MATHS

BOOKS - CENGAGE MATHS (ENGLISH)

JEE 2019

Chapter 1 Coordinate System

1. If two vertices of a triangle are $(0, 2)$ and $(4, 3)$ and its orthocentre is $(0, 0)$ then the third vertex of the triangle lies in (a) I^{st} quadrant (b) $2^{nd}\ rant(c)$3^(rd)quadrant (d) `4^(th)quadrant

A. Fourth

B. Second

C. Third

D. First

Answer: B

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2. If the straight line $2x - 3y + 17 = 0$ is perpendicular to the line passing through the points $(7, 17)$ and $(15, \beta)$, then β equals

A. -5

B. $-\frac{35}{3}$

C. $\frac{35}{3}$

D. 5

Answer: D

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Chapter 2 Straight Lines

1. Consider the set of all lines px+qy+r=0 such that 3p+2q+4r=0. Which one of the following statements is true ?

A. The lines are all parallel.

B. Each line passes through the origin.

C. The lines are not concurrent.

D. The lines are concurrent at the point $\left(\frac{3}{4}, \frac{1}{2}\right)$.

Answer: D

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2. The equations of two sides of a triangle are $3x - 2y + 6 = 0 \, and \, 4x + 5y - 20 \, and$ the orthocentre is (1,1). Find the equation of the third side.

A. 122t-26x-1675=0

B. 26x+61y+1675=0

C. 122y+26x+1675=0

D. 26x-122y-1675=0

Answer: D

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3. A line $4x + 3y = 24$ cut the x-axis at point A and cut the y-axis at point B then incentre of triangle OAB is (a) $(4, 4)$ (b) $(4, 3)$ (c) $(3, 4)$ (d) $(2, 2)$

A. (3, 4)

B. (2,2)

C. (4,4)

D. (4,3)

Answer: B

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4. A point P moves on line $2x - 3y + 4 = 0$ If $Q(1, 4)$ and $R(3, -2)$ are fixed points, then the locus of the centroid of $\triangle PQR$ is a line: (a) with slope $\frac{3}{2}$ (b) parallel to y-axis (c) with slope $\frac{2}{3}$ (d) parallel to x-axis

A. parallel to x-axis

B. with slope $\frac{2}{3}$

C. with slope $\frac{3}{2}$

D. parallel to y-axis

Answer: B

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5. Two sides of a parallelogram are along the lines x+y=3 and x=y+3. If its diagonals intersect at (2, 4) , then one of its vertices is

A. (2, 6)

B. (2,1)

C. (3, 5)

D. (6,3)

Answer: D

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6. If in parallelogram $ABDC$, the coordinate of A, B and C are respectively $(1, 2)$, $(3, 4)$ and $(2, 5)$, then the equation of the diagonal AD is

A. 5x+3y-11=0

B. 3x-5y+7=0

C. 3x+5y-13=0

D. 5x-3y+1=0

Answer: D

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7. If a straight line passing through the point P(-3,4) is such that its intercepted portion between the coordinate axes is bisected a P, then its equation is

A. x-y+7=0

B. 3x-4y+25=0

C. 4x+3y=0

D. 4x-3y+24=0

Answer: D

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8. The tangent to the curve $y = x^2 - 5x + 5.$ parallel to the line $2y = 4x + 1,$ also passes through the point :

A. $\left(\frac{1}{4}, \frac{7}{2}\right)$

B. $\left(\frac{7}{2}, \frac{1}{4}\right)$

C. $\left(-\frac{1}{8}, 7\right)$

D. $\left(\frac{1}{8}, -7\right)$

Answer: D

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Chapter 4 Circle

1. 3 circles of radii `a,b,c (a

A. $(1)\left(\sqrt{a}\right) = \frac{1}{\sqrt{b}} + \frac{1}{\sqrt{c}}$

B. a, b, c are in A.P.

C. $\sqrt{a}, \sqrt{b}, \sqrt{c}$ are in A. P.

D. $\frac{1}{\sqrt{b}} = \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{c}}$

Answer: A

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2. If the circles $x^2 + y^2 - 16x - 20y + 164 = r^2$ and $(x - 4)^2 + (y - 7)^2 = 36$ intersect at two points then (a) $1 < r < 11$ (b) $r = 11$ (c) $r > 11$ (d) $0 < r < 1$

A. $0 < r < 1$

B. $1 < r < 11$

C. $r < 11$

D. $r = 11$

Answer: B

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3. If a circle C passing through $(4, 0)$ touches the circle $x^2 + y^2 + 4x - 6y - 12 = 0$ externally at a point $(1, -1)$, then the radius of the circle C is :-

A. $\sqrt{57}$

B. 4

C. $2\sqrt{5}$

D. 5

Answer: D

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4. A square is incribed in a circle $x^2 + y^2 - 6x + 8y - 103 = 0$ such that its sides are parallel to co-ordinate axis then the distance of the nearest vertex to origin, is equal to (A) 13 (B) $\sqrt{127}$ (C) $\sqrt{41}$ (D) 1

A. 13

B. $\sqrt{137}$

C. 6

D. $\sqrt{41}$

Answer: D

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5. A line $2x + y = 1$ intersect co-ordinate axis at points A and B. A circle is drawn passing through origin and point A & B. If perpendicular from point A and B are drawn on tangent to the circle at origin then sum of perpendicular distance is (A) $\frac{5}{\sqrt{2}}$ (B) $\frac{\sqrt{5}}{2}$ (C) $\frac{\sqrt{5}}{4}$ (D) $\frac{5}{2}$

A. $\frac{\sqrt{5}}{4}$

B. $\frac{\sqrt{5}}{2}$

C. $2\sqrt{5}$

D. $24\sqrt{5}$

Answer: B

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6. Two circles with equal radii are intersecting at the points (0, 1) and (0,-1). The tangent at the point (0,1) to one of the circles passes through the centre of the other circle. Then the distance between the centres of these circles is.

A. 1

B. $\sqrt{2}$

C. $2\sqrt{2}$

D. 2

Answer: D

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7. A circle cuts the chord on x-axis of length $4a$. If this circle cuts the y-axis at a point whose distance from origin is $2b$. Locus of its centre is (A) Ellipse (B) Parabola (C) Hyperbola (D) Straight line

A. A hyperbola

B. A parabola

C. A straight line

D. An ellipse

Answer: B

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8. If a variable line $3x+4y-\lambda=0$ is such that the two circles $x^2+y^2-2x-2y+1=0$ and$x^2+y^2-18x-2y+78=0$ are on its opposite sides, then the set of all values of λ is the interval (a) [12,21] (b) (2, 17) (c) (23, 31) (d) [13, 23]

A. [12,21]

B. (2, 17)

C. (23, 31)

D. [13, 23]

Answer: A

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9. Let $x^2 + y^2 - 2x - 2y - 2 = 0$ and $x^2 + y^2 - 6x - 6y + 14 = 0$ are two circles C_1, C_2 are the centre of circles and circles intersect at P, Q find the area of quadrilateral C_1PC_2Q (A) 12 (B) 6 (C) 8 (D) 4

A. 8

B. 6

C. 9

D. 4

Answer: D

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10. A circle of radius 'r' passes through the origin O and cuts the axes at A and B,Locus of the centroid of triangle OAB is

A. $\left(x^2+y^2\right)^2=4Rx^2y^2$

B. $\left(x^2+y^2\right)(x+y)=R^2xy$

C. $\left(x^2+y^2\right)^3=4R^2x^2y^2$

D. $\left(x^2+y^2\right)^2=4R^2x^2y^2$

Answer: C

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Chapter 5 Parabola

1. Equation of a common tangent to the circle $x^2+y^2-6x=0$ and the parabola $y^2=4x$ is

A. $2\sqrt{3}y=12x+1$

B. $2\sqrt{3}y = -x - 12$

C. $\sqrt{3}y = x + 3$

D. $\sqrt{3}y = 3x + 1$

Answer: C

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2. Axis of a parabola lies along x-axis. If its vertex and focus are at distances 2 and 4 respectively from the origin, on the positive x-axis then which of the following points does not lie on it?

A. (4, -4)

B. $(5, 2\sqrt{6})$

C. (8, 6)

D. $(6, 4\sqrt{2})$

Answer: C

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3. Let $A(4, -4)$ and B(9,6) be points on the parabola $y^2 = 4x$. Let C be chosen on the on the arc AOB of the parabola where O is the origin such that the area of ΔACB is maximum. Then the area (in sq. units) of ΔACB is :

A. $31\frac{3}{4}$

B. 32

C. $30\frac{1}{2}$

D. $31\frac{1}{4}$

Answer: D

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4. If $y^2 = 4b(x - c)$ and $y^2 = 8ax$ having common normal then (a, b, c) is (a) $\left(\frac{1}{2}, 2, 0\right)$ (b) $(1, 1, 3)$ (c) $(1, 1, 1)$ (d) $(1, 3, 2)$

A. (1, 1, 0)

B. $\left(\frac{1}{2}, 2, 3\right)$

C. $\left(\frac{1}{2}, 2, 0\right)$

D. (1,1,3)

Answer: D

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5. The length of the common chord of the two circles $x^2+y^2-4y=0$ and $x^2+y^2-8x-4y+11=0$ is

A. $2\sqrt{11}$

B. $3\sqrt{2}$

C. $6\sqrt{3}$

D. $8\sqrt{2}$

Answer: C

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6. If the area of the triangle whose one vertex is at the vertex of the parabola, $y^2 + 4(x - a^2) = 0$ and the other two vertices are the points of intersection of the parabola and Y-axis, is 250 sq units, then a value of 'a' is

A. $5\sqrt{5}$

B. $(10)^{2/3}$

C. $5\left(2^{1/3}\right)$

D. 5

Answer: D

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7. Let A (4, -4) and B (9, 6) be points on the parabola, $y^2 = 4x$. Let C be chosen on the are AOB of the parabola, where O is the origin, such that the area of ΔACB is maximum. Then, the area (in sq. units) of ΔACB is

A. $\dfrac{125}{4}$

B. $\dfrac{125}{2}$

C. $\dfrac{625}{4}$

D. $\dfrac{75}{2}$

Answer: A

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8. A tangent is drawn to parabola $y^2 = 8x$ which makes angle θ with positive direction of x-axis. The equation of tangent is

A. $x = y\cot\theta + 2\tan\theta$

B. $x = y\cot\theta - 2\tan\theta$

C. $y = x\tan\theta - 2\cot\theta$

D. $y = x\tan\theta + 2\cot\theta$

Answer: A

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Chapter 6 Ellipse

1. If normals are drawn to the ellipse $x^2 + 2y^2 = 2$ from the point $(2, 3)$. then the co-normal points lie on the curve

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2. Let the length of latus rectum of an ellipse with its major axis along x-axis and center at the origin, be 8. If the distance between the foci of this ellipse is equal to the length of the minor axis , then which of the following points lies on it: (a) $(4\sqrt{2}, 2\sqrt{2})$ (b) $(4\sqrt{3}, 2\sqrt{2})$ (c) $(4\sqrt{3}, 2\sqrt{3})$ (d) $(4\sqrt{2}, 2\sqrt{3})$

A. $(4\sqrt{3}, 2\sqrt{3})$

B. $(4sart(3), 2\sqrt{2})$

C. $(4\sqrt{2}, 2\sqrt{2})$

D. $(4\sqrt{2}, 2\sqrt{3})$

Answer: B

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3. Let S and S' be the foci of the ellipse and B be any one of the extremities of its minor axis. If $\Delta S'BS = 8sq.$ units, then the length of a latus rectum of the ellipse is

A. $2\sqrt{2}$

B. 2

C. 4

D. $4\sqrt{2}$

Answer: C

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Chapter 7 Hyperbola

1. If eccentricity of the hyperbola $\frac{x^2}{\cos^2\theta}-\frac{y^2}{\sin^2\theta}=1$ is move than 2 when $\theta\in\left(0,\frac{\pi}{2}\right)$. Find the possible values of length of latus rectum (a) $(3,\infty)$ (b) $1,3/2)$ (c) $(2,3)$ (d) $(-3,-2)$

A. (2,3)

B. $(3,\infty)$

C. $(3/2,2)$

D. $(1,3/2)$

Answer: B

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2. A hyperbola has its centre at the origin, passes through the point (4, 2) and has transverse axis of length 4 along the x-axis. Then the eccentricity of the hyperbola is

A. $\frac{2}{\sqrt{3}}$

B. $\frac{3}{2}$

C. $\sqrt{3}$

D. 2

Answer: A

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3. The equation of tangent to hyperbola $4x^2 - 5y^2 = 20$ which is parallel to $x - y = 2$ is (a) $x - y + 3 = 0$ (b) $x - y + 1 = 0$ (c) $x - y = 0$ (d) $x - y - 3 = 0$

A. x-y+9=0

B. x-y+7=0

C. x-y+1=0

D. x-y-3=0

Answer: C

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4. Let $S=\left\{(x, y) \in R^2: \frac{y^2}{1+r}-\frac{x^2}{1-r}=1\right\}$, where $r \neq \pm 1$. Then S represents:

A. A hyperbolawhose eccentricity is $\frac{2}{\sqrt{r+1}}$, where $0<r<1$.

B. An ellipse whose eccentricity is $\frac{1}{\sqrt{r+1}}$, $\quad$ where$r>1$

C. A hyperbola whose eccentricity is $\frac{2}{\sqrt{1-r}}$, $\quad$ where $\quad 0<r<1$.

D. An ellipse whose eccentricity is $\sqrt{\frac{2}{r+1}}$, $\quad$ where $\quad r>1$.

Answer: D

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5. Equation of a common tangent to the parabola $y^2 = 4x$ and the hyperbola xy=2 is

A. x+2y+4=0

B. x-2y+4=0

C. x+y+1=0

D. 4x+2y+1=0

Answer: A

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6. If a hyperbola has length of its conjugate axis equal to 5 and the distance between its foci is 13, then the eccentricity of the hyperbola is

A. 2

B. $\frac{13}{6}$

C. $\frac{13}{8}$

D. $\frac{13}{12}$

Answer: D

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7. If the vertices of the parabola be at $(-2, 0)$ and $(2, 0)$ and one of the foci be at $(-3, 0)$ then which one of the following points does not lie on the hyperbola? (a) $(-6, 2\sqrt{10})$ (b) $(2\sqrt{6}, 5)$ (c) $(4, \sqrt{15})$ (d) $(6, 5\sqrt{2})$

A. $(4, \sqrt{15})$

B. $(-6, 2\sqrt{10})$

C. $(6, 5\sqrt{2})$

D. $(2, \sqrt{6}, 5)$

Answer: B

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Matching Coluumn Type

1. Match the following Column I to Column II

1.

Column I	Column II
(a) In a triangle ΔXYZ, let a, b and c be the lengths of the sides opposite to the angles X, Y and Z respectively. If $2(a^2 - b^2) = c^2$ and $\lambda = \frac{\sin(X-Y)}{\sin Z}$ then possible values of n for which $\cos(n\pi\lambda) = 0$ is (are)	(p) 1
(b) In a triangle ΔXYZ, let a, b and c be the lengths of the sides opposite to the angles X, Y and Z, respectively. If $1 + \cos 2X - 2\cos 2Y = 2\sin X \sin Y$, then possible value(s) of $\frac{a}{b}$ is (are)	(q) 2
(c) In R^2, let $\sqrt{3}\hat{i} + \hat{j}, \hat{i} + \sqrt{3}\hat{j}$ and $\beta\hat{i} + (1-\beta)\hat{j}$ be the position vectors of X, Y and Z with respect of the origin O, respectively. If the distance of Z from the bisector of the acute angle of $\overrightarrow{OX}$ and $\overrightarrow{OY}$ is $\frac{3}{\sqrt{2}}$, then possible value(s) of $\lvert\beta\rvert$ is (are)	(r) 3
(d) Suppose that $F(\alpha)$ denotes the area of the region bounded by $x = 0$, $x = 2$, $y^2 = 4x$ and $y = \lvert\alpha x - 1\rvert + \lvert\alpha x - 2\rvert + \alpha x$, where $\alpha \in \{0, 1\}$. Then the value(s) of $F(\alpha) + \frac{8}{3}\sqrt{2}$, when $\alpha = 0$ and $\alpha = 1$, is (are)	(s) 5
	(t) 6

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Integer Answer Type

1. Suppose that $\vec{p}, \vec{q}$ and $\vec{r}$ are three non- coplaner in R^3 ,Let the components of a vector $\vec{s}$ along $\vec{p}, \vec{q}$ and $\vec{r}$ be 4,3, and 5, respectively , if the components this vector $\vec{s}$ along $\left(-\vec{p}+\vec{q}+\vec{r}\right), \left(\vec{p}-\vec{q}+\vec{r}\right)$ and $\left(-\vec{p}-\vec{q}+\vec{r}\right)$ are x, y and z , respectively , then the value of $2x+y+z$ is

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2. Let $\vec{a}, \vec{b},$ and $\vec{c}$ be three non coplanar unit vectors such that the angle between every pair of them is $\frac{\pi}{3}$. If $\vec{a} \times \vec{b} + \vec{b} \times \vec{x} = p\vec{a} + q\vec{b} + r\vec{c}$ where p,q,r are scalars then the value of $\frac{p^2+2q^2+r^2}{q^2}$ is

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Chapter 2 Multiple Correct Answers Type

1. Let $\overrightarrow{x}, \overrightarrow{y}$ and $\overrightarrow{z}$ be three vectors each of magnitude $\sqrt{2}$ and the angle between each pair of them is $\frac{\pi}{3}$ if $\overrightarrow{a}$ is a non-zero vector perpendicular to $\overrightarrow{x}$ and $\overrightarrow{y} \times \overrightarrow{z}$ and $\overrightarrow{b}$ is a non-zero vector perpendicular to $\overrightarrow{y}$ and $\overrightarrow{z} \times \overrightarrow{x}$, then

A. $\overrightarrow{b} = \left(\overrightarrow{b}.\overrightarrow{z}\right)\left(\overrightarrow{z} - \overrightarrow{x}\right)$

B. $\overrightarrow{a} = \left(\overrightarrow{a}.\overrightarrow{y}\right)\left(\overrightarrow{y} - \overrightarrow{z}\right)$

C. $\overrightarrow{a}.\overrightarrow{b} = -\left(\overrightarrow{a}.\overrightarrow{y}\right)\left(\overrightarrow{b}.\overrightarrow{z}\right)$

D. $\overrightarrow{a} = \left(\overrightarrow{a}.\overrightarrow{y}\right)\left(\overrightarrow{z} - \overrightarrow{y}\right)$

Answer: A::B::C

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2. Let PQR be a triangle . Let $\overrightarrow{a} = \overline{QR}, \overrightarrow{b} = \overline{RP}$ and $\overrightarrow{c} = \overline{PQ}$. if $\left|\overrightarrow{a}\right| = 12, \left|\overrightarrow{b}\right| = 4\sqrt{3}$ and $\overrightarrow{b}.\overrightarrow{c}$

then which of the following is (are) true ?

A. $\dfrac{\left|\vec{c}\right|^2}{2} - \left|\vec{a}\right| = 12$

B. $\dfrac{\left|\vec{c}\right|^2}{2} - \left|\vec{a}\right| = 30$

C. $\left|\vec{a} \times \vec{b} + \vec{c} \times \vec{a}\right| = 48\sqrt{3}$

D. $\vec{a} \, . \, \vec{b} = -72$

Answer: A::C::D

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Matching Column Type

1.

Column I	Column II
(p) Let $y(x) = \cos(3\cos^{-1}x)$, $x \in [-1, 1]$, $x \neq \pm \frac{\sqrt{3}}{2}$. Then $\frac{1}{y(x)}\left\{(x^2-1)\frac{d^2y(x)}{dx^2} + x\frac{dy(x)}{dx}\right\}$ equals	(1) 1
(q) Let $A_1, A_2, \ldots, A_n$ $(n > 2)$ be the vertices of a regular polygon of n sides with its centre at the origin. Let $\vec{a_k}$ be the position vector of the point A_k, $k = 1, 2, \ldots, n$. If $\left\lvert\sum_{k=1}^{n-1}(\vec{a_k} \times \vec{a_{k+1}})\right\rvert = \left\lvert\sum_{k=1}^{n-1}(\vec{a_k} \cdot \vec{a_{k+1}})\right\rvert$, then the minimum value of n is	(2) 2
(r) If the normal from the point $P(h, 1)$ on the ellipse $\frac{x^2}{6} + \frac{y^2}{3} = 1$ is perpendicular to the line $x + y = 8$, then the value of h is	(3) 8
(s) Number of positive solutions satisfying the equation $\tan^{-1}\left(\frac{1}{2x+1}\right) + \tan^{-1}\left(\frac{1}{4x+1}\right) = \tan^{-1}\left(\frac{2}{x^2}\right)$ is	(4) 9

A. $\begin{matrix} (p) & (q) & (r) & (s) \\ (4) & (3) & (2) & (1) \end{matrix}$

B. $\begin{matrix} (p) & (q) & (r) & (s) \\ (2) & (4) & (3) & (1) \end{matrix}$

C. $\begin{matrix} (p) & (q) & (r) & (s) \\ (4) & (3) & (1) & (2) \end{matrix}$

D. $\begin{matrix} (p) & (q) & (r) & (s) \\ (2) & (4) & (1) & (3) \end{matrix}$

Answer: A

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2.

Column I	Column II
(a) In R^2, if the magnitude of the projection vector of the vector $\alpha\hat{i} + \beta\hat{j}$ on $\sqrt{3}\hat{i} + \hat{j}$ is $\sqrt{3}$ and if $\alpha = 2 + \sqrt{3}\beta$, then possible value(s) of $\lvert\alpha\rvert$ is (are)	(p) 1
(b) Let a and b be real numbers such that the function $f(x) = \begin{cases} -3ax^2 - 2, & x < 1 \\ bx + a^2, & x \geq 1 \end{cases}$ is differentiable for all $x \in R$. Then possible value(s) of α is (are)	(q) 2
(c) Let $\omega \neq 1$ be a complex cube root of unity. If $(3 - 3\omega + 2\omega^2)^{4n+3} + (2 + 3\omega - 3\omega^2)^{4n+3} + (-3 + 2\omega + 3\omega^2)^{4n+3} = 0$, then possible value(s) of n is (are)	(r) 3
(d) Let the harmonic mean of two positive real numbers a and b be 4. If q is a positive real number such that $a, 5, q, b$ is an arithmetic progression, then the value(s) of $\lvert q - a\rvert$ is (are)	(s) 4
	(t) 5

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Chapter 3 Multiple Correct Answers Type

1. let L be a straight line passing through the origin. Suppose that all the points on L are at a constant distance from the two planes $P_1 : x + 2y - z + 1 = 0$ and $P_2 : 2x - y + z - 1 = 0$, Let M be the locus of the feet of the perpendiculars drawn from the points on L to the plane P_1. Which of the following points lie(s) on M?

A. $\left(0, -\frac{5}{6}, -\frac{2}{3}\right)$

B. $\left(-\frac{1}{6}, -\frac{1}{3}, \frac{1}{6}\right)$

C. $\left(-\frac{5}{6}, 0, \frac{1}{6}\right)$

D. $\left(-\frac{1}{3}, 0, \frac{2}{3}\right)$

Answer: A::B

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2. In R^3, consider the planes $P_1 : y = 0$ and $P_2, x + z = 1$. Let P_3 be a plane, different from P_1 and P_2 which passes through the intersection of P_1 and P_2, If the distance of the point (0,1,0) from P_3 is 1 and the distance of a point (α, β, γ) from P_3 is 2, then which of the following relation(s) is/are true?

A. $2\alpha + \beta + 2\gamma + 2 = 0$

B. $2\alpha + \beta + 2\gamma + 4 = 0$

C. $2\alpha + \beta + 2\gamma - 10 = 0$

D. $2\alpha + \beta + 2\gamma - 8 = 0$

Answer: B::D

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Chapter 2

1. Let $\overrightarrow{u} = u_1\hat{i} + u_2\hat{j} + u_3\hat{k}$ be a unit vector in R^3 and $\overrightarrow{w} = \frac{1}{\sqrt{6}}\left(\hat{i} + \hat{j} + 2\hat{k}\right)$, Given that there exists a vector $\overrightarrow{v}$ in R^3 such that $\left|\overrightarrow{u} \times \overrightarrow{v}\right| = 1$ and $\overrightarrow{w}.\left(\overrightarrow{u} \times \overrightarrow{v}\right) = 1$ which of the following statements is correct ? (a)there is exactly one choice for such $\overrightarrow{v}$ (b)there are infinitely many choices for such $\overrightarrow{v}$ (c)if $\widehat{u}$ lies in the xy - plane then $|u_1| = |u_2|$ (d)if $\widehat{u}$ lies in the xz-plane then $2|u_1| = |u_3|$

A. there is exactly one choice for such $\overrightarrow{v}$

B. there are infinitely many choices for such $\overrightarrow{v}$

C. if $\widehat{u}$ lies in the xy - plane then $|u_1| = |u_2|$

D. if $\widehat{u}$ lies in the xz-plane then $2|u_1| = |u_3|$

Answer: B::C

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Single Correct Answer Type

1. the mirror image of point $(3, 1, 7)$ with respect to the plane $x - y + z = 3$ is P. then equation plane which is passes through the point P and contains the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{1}$.

A. $x + y - 3z = 0$

B. $3x + z = 0$

C. $x - 4y + 7z = 0$

D. $2x - y = 0$

Answer: C

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Multiple Correct Answers Type

1. Consider a pyramid OPQRS located in the first octant $(x \geq 0, y \geq 0, z \geq 0)$ with O as origin and OP and OR along the X-axis and the Y-axis , respectively. The base OPQRS of the pyramid is a square with OP=3. The point S is directly above the mid point T of diagonal OQ such that TS=3. Then,

A. the acute angle between OQ and OS is $\pi / 3$

B. the equataion of the plane containing th etriangle OQS is x-y=0

C. the length of the perpendicular from P to the plane containing the triagle OQS is $\frac{3}{\sqrt{2}}$

D. the perpendcular distance from O to the straight line containing RS is $\sqrt{\frac{15}{2}}$

Answer: b.,c.,d

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Chapter 2

1. Let O be the origin and let PQR be an arbitrary triangle. The point S is such that

$$\overline{OP}\cdot\overline{OQ}+\overline{OR}\cdot\overline{OS}=\overline{OR}\cdot\overline{OP}+\overline{OQ}\cdot\overline{OS}=\overline{OQ}\cdot\overline{OR}+\overline{OP}\cdot\overline{OS}$$

Then the triangle PQR has S as its

A. centriod

B. circumectre

C. incente

D. orthocenter

Answer: D

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Linked Comprehension Type

1. Let O be the origin and$\overrightarrow{OX}, \overrightarrow{OY}, \overrightarrow{OZ}$ be three unit vector in the directions of the sides $\overrightarrow{QR}, \overrightarrow{RP}, \overrightarrow{PQ}$ respectively , of a triangle PQR.

$\left|\overrightarrow{OX} \times \overrightarrow{OY}\right| =$

A. sin (P + Q)

B. sin 2R

C. sin (P+R)

D. sin (Q+R)

Answer: A

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2. Let O be the origin, and $OXxOY, OZ$ be three unit vectors in the direction of the sides QR , RP , PQ , respectively of a triangle PQR. If the triangle PQR varies, then the minimum value of $\cos(P+Q)+\cos(Q+R)+\cos(R+P)$ is: $-\frac{3}{2}$ (b) $\frac{5}{3}$ (c) $\frac{3}{2}$ (d) $-\frac{5}{3}$

A. $-\frac{5}{3}$

B. $-\frac{3}{2}$

C. $\frac{3}{2}$

D. $\frac{5}{3}$

Answer: B

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Chapter 3

1. The equation of the plane passing through the point (1,1,1) and perpendicular to the planes $2x + y - 2z = 5$ and $3x - 6y - 2z = 7$

A. $14x + 2y + 15x = 31$

B. $14x + 2y - 15z = 1$

C. $14x + 2y + 15x = 3$

D. $14x - 2y + 15z = 27$

Answer: A

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Mcq

1. In a class 140 students numbered 1 to 140, all even numbered students opted Mathematics course, those whose number is divisible by 3 opted Physics course and those whose number is divisible by 5 opted Chemistry course. Then the number of students who did not opt for any of the three courses is (a) 38 (b) 1 (c)42 (d) 102

A. 102

B. 42

C. 1

D. 38

Answer: D

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2. Let α and β be two roots of the equation $x^2 + 2x + 2 = 0$. Then $\alpha^{15} + \beta^{15}$ is equal to

A. 512

B. -512

C. -256

D. 256

Answer: C

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3. If both the roots of the quadratic equation $x^2 - mx + 4 = 0$ are real and distinct and they lie in the interval [1, 5], then m lies in the interval

A. (4, 5]

B. (3, 4)

C. (5, 6)

D. $(-5, -4)$

Answer: A

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4. The number of all possible positive integral values of α for which the roots of the quadratic equation $6x^2 - 11x + \alpha = 0$ are rational numbers is : (a) 3 (b) 2 (c) 4 (d) 5

A. 2

B. 5

C. 3

D. 4

Answer: C

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5. Consider the quadratic equation $(c-5)x^2 - 2cx + (c-4) = 0, c \neq 5$. Let S be the set of all integral values of c for which one root of the equation lies in the interval (0, 2) and its other root lies in the interval (2, 3). Then the number of elements in S is a. 11 b. 18 c. 10 d. 12

A. 11

B. 18

C. 10

D. 12

Answer: A

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6. The values of λ such that sum of the squares of the roots of the quadratic equation $x^2 + (3 - \lambda)x + 2 = \lambda$ has the least value is

A. 2

B. $\frac{4}{9}$

C. $\frac{15}{8}$

D. 1

Answer: A

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7. If one root is cube of the other of equation $81x^2 + kx + 256 = 0$ then value of k is equal to (A) 100 (B) -300 (C) -81 (D) 400

A. 100

B. -300

C. -81

D. 400

Answer: b

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8. Let α and β be the roots of the quadratic equation x^2 sin $\theta - x(\sin\theta\cos\theta + 1) + \cos\theta = 0(0 < \theta < 45^\circ)$, and $\alpha < \beta$.

Then $\Sigma_{n=0}^{\infty}\left(\alpha^n + \frac{(-1)^n}{\beta^n}\right)$ is equal to

A. $\frac{1}{1-\cos\theta} + \frac{1}{1+\sin\theta}$

B. $\frac{1}{1+\cos\theta} + \frac{1}{1-\sin\theta}$

C. $\frac{1}{1-\cos\theta} - \frac{1}{1+\sin\theta}$

D. $\frac{1}{1+\cos\theta} - \frac{1}{1-\sin\theta}$

Answer: A

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9. If ratio of the roots of the quadratic equation $3m^2x^2 + m(m-4)x + 2 = 0$ is λ such that $\lambda + \frac{1}{\lambda} = 1$ then least value of m is (A) $-2 - 2\sqrt{3}$ (B) $-2 + 2\sqrt{3}$ (C) $4 + 3\sqrt{2}$ (D) $4 - 3\sqrt{2}$

A. $2 - \sqrt{3}$

B. $4 - 3\sqrt{2}$

C. $-2 + \sqrt{2}$

D. $4 - 2\sqrt{3}$

Answer: B

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10. The number of integral values of m for which the quadratic expression $(1+2m)x^2 - 2(1+3m)x + 4(1+m), x \in R$, is always positive is

A. 8

B. 7

C. 6

D. 3

Answer: B

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11. Let $A = \left\{\theta \in \left(-\frac{\pi}{2}, \pi\right) : \frac{3 + 2i\sin\theta}{1 - 2i\sin\theta}\right.$ is purely imaginary }

Then the sum of the elements in A is

A. $\frac{5\pi}{6}$

B. $\frac{2\pi}{3}$

C. $\frac{3\pi}{4}$

D. π

Answer: B

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12. Let Z_0 is the root of equation $x^2+x+1=0$ and $Z=3+6i(Z_0)^{81}-3i(Z_0)^{93}$ Then arg (Z) is equal to (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) π (d) $\frac{\pi}{6}$

A. $\frac{\pi}{4}$

B. $\frac{\pi}{3}$

C. 0

D. $\frac{\pi}{6}$

Answer: A

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13. Let z_1 and z_2 be any two non-zero complex numbers such that $3|z_1|=2|z_2|$. If $z=\frac{3z_1}{2z_2}+\frac{2z_2}{3z_1}$, then

A. $|z|=\frac{1}{2}\sqrt{\frac{17}{2}}$

B. Re(z) = 0

C. $|z| = \sqrt{\frac{5}{2}}$

D. Im(z) = 0

Answer: D

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14. If $z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5$, then prove that $Im(z) = 0$

A. R(z) > 0 and $I(z) > 0$

B. $R(z) < 0$ and $I(z) > 0$

C. $R(z) = -3$

D. I(z) = 0

Answer: D

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15. Let $\left(-2-\frac{1}{3}i\right)^3=\frac{x+iy}{27}(i=\sqrt{-1})$ where x and y are real numbers then y-x equals

A. 85

B. 85

C. -91

D. 91

Answer: D

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16. Let $\frac{z-\alpha}{z+\alpha}$ is purely imaginary and $|z|=2, \alpha \varepsilon R$ then α is equal to (A) 2 (B) 1 (C) $\sqrt{2}$ (D) $\sqrt{3}$

A. 1

B. 2

C. $\sqrt{2}$

D. $\frac{1}{2}$

Answer: B

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17. Let Z_1 and Z_2 be two complex numbers satisfying $|Z_1| = 9$ and $|Z_2 - 3 - 4i| = 4$. Then the minimum value of $|Z_1 - Z_2|$ is

A. (a) 0

B. (b) 1

C. (c) $\sqrt{2}$

D. (d) 2

Answer: A

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18. Consider the statement : $P(n): n^2 - n + 41$ is prime." Then, which one of the following is true?

A. P(5) is false but P(3) is true

B. Both P(3) and P(5) are false

C. P(3) is false but P(5) is true

D. Both P(3) and P(5) are true

Answer: D

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19. If a, b, c are three distinct real numbers in G.P. and $a + b + c = xb$, then prove that either $x \langle -1 \text{ or } x \rangle 3.$

A. 4

B. -3

C. -2

D. 2

Answer: D

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20. Let $a_1, a_2, \ldots, a_{30}$ be an AP, $S = \sum_{i=1}^{30} a_i$ and $T = \sum_{i=1}^{15} a_{2i-1}$ If $a_5 = 27$ and $S - 2T = 75$ then a_{10} is equal to (a) 57 (b) 42 (c) 52 (4) 47

A. 57

B. 47

C. 42

D. 52

Answer: D

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21. The sum of series

$$1+6+\frac{9\left(1^2+2^2+3^2\right)}{7}+\frac{12\left(1^2+2^2+3^2+4^2\right)}{9}+\frac{15\left(1^2+2^2+\ldots+5\right.}{11}$$

up to 15 terms is

A. 7820

B. 7830

C. 7520

D. 7510

Answer: A

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22. Let a, b and c be the 7th, 11th and 13th terms, respectively, of a non-constant A.P.. If these are also the three consecutive terms of a G.P., then $\frac{a}{c}$ is equal to

A. $1/2$

B. 4

C. 2

D. $7/13$

Answer: B

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23. The sum of all two digit positive numbers which when divided by 7 yield 2 or 5 as remainder is:

A. 1365

B. 1256

C. 1465

D. 1356

Answer: D

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24. If 5, 5r and $5r^2$ are the lengths of the sides of a triangle, then r cannot be equal to

A. $\frac{3}{2}$

B. $\frac{3}{4}$

C. $\frac{5}{4}$

D. $\frac{7}{4}$

Answer: D

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25. The sum of an infinite geometric series with positive terms is 3 and the sums of the cubes of its terms is $\frac{27}{19}$. Then the common ratio of this series is

A. $\frac{4}{9}$

B. $\frac{2}{9}$

C. $\frac{2}{3}$

D. $\frac{1}{3}$

Answer: C

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26. Let $a_1, a_2, a_3, ?\, a_{10}$are in G.P. if $\frac{a_3}{a_1} = 25$ then $\frac{a_9}{a_5}$ is equal to

(A) 5^4

(B) 4.5^4

(C) 4.5^3

(D) 5^3

A. $2(5^2)$

B. $4(5^2)$

C. 5^4

D. 5^3

Answer: C

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27. If 19^{th} term of a non-zero A.P. is zero, then $(49^{th}$ term) : $(29^{th}$ term) is

A. $3:1$

B. $4:1$

C. $2:1$

D. $1:3$

Answer: A

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28. The product of three consecutive terms of a GP is 512. If 4 is added to each of the first and the second of these terms, the three terms now form an AP. Then the sum of the original three terms of the given GP is: (a) 36 (b) 32 (c) 24 (d) 28

A. 36

B. 24

C. 32

D. 28

Answer: D

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29. Let $S_k = \dfrac{1+2+3+\ldots+k}{k}$. If $S_1^2 + s_2^2 + \ldots + S_{10}^2 = \dfrac{5}{12}A$, then A is equal to

A. 303

B. 283

C. 156

D. 301

Answer: A

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30. If the sum of the first 15 terms of the series $\left(\frac{3}{4}\right)^3+\left(1\frac{1}{2}\right)^3+\left(2\frac{1}{4}\right)^3+3^3+\left(3\frac{3}{4}\right)^3+\ldots$ is equal to 225k, then k is equal to

A. 9

B. 27

C. 108

D. 54

Answer: B

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31. Let x, y be positive real numbers and m, n be positive integers, The maximum value of the expression

$$\frac{x^m y^n}{(1+x^{2m})(1+y^{2n})}$$ is

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32. Consider a class of 5 girls and 7 boys. The number of different teams consisting of 2 girls and 3 boys that can be formed from this class, if there are two specific boys A and B, who refuse to be the members of the same team, is

A. (a) 200

B. (b) 300

C. (c) 500

D. (d) 350

Answer: B

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33. Let S be the set of all triangles in the xy-plane, each having one vertex at the origin and the other two vertices lie on coordinate axes with integral coordinates. If each triangle in S has area 50 eq. units, then the number of elements in the set S is

A. 9

B. 18

C. 32

D. 36

Answer: D

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34. The number of natural numbers less than 7,000 which can be formed by using the digits 0, 1, 3, 7, 9 (repetition of digits allowed) is equal to

A. 250

B. 374

C. 372

D. 375

Answer: B

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35. If set $A = \{1, 2, 3, \ ?2, \}$, then the find the number of onto functions from A to A such that $f(k)$ is a multiple of 3, whenever k is a multiple of 4. (A) $6^5 \times 15!$ (B) $5^6 \times 15!$ (C) $6! \times 5!$ (D) $6! \times 15!$

A. $(15)! \times 6!$

B. $5^6 \times 15$

C. $5! Xx6!$

D. $6^5 \times (15)!$

Answer: A

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36. Let $S = \{1, 2, 3, \ldots, 100\}$. The number of non-empty subsets A to S such that the product of elements in A is even is

A. $2^{50}(2^{50} - 1)$

B. $2^{100} - 1$

C. $2^{50} - 1$

D. $2^{50} + 1$

Answer: A

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37. Consider three boxes, each containing 10 balls labelled 1, 2, ...,10. Suppose one ball is randomly drawn from each of the boxes. Denote by n_i ,the label of the ball drawn from the i^{th} box, $(i = 1, 2, 3)$. Then, the number of ways in which the balls can be chosen such that $n_1 < n_2 < n_3$ is :

A. 82

B. 240

C. 164

D. 120

Answer: D

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38. Let Z be the set of integers. If A = $\left\{x \in Z : 2^{(x+2)\,(x^2-5x+6)} = 1\right.$ and $B = \{x \in Z : -3 < 2x - 1 < 9\}$, then the number of subsets of the set A $\times$ B is

A. 2^{18}

B. 2^{10}

C. 2^{15}

D. 2^{12}

Answer: C

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39. There are m men and two women participating in a chess tournament. Each participant plays two games with every other participant. If the number of games played by the men between themselves exceeds the number of games played between the men and the women by 84, then teh value of m is

A. 9

B. 11

C. 12

D. 7

Answer: C

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40. If the fractional part of the number $\frac{2^{403}}{15}$ is $\frac{k}{15}$ then k is equal to

A. 14

B. 6

C. 4

D. 8

Answer: D

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41. The coefficient of t^4 in $\left(\frac{1-t^6}{1-t}\right)^3$ (a) 18 (b) 12 (c) 9 (d) 15

A. 12

B. 15

C. 10

D. 14

Answer: B

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42. If $\Sigma_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{20}C_i + {}^{20}C_{i-1}}\right)^3 = \frac{k}{21}$, then k equals

A. 200

B. 50

C. 100

D. 400

Answer: C

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43. If the third term in expansion of $\left(1 + x^{\log_2 x}\right)^5$ is 2560 then x is equal to (a) $2\sqrt{2}$ (b) $\frac{1}{8}$ (c) $\frac{1}{4}$ (d) $4\sqrt{2}$

A. $2\sqrt{2}$

B. $\frac{1}{8}$

C. $4\sqrt{2}$

D. $\frac{1}{4}$

Answer: D

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44. The positive value of λ for which the coefficient of x^2 in the expression $x^2\left(\sqrt{x} + \frac{\lambda}{x^2}\right)^{10}$ is 720 is

A. $\sqrt{5}$

B. 4

C. $2\sqrt{2}$

D. 3

Answer: B

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45. If $\Sigma_{r=0}^{25}\left({}^{50}C_r\, {}^{50-r}C_{25-r}\right) = K\left({}^{50}C_{25}\right)$, then K is equal to

A. $2^{25}-1$

B. $(25)^2$

C. 2^{25}

D. 2^{24}

Answer: C

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46. If the middle term of the expansion of $\left(\frac{x^3}{3}+\frac{3}{x}\right)^8$ is 5670 then sum of all real values of x is equal to (A) 6 (B) 3 (C) 0 (D) 2

A. 6

B. 8

C. 0

D. 4

Answer: C

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47. The value of r for which

$.^{20}C_r, .^{20}C_{r-1}.^{20}C_1+.^{20}C_2+\ldots\ldots+.^{20}C_0.^{20}C_r$ is maximum, is

A. 20

B. 15

C. 11

D. 10

Answer: A

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48. Let $(x+10)^{50}+(x-10)^{50}=a_0+a_1x+a_2x^2+\ldots+a_{50}x^{50}$ for all $x \in R$, then $\frac{a_2}{a_0}$ is equal to

A. 12.5

B. 12

C. 12.75

D. 12.25

Answer: D

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49. Let $S_n = 1 + q + q^2 + \dots + q^n$ and $T_n = 1 + \left(\frac{q+1}{2}\right) + \left(\frac{q+1}{2}\right)^2 + \dots \left(\frac{q+1}{2}\right)^n$ If $\alpha T_{100} = {}^{101}C_1 + {}^{101}C_2$ x $S_1 \dots + {}^{101}C_{101}$ xS_{100}, then the value of α is equal to (A) 2^{99} (B) 2^{101} (C) 2^{100} (D) -2^{100}

A. 2^{100}

B. 200

C. 2^{99}

D. 202

Answer: A

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50. Ratio of the 5^{th} term from the beginning to the 5^{th} term from the end in the binomial expansion of $\left(2^{1/3} + \frac{1}{2(3)^{1/3}}\right)^{10}$ is

A. $1 : 4(16)^{\frac{1}{3}}$

B. $1 : 2(6)^{\frac{1}{3}}$

C. $2(36)^{\frac{1}{3}} : 1$

D. $4(36)^{\frac{1}{3}} : 1$

Answer: D

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51. If ${}^{n}C_4, {}^{n}C_5$ and ${}^{n}C_6$ are in A.P. then the value of n is

A. 14

B. 11

C. 9

D. 12

Answer: A

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52. Number of irrational terms in expansion of $\left(2^{\frac{1}{5}} + 3^{\frac{1}{10}}\right)^{60}$ is

A. 55

B. 49

C. 48

D. 54

Answer: D

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53. Two integers are selected at random from the set {1, 2, ..., 11}. Given that the sum of selected numbers is even, the conditional probability that both the numbers are even is

A. $\frac{2}{5}$

B. $\frac{1}{2}$

C. $\frac{3}{5}$

D. $\frac{7}{10}$

Answer: A

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54. Let $S = \{1, 2, \ldots, 20\}$ A subset B of S is said to be nice, if the sum of the elements of B is 203. Then the probability that a randomly chosen subset of S is nice is: (a) $\frac{7}{2^{20}}$ (b) $\frac{5}{2^{20}}$ (c) $\frac{4}{2^{20}}$ (d) $\frac{6}{2^{20}}$

A. $\frac{6}{2^{20}}$

B. $\frac{5}{2^{20}}$

C. $\frac{4}{2^{20}}$

D. $\frac{7}{2^{20}}$

Answer: B

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55. In a class of 60 students, 40 opted for NCC, 30 opted for NSS and 20 opted for both NCC and NSS. If one of these students is selected at random, then the probability that the student selected has opted neither for NCC nor for NSS is

A. $\frac{2}{3}$

B. $\frac{1}{6}$

C. $\frac{1}{3}$

D. $\frac{5}{6}$

Answer: B

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56. In a game, a man wins Rs 100 if he gets 5 or 6 on a throw of a fair die and loses Rs 50 for getting any other number on the die. If he decides to throw the die either till he gets a five or a six or to a maximum of three throws, then his expected gain/loss (in rupees) is: (a) $\frac{400}{3}$ gain (b) $\frac{400}{9}$ loss (c) 0 (d) $\frac{400}{3}$ loss

A. $\frac{400}{3}$ gain

B. $\frac{400}{3}$ loss

C. 0

D. $\frac{400}{9}$ loss

Answer: C

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57. If the Boolean expression $(p \oplus q) \wedge (\sim p \Theta q)$ is equivalent to $p \wedge q$, where $\oplus, \Theta \in \{\vee, \wedge\}$, then the ordered pair $(\oplus, \Theta)$` is

A. $(\wedge, \vee)$

B. $(\vee, \vee)$

C. $(\wedge, \wedge)$

D. $(\vee, \wedge)$

Answer: A

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58. The logical statement $[\sim(\sim p \vee q) \vee (p \wedge r)] \wedge (\sim q \wedge r)$ is equivalent to (a) $(\sim p \wedge \sim q) \wedge r$ (b) $\sim p \vee r$ (c) $(p \wedge r) \wedge \sim q$ (d) $(p \wedge \sim q) \vee r$

A. $(p \wedge r) \wedge \sim q$

B. $(\sim p \wedge \sim q) \wedge r$

C. $\sim p \vee r$

D. $(p \wedge \sim q) \vee r$

Answer: A

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59. Given three statements P: 5 is a prime number, Q:7 is a factor of 192, R:The LCM of 5 & 7 is 35 Then which of the following statements are true (a) $Pv(\sim Q \wedge R)$ (b) $\sim P \wedge (\sim Q \wedge R)$ (c) $(PvQ) \wedge \sim R$ (d) $\sim P \wedge (\sim Q \wedge R)$

A. $(p \wedge q) \vee (\sim r)$

B. $(\sim p) \wedge (\sim q \wedge r)$

C. $(\sim p) \vee (q \wedge r)$

D. $p \vee (\sim q \wedge r)$

Answer: D

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60. If q is false and $(p \wedge q)$harr r $isalsotruethenwhichofthefollow \in gare\tau \rightarrow \log y(A)$(pvvr)-gt(p^^r) (B)(pvvr)(C)(p^^r)-gt(pvvr)(D)p^^r`

A. $(p \vee r) \rightarrow (p \wedge r)$

B. $p \vee r$

C. $p \wedge r$

D. $(p \wedge r) \rightarrow (p \vee r)$

Answer: D

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61. The Boolean expression $\sim(p \vee q) \vee (\sim p \wedge q)$ is equivalent to (1) $\sim p$ (2) p (3) q (4) $\sim q$

A. $p \wedge (\sim q)$

B. $p \vee (\sim q)$

C. $(\sim p) \wedge (\sim q)$

D. $p \wedge q$

Answer: C

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62. $(\sim p \vee \sim q)$ is logically equivalent to

A. $\sim p \wedge \sim q$

B. $p \wedge q$

C. $\sim(p \wedge q)$

D. $p \wedge \sim q$

Answer: A

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63. Average height & variance of 5 students in a class is 150 and 18 respectively. A new student whose height is $156cm$ is added to the group. Find new variance. (a) 20 (b) 22 (c) 16 (d) 14

A. 22

B. 20

C. 16

D. 18

Answer: B

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64. A data consists of n observations $x_1, x_2, \ldots, x_n$. $If \Sigma_{i=1}^{n}(x_i+1)^2 = 9n$ and $\Sigma_{i=1}^{n}(x_i-1)^2 = 5n$, then the standard deviation of this data is

A. 5

B. $\sqrt{5}$

C. $\sqrt{7}$

D. 2

Answer: B

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65. The mean of five observations is 5 and their variance is 9.20. If three of the given five observations are 1,3 and 8, then a ratio of other two observations is

A. $4 : 9$

B. $6 : 7$

C. $5 : 8$

D. $10 : 3$

Answer: A

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66. The mean and standart deviation of five observations x_1, x_2, x_3, x_4, x_5 and are 10 and 3 respectively, then variance of the observation $x_1, x_2, x_3, x_4, x_5, \; - 50$ is equal to (a) 437.5 (b) 507.5 (c) 537.5 (d) 487.5

A. 582.5

B. 507.5

C. 586.5

D. 509.5

Answer: B

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67. The outcome of each of 30 items was observed , 10 items gave an outcome $\frac{1}{2} - d$ each, 10 items gave outcome $\frac{1}{2}$ each and the remaining 10 items gave outcome $\frac{1}{2} + d$ each. If the variance of this outcome data is $\frac{4}{3}$, then |d| equals

A. 2

B. $\frac{\sqrt{5}}{2}$

C. $\frac{2}{3}$

D. $\sqrt{2}$

Answer: D

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68. Contrapositive of the statement "If two numbers are not equal, then their squares are not equal." is

A. If the squares of two numbers are equal, then the numbers are equal.

B. If the squares of two numbers are equal, then the numbers are not equal.

C. If the squares of two numbers are not equal, then the numbers are equal

D. If the squares of two numbers are not equal, then the numbers are not equal.

Answer: A

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69. There are 30 white balls and 10 red balls in bag. 16 balls are drawn with replacement from the bag. If X be the number of white balls drawn then the value of $\frac{mean(X)}{s\tan darddeviation(X)}$ is equal to (A) $4\sqrt{3}$ (B) $2\sqrt{3}$ (C) $3\sqrt{3}$ (D) $3\sqrt{2}$

A. 4

B. $\frac{4\sqrt{3}}{3}$

C. $4\sqrt{3}$

D. $3\sqrt{2}$

Answer: C

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70. If the sum of the deviations of 50 observations from 30 is 50, then the mean of these observations is

A. 50

B. 51

C. 30

D. 31

Answer: D

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71. Mean and variance of five observations are 4 and 5.2 respectively. If three of these observations are $3, 4, 4$ then find absolute difference between the other two observations (A) 3 (B) 7 (C) 2 (D) 5

A. 1

B. 3

C. 7

D. 5

Answer: C

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72. The system of linear equations

x + y + z = 2

2x + 3y + 2z = 5

$2x + 3y + (a^2 - 1)z = a + 1$

A. has infinitely many solutions for a = 4

B. is inconsisten when $|a| = \sqrt{3}$

C. is inconsistent when a = 4

D. has a unique solution for $|a| = \sqrt{3}$

Answer: B

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73. If the system of linear equations $x - 4y + 7z = g,\ 3y - 5z = h,$ $-2x + 5y - 9z = k$ is consistent, then

A. g + h + k = 0

B. 2g + h + k = 0

C. g + h + 2k = 0

D. g + 2h + k = 0

Answer: B

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74. If the system fo equations

x+y+z = 5

x + 2y + 3z = 9

$x + 3y + \alpha z = \beta$

has infinitely many solution, then $\beta - \alpha$ equals

A. 5

B. 18

C. 21

D. 8

Answer: D

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75. Let $a_1, a_2, a_3, \ldots, a_{10}$ be in G.P. with $a_i > 0$ for i=1, 2, ..., 10 and S be te set of pairs (r, k), r, k $\in$ N (the set of natural numbers) for which $\begin{vmatrix} \log_e a_1^r a_2^k & \log_e a_2^r a_3^k & \log_e a_3^r a_4^k \\ \log_e a_4^r a_5^k & \log_e a_5^r a_6^k & \log_e a_6^r a_7^k \\ \log_e a_7^r a_8^k & \log_e a_8^r a_9^k & \log_e a_9^r a_{10}^k \end{vmatrix}$ = 0. Then the number of elements in S is

A. Infinitely many

B. 4

C. 10

D. 2

Answer: A

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76. If the system of linear equations

$2x+2y+3z=a$

$3x-y+5z=b$

$x-3y+2z=c$

where a,b and c are non-zero real numbers, has more than one solution, then

A. $b - c - a = 0$

B. $a + b + c = 0$

C. $b + c - a = 0$

D. $b - c + a = 0$.

Answer: A

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77. prove that $\begin{bmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{bmatrix}$ = $(a+b+c)^3$

A. $-(a+b+c)$

B. $2(a+b+c)$

C. abc

D. $-2(a+b+c)$

Answer: D

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78. An ordered pair (α, β) for which the system of linear equations $(1+\alpha)x+\beta y+z=2, \alpha x+(1+\beta)y+z=3$ and $\alpha x+\beta y+2z=2$ has unique solution is: (a) (2,4) (b) (-3,1) (c) (-4,2) (d) (1,-3)

A. (1, -3)

B. (-3, 1)

C. (2, 4)

D. (-4, 2)

Answer: C

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79. The set of all values of λ for which the system of linear equations

$x - 2y - 2z = \lambda x$

$x + 2y + z = \lambda y$

$-x - y = \lambda z$

has a non-trivial solution

A. contains more than two elements

B. is a singleton

C. is an empty set

D. contains exactly two elements

Answer: B

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80. If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then the matrix A^{-50}, when $\theta = \frac{\pi}{12}$, is equal to

A. $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

B. $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

C. $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

D. $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

Answer: A

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81. Matrix $= \begin{bmatrix} e^t & e^{-t}(\sin t - 2\cos t) & e^{-t}(-2\sin t - \cos t) \\ e^t & -e^{-t}(2\sin t + \cos t) & e^{-t}(\sin t - 2\cos t) \\ e^t & e^t \cos t & e^{-t}\sin t \end{bmatrix}$ is invertible. (a) only if $t = \frac{\pi}{2}$ (b) only $t = \pi$ (c) $t \varepsilon R$ (d) $t \notin R$

A. invertible only if $t = \frac{\pi}{2}$

B. not invertible for any $t \in R$

C. invertible for all $t \in R$

D. invertible only if $t = \pi$

Answer: C

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82. Let $d \in R$ and $A = \begin{pmatrix} -2 & 4+d & \sin\theta - 2 \\ 1 & \sin\theta + 2 & d \\ 5 & 2\sin\theta - d & (-\sin\theta) + 2 + 2d \end{pmatrix}$ where $\theta \in [0, \pi]$. If the minimum value of $\det(A)$ is 8, then the value of d is (a) -7 (b) -5 (c) $2(\sqrt{2}+1)$ (d) $2(\sqrt{2}+2)$

A. -7

B. $2(\sqrt{2}+2)$

C. -5

D. $2(\sqrt{2}+1)$

Answer: C

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83. Let $A=\begin{bmatrix}2 & b & 1 \\ b & b^2+1 & b \\ 1 & b & 2\end{bmatrix}$ where $b>0$. Then the minimum value of $\dfrac{\det.(\mathrm{A})}{b}$ is

A. $\sqrt{3}$

B. $-\sqrt{3}$

C. $-2\sqrt{3}$

D. $2\sqrt{3}$

Answer: D

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84. Let $A = \begin{bmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{bmatrix}$ If $AA^T = I_3$ then $|p| =$

A. $\frac{1}{\sqrt{2}}$

B. $\frac{1}{\sqrt{5}}$

C. $\frac{1}{\sqrt{6}}$

D. $\frac{1}{\sqrt{3}}$

Answer: A

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85. Let A and B be two invertible matrices of order 3×3. If det. (ABA^T) = 8 and det. (AB^{-1}) = 8, then det. $(BA^{-1}B^T)$ is equal to

A. 16

B. $\frac{1}{16}$

C. $\frac{1}{4}$

D. 1

Answer: B

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86. Let $P = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix}$ and I be the identity matrix of order 3. If $Q = [qij]$ is a matrix, such that $P^{50} - Q = I$, then $\frac{q_{31} + q_{32}}{q_{21}}$ equals

A. 52

B. 103

C. 201

D. 205

Answer: B

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87. If $A = \begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$, then for all $\theta \in \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right)$, det. (A) lies in the interval

A. $\left[\frac{5}{2}, 4\right)$

B. $\left(\frac{3}{2}, 3\right]$

C. $\left(0, \frac{3}{2}\right]$

D. $\left(1, \frac{5}{2}\right]$

Answer: B

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88. Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Let X denote the random variable of number of aces obtained in the two drawn cards. Then P(X = 1) + P(X = 2) equals

A. $52/169$

B. $25/169$

C. $49/169$

D. $24/169$

Answer: B

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89. An urn contains 5 red and 2 green balls. A ball is drawn at random from the urn. If the drawn ball is green, then a red ball is added to the urn and if the drawn ball is red, then a green ball is added to the urn, the original ball is not returned to the urn. Now, a second ball is drawn at random from it. The probability that the second ball is red is

A. $\frac{26}{49}$

B. $\frac{32}{49}$

C. $\frac{27}{49}$

D. $\frac{21}{49}$

Answer: B

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90. An unbiased coin is tossed. If the result is a head, a pair of unbiased dice is rolled and the number obtained by adding the numbers on two faces is noted. If the result is a tail, a card from a well-shuffled pack of 11 cards numbered 2, 3, 4, ..., 12 is picked and the number on the card is noted. What is the probability that the noted number is either 7 or 8?

A. $\frac{13}{36}$

B. $\frac{19}{36}$

C. $\frac{19}{72}$

D. $\frac{15}{72}$

Answer: C

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91. If the probability of hitting a target by a shooter, in any shot is 1/3, then the minimum number of independent shots at the target required by him so that the probability of hitting the target at least once is greater than $\frac{5}{6}$ is

A. 6

B. 5

C. 4

D. 3

Answer: B

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92. In a random experiment, a fair die is rolled until two fours are obtained in succession.The probability that the experiment will end in the fifth throw of the die is equal to

A. $\frac{150}{6^5}$

B. $\frac{175}{6^5}$

C. $\frac{200}{6^5}$

D. $\frac{225}{6^5}$

Answer: B

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Chapter 1

1. For $x \in \mathbb{R} - \{0, 1\}$, let $f_1(x) = \frac{1}{x}$, $f_2(x) = 1 - x$ and $f_3(x) = \frac{1}{1 - x}$ be three given functions. If a function, $J(x)$ satisfies $(f_2 o J_o f_1)(x) = f_3(x)$ then $J(x)$ is equal to :

A. $f_3(x)$

B. $f_1(x)$

C. $f_2(x)$

D. $\frac{1}{x}f_3(x)$

Answer: A

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2. Let $A = \{\xi nR : x \text{ is not a positive integer }\}$define a function $f: A \rightarrow R \text{ such that } f(x) = \frac{2x}{x-1}$. Then f is

A. injective but not surjective

B. not injective

C. surjective but not injective

D. neither inhective nor surjective

Answer: A

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3. Let N be the set of natural numbers and two functions f and g be defined as f, g : $N \to N$ such that : $f(n) = \begin{cases} \frac{n+1}{2} & \text{if n is odd} \\ \frac{n}{2} & \text{if n is even} \end{cases}$ and $g(n) = n - (-1)^n$. The fog is :

A. both one-one and onto

B. one-one but not onto

C. neither one-one nor onto

D. onto but not one-one

Answer: D

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4. Let $f: R \to R$ be defined by $f(x) = \frac{x}{1+x^2}, x \in R$. Then the range of f is

A. (-1,1)-{0}

B. $\left[-\frac{1}{2}, \frac{1}{2}\right]$

C. $R - \left[-\frac{1}{2}, \frac{1}{2}\right]$

D. R-[-1,1]

Answer: B

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5. Let a function $f:(0,\infty) \rightarrow [0,\infty)$ be defined by $f(x) = \left|1 - \frac{1}{x}\right|$.

Then f is

A. injective only

B. not injective but it is surjective

C. both injective nor surjective

D. injective only

Answer: B

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1. $\lim_{y \to 0} \frac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4}$

A. exists and equals $\frac{1}{4\sqrt{2}}$

B. does not exist

C. exists and equals $\frac{1}{2\sqrt{2}}$

D. exists and equals $\frac{1}{2\sqrt{2}\left(\sqrt{2}+1\right)}$

Answer: A

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2. For each $x \in R$, let [x]be the greatest integer less than or equal to x.

Then $\lim_{x \to 1^+} \frac{x([x]+|x|)\sin[x]}{|x|}$ is equal to

A. $-2\sin 1$

B. 0

C. 1

D. 2sin 1

Answer: A

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3. For each t $\in R$,let[t]be the greatest integer less than or equal to t. Then

$$\lim_{x \to 1^+} \frac{(1 - |x| + \sin|1 - x|)\sin\left(\frac{\pi}{2}[1 - x]\right)}{|1 - x|[1 - x]}$$

A. equals-1

B. equals 1

C. does not exist

D. equals 0

Answer: D

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4. let [x] denote the greatest integer less than or equal to x.

Then $\lim_{x \to 0} \frac{\tan\left(\pi \sin^2 x\right) + (|.|x - \sin(x[x]))^2}{x^2}$

A. equals π

B. equals 0

C. equals $\pi + 1$

D. does not exist

Answer: D

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5. $\lim_{x \to 0} \frac{x \cot(4x)}{\sin^2 x \cot^2(2x)}$ is equal to

A. 2

B. 0

C. 4

D. 1

Answer: D

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6. $\lim_{x \to \frac{\pi}{4}} \frac{\cot^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$ is

A. 4

B. $8\sqrt{2}$

C. 8

D. $4\sqrt{2}$

Answer: C

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7. $\lim_{x \to 1^-} \frac{\sqrt{\pi} - \sqrt{2\sin^{-1}x}}{\sqrt{1-x}}$ is equal to

A. $\frac{1}{\sqrt{2\pi}}$

B. $\frac{\sqrt{\pi}}{2}$

C. $\sqrt{\frac{2}{\pi}}$

D. $\sqrt{\pi}$

Answer: C

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Chapter 3

1. If x = 3 tant and y = 3 sec t, then the value of $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$ is

A. $\frac{3}{2\sqrt{2}}$

B. $\frac{1}{3\sqrt{2}}$

C. $\frac{1}{6}$

D. $\frac{1}{6\sqrt{2}}$

Answer: D

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2. Let $f: R \rightarrow R$ be a function such that $f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3), x \in R$. Then f(2) equals

A. 8

B. -2

C. -4

D. 30

Answer: B

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3. If $x\log_e(\log_e x) - x^2 + y^2 = 4 (y > 0)$, then dy/dx at $x = e$ is equal to

A. $\dfrac{e}{4+e^2}$

B. $\dfrac{(1+2e)}{2\sqrt{4+e^2}}$

C. $\dfrac{(2e-1)}{2\sqrt{4+e^2}}$

D. $\dfrac{(1+2e)}{\sqrt{4+e^2}}$

Answer: C

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4. for $x > 1$ if $(2x)^{2y} = 4e^{2x-2y}$ then $(1+\log_e 2x)^2 \dfrac{dy}{dx}$

A. $\log_e 2x$

B. $\dfrac{x\log_e 2x + \log_e 2}{x}$

C. $x\log_e 2x$

D. $\dfrac{x\log_e 2x - \log_e 2}{x}$

Answer: D

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5. Let f be a differentiable function such that f(1) = 2 and f'(x) = f (x) for all $x \in R$. If h(x)=f(f(x)), then h'(1) is equal to

A. 4e

B. $4e^2$

C. 2e

D. $2e^2$

Answer: A

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Chapter 4

1. $f(x) = \begin{cases} 5 & x \leq 1 \\ a+bx & 1 < x < 3 \\ b+5x & 3 \leq x < 5 \\ 30 & x \geq 5 \end{cases}$ then (a) $f(x)$ is discontiuous $\forall$ a $\in R, b \in R$ (b) $f(x)$ is discontiuous if $a = 0$ $\&b = 5$ (c) $f(x)$ is discontiuous if $a = 5$ $\&b = 0$ (d) $f(x)$ is discontiuous if $a = -5$ & $b = 10$

A. continuous if a =5 and b = 5

B. continuous if a =-5 and b = 10

C. continuous if a = 0 and b = 5

D. not continuous for any values of a and b

Answer: D

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2. Let $f(x)\begin{cases} \max . \{|x|, x^2\}, & |x| \leq 2 \\ 8-2|x|, & 2 < |x| \leq 4 \end{cases}$ .Let S be the set of points in the intercal (-4,4) at which f is not differentible. Then S

A. is an empty set

B. equals {-2,-1,1,2}

C. equals {-2,-1,0,1,2}

D. equals {-2,2}

Answer: C

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3. Let $f:(-1,1) \to R$ be a function defined by $f(x) = \max\left\{-|x|, -\sqrt{1-x^2}\right\}$. If K be the set of all points at which f is not differentiable, then K has exactly :

A. three elements

B. one element

C. five elements

D. two elements

Answer: A

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4. Let $f(x) = \begin{cases} - & 1 & -2 \leq x < 0 \\ x^2 & -1 & 0 \leq x < 2 \end{cases}$ if $g(x) = |f(x)| + f(|x|)$ then $g(x)$ in $(-2, 2)$ is

(A) not continuous is (B) not differential at one point (C) differential at all points (D) not differential at two points

A. Differentiable at all points

B. not differentiable at two points

C. Not continuous

D. not differentiable at one point

Answer: D

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5. Let K be the set of all values of x, where the function $f(x) = \sin|x| - |x| + 2(x - \pi)\cos|x|$ is not differentiable.

Then, the set K is equal to

A. $\{\pi\}$

B. $\{0\}$

C. ϕ(an empty set)

D. $\{0, \pi\}$

Answer: C

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6. Let S be the set of all points in $(-\pi, \pi)$ at which the f(x)=min(sinx ,cosx) is not differentiable Then, S is a subset of which of the following?

A. $\left\{-\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{4}\right\}$

B. $\left\{-\frac{3\pi}{4}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{4}\right\}$

C. $\left\{-\frac{\pi}{2}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{\pi}{2}\right\}$

D. $\left\{-\frac{\pi}{4}, 0, \frac{\pi}{4}\right\}$

Answer: A

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Chapter 5

1. if θ denotes the acute angle between the curves, $y = 10 - x^2$ and $y = 2 + x^2$ at a point of their intersection, then $|\tan\theta|$ is equal to

A. $4/9$

B. $7/17$

C. $8/17$

D. $8/15$

Answer: D

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2. The tangent to the curve $y = xe^{x^2}$ passing through the point (1,e) also passes through the point

A. $\left(\frac{4}{3}, 2e\right)$

B. (2, 3e)

C. $\left(\frac{5}{3}, 2e\right)$

D. (3, 6e)

Answer: A

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3. A helicopter flying along the path $y = 7 + x^{\frac{3}{2}}$, A soldier standint at point $\left(\frac{1}{2}, 7\right)$ wants to hit the helicopter when it is closest from him, then minimum distance is equal to

A. a. $\frac{1}{2}$

B. b. $\frac{1}{3}\sqrt{\frac{7}{3}}$

C. c. $\frac{1}{6}\sqrt{\frac{7}{3}}$

D. d. $\frac{\sqrt{5}}{6}$

Answer: C

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Chapter 6

1. The maximum volume (in cu.m) of the right circular cone having slant height 3 m is

A. $3\sqrt{3}\pi$

B. 6π

C. $2\sqrt{3}\pi$

D. $\frac{4}{3}\pi$

Answer: C

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2. The shortest distance between the point $\left(\frac{3}{2}, 0\right)$ and the curve $y = \sqrt{x}, (x > 0)$, is

A. $\frac{\sqrt{5}}{2}$

B. $\frac{5}{4}$

C. $\frac{3}{2}$

D. $\frac{\sqrt{3}}{2}$

Answer: A

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3. If x satisfies the condition $f(x)=\{x : x^2+30 \le 11x\}$ then maximum value of function $f(x)=3x^3-18x^2-27x-40$ is equal to (A) -122 (B) 122 (C) 222 (D) -222

A. 122

B. -222

C. -122

D. 222

Answer: A

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4. Let $f(x)=\dfrac{x}{\sqrt{a^2+x^2}}-\dfrac{d-x}{\sqrt{b^2+(d-x)^2}}, x \in R,$ where a, b and d are non-zero real constants. Then,

A. f is a decreasing function of x

B. f is neither increasing nor decreasing function of x

C. f' is not a continuous function of x

D. f is an increasing function of x

Answer: D

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5. Let a parabola be $y = 12 - x^2$. Find the maximum area of rectangle whose base lie on x-axis and two points lie on parabola. (A) 8 (B) 4 (C) 32 (D) 34

A. $20\sqrt{2}$

B. $18\sqrt{2}$

C. 32

D. 36

Answer: C

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6. Let $f(x) = x^3 - 3(a-2)x^2 + 3ax + 7$ and $f(x)$ is increasing in $(0, 1]$ and decreasing is $[1, 5)$, then roots of the equation $\frac{f(x) - 14}{(x-1)^2} = 0$ is (A) 1 (B) 3 (C) 7 (D) -2

A. 6

B. 5

C. 7

D. -7

Answer: C

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Chapter 7

1. if $x^2 \neq n\pi + 1$, ninN$then$int x sqrt((2sin(x^(2)-1)-sin2(x^(2)-1))/(2sin(x^(2)-1)+sin2(x^(2)-1))) dx$isequal \to (a)$In cos ((x^(2)-1)/(2))+c(b)(1)/(2)In cos ((x^(2)-1)/(2))+c(c)In sec ((x^(2)-1)/(2))+c(d)

(1)/(2)In sec ((x^(2)-1)/(2))+c`

A. $\log_e\left|\left(\sec.\ \frac{x^2-1}{2}\right)\right|+c$

B. $\log_e\left|\frac{1}{2}\sec^2.\ (x^2-1)\right|+c$

C. $\frac{1}{2}\log_e\left|\sec^2.\ \left(\frac{x^2-1}{2}\right)\right|+c$

D. $\frac{1}{2}\log_e\left|\sec.\ (x^2-1)\right|+c$

Answer: A

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2. If $f(x)=\int\frac{5x^8+7x^6}{(x^2+1+2x^7)^2}dx,\ (x\geq 0)$, and f(0) = 0, then the value of f(1) is

A. $-\frac{1}{2}$

B. $\frac{1}{2}$

C. $-\frac{1}{4}$

D. $\frac{1}{4}$

Answer: D

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3. Let $n \geq 2$ be a natural number and $0 < \theta < \frac{\pi}{2}$, Then, $\int \frac{(\sin^n \theta - \sin \theta)^{\frac{1}{n}} \cos \theta}{\sin^{n+1} \theta} d\theta$ is equal to (where C is a constant of integration)

A. $\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n+1}\theta}\right)^{\frac{n+1}{n}}+C$

B. $\frac{n}{n^2+1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$

C. $\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$

D. $\frac{n}{n^2-1}\left(1+\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$

Answer: C

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4. If $\int x^5 e^{-4x^3} dx = \frac{1}{48} e^{-4x^3}(f(x)) + c$, where c is contant of intergration then $f(x)$ equals to (a) $-4x^3 - 1$ (b) $-1 - 2x^3$ (c) $4x^3 + 1$ (d) $1 - 2x^3$

A. $-4x^3 - 1$

B. $4x^3 + 1$

C. $-2x^3 - 1$

D. $-2x^3 + 1$

Answer: A

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5. If $\int \frac{\sqrt{1-x^2}}{x^4} dx = A(x)\left(\sqrt{1-x^2}\right)^m + C$,for a suitable chosen integer m and a function A(x), where C is a constant of integration, then $(A(x))^m$ equals

A. $\frac{-1}{3x^3}$

B. $\frac{-1}{27x^9}$

C. $\frac{1}{9x^4}$

D. $\frac{1}{27x^6}$

Answer: B

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6. If $\int \frac{x+1}{\sqrt{2x-1}}dx = f(x)\sqrt{2x-1} + C$, where C is a constant of integration, then f(x) is equal to

A. $\frac{1}{3}(x+4)$

B. $\frac{1}{3}(x+1)$

C. $\frac{2}{3}(x+2)$

D. $\frac{2}{3}(x-4)$

Answer: A

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7. The integral $\int \cos(\log_e x) dx$ is equal to: (where C is a constant of integration)

A. $\frac{x}{2}[\sin(\log_e x - \cos(\log_e x)] + C$

B. $\frac{x}{2}[\cos(\log_e x + \sin(\log_e x)] + C$

C. $x[\cos(\log_e x + \sin(\log_e x)] + C$

D. $x[\cos(\log_e x - \sin(\log_e x)] + C$

Answer: B

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8. $\int \frac{3x^{13} + 2x^{11}}{(2x^4 + 3x^2 + 1)^4} dx$

A. $\frac{x^4}{(2x^4 + 3x^2 + 1)^3} + C$

B. $\frac{x^{12}}{6(2x^4 + 3x^2 + 1)^3} + C$

C. $\frac{x^4}{6(2x^4 + 3x^2 + 1)^3} + C$

D. $\frac{x^{12}}{(2x^4+3x^2+1)^3}+C$

Answer: B

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Chapter 8

1. The value of $\int_0^{\pi}|\cos x|^3dx$ is

A. $2/3$

B. 0

C. $-4/3$

D. $4/3$

Answer: D

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2. If $|f(x) - f(y)| \leq 2|x - y|^{\frac{3}{2}}\ \forall x, y \in R$ and $f(0) = 1$ then value of $\int_0^1 f^2(x)dx$ is equal to (a) 1 (b) 2 (c) $\sqrt{2}$ (d) 4

A. 0

B. $\frac{1}{2}$

C. 2

D. 1

Answer: D

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3. $\int_0^{\frac{\pi}{3}} \frac{\tan\theta}{\sqrt{2k\sec\theta}} d\theta = 1 - \frac{1}{\sqrt{2}}, (k > 0)$, then the value of k is

A. 2

B. $\frac{1}{2}$

C. 4

D. 1

Answer: A

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4. Let $I=\int_a^b \left(x^4-2x^2\right)dx$. If I is minimum, then the ordered pair (a, b) is

A. $\left(-\sqrt{2}, 0\right)$

B. $\left(-\sqrt{2}, \sqrt{2}\right)$

C. $\left(0, \sqrt{2}\right)$

D. $\left(\sqrt{2}, -\sqrt{2}\right)$

Answer: B

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5. The value of $\int_{-\pi/2}^{\pi/2} \frac{dx}{[x]+[\sin x]+4}$ where [t] denotes the greatest integer less or equal to t, is

A. $\frac{1}{12}(7\pi + 5)$

B. $\frac{3}{10}(4\pi - 3)$

C. $\frac{1}{12}(7\pi - 5)$

D. $\frac{3}{20}(4\pi - 3)$

Answer: D

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6. If $\int_0^x f(t)dt = x^2 + \int +_x^1 t^2 f(t)dt$, then $f\left(\frac{1}{2}\right)$ is equal to (a) $\frac{24}{25}$ (b) $\frac{4}{25}$ (c) $\frac{4}{5}$ (d) $\frac{2}{5}$

A. $\frac{6}{25}$

B. $\frac{24}{25}$

C. $\frac{18}{25}$

D. $\frac{4}{5}$

Answer: B

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7. The value of the integral $\int_{-2}^{2} \frac{\sin^2 x}{-2\left[\frac{x}{\pi}\right]+\frac{1}{2}} dx$ (where [x] denotes the greatest integer less then or equal to x) is

A. 4

B. $4-\sin 4$

C. $\sin 4$

D. 0

Answer: D

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8. The integral $\int_{\pi/6}^{\pi/4} \frac{dx}{\sin 2x\left(\tan^5 x+\cot^5 x\right)}$ equals

A. $\frac{1}{10}\left(\frac{\pi}{4}-\tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)\right)$

B. $\frac{1}{5}\left(\frac{\pi}{4}-\tan^{-1}\left(\frac{1}{3\sqrt{3}}\right)\right)$

C. $\frac{\pi}{10}$

D. $\frac{1}{20}-\tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)$

Answer: A

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9. Let f and g be continuous fuctions on [0, a] such that $f(x)=f(a-x)$ and $g(x)+g(a-x)=4$ then $\int_0^a f(x)g(x)dx$ is equal to

A. $4\int_0^a f(x)dx$

B. $2\int_0^a f(x)dx$

C. $-3\int_0^a f(x)dx$

D. $\int_0^a f(x)dx$

Answer: B

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10. The integral $\int_1^e \left\{\left(\frac{x}{e}\right)^{2x} - \left(\frac{e}{x}\right)^x\right\}\log_e x dx$ is equal to

A. $\frac{1}{2} - e - \frac{1}{e^2}$

B. $\frac{3}{2} - \frac{1}{e} - \frac{1}{2e^2}$

C. $-\frac{1}{2} + \frac{1}{e} - \frac{1}{2e^2}$

D. $\frac{3}{2} - e - \frac{1}{2e^2}$

Answer: D

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11. $\lim_{n\to\infty}\left(\frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + \frac{n}{n^2+3^2} + \dots + \frac{n}{5}n^2\right)$ is equal to

A. $\frac{\pi}{4}$

B. $\tan^{-1}(2)$

C. $\tan^{-1}(3)$

D. $\frac{\pi}{2}$

Answer: B

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Chapter 9

1. The area (in sq. units) bounded by the parabola $y = x^2 - 1$, the tangent at the point (2,3) to it and the y-axis is

A. $\frac{14}{3}$

B. $\frac{56}{3}$

C. $\frac{8}{3}$

D. $\frac{32}{3}$

Answer: C

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2. The area (in sq. units) of the region

$A = [(x, y): 0 \leq y \leq x|x| + 1 \text{ and } -1 \leq x \leq x]$ is

A. $\frac{1}{3}$

B. $\frac{1}{3}$

C. 2

D. $\frac{4}{3}$

Answer: C

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3. If the area enclosed between the curves $y = kx^2$ and $x = ky^2$, where $k > 0$, is 1 square unit. Then k is: (a) $\frac{1}{\sqrt{3}}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{2}{\sqrt{3}}$ (d) $\sqrt{3}$

A. $\dfrac{1}{\sqrt{3}}$

B. $\dfrac{2}{\sqrt{3}}$

C. $\dfrac{\sqrt{3}}{2}$

D. $\sqrt{3}$

Answer: A

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4. The area of the region bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$ is

A. $\dfrac{5}{4}$

B. $\dfrac{9}{8}$

C. $\dfrac{3}{4}$

D. $\dfrac{7}{8}$

Answer: B

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5. The area (in sq. units) in the first quadrant bounded by the parabola $y = x^2 + 1$, the tangent to it at the point (2, 5) and the coordinate axes is

A. $\frac{14}{3}$

B. $\frac{187}{24}$

C. $\frac{37}{24}$

D. $\frac{8}{3}$

Answer: C

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6. The area (in sq. units) of the region bounded by the parabola $y = x^2 + 2$ and the lines $y = x + 1, x = 0$ and $x = 3$, is

A. $\frac{15}{4}$

B. $\frac{15}{2}$

C. $\frac{21}{2}$

D. $\frac{17}{4}$

Answer: B

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Chapter 10

1. If y = y(x) is the solution of the differential equation, $x\frac{dy}{dx} + 2y = x^2$ satisfying y(1) = 1, then $y\left(\frac{1}{2}\right)$ is equal to

A. $\frac{4}{64}$

B. $\frac{13}{16}$

C. $\frac{49}{16}$

D. $\frac{1}{4}$

Answer: C

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2. Let $f: [0, 1] \to R$ be such that $f(xy) = f(x).\ f(y),$ for all $x, y \in [0, 1]$ and $f(0) \neq 0.$ If $y = y(x)$ satisfies the differential equation, $\frac{dy}{dx} = f(x)$ with $y(0) = 1,$ then $y\left(\frac{1}{4}\right) + y\left(\frac{3}{4}\right)$ is equql to

A. 4

B. 3

C. 5

D. 2

Answer: B

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3. If

$$\frac{dy}{dx}+\frac{3}{\cos^2 x}y=\frac{1}{\cos^2 x}, x\in\left(\frac{-\pi}{3},\frac{\pi}{3}\right) \text{ and } y\left(\frac{\pi}{4}\right)=\frac{4}{3}, \text{ then } y\Big(-$$

equals

A. $\frac{1}{3}+e^6$

B. $\frac{1}{3}$

C. $-\frac{1}{4}$

D. $\frac{1}{3}+e^3$

Answer: A

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4. Let f be differentiable function such that

$$f'(x)=7-\frac{3}{4}\frac{f(x)}{x}, (x>0) \text{ and } f(1)\neq 4 \text{ Then } \lim_{x\to 0^+} xf\left(\frac{1}{x}\right)$$

A. exists abd equals 4

B. does not exist

C. exists and equals 0

D. exists and equals $4/7$

Answer: A

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5. The curve amongst the family of curves, represented by the differential equation $(x^2 - y^2)dx + 2xydy = 0$ which passes through (1,1) is

A. a circle with centre on the y-axis

B. a circle with centre on the x-axis

C. an ellipse with major axis along the y-axis

D. a hyperbola with transverse axis along the

Answer: B

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6. If $y(x)$ is solution of differential equation satisfying $\frac{dy}{dx}+\left(\frac{2x+1}{x}\right)y=e^{-2x}, y(1)=\frac{1}{2}e^{-2}$ then (A) $y(\log_e 2)=\log_e 2$ (B) $y(\log_e 2)=\frac{\log_e 2}{4}$ (C) $y(x)$ is decreasing is $(0,1)$ (D) $y(x)$ is decreasing is $\left(\frac{1}{2},1\right)$

A. y(x) is decreasing in (0,1)

B. y(x) is decreasing in $\left(\frac{1}{2},1\right)$

C. $y(\log_e 2)=\frac{\log_e 2}{4}$

D. $y(\log_e 2)=\log_2 4$

Answer: B

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7. The solution of the differential equation, $\frac{dy}{dx}=(x-y)^2$,

when $y(1)=1,$ is

A. $\log_e\left|\frac{2-y}{2-x}\right|=2(y-1)$

B. $\log_e\left|\frac{2-x}{2-y}\right| = x - y$

C. $-\log_e\left|\frac{1+x-y}{1-x+y}\right| = x + y - 2$

D. $-\log_e\left|\frac{1-x+y}{1+x-y}\right| = 2(x-1)$

Answer: D

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8. Let y = y(x) be the solution of the differential equation $x\frac{dy}{dx} + y = x\log_e x, (x > 1)$. If $2y(2) = \log_e 4 - 1$, then $y(e)$ is equal to

A. $\frac{e^2}{4}$

B. $\frac{e}{4}$

C. $-\frac{e}{2}$

D. $-\frac{e^2}{2}$

Answer: B

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9. If a curve passes through the point (1, -2) and has slope of the tangent at any point (x,y) on it as $\frac{x^2 - 2y}{x}$, then the curve also passes through the point

A. $\left(-\sqrt{2}, 1\right)$

B. $\left(\sqrt{3}, 0\right)$

C. $\left(-1, 2\right)$

D. (3, 0)

Answer: B

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