

MATHS

BOOKS - CENGAGE MATHS (ENGLISH)

PRINCIPLE OF MATHEMATICAL INDUCTION

Sovled Examples

1. Prove the following by using the principle of mathematical induction $(1 + 1)^{2}$

for all
$$n \in N$$
: $1^3 + 2^3 + 3^3 + + n^3 = \left(rac{n(n+1)}{2}
ight)^2$

2. Using the principle of mathematical induction prove that
$$1+rac{1}{1+2}+rac{1}{1+2+3}+rac{1}{1+2+3+4}++rac{1}{1+2+3++n}=rac{2n}{n+1}$$
 for all $n\in N$

3. Using the principle of mathematical induction, prove that $1.3+2.3^2+3.3^2+...+n.3^n=rac{(2n-1)(3)^{n+1}+3}{4}$ for all $n\in N.$

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4. Using principle of mathematical induction, prove that for all $n\in N, n(n+1)(n+5)$ is a multiple of 3.



5. Prove the following by the principle of mathematical induction: $3^{2n+2}-8n-9$ is divisible 8 for all $n\in N$.

6. Using the principle of mathematical induction prove that $41^n - 14^n$ is a

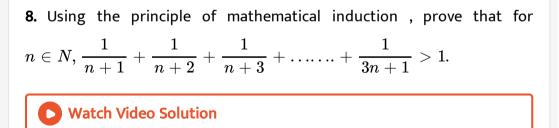
multiple of 27.



7. Prove the following by using the principle of mathematical induction

for all $n\in N\!:\left(2n+7
ight)<\left(n+3
ight)^{2}.$

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9. A sequence a_1, a_2, a_3, \ldots is defined by letting $a_1 = 3$ and $a_k = 7a_{k-1}$, for all natural numbers $k \le 2$. Show that $a_n = 3 \cdot 7^{n-1}$ for natural

numbers.

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10. Let

$$A_n = a_1 + a_2 + \dots + a_n, B_n = b_1 + b_2 + b_3 + \dots + b_n, D_n = c_1 + c_2$$

and $c_n = a_1b_n + a_2b_{n-1} + \dots + a_nb_1Aan \in N$. Using mathematical
induction , prove that
(a)
 $D_n = a_1B_n + a_2B_{n-1} + \dots + a_nB_1 = b_1A_n + b_2A_{n-1} + \dots + b_nA_1 \forall n$
(b) $D_1 + D_2 + \dots + D_n = A_1B_n + A_2B_{n-1} + \dots + A_nB_1 \forall n \in N$
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11. Let $U_1 = 1, U_2 = 1 \text{ and } U_{n+2} = U_{n+1} + U_n f \text{ or } n \ge 1$. use
mathematical induction to show that:
 $1 + ((1 + \sqrt{n})^n + (1 - \sqrt{n})^n)$

$$U_n=rac{1}{\sqrt{5}}igg\{\left(rac{1+\sqrt{5}}{2}
ight)^n-\ \left(rac{1-\sqrt{5}}{2}
ight)^nigg\}\ f \ ext{or} \ \ all\ n\geq 1.$$

12. If p is a fixed positive integer, prove by induction that $p^{n+1}+(p+1)^{2n-1}$ is divisible by P^2+p+1 for all $n\in N.$

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13. Let $0 < A_i < \pi$ for $i = 1, 2, \ldots .n$. Use mathematical induction to prove that $\sin A_1 + \sin A_2 + \ldots + \sin A_n \le n \sin \left(\frac{A_1 + A_2 + \ldots + A_n}{n} \right)$ where $n \ge 1$ is a natural number.

[You may use the fact that $p \sin x + (1-p) \sin y \leq \sin[px + (1-p)y],$ where $0 \leq p \leq 1$ and $0 \leq x, y \leq \pi$]

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14. Prove the following by the principle of mathematical induction: $1.3 + 2.4 + 3.5 + + (2n - 1)(2n + 1) = \frac{n(4n^2 + 6n - 1)}{3}$



15. Using the principle of mathematical induction prove that $\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$ for all $n \in N$

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16. Using the principle of mathematical induction, prove that $\left(2^{3n}-1
ight)$ is

divisible by 7 for all $n \in N_{\cdot}$

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17. Using the principle of mathematical induction. Prove that $(x^n - y^n)$ is

divisible by (x-y) for all $n \in N$.



18. Using principle of mathematical induction prove that
$$\sqrt{n} < \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \dots + \frac{1}{\sqrt{n}}$$
 for all natural numbers $n \ge 2$.
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19. Show that
$$rac{n^5}{5}+rac{n^3}{3}+rac{7n}{15}$$
 is a natural number, for all $\mathsf{n}\ \in N$

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20. Using principle of mathematical induction, prove that $7^{4^n} - 1$ is divisible by 2^{2n+3} for any natural number n.

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21. Prove by mathematical induction that n^5 and n have the same unit digit for any natural number n.

22. A sequence b_0, b_1, b_2, \ldots is defined by letting $b_0 = 5$ and $b_k = 4 + b_{k-1}$, for all natural number k. Show that $b_n = 5 + 4n$, for all natural number n using mathematical induction.