hat{k}$ form a tetrahedron in spacel.

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## Illustration 39

1. If $\vec{a}$ and $\vec{b}$ are two non-collinear vectors, show that points $l_1\vec{a} + m_1\vec{b}, l_2\vec{a} + m_2\vec{b}$ and $l_3\vec{a} + m_3\vec{b}$ are collinear if $|l_1 l_2 l_3 m_1 m_2 m_3 111| = 0.$

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## Illustration 40

1. The vectors $\vec{a}$ and $\vec{b}$ are non collinear. Find for what value of x the vectors $\vec{c} = (x-2)\vec{a} + \vec{b}$ and $\vec{d} = (2x+1)\vec{a} - \vec{b}$ are collinear.?

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## Illustration 41

1. The median AD of the triangle ABC is bisected at E and BE meets AC at F. Find AF:FC.

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## Illustration 42

**1.** Prove that the necessary and sufficient condition for any four points in three-dimensional space to be coplanar is that there exists a liner relation connecting their position vectors such that the algebraic sum of the coefficients (not all zero) in it is zero.

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## Illustration 43

**1.** i. If $\vec{a}, \vec{b}$ and $\vec{c}$ are non-coplanar vectors, prove that vectors $3\vec{a} - 7\vec{b} - 4\vec{c}, 3\vec{a} - 2\vec{b} + \vec{c}$ and $\vec{a} + \vec{b} + 2\vec{c}$ are coplanar.

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## Illustration 44

**1.** If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are non-coplanar vectors, prove that the four points $2\vec{a}+3\vec{b}-\vec{c}$, $\vec{a}-2\vec{b}+3\vec{c}$, $3\vec{a}+4\vec{b}-2\vec{c}$ and $\vec{a}-6\vec{b}+6\vec{c}$ are coplanar.

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## Illustration 45

**1.** let $P$ an interioer point of a triangle $ABC$ and $AP, BP, CP$ meets the sides $BC, CA, AB$ in $D, E, F$, respectively, Show that $\frac{AP}{PD}=\frac{AF}{FB}+\frac{AE}{EC}$.

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## Illustration 46

**1.** Points $A\left(\vec{a}\right), B\left(\vec{b}\right), C\left(\vec{c}\right) and D\left(\vec{d}\right)$ are relates as $x\vec{a} + y\vec{b} + z\vec{c} + w\vec{d} = 0$ and $x + y + z + w = 0, where x, y, z, and w$ are scalars (sum of any two of $x, y, znadw$ is not zero). Prove that if $A, B, C and D$ are concylic, then $|xy|\left|\vec{a} - \vec{b}\right|^2 = |wz|\left|\vec{c} - \vec{d}\right|^2$.

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## Concept Application Exercise 1 1

**1.** Find the unit vector in the direction of the vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$.

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**2.** Find the direction cosines of the vector $\hat{i} + 2\hat{j} + 3\hat{k}$.

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**3.** Find the direction cosines of the vector joining the points $A(1, 2, -3)$ and $B(-1, -2, 1)$ directed from $A$ to $B$.

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**4.** The position vectors of $P$ and $Q$ are $5\hat{i} + 4\hat{j} + a\hat{k}$ and $-\hat{i} + 2\hat{j} - 2\hat{k}$, respectively. If the distance between them is 7, then find the value of $a$.

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**5.** Given three points are $A(-3, -2, 0), B(3, -3, 1) and C(5, 0, 2)$. Then find a vector having the same direction as that of $\vec{A}B$ and magnitude equal to $\left|\vec{A}C\right|$.

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**6.** Find a vector of magnitude 5 units, and parallel to the resultant of the vectors

$$\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k} \text{ and } \vec{b} = \hat{i} - 2\hat{j} + \hat{k}.$$

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**7.** Show that the points $A(1, -2, -8), B(5, 0, -2)$ and $C(11, 3, 7)$ are collinear, and find the ratio in which B divides AC.

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**8.** If $ABCD$ is a rhombus whose diagonals cut at the origin $O$, then proved that $\vec{O}A + \vec{O}B + \vec{O}C + \vec{O}D + \vec{O}.$

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**9.** Let $D, EandF$ be the middle points of the sides $BC, CAandAB$, respectively of a triangle $ABC$. Then prove that $\vec{A}D + \vec{B}E + \vec{C}F = \vec{0}$.

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**10.** Let $ABCD$ be a p[arallelogram whose diagonals intersect at $P$ and let $O$ be the origin. Then prove that $\vec{O}A + \vec{O}B + \vec{O}C + \vec{O}D = 4\vec{O}P$.

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**11.** If $ABCD$ is quadrilateral and $EandF$ are the mid-points of $ACandBD$ respectively, prove that $\vec{A}B + \vec{A}D$ +$\vec{C}B$ +$\vec{C}D$ =4 $\vec{E}F$.

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**12.** If $\vec{A}O + \vec{O}B = \vec{B}O + \vec{O}C$ , then $A, BnadC$ are (where $O$ is the origin) a. coplanar b. collinear c. non-collinear d. none of these

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**13.** If the sides of an angle are given by vectors $\vec{a} = \hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} + 2\hat{k}$, then find the internal bisector of the angle.

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**14.** $ABCD$ is a parallelogram. If $LandM$ are the mid-points of $BCandDC$ respectively, then express $\vec{A}Land\vec{A}M$ in terms of $\vec{A}Band\vec{A}D$ . Also, prove that $\vec{A}L + \vec{A}M = \frac{3}{2}\vec{A}C\cdot$

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**15.** $ABCD$ is a quadrilateral and $E$ and the point intersection of the lines joining the middle points of opposite side. Show that the resultant of $\overrightarrow{O}A, \overrightarrow{O}B, \overrightarrow{O}C and \overrightarrow{O}D$ is equal to 4 $\overrightarrow{O}E,$ where $O$ is any point.

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**16.** What is the unit vector parallel to $\overrightarrow{a} = 3\hat{i} + 4\hat{j} - 2\hat{k}$ ? What vector should be added to $\overrightarrow{a}$ so that the resultant is the unit vector $\hat{i}$?

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**17.** The position vectors of points A and B w.r.t. the origin are $\overrightarrow{a} = \hat{i} + 3\hat{j} - 2\hat{k}$ and $\overrightarrow{b} = 3\hat{i} + \hat{j} - 2\hat{k}$, respectively. Determine vector $\overrightarrow{OP}$ which bisects angle $AOB$, where P is a point on AB.

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**18.** If $\overrightarrow{r}_1, \overrightarrow{r}_2, \overrightarrow{r}_3$ are the position vectors off thee collinear points and scalar $pandq$ exist such that $\overrightarrow{r}_3 = p\overrightarrow{r}_1 + q\overrightarrow{r}_2,$ then show that $p + q = 1.$

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**19.** If $\overrightarrow{a}$ and $\overrightarrow{b}$ are two vectors of magnitude 1 inclined at $120^\circ$, then find the angle between $\overrightarrow{b}$ and $\overrightarrow{b} - \overrightarrow{a}$.

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**20.** Find the vector of magnitude 3, bisecting the angle between the vectors $\overrightarrow{a} = 2\hat{i} + \hat{j} - \hat{k}$ and $\overrightarrow{b} = \hat{i} - 2\hat{j} + \hat{k}.$

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## Concept Application Exercise 1 2

**1.** If $\vec{a}, \vec{b}, \vec{c}$ $and \vec{d}$ are four vectors in three-dimensional space with the same initial point and such that $3\vec{a} + 2\vec{b} + \vec{c} - 2\vec{d} = 0$, Find the point at which $AC and BD$ meet. Find the ratio in which $P$ divides $AC and BD$.

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**2.** Show that the vectors $2\vec{a} - \vec{b} + 3\vec{c}, \vec{a} + \vec{b} - 2\vec{c}$ and $\vec{a} + \vec{b} - 3\vec{c}$ are non-coplanar vectors (where $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors).

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**3.** Examine the following vectors for linear independence :

i. $\vec{i} + \vec{j} + \vec{k}, 2\vec{i} + \vec{j} - \vec{k}, -\vec{i} - 2\vec{j} + 2\vec{k}$

ii. $3\vec{i} + \vec{j} - \vec{k}, 2\vec{i} - \vec{j} + 7\vec{k}, 7\vec{i} - \vec{j} + 13\vec{k}$

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**4.** If $\vec{a}$ and $\vec{b}$ are non-collinear vectors and $\vec{A} = (p + 4q)\vec{a} + (2p + q + 1)\vec{b}$ and $\vec{B} = (-2p + q + 2)\vec{a} + (2p - 3$

, and if $3\vec{A} = 2\vec{B}$, then determine $p$ and $q$.

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**5.** If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are any three non-coplanar vectors, then prove that points

$l_1\vec{a} + m_1\vec{b} + n_1\vec{c}, l_2\vec{a} + m_2\vec{b} + n_2\vec{c}, l_3\vec{a} + m_3\vec{b} + n_3\vec{c}, l_4\vec{a} + m_4$

are coplanar if $\begin{vmatrix} l_1 & l_2 & l_3 & l_4 \\ m_1 & m_2 & m_3 & m_4 \\ n_1 & n_2 & n_3 & n_4 \\ 1 & 1 & 1 & 1 \end{vmatrix} = 0$

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**6.** If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three non-zero, non-coplanar vectors,then find the linear relation between the following four vectors :

$\overrightarrow{a} - 2\overrightarrow{b} + 3\overrightarrow{c}, 2\overrightarrow{a} - 3\overrightarrow{b} + 4\overrightarrow{c}, 3\overrightarrow{a} - 4\overrightarrow{b} + 5\overrightarrow{c}, 7\overrightarrow{a} - 11\overrightarrow{b} + 15\overrightarrow{c}$

.

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**7.** Let $a, b, c$ be distinct non-negative numbers and the vectors $a\hat{i} + a\hat{j} + c\hat{k}, \hat{i} + \hat{k}, c\hat{i} + c\hat{j} + b\hat{k}$ lie in a plane, and then prove that the quadratic equation $ax^2 + 2cx + b = 0$ has equal roots.

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## Subjective

**1.** The position vectors of the vertices $A, B$ and $C$ of triangle are $\hat{i} + \hat{j}, \hat{j} + \hat{k}$ and $\hat{i} + \hat{k}$, respectively. Find the unit vectors $\hat{r}$ lying in the plane of $ABC$ and perpendicular to $IA$, where I is the incentre of the triangle.

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**2.** A ship is sailing towards the north at a speed of 1.25 m/s. The current is taking it towards the east at the rate of 1 m/s and a sailor is climbing a vertical pole on the ship at the rate of 0.5 m/s. Find the velocity of the sailor in space.

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**3.** Given four points $P_1, P_2, P_3 and P_4$ on the coordinate plane with origin $O$ which satisfy the condition $\left(\overrightarrow{OP}\right)_{n-1} + \left(\overrightarrow{OP}\right)_{n+1} = \frac{3}{2}\overrightarrow{OP}_n$. If P1 and P2 lie on the curve xy=1 , then prove that P3 does not lie on the curve

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**4.** $ABCD$ is a tetrahedron and $O$ is any point. If the lines joining $O$ to the vertices meet the opposite faces at $P, Q, R and S,$ prove that $\frac{OP}{AP} + \frac{OQ}{BQ} + \frac{OR}{CR} + \frac{OS}{DS} = 1.$

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**5.** A pyramid with vertex at point $P$ has a regular hexagonal base $ABCDEF$ , Position vector of points A and B are $\hat{i}$ and $\hat{i} + 2\hat{j}$ The centre of base has the position vector $\hat{i} + \hat{j} + \sqrt{3}\hat{k}$. Altitude drawn from $P$ on the base meets the diagonal $AD$ at point $G$. find the all possible position vectors of $G$. It is given that the volume of the pyramid is $6\sqrt{3}$ cubic units and $AP$ is 5 units.

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**6.** A straight line $L$ cuts the lines $AB, ACandAD$ of a parallelogram $ABCD$ at points $B_1, C_1andD_1,$ respectively. If $\left(\vec{A}B\right)_1, \lambda_1\vec{A}B, \left(\vec{A}D\right)_1 = \lambda_2\vec{A}Dand\left(\vec{A}C\right)_1 = \lambda_3\vec{A}C,$ then prove that $\dfrac{1}{\lambda_3} = \dfrac{1}{\lambda_1} + \dfrac{1}{\lambda_2}$.

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**7.** The position vectors of the points P and Q are $5\hat{i} + 7\hat{j} - 2\hat{k}$ and $-3\hat{i} + 3\hat{j} + 6\hat{k}$, respectively. Vector $\vec{A} = 3\hat{i} - \hat{j} + \hat{k}$ passes through point P and vector $\vec{B} = -3\hat{i} + 2\hat{j} + 4\hat{k}$ passes through point Q. A third vector $2\hat{i} + 7\hat{j} - 5\hat{k}$ intersects vectors A and B. Find the position vectors of points of intersection.

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**8.** Sow that $x_1\hat{i} + y_1\hat{j} + z_1\hat{k}, x_2\hat{i} + y_2\hat{j} + z_2\hat{k}, and x_3\hat{i} + y_3\hat{j} + z_3\hat{k},$ are non-coplanar if

$|x_1| > |y_1| + |z_1|, |y_2| > |x_2| + |z_2| and |z_3| > |x_3| + |y_3|$ .

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**9.** If $\vec{A} nd \vec{B}$ are two vectors and $k$ any scalar quantity greater than zero, then prove that $\left|\vec{A} + \vec{B}\right|^2 \leq (1+k)\left|\vec{A}\right|^2 + \left(1 + \frac{1}{k}\right)\left|\vec{B}\right|^2$.

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**10.** Consider the vectors $\hat{i} + \cos(\beta - \alpha)\hat{j} + \cos(\gamma - \alpha)\hat{k}, \cos(\alpha - \beta)\hat{i} + \hat{j} + \cos(\gamma - \beta)\hat{k}$ and $\cos(\alpha - \gamma)\hat{i} + \cos(\beta - \gamma)\hat{k} + a\hat{k}$where $\alpha, \beta$, and $\gamma$ are different angles. If these vectors are coplanar, show that $a$ is independent of $\alpha, \beta$ and $\gamma$

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**11.** In a triangle $PQR, SandT$ are points on $QRandPR$, respectively, such that $QS = 3SRandPT = 4TR$. Let $M$ be the point of intersection of $PSandQT$. Determine the ratio $QM : MT$ using the vector method .

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**12.** A boat moves in still water with a velocity which is $k$ times less than the river flow velocity. Find the angle to the stream direction at which the boat should be rowed to minimize drifting.

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**13.** If $D, E$ and $F$ are three points on the sides $BC, CA$ and $AB$, respectively, of a triangle $ABC$ show that the $\frac{BD}{CD} = \frac{CE}{AE} = \frac{AF}{BF} = -1$

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**14.** In a quadrilateral $PQRS, \vec{P}Q = \vec{a}, \vec{Q}R = \vec{b}, \vec{S}P = \vec{a} - \vec{b}, M$ is the midpoint of $\vec{Q}RandX$ is a point on $SM$ such that $SX = \frac{4}{5}SM$. Prove that $P, XandR$ are collinear.

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## Single Correct Answer Type

**1.** Four non zero vectors will always be a. linearly dependent b. linearly independent c. either a or b d. none of these

A. linearly dependent

B. linearly independent

C. either a or b

D. none of these

**Answer: A**

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**2.** Let $\vec{a}, \vec{b}, \vec{c}$ be three unit vectors such that $3\vec{a} + 4\vec{b} + 5\vec{c} = \vec{0}$. Then which of the following statements is true? (A) $\vec{a}$ is parrallel to vecb $(B)$veca$isperpendic\underline{a}r \rightarrow \vec{b}$ (C) $\vec{a}$ is neither parralel nor perpendicular to $\vec{b}$ (D) $\vec{a}, \vec{b}, \vec{c}$ are copalanar

A. $\vec{a}$ is parallel to $\vec{b}$

B. $\vec{a}$ is perpendicular to $\vec{b}$

C. $\vec{a}$ is neither parallel nor perpendicular to $\vec{b}$

D. none of these

**Answer: D**

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**3.** Let ABC be a triangle the position vectors of whose vertices are respectively $\hat{i} + 2\hat{j} + 4\hat{k},\ -2\hat{i} + 2\hat{j} + \hat{k}$ and $2\hat{i} + 4\hat{j} - 3\hat{k}$. Then the $\triangle ABC$ is (A) isosceles (B) equilateral (C) righat angled (D) none of these

A. isosceles

B. equilateral

C. right angled

D. none of these

**Answer: C**

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**4.** If $\left|\vec{a}+\vec{b}\right|<\left|\vec{a}-\vec{b}\right|$, then the angle between $\vec{a}$ and $\vec{b}$ can lie in the interval

A. $(-\pi/2, \pi/2)$

B. $(0, \pi)$

C. $(\pi/2, 3\pi/2)$

D. $(0, 2\pi)$

**Answer: C**

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**5.** A point $O$ is the centre of a circle circumscribed about a triangle $ABC$. Then $\vec{O}A\sin 2A+\vec{O}B\sin 2B+\vec{O}C\sin 2C$ is equal to a. $\left(\vec{O}A+\vec{O}B+\vec{O}C\right)\sin 2A$ b. $3\vec{O}G, where G$ is the centroid of triangle $ABC$ c. $\vec{0}$ d. none of these

A. $\left(\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}\right)\sin 2A$

B. $3\overrightarrow{OG}$, where G is the centroid of triangle ABC

C. $\overrightarrow{0}$

D. none of these

**Answer: C**

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6. If $G$ is the centroid of a triangle $ABC$, prove that $\overrightarrow{G}A + \overrightarrow{G}B + \overrightarrow{G}C = \overrightarrow{0}$.

A. $\overrightarrow{0}$

B. $3\overrightarrow{GA}$

C. $3\overrightarrow{GB}$

D. $3\overrightarrow{GC}$

**Answer: A**

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**7.** If $\overrightarrow{a}$ is a non zero vecrtor iof modulus $\overrightarrow{a}$ $and\, m$ is a non zero scalar such that $ma$ is a unit vector, then`

A. $m = \pm 1$

B. $a = |m|$

C. $a = 1/|m|$

D. $a = \frac{1}{m}$

**Answer: C**

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**8.** $ABCD$ parallelogram, and $A_1 and B_1$ are the midpoints of sides $BC and CD$, respectivley . If $\overrightarrow{A} A_1 + \overrightarrow{A} B_1 = \lambda \overrightarrow{A} C, then \lambda$ is equal to a. $\frac{1}{2}$ b. $1$ c. $\frac{3}{2}$ d. $2$ e. $\frac{2}{3}$

A. $\frac{1}{2}$

B. $1$

C. $\frac{3}{2}$

D. $2$

**Answer: C**

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**9.** The position vectors of the points P and Q with respect to the origin O are $\vec{a} = \hat{i} + 3\hat{j} - 2\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} - 2\hat{k}$, respectively. If M is a point on PQ, such that OM is the bisector of POQ, then $\overrightarrow{OM}$ is

A. $2\left(\hat{i} - \hat{j} + \hat{k}\right)$

B. $2\hat{i} + \hat{j} - 2\hat{k}$

C. $2\left(-\hat{i} + \hat{j} - \hat{k}\right)$

D. $2\left(\hat{i} + \hat{j} + \hat{k}\right)$

**Answer: B**

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**10.** $ABCD$ is a quadrilateral. $E$ is the point of intersection of the line joining the midpoints of the opposite sides. If $O$ is any point and $\vec{O}A + \vec{O}B + \vec{O}C + \vec{O}D = x\vec{O}E, thenx$ is equal to a. $3$ b. $9$ c. $7$ d. $4$

A. 3

B. 9

C. 7

D. 4

**Answer: D**

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**11.** The vector $\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are sides of a triangle ABC. The length of the median through A is (A) $\sqrt{18}$ (B) $\sqrt{72}$ (C) $\sqrt{33}$ (D) $\sqrt{288}$

A. $\sqrt{14}$

B. $\sqrt{18}$

C. $\sqrt{29}$

D. $5$

**Answer: B**

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**12.** A, B, C and D have position vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$, repectively, such that $\vec{a} - \vec{b} = 2\left(\vec{d} - \vec{c}\right)$. Then

A. AB and CD bisect each other

B. BD and AC bisect each other

C. AB and CD trisect each other

D. BD and AC trisect each other

**Answer: D**

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**13.** If $\overrightarrow{a}$ and $\overrightarrow{b}$ are two unit vectors and $\theta$ is the angle between them, then the unit vector along the angular bisector of $\overrightarrow{a}$ and $\overrightarrow{b}$ will be given by

A. $\dfrac{\overrightarrow{a} - \overrightarrow{b}}{2\cos(\theta/2)}$

B. $\dfrac{\overrightarrow{a} + \overrightarrow{b}}{2\cos(\theta/2)}$

C. $\dfrac{\overrightarrow{a} - \overrightarrow{b}}{\cos(\theta/2)}$

D. none of these

**Answer: B**

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**14.** let us define , the length of a vector as $|a| + |b| + |c|$. this definition coincides with the usual definition of the length of a vector $a\hat{i} + b\hat{j} + c\hat{k}$

if

A. $a = b = c = 0$

B. any two of a, b and c are zero

C. any one of a, b and c is zero

D. $a + b + c = 0$

**Answer: B**

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**15.** Given three vectors $\vec{a} = 6\hat{i} - 3\hat{j}, \vec{b} = 2\hat{i} - 6\hat{j}$ and $\vec{c} = -2\hat{i} + 21\hat{j}$ such that $\vec{\alpha} = \vec{a} + \vec{b} + \vec{c}$. Then the resolution of te vector $\vec{\alpha}$ into components with respect to $\vec{a}$ and $\vec{b}$ is given by (A) $3\vec{a} - 2\vec{b}$ (B) $2\vec{a} - 3\vec{b}$ (C) $3\vec{b} - 2\vec{a}$ (D) none of these

A. $3\vec{a} - 2\vec{b}$

B. $3\vec{b} - 2\vec{a}$

C. $2\overrightarrow{a} - 3\overrightarrow{b}$

D. $\overrightarrow{a} - 2\overrightarrow{b}$

**Answer: C**

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**16.** If $\overrightarrow{\alpha} + \overrightarrow{\beta} + \overrightarrow{\gamma} = a\overrightarrow{\delta}\, and\, \overrightarrow{\beta} + \overrightarrow{\gamma} + \overrightarrow{\delta} = b\overrightarrow{\alpha}, \overrightarrow{\alpha}\, and\, \overrightarrow{\delta}$ are non-colliner, then $\overrightarrow{\alpha} + \overrightarrow{\beta} + \overrightarrow{\gamma} + \overrightarrow{\delta}$ equals a. $a\overrightarrow{\alpha}$ b. $b\overrightarrow{\delta}$ c. $0$ d. $(a+b)\overrightarrow{\gamma}$

A. $a\overrightarrow{\alpha}$

B. $b\overrightarrow{\delta}$

C. $0$

D. $(a+b)\overrightarrow{\gamma}$

**Answer: C**

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**17.** In triangle $ABC, \angle A = 30^0, H$ is the orthocenter and $D$ is the midpoint of $BC$. Segment $HD$ is produced to $T$ such that $HD = DT$. The length $AT$ is equal to a. $2BC$ b. $3BC$ c. $\frac{4}{2}BC$ d. none of these

A. 2 BC

B. 3 BC

C. $\frac{4}{3}BC$

D. none of these

**Answer: A**

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**18.** Let $\vec{r}_1, \vec{r}_2, \vec{r}_3, , \vec{r}_n$ be the position vectors of points $P_1, P_2, P_3, P_n$ relative to the origin $O$. If the vector equation $a_1\vec{r}_1 + a_2\vec{r}_2 + \; + a_n\vec{r}_n = 0$ hold, then a similar equation will also hold w.r.t. to any other origin provided a. $a_1 + a_2 + \dot{+} a_n = n$ b. $a_1 + a_2 + \dot{+} a_n = 1$ c. $a_1 + a_2 + \dot{+} a_n = 0$ d. $a_1 = a_2 = a_3 \dot{+} a_n = 0$

A. $a_1 + a_2 + \ldots. + a_n = n$

B. $a_1 + a_2 + \ldots. + a_n = 1$

C. $a_1 + a_2 + \ldots + a_n = 0$

D. $a_1 = a_2 = a_3 = \ldots. = a_n = 0$

**Answer: C**

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**19.** Given three non-zero, non-coplanar vectors $\vec{a}, \vec{b}$ and $\vec{c}$. $\vec{r}_1 = p\vec{a} + q\vec{b} + \vec{c}$ and $\vec{r}_2 = \vec{a} + p\vec{b} + q\vec{c}$. If the vectors $\vec{r}_1 + 2\vec{r}_2$ and $2\vec{r}_1 + \vec{r}_2$ are collinear, then $(p, q)$ is

A. $(0, 0)$

B. $(1, -1)$

C. $(-1, 1)$

D. $(1, 1)$

**Answer: D**

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**20.** If the vectors $\vec{a}$ and $\vec{b}$ are linearly independent and satisfying $(\sqrt{3}\tan\theta + 1)\vec{a} + (\sqrt{3}\sec\theta - 2)\vec{b} = \vec{0}$, then the most general values of $\theta$ are:

A. $n\pi - \frac{\pi}{6}, n \in Z$

B. $2n\pi \pm \frac{11\pi}{6}, n \in Z$

C. $n\pi \pm \frac{\pi}{6}, n \in Z$

D. $2n\pi + \frac{11\pi}{6}, n \in Z$

**Answer: D**

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**21.** In a trapezium ABCD the vector $\overrightarrow{BC} = \alpha \overrightarrow{AD}$. If $\overrightarrow{p} = \overrightarrow{AC} + \overrightarrow{BD}$ is coillinear with $\overrightarrow{AD}$ such that $\overrightarrow{p} = \mu \overrightarrow{AD}$, then

A. $\mu = \alpha + 2$

B. $\mu + \alpha = 1$

C. $\alpha = \mu + 1$

D. $\mu = \alpha + 1$

**Answer: D**

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**22.** Vectors $\overrightarrow{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\overrightarrow{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $\overrightarrow{c} = 3\hat{i} + \hat{j} + 4\hat{k}$ are so placed that the end point of one vector is the starting point of the next vector. Then the vectors are

A. not coplanar

B. coplanar but cannot form a triangle

C. coplanar and form a triangle

D. coplanar and can form a right-angled triangle

**Answer: B**

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**23.** Vectors $\overrightarrow{a} = -4\hat{i} + 3\hat{k}; \overrightarrow{b} = 14\hat{i} + 2\hat{j} - 5\hat{k}$ are laid off from one point. Vector $\hat{d}$ , which is being laid of from the same point dividing the angle between vectors $\overrightarrow{a} \, and \, \overrightarrow{b}$ in equal halves and having the magnitude $\sqrt{6}$, is a. $\hat{i} + \hat{j} + 2\hat{k}$ b. $\hat{i} - \hat{j} + 2\hat{k}$ c. $\hat{i} + \hat{j} - 2\hat{k}$ d. $2\hat{i} - \hat{j} - 2\hat{k}$

A. $\hat{i} + \hat{j} + 2\hat{k}$

B. $\hat{i} - \hat{j} + 2\hat{k}$

C. $\hat{i} + \hat{j} - 2\hat{k}$

D. $2\hat{i} - \hat{j} - 2\hat{k}$

**Answer: A**

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**24.** If $\hat{i} - 3\hat{j} + 5\hat{k}$ bisects the angle between $\widehat{a}$ and $-\hat{i} + 2\hat{j} + 2\hat{k}$, where $\widehat{a}$ is a unit vector, then

A. $\widehat{a} = \frac{1}{150}\left(41\hat{i} + 88\hat{j} - 40\hat{k}\right)$

B. $\widehat{a} = \frac{1}{105}\left(41\hat{i} + 88\hat{j} + 40\hat{k}\right)$

C. $\widehat{a} = \frac{1}{105}\left(-41\hat{i} + 88\hat{j} - 40\hat{k}\right)$

D. $\widehat{a} = \frac{1}{105}\left(41\hat{i} - 88\hat{j} - 40\hat{k}\right)$

**Answer: D**

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**25.** If $4\hat{i} + 7\hat{j} + 8\hat{k}, 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $2\hat{i} + 5\hat{j} + 7\hat{k}$ are the position vectors of the vertices A, B and C, respectively, of triangle ABC, then the position vector of the point where the bisector of angle A meets BC is

A. $\frac{2}{3}\left(-6\hat{i}-8\hat{j}-6\hat{k}\right)$

B. $\frac{2}{3}\left(6\hat{i}+8\hat{j}+6\hat{k}\right)$

C. $\frac{1}{3}\left(6\hat{i}+13\hat{j}+18\hat{k}\right)$

D. $\frac{1}{3}\left(5\hat{j}+12\hat{k}\right)$

**Answer: C**

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**26.** If $\overrightarrow{b}$ is a vector whose initial point divides thejoin of $5\hat{i} and 5\hat{j}$ in the ratio $k:1$ and whose terminal point is the origin and $\left|\overrightarrow{b}\right| \leq \sqrt{37}, thenk$ lies in the interval

A. $[-6, -1/16]$

B. $(-\infty, -6] \cup [-1/6, \infty)$

C. $[0, 6]$

D. none of these

**Answer: B**

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**27.** The value of the$\lambda$ so that P, Q, R, S on the sides OA, OB, OC and AB of a regular tetrahedron are coplanar. When $\frac{OP}{OA}=\frac{1}{3}; \frac{OQ}{OB}=\frac{1}{2}$ and $\frac{OS}{AB}=\lambda$ is (A) $\lambda=\frac{1}{2}$ (B) $\lambda=-1$ (C) $\lambda=0$ (D) $\lambda=2$

A. $\lambda=\frac{1}{2}$

B. $\lambda=-1$

C. $\lambda=0$

D. for no value of $\lambda$

**Answer: B**

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**28.** $'I'$ is the incentre of triangle $ABC$ whose corresponding sides are $a, b, c,$ rspectively. $a\overrightarrow{I}A + b\overrightarrow{I}B + c\overrightarrow{I}C$ is always equal to a. $\overrightarrow{0}$ b. $(a+b+c)\overrightarrow{B}C$ c. $\left(\overrightarrow{a} + \overrightarrow{b} + \overrightarrow{c}\right)\overrightarrow{A}C$ d. $(a+b+c)\overrightarrow{A}B$

A. $\overrightarrow{0}$

B. $(a+b+c)\overrightarrow{BC}$

C. $\left(\overrightarrow{a} + \overrightarrow{b} + \overrightarrow{c}\right)\overrightarrow{AC}$

D. $(a+b+c)\overrightarrow{AB}$

**Answer: A**

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**29.** Let $x^2 + 3y^2 = 3$ be the equation of an ellipse in the $x - y$ plane. $AandB$ are two points whose position vectors are $-\sqrt{3}\hat{i}and - \sqrt{3}\hat{i} + 2\hat{k}$. Then the position vector of a point $P$ on the ellipse such that $\angle APB = \pi/4$ is a. $\pm\hat{j}$ b. $\pm\left(\hat{i} + \hat{j}\right)$ c. $\pm\hat{i}$ d. none of these

A. $\pm \hat{j}$

B. $\pm\left(\hat{i}+\hat{j}\right)$

C. $\pm \hat{i}$

D. none of these

**Answer: A**

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**30.** Locus of the point P, for which $\overrightarrow{OP}$ represents a vector with direction cosine $\cos\alpha = \frac{1}{2}$ (where O is the origin) is

A. a circle parallel to the y-z plane with centre on the x-axis

B. a conic concentric with the positive x-axis having vertex at the origin and slant height equal to the magnitude of the vector

C. a ray emanating from the origin and making an angle of $60^\circ$ with the x-axis

D. a dise parallel to the y-z plane with centre on the x-axis and radius equal to $\left|\overrightarrow{OP}\right|\sin 60^\circ$.

**Answer: B**

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**31.** If $\overrightarrow{x}\, and\, \overrightarrow{y}$ are two non-collinear vectors and $ABC$ isa triangle with side lengths $a, b, andc$ satisfying $(20a-15b)\overrightarrow{x}+(15b-12c)\overrightarrow{y}+(12c-20a)\left(\overrightarrow{x}\times\overrightarrow{y}\right)=0,$ then triangle $ABC$ is a. an acute-angled triangle b. an obtuse-angled triangle c. a right-angled triangle d. an isosceles triangle

A. an acute-angled triangle

B. an obtuse-angled triangle

C. a right-angled triangle

D. an isosceles triangle

**Answer: C**

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**32.** A uni-modular tangent vector on the curve $x = t^2 + 2, y = 4t - 5, z = 2t^2 - 6t = 2$ is a. $\frac{1}{3}\left(2\hat{i} + 2\hat{j} + \hat{k}\right)$ b. $\frac{1}{3}\left(\hat{i} - \hat{j} - \hat{k}\right)$ c. $\frac{1}{6}\left(2\hat{i} + \hat{j} + \hat{k}\right)$ d. $\frac{2}{3}\left(\hat{i} + \hat{j} + \hat{k}\right)$

A. $\frac{1}{3}\left(2\hat{i} + 2\hat{j} + \hat{k}\right)$

B. $\frac{1}{3}\left(\hat{i} - \hat{j} - \hat{k}\right)$

C. $\frac{1}{6}\left(2\hat{i} + \hat{j} + \hat{k}\right)$

D. $\frac{2}{3}\left(\hat{i} + \hat{j} + \hat{k}\right)$

**Answer: A**

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**33.** If $\vec{x}\, and\, \vec{y}$ are two non-collinear vectors and a, b, and c represent the sides of a $ABC$ satisfying $(a-b)\vec{x} + (b-c)\vec{y} + (c-a)\left(\vec{x} \times \vec{y}\right) = 0,$ then $ABC$ is (where

$\overrightarrow{\times}\ x\overrightarrow{y}$ is perpendicular to the plane of $x and y$ ) a. an acute-angled triangle b. an obtuse-angled triangle c. a right-angled triangle d. a scalene triangle

A. an acute-angled triangle

B. an obtuse-angled triangle

C. a right-angled triangle

D. a scalene triangle

**Answer: A**

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**34.** $\overrightarrow{A}$ isa vector with direction cosines $\cos\alpha, \cos\beta and \cos\gamma$. Assuming the $y-z$ plane as a mirror, the directin cosines of the reflected image of $\overrightarrow{A}$ in the plane are a. $\cos\alpha, \cos\beta, \cos\gamma$ b. $\cos\alpha, -\cos\beta, \cos\gamma$ c. $-\cos\alpha, \cos\beta, \cos\gamma$ d. $-\cos\alpha, -\cos\beta, -\cos\gamma$

A. $\cos\alpha, \cos\beta, \cos\gamma$

B. $\cos\alpha, \ -\cos\beta, \cos\gamma$

C. $-\cos\alpha, \cos\beta, \cos\gamma$

D. $-\cos\alpha, \ -\cos\beta, \ -\cos\gamma$

**Answer: C**

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**35.** The points with position vectors $60\hat{i}+3\hat{j}, 40\hat{i}-8\hat{j}, a\hat{i}-52\hat{j}$ are collinear if (A) $a=\ -40$ (B) $a=40$ (C) $a=20$ (D) none of these

A. $a=\ -40$

B. $a=40$

C. $a=20$

D. none of these

**Answer: A**

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**36.** Let $a, b$ and $c$ be distinct non-negative numbers. If vectos $a\hat{i} + a\hat{j} + c\hat{k}, \hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ are coplanar, then c is

A. the arithmetic mean of a and b

B. the geometric mean of a and b

C. the harmonic mean of a and b

D. equal to zero

**Answer: B**

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**37.** Let $\vec{a} = \vec{i} - \vec{k}, \vec{b} = x\vec{i} + \vec{j} + (1-x)\vec{k}$ and $\vec{c} = y\vec{i} + x\vec{j} + (1+x$

. Then $\vec{a}, \vec{b}$ and $\vec{c}$ are non-coplanar for

A. some values of x

B. some values of y

C. no values of x and y

D. for all values of x and y

**Answer: D**

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**38.** Let $\alpha, \beta$ and $\gamma$ be distinct real numbers. The points whose position vector's are $\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}; \beta\hat{i}+\gamma\hat{j}+\alpha\hat{k}$ and $\gamma\hat{i}+\alpha\hat{j}+\beta\hat{k}$ a. are collinear. b. forms an equilateral triangle. c. forms a scalene triangle. d. forms a right angled triangle.

A. are collinear

B. form an equilateral triangle

C. form a scalene triangle

D. form a right-angled triangle

**Answer: B**

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**39.** The number of distinct values of $\lambda$, for which the vectors $-\lambda^2\hat{i}+\hat{j}+\hat{k}, \hat{i}-\lambda^2\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}-\lambda^2\hat{k}$ are coplanar, is

A. zero

B. one

C. two

D. three

**Answer: C**

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**40.** If $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=4\hat{i}+3\hat{j}+4\hat{k}$ and $\vec{c}=\hat{i}+\alpha\hat{j}+\beta\hat{k}$ are linearly dependent vectors and $\left|\vec{c}\right|=\sqrt{3}$ then:

A. $\alpha = 1, \beta = -1$

B. $\alpha = 1, \beta = \pm 1$

C. $\alpha = -1, \beta = \pm 1$

D. $\alpha = \pm 1, \beta = 1$

**Answer: D**

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## Multiple Correct Answer Type

**1.** The vectors $x\hat{i} + (x+1)\hat{j} + (x+2)\hat{k}, (x+3)\hat{i} + (x+4)\hat{j} + (x+5)\hat{k}$ and $(x+6)\hat{i}$

are coplanar if x is equal to

A. 1

B. -3

C. 4

D. 0

**Answer: A::B::C::D**

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**2.** The sides of a parallelogram are $2\hat{i}+4\hat{j}-5\hat{k}$ and $\hat{i}+2\hat{j}+3\hat{k}$. The unit vector parallel to one of the diagonals is

A. $\frac{1}{7}\left(3\hat{i}+6\hat{j}-2\hat{k}\right)$

B. $\frac{1}{7}\left(3\hat{i}-6\hat{j}-2\hat{k}\right)$

C. $\frac{1}{\sqrt{69}}\left(\hat{i}+2\hat{j}+8\hat{k}\right)$

D. $\frac{1}{\sqrt{69}}\left(-\hat{i}-2\hat{j}+8\hat{k}\right)$

**Answer: A::D**

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**3.** The vector $\overrightarrow{a}$ has the components $2p$ and 1 w.r.t. a rectangular Cartesian system. This system is rotated through a certain angle about the origin in the counterclockwise sense. If, with respect to a new system, $\overrightarrow{a}$ has components $(p+1) and 1$, then $p$ is equal to

a. $-4$ b. $-1/3$ c. $1$ d. $2$

A. $-1$

B. $-1/3$

C. $1$

D. $2$

**Answer: B::C**

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**4.** If points $\hat{i}+\hat{j}$, $\hat{i}-\hat{j}$ and $p\hat{i}+q\hat{j}+r\hat{k}$ are collinear, then

A. $p=1$

B. $r = 0$

C. $q \in R$

D. $q \neq 1$

**Answer: A::B::D**

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**5.** If $\vec{a}, \vec{b}, \vec{c}$ are non coplanar vectors and $\lambda$ is a real number, then the vectors $\vec{a} + 2\vec{b} + 3\vec{c}, \lambda\vec{b} + 4\vec{}$ and $(2\lambda - 1)\vec{c}$ are non coplanar for

A. $\mu \in R$

B. $\lambda = \frac{1}{2}$

C. $\lambda = 0$

D. no value of $\lambda$

**Answer: A::B::C**

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6. If the resultant of three forces $\overrightarrow{F}_1 = p\hat{i} + 3\hat{j} - \hat{k}, \overrightarrow{F}_2 = 6\hat{i} - \hat{k}$ and $\overrightarrow{F}_3 = -5\hat{i} + \hat{j} + 2\hat{k}$ acting on a particle has a magnitude equal to 5 units, then the value of $p$ is

A. $-6$

B. $-4$

C. 2

D. 4

**Answer: B::C**

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7. If the vectors $\hat{i} - \hat{j}, \hat{j} + \hat{k}$ and $\overrightarrow{a}$ form a triangle then $\overrightarrow{a}$ may be (A) $-\hat{i} - \hat{k}$ (B) $\hat{i} - 2\hat{j} - \hat{k}$ (C) $2\hat{i} + \hat{j} + \hat{j}k$ (D) $\hat{i} + \hat{k}$

A. $-\hat{i} - \hat{k}$

B. $\hat{i} - 2\hat{j} - \hat{k}$

C. $2\hat{i} + \hat{j} + \hat{k}$

D. $\hat{i} + \hat{k}$

**Answer: A::B::D**

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**8.** The vector $\hat{i} + x\hat{j} + 3\hat{k}$ is rotated through an angle $\theta$ and doubled in magnitude, then it becomes $4\hat{i} + (4x - 2)\hat{j} + 2\hat{k}$. Then values of x are (A) $-\frac{2}{3}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) 2

A. $1$

B. $-2/3$

C. $2$

D. $4/3$

**Answer: B::C**

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**9.** $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three coplanar vectors such that $\vec{a} + \vec{b} + \vec{c} = 0.$ If three vectors $\vec{p}$, $\vec{q}$ and $\vec{r}$ are parallel to $\vec{a}$, $\vec{b}$ and $\vec{c}$, respectively, and have integral but different magnitudes, then among the following options, $\left|\vec{p} + \vec{q} + \vec{r}\right|$ can take a value equal to

A. $1$

B. $0$

C. $\sqrt{3}$

D. $2$

**Answer: C::D**

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**10.** If non-zero vectors a annd b are equally inclined to coplanar vector c, then c can be

A. $\dfrac{\left|\vec{a}\right|}{\left|\vec{a}\right|+2\left|\vec{b}\right|}\vec{a}+\dfrac{\left|\vec{b}\right|}{\left|\vec{a}\right|+\left|\vec{b}\right|}\vec{b}$

B. $\dfrac{\left|\vec{b}\right|}{\left|\vec{a}\right|+\left|\vec{b}\right|}\vec{a}+\dfrac{\left|\vec{a}\right|}{\left|\vec{a}\right|+\left|\vec{b}\right|}\vec{b}$

C. $\dfrac{\left|\vec{a}\right|}{\left|\vec{a}\right|+2\left|\vec{b}\right|}\vec{a}+\dfrac{\left|\vec{b}\right|}{\left|\vec{a}\right|+2\left|\vec{b}\right|}\vec{b}$

D. $\dfrac{\left|\vec{b}\right|}{2\left|\vec{a}\right|+\left|\vec{b}\right|}\vec{a}+\dfrac{\left|\vec{a}\right|}{2\left|\vec{a}\right|+\left|\vec{b}\right|}\vec{b}$

**Answer: B::D**

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**11.** If $A(-4, 0, 3) and B(14, 2, -5)$, then which one of the following points lie on the bisector of the angle between $\vec{O}A and \vec{O}B(O$ is the origin of reference )? a. $(2, 2, 4)$ b. $(2, 11, 5)$ c. $(-3, -3, -6)$ d. $(1, 1, 2)$

A. $(2, 2, 4)$

B. $(2, 11, 5)$

C. $(-3, -3, -6)$

D. $(1, 1, 2)$

**Answer: A::C::D**

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**12.** In a four-dimensional space where unit vectors along the axes are $\hat{i}, \hat{j}, \hat{k}$ and $\hat{l}$, and $\vec{a}_1, \vec{a}_2, \vec{a}_3, \vec{a}_4$ are four non-zero vectors such that no vector can be expressed as a linear combination of others and $(\lambda - 1)\left(\vec{a}_1 - \vec{a}_2\right) + \mu\left(\vec{a}_2 + \vec{a}_3\right) + \gamma\left(\vec{a}_3 + \vec{a}_4 - 2\vec{a}_2\right) + \vec{a}_3 + \delta\vec{a}$

then

A. $\lambda = 1$

B. $\mu = -2/3$

C. $\gamma = 2/3$

D. $\delta = 1/3$

**Answer: A::B::D**

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**13.** Let ABC be a triangle, the position vectors of whose vertices are respectively

$7\hat{j}+10\hat{k}, -\hat{i}+6\hat{j}+6\hat{k}$ and $-4\hat{i}+9\hat{j}+6\hat{k}$. Then, $\Delta ABC$ is

A. isosceles

B. equilateral

C. right angled

D. none of these

**Answer: A::C**

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## Reasoning Type

**1.** A vector has components $p$ and 1 with respect to a rectangular Cartesian system. The axes are rotated through an angle $\alpha$ about the origin in the anticlockwise sense.

Statement 1: If the vector has component $p+2$ and 1 with respect to the new system, then $p = -1$.

Statement 2: Magnitude of the original vector and new vector remains the same.

A. Both the statements are true, and Statement 2 is the correct explanation for Statement 1.

B. Both the statements are true, but Statement 2 is not the correct explanation for Statement 1.

C. Statement 1 is true and Statement 2 is false.

D. Statement 1 is false and Statement 2 is true.

**Answer: A**

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**2.** Statement 1: if three points $P, QandR$ have position vectors $\overrightarrow{a}, \overrightarrow{b}, and \overrightarrow{c}$, respectively, and $2\overrightarrow{a}+3\overrightarrow{b}-5\overrightarrow{c}=0$, then the points $P, Q, andR$ must be collinear. Statement 2: If for three points $A, B, andC, \overrightarrow{A}B=\lambda\overrightarrow{A}C$, then points $A, B, andC$ must be collinear.

A. Both the statements are true, and Statement 2 is the correct explanation for Statement 1.

B. Both the statements are true, but Statement 2 is not the correct explanation for Statement 1.

C. Statement 1 is true and Statement 2 is false.

D. Statement 1 is false and Statement 2 is true.

**Answer: A**

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**3.** Statement 1: If $\vec{u} \, and \, \vec{v}$ are unit vectors inclined at an angle $\alpha and \vec{x}$ is a unit vector bisecting the angle between them, then $\vec{x} = \left(\vec{u} + \vec{v}\right) / (2 \sin(\alpha / 2)$. Statement 2: If $\mathrm{Delta}ABC$ is an isosceles triangle with $AB = AC = 1,$ then the vector representing the bisector of angel $A$ is given by $\vec{A}D = \left(\vec{A}B + \vec{A}C\right) / 2.$

A. Both the statements are true, and Statement 2 is the correct explanation for Statement 1.

B. Both the statements are true, but Statement 2 is not the correct explanation for Statement 1.

C. Statement 1 is true and Statement 2 is false.

D. Statement 1 is false and Statement 2 is true.

**Answer: D**

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**4.** Statement 1: If $\cos\alpha, \cos\beta, and \cos\gamma$ are the direction cosines of any line segment, then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$. Statement 2: If $\cos\alpha, \cos\beta, and \cos\gamma$ are the direction cosines of any line segment, then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.

A. Both the statements are true, and Statement 2 is the correct explanation for Statement 1.

B. Both the statements are true, but Statement 2 is not the correct explanation for Statement 1.

C. Statement 1 is true and Statement 2 is false.

D. Statement 1 is false and Statement 2 is true.

**Answer: B**

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**5.** Statement 1: The direction cosines of one of the angular bisectors of two intersecting line having direction cosines as $l_1, m_1, n_1 and l_2, m_2, n_2$ are proportional to $l_1 + l_2, m_1 + m_2, n_1 + n_2$. Statement 2: The angle between the two intersection lines having direction cosines as $l_1, m_1, n_1 and l_2, m_2, n_2$ is given by $\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$.

A. Both the statements are true, and Statement 2 is the correct explanation for Statement 1.

B. Both the statements are true, but Statement 2 is not the correct explanation for Statement 1.

C. Statement 1 is true and Statement 2 is false.

D. Statement 1 is false and Statement 2 is true.

**Answer: B**

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6. Statement 1: In $\mathrm{Delta}ABC, \vec{A}B + \vec{A}B + \vec{C}A = 0$ Statement 2: If $\vec{O}A = \vec{a}, \vec{O}B = \vec{b}, then\vec{A}B = \vec{a} + \vec{b}$

A. Both the statements are true, and Statement 2 is the correct explanation for Statement 1.

B. Both the statements are true, but Statement 2 is not the correct explanation for Statement 1.

C. Statement 1 is true and Statement 2 is false.

D. Statement 1 is false and Statement 2 is true.

**Answer: C**

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7. Statement 1 : $\vec{a} = 3\vec{i} + p\vec{j} + 3\vec{k}$ and $\vec{b} = 2\vec{i} + 3\vec{j} + q\vec{k}$ are parallel vectors if $p = 9/2$ and $q = 2$.

Statement 2 : If

$\overrightarrow{a} = a_1 \overrightarrow{i} + a_2 \overrightarrow{j} + a_3 \overrightarrow{k}$ and $\overrightarrow{b} = b_1 \overrightarrow{i} + b_2 \overrightarrow{j} + b_3 \overrightarrow{k}$ are parallel, then $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$.

A. Both the statements are true, and Statement 2 is the correct explanation for Statement 1.

B. Both the statements are true, but Statement 2 is not the correct explanation for Statement 1.

C. Statement 1 is true and Statement 2 is false.

D. Statement 1 is false and Statement 2 is true.

**Answer: A**

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**8.** Statement 1 : If $\left|\overrightarrow{a} + \overrightarrow{b}\right| = \left|\overrightarrow{a} - \overrightarrow{b}\right|$, then $\overrightarrow{a}$ and $\overrightarrow{b}$ are perpendicular to each other.

Statement 2 : If the diagonals of a parallelogram are equal in magnitude, then the parallelogram is a rectangle.

A. Both the statements are true, and Statement 2 is the correct explanation for Statement 1.

B. Both the statements are true, but Statement 2 is not the correct explanation for Statement 1.

C. Statement 1 is true and Statement 2 is false.

D. Statement 1 is false and Statement 2 is true.

**Answer: A**

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**9.** Statement 1 : Let $A\left(\overrightarrow{a}\right), B\left(\overrightarrow{b}\right)$ and $C\left(\overrightarrow{c}\right)$ be three points such that $\overrightarrow{a} = 2\hat{i} + \hat{k}, \overrightarrow{b} = 3\hat{i} - \hat{j} + 3\hat{k}$ and $\overrightarrow{c} = -\hat{i} + 7\hat{j} - 5\hat{k}$. Then OABC is tetrahedron.

Statement 2 : Let $A\left(\overrightarrow{a}\right), B\left(\overrightarrow{b}\right)$ and $C\left(\overrightarrow{c}\right)$ be three points such that vectors $\overrightarrow{a}, \overrightarrow{b}$ and $\overrightarrow{c}$ are non-coplanar. Then OABC is a tetrahedron, where O is the origin.

A. Both the statements are true, and Statement 2 is the correct explanation for Statement 1.

B. Both the statements are true, but Statement 2 is not the correct explanation for Statement 1.

C. Statement 1 is true and Statement 2 is false.

D. Statement 1 is false and Statement 2 is true.

**Answer: A**

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**10.** Statement 1: Let $\vec{a}, \vec{b}, \vec{c} and \vec{d}$ be the position vectors of four points $A, B, C and D$ and $3\vec{a} - 2\vec{b} + 5\vec{c} - 6\vec{d} = 0$. Then points $A, B, C, and D$ are coplanar. Statement 2: Three non-zero, linearly dependent coinitial vector $\left(\vec{P}Q, \vec{P}R and \vec{P}S\right)$ are coplanar. Then $\vec{P}Q = \lambda\vec{P}R + \mu\vec{P}S, where \lambda and \mu$ are scalars.

A. Both the statements are true, and Statement 2 is the correct explanation for Statement 1.

B. Both the statements are true, but Statement 2 is not the correct explanation for Statement 1.

C. Statement 1 is true and Statement 2 is false.

D. Statement 1 is false and Statement 2 is true.

**Answer: A**

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**11.** Statement 1 : If $\left|\overrightarrow{a}\right| = 3, \left|\overrightarrow{b}\right| = 4$ and $\left|\overrightarrow{a} + \overrightarrow{b}\right| = 5,$ then $\left|\overrightarrow{a} - \overrightarrow{b}\right| = 5.$

Statement 2 : The length of the diagonals of a rectangle is the same.

A. Both the statements are true, and Statement 2 is the correct explanation for Statement 1.

B. Both the statements are true, but Statement 2 is not the correct explanation for Statement 1.

C. Statement 1 is true and Statement 2 is false.

D. Statement 1 is false and Statement 2 is true.

**Answer: A**

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## Linked Comprehension Type

**1.** ABCD is a parallelogram. L is a point on BC which divides BC in the ratio $1:2$. AL intersects BD at P. M is a point on DC which divides DC in the ratio $1:2$ and AM intersects BD in Q.

Point P divides AL in the ratio

A. $1:2$

B. $1:3$

C. $3:1$

D. $2:1$

**Answer: C**

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**2.** ABCD is a parallelogram. L is a point on BC which divides BC in the ratio $1:2$. AL intersects BD at P.M is a point on DC which divides DC in the ratio $1:2$ and AM intersects BD in Q.

Point Q divides DB in the ratio

A. $1:2$

B. $1:3$

C. $3:1$

D. $2:1$

**Answer: B**

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**3.** ABCD is a parallelogramm. L is a point on BC which divides BC in the ratio 1:2 AL intersects BD at P.M is a point onn DC which divides DC in the ratio 1:2 and AM intersects BD in Q.

Q. $PQ : DB$ is equal to

A. $2/3$

B. $1/3$

C. $1/2$

D. $3/4$

**Answer: C**

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**4.** Let OABCD be a pentagon in which the sides OA and CB are parallel and the sides OD and AB are parallel as shown in figure. Also, OA:CB=2:1 and

OD:AB=1:3. if the diagonals OC and AD meet at x, find OX:OC.

A. $3/4$

B. $1/3$

C. $2/5$

D. $1/2$

**Answer: C**

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**5.** Let OABCD be a pentagon in which the sides OA and CB are parallel and the sides OD and AB are parallel as shown in figure. Also, OA:CB=2:1 and OD:AB=1:3. if the diagonals OC and AD meet at x, find OX:OC.

A. $5/2$

B. $6$

C. $7/3$

D. $4$

**Answer: B**

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**6.** If ABCDEF is regular hexagon, then AD+EB+FC is

A. 2 $\overrightarrow{AB}$

B. 3 $\overrightarrow{AB}$

C. 4$A\overrightarrow{AB}$

D. none of these

**Answer: C**

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**7.** Consider the regular hexagon ABCDEF with centre at O (origin).

Q. Five forces AB,AC,AD,AE,AF act at the vertex A of a regular hexagon ABCDEF. Then, their resultant is (a)3AO (b)2AO (c)4AO (d)6AO

A. $3\overrightarrow{AO}$

B. $2\overrightarrow{AO}$

C. $4\overrightarrow{AO}$

D. $6\overrightarrow{AO}$

**Answer: D**

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**8.** Let A,B,C,D,E represent vertices of a regular pentangon ABCDE. Given the position vector of these vertices be a,a+b,b,$\lambda a$ and $\lambda b$ respectively.

Q. AD divides EC in the ratio

A. $1 - \cos\frac{3\pi}{5} : \cos\frac{3\pi}{5}$

B. $1+2\cos\frac{2\pi}{5}:\cos\frac{\pi}{5}$

C. $1+2\cos\frac{\pi}{5}:2\cos\frac{\pi}{5}$

D. None of these

**Answer: C**

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**9.** Let A,B,C,D,E represent vertices of a regular pentangon ABCDE. Given the position vector of these vertices be a,a+b,b,$\lambda a$ and $\lambda b$ respectively.

Q. AD divides EC in the ratio

A. $\cos\frac{2\pi}{5}:1$

B. $\cos\frac{3\pi}{5}:1$

C. $1:2\cos\frac{2\pi}{5}$

D. $1:2$

**Answer: C**

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**10.** In a parallelogram OABC vectors a,b,c respectively, THE POSITION VECTORS OF VERTICES A,B,C with reference to O as origin. A point E is taken on the side BC which divides it in the ratio of 2:1 also, the line segment AE intersects the line bisecting the angle $\angle AOC$ internally at point P. if CP when extended meets AB in points F, then

Q. The position vector of point P is

A. $\dfrac{\left|\vec{a}\right|\left|\vec{c}\right|}{3\left|\vec{c}\right|+2\left|\vec{a}\right|}\left(\dfrac{\vec{a}}{\left|\vec{a}\right|}+\dfrac{\vec{c}}{\left|\vec{c}\right|}\right)$

B. $\dfrac{3\left|\vec{a}\right|\left|\vec{c}\right|}{3\left|\vec{c}\right|+2\left|\vec{a}\right|}\left(\dfrac{\vec{a}}{\left|\vec{a}\right|}+\dfrac{\vec{c}}{\left|\vec{c}\right|}\right)$

C. $\dfrac{2\left|\vec{a}\right|\left|\vec{c}\right|}{3\left|\vec{c}\right|+2\left|\vec{a}\right|}\left(\dfrac{\vec{a}}{\left|\vec{a}\right|}+\dfrac{\vec{c}}{\left|\vec{c}\right|}\right)$

D. None of these

**Answer: B**

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**11.** In a parallelogram OABC vectors a,b,c respectively, THE POSITION VECTORS OF VERTICES A,B,C with reference to O as origin. A point E is taken on the side BC which divides it in the ratio of 2:1 also, the line segment AE intersects the line bisecting the angle $\angle AOC$ internally at point P. if CP when extended meets AB in points F, then

Q. The position vector of point P is

A. $\dfrac{2\left|\vec{a}\right|}{\left|\left|\vec{a}-3\right|\vec{c}\right|\right|}$

B. $\dfrac{\left|\vec{a}\right|}{\left|\left|\vec{a}\right|-3\left|\vec{c}\;\right|\;\right|}$

C. $\dfrac{3\left|\vec{a}\right|}{\left|\left|\vec{a}\right|-3\left|\vec{c}\;\right|\;\right|}$

D. $\dfrac{3\left|\vec{c}\right|}{3\left|\vec{c}\right|-\left|\vec{a}\right|}$

**Answer: D**

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Integer Type

**1.** Let ABC be a triangle whose centroid is G, orthocentre is H and circumcentre is the origin 'O'. If D is any point in the plane of the triangle such that no three of O,A,C and D are collinear satisfying the relation.

AD+BD+CH+3HB=$\lambda HD$, then what is the value of the scalar $\Delta$.

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**2.** If the resultant of three forces $\overrightarrow{F}_1 = p\hat{i} + 3\hat{j} - \hat{k}, \overrightarrow{F}_2 = -5\hat{i} + \hat{j} + 2\hat{k}$ and $\overrightarrow{F}_3 = 6\hat{i} - \hat{k}$ acting on a particle has a magnitude equal to 5 units, then what is difference in the values of $p$ ?

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**3.** Let $\overrightarrow{a}, \overrightarrow{b}$ and $\overrightarrow{c}$ be unit vector such that $\overrightarrow{a} + \overrightarrow{b} - \overrightarrow{c} = 0$. If the area of triangle formed by vectors $\overrightarrow{a}$ and $\overrightarrow{b}$ is A, then what is the value of $4A^2$ ?

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**4.** Find the least positive integral value of $x$ for which the angle between vectors $\overrightarrow{a} = x\hat{i} - 3\hat{j} - \hat{k}$ and $\overrightarrow{b} = 2x\hat{i} + x\hat{j} - \hat{k}$ is acute.

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**5.** Vectors along the adjacent sides of parallelogram are $\overrightarrow{a} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\overrightarrow{b} = \hat{i} + 2\hat{j} + 3\hat{k}$. Find the length of the longer diagonal of the parallelogram.

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**6.** If vectors $\overrightarrow{a} = \hat{i} + 2\hat{j} - \hat{k}, \overrightarrow{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $\overrightarrow{c} = \lambda\hat{i} + \hat{j} + 2\hat{k}$ are coplanar, then find the value of $\lambda$.

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**7.** Consider the set of eight vector $V = \left\{a\hat{i} + b\hat{j} + c\hat{k}; a, bc \in \{-1, 1\}\right\}$. Three non-coplanar vectors can be chosen from $V$ is $2^p$ ways. Then $p$ is______.

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**8.** Suppose that $\vec{p}$, $\vec{q}$ and $\vec{r}$ are three non- coplaner in $R^3$ ,Let the components of a vector $\vec{s}$ along $\vec{p}$, $\vec{q}$ and $\vec{r}$ be 4,3, and 5, respectively , if the components this vector $\vec{s}$ along $\left(-\vec{p} + \vec{q} + \vec{r}\right), \left(\vec{p} - \vec{q} + \vec{r}\right)$ and $\left(-\vec{p} - \vec{q} + \vec{r}\right)$ are x, y and z , respectively , then the value of $2x + y + z$ is

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## Archives Subjective Type

**1.** Find the all the values of lamda such that $(x, y, z) \neq (0, 0, 0)$and $x\left(\hat{i} + \hat{j} + 3\hat{k}\right) + y\left(3\hat{i} - 3\hat{j} + \hat{k}\right) + z\left(-4\hat{i} + 5\hat{j}\right) = \lambda\left(x\hat{i} + y\hat{j} + z\hat{k}\right)$

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**2.** A vector a has components $a_1, a_2$ and $a_3$ in a right handed rectangular cartesian system OXYZ. The coordinate system is rotated about Z-axis through angle $\frac{\pi}{2}$. Find components of a in the new system.

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**3.** The position vectors of the points A,B, C and D are $3\hat{i}-2\hat{j}-\hat{k}, 2\hat{i}+3\hat{j}-34\hat{k}, -\hat{i}+\hat{j}+2\hat{k}$ and $4\hat{i}+5\hat{j}+\lambda\hat{k}$ respectively. If the points A, B ,C and D lie on a plane, find the value of $\lambda$.

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**4.** Let OACB be a parallelogram with O at the origin and OC a diagonal. Let D be the mid-point of OA. Using vector methods prove that BD and CO intersects in the same ratio. Determine this ratio.

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**5.** In a triangle $ABC, DandE$ are points on $BCandAC$, respectivley, such that $BD = 2DCandAE = 3EC$. Let $P$ be the point of intersection of $ADandBE$. Find $BP/PE$ using the vector method.

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**6.** Prove, by vector method or otherwise, that the point of intersection of the diagonals of a trapezium lies on the line passing through the midpoint of the parallel sides (you may assume that the trapezium is not a parallelogram).

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**8.** Let $\overrightarrow{A}(t) = f_1(t)\hat{i} + f_2(t)\hat{j}$ and $\overrightarrow{B}(t) = g(t)\hat{i} + g_2(t)\hat{j}, t \in [0, 1], f_1, f_2, g_1 g_2$ are continuous functions. If $\overrightarrow{A}(t)$ and $\overrightarrow{B}(t)$ are non-zero vectors for all $t$ and $\overrightarrow{A}(0) = 2\hat{i} + 3\hat{j}, \overrightarrow{A}(1) = 6\hat{i} + 2\hat{j}, \overrightarrow{B}(0) = 3\hat{i} + 2\hat{i}$ and $\overrightarrow{B}(1) = 2\hat{i}$ Then,show that $\overrightarrow{A}(t)$ and $\overrightarrow{B}(t)$ are parallel for some $t$.

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## Fill In The Blanks

**1.** If a,b, and c are all different and if $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix}$ =0 Prove that abc =-1.

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**2.** If the vectors $a\hat{i}+\hat{j}+\hat{k}, \hat{i}+b\hat{j}+\hat{k}, \hat{i}+\hat{j}+c\hat{k}(a\neq 1, b\neq 1, c\neq 1)$ are coplanar then the value of $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}$ is (A) 0 (B) 1 (C) -1 (D) 2

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## True Or False

**1.** The points with position vectors $\vec{x}+\vec{y}, \vec{x}-\vec{y}$ and $\vec{x}+\lambda\vec{y}$ are collinear for all real values of λ.

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## Matrix Match Type

**1.** Refer to the following diagram :

| | Column I | | Column II |
|---|---|---|---|
| $a.$ | Collinear vectors | $p.$ | $\overrightarrow{a}$ |
| $b.$ | Coinitial vectors | $q.$ | $\overrightarrow{b}$ |
| $c.$ | Equal vectors | $r.$ | $\overrightarrow{c}$ |
| $d.$ | Unlike vectors (same initial point) | $s.$ | $\overrightarrow{d}$ |

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**2.** a and b form the consecutive sides of a regular hexagon ABCDEF.

| | Column I | | Column II |
|---|---|---|---|
| a. | If $\mathbf{CD} = x\mathbf{a} + y\mathbf{b}$, then | p. | $x = -2$ |
| b. | If $\mathbf{CE} = x\mathbf{a} + y\mathbf{b}$, then | q. | $x = -1$ |
| c. | If $\mathbf{AE} = x\mathbf{a} + y\mathbf{b}$, then | r. | $y = 1$ |
| d. | If $\mathbf{AD} = -x\mathbf{b}$, then | s. | $y = 2$ |

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3.

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