

# MATHS

# **BOOKS - CENGAGE MATHS (ENGLISH)**

# **PROPERTIES AND SOLUTIONS OF TRIANGLE**

#### Example

1. In triangle ABC< D is on AC such that AD=BC and BD=DC,  $\angle DBC = 2x$ 

and  $\angle BAD = 3x$  where each angle is in degree. Then find x

Watch Video Solution

2. In a circle of radius r, chords of length aandbcm subtend angles heta and 3 heta, respectively, at the center. Show that  $r=a\sqrt{rac{a}{3a-b}}cm$ 

3. perpendiculars are drawn from the angles A, BandC of an acuteangled triangle on the opposite sides, and produced to meet the circumscribing circle. If these produced parts are  $\alpha$ ,  $\beta$ ,  $\gamma$ , respectively, then show that  $\frac{a}{\alpha} + \frac{b}{\beta} + \frac{c}{\gamma} = 2(\tan A + \tan B + tanC)$ .

Watch Video Solution

**4.** D, E, F are three points on the sides BC, CA, AB, respectively, such that  $\angle ADB = \angle BEC = \angle CFA = \theta$ . A', B', C' are the points of intersections of the lines AD, BE, CF inside the triangle. Show that area of  $\Delta A'B'C' = 4\Delta\cos^2\theta$ , where  $\Delta$  is the area of  $\Delta ABC$ 

# View Text Solution

**5.** In *ABC*, as semicircle is inscribed, which lies on the side  $\cdot$  If x is the length of the angle bisector through angle C, then prove that the radius of the semicircle is  $x \sin\left(\frac{C}{2}\right)$ .

6. Given the base of a triangle, the opposite angle A, and the product  $k^2$ of the other two sides, show that it is not possible for a to be less than  $2k\frac{\sin A}{2}$ 

**Watch Video Solution** 

7. If in a triangle of base 'a', the ratio of the other two sides is r ( <1). Show

that the altitude of the triangle is less than or equal to  $\displaystyle rac{ar}{1-r^2}$ 

Watch Video Solution

8. Let ABC be a triangle with incentre I. If P and Q are the feet of the perpendiculars from A to BI and CI, respectively, then prove that  $\frac{AP}{BI} + \frac{AQ}{Cl} = \cot. \frac{A}{2}$ 

9. Let O be the circumcentre and H be the orthocentre of an acute angled triangle ABC. If A > B > C, then show that  $Ar(\Delta BOH) = Ar(\Delta AOH) + Ar(\Delta COH)$ 

Watch Video Solution

Watch Video Solution

10. If I is the incenter of  $\Delta ABC$  and  $R_1, R_2$ , and  $R_3$  are, respectively, the radii of the circumcircle of the triangle IBC, ICA, and IAB, then prove that  $R_1R_2R_3 = 2rR^2$ 



12. In a  $\Delta ABC$ , the median to the side BC is of length  $\frac{1}{\sqrt{11-6\sqrt{3}}}$  and

it divides the  $\angle A$  into angles  $30\,^\circ\,$  and  $45\,\circ\,.\,$  Find the length of the side BC.



13. Three circles touch each other externally. The tangents at their point of contact meet at a point whose distance from a point of contact is 4.Then, the ratio of their product of radii to the sum of the radii is

## Watch Video Solution

14. Let ABC be a triangle with incentre I and inradius r. Let D, E, F be the feet of the perpendiculars from I to the sides BC, CA and AB, respectively, If  $r_2$  and  $r_3$  are the radii of circles inscribed in the quadrilaterIs AFIE, BDIF and CEID respectively, then prove that  $\frac{r_1}{r-r_1} + \frac{r_2}{r-r_2} + \frac{r_3}{r-r_3} = \frac{r_1r_2r_3}{(r-r_1)(r-r_2)(r-r_3)}$ 

15. In convex quadrilateral ABCD, AB = a, BC = b, CD = c, DA = d

. This quadrilateral is such that a circle can be inscribed in it and a circle

can also be circumscribed about it. Prove that  $rac{ an^2 A}{2} = rac{bc}{ad}$ .

Watch Video Solution

#### Illustration

1. If an a triangle  $ABC, b = 3cand \ C - B = 90^0$ , then find the value of

 $\tan B$ 

Watch Video Solution

2. In a triangle ABC if BC = 1 and AC = 2, then what is the

maximum possible value of angle A?

**3.** The perimeter of a triangle ABC is saix times the arithmetic mean of the

sines of its angles. If the side ais1 then find angle  $A_{\cdot}$ 



**4.** If 
$$A=75^0,\,b=45^0,\,$$
 then prove that  $b+c\sqrt{2}=2a$ 

Watch Video Solution

5. If the base angles of triangle are  $\frac{22}{12}and112\frac{1}{2^0}$ , then prove that the altitude of the triangle is equal to  $\frac{1}{2}$  of its base.

#### Watch Video Solution

**6.** If  $a^2, b^2, c^2$  are in A.P., then prove that  $\tan A, \tan B, \tan C$  are in H.P.

7. Prove that 
$$rac{a^2\sin(B-C)}{\sin b+\sin C}+rac{b^2\sin(C-A)}{\sin C+\sin A}+rac{c^2\sin(A-B)}{\sin A+\sin B}=0$$

#### Watch Video Solution

8. In any triangle. if 
$$\frac{a^2-b^2}{a^2+b^2}=\frac{\sin(A-B)}{\sin(A+B)}$$
, then prove that the

triangle is either right angled or isosceles.

Watch Video Solution

**9.** ABCD is a trapezium such that  $AB \mid |CDandCB$  is perpendicular to

them. If  $\angle ADB = \theta, BC = p, andCD = q$  , show that  $AB = rac{(p^2 + q^2)\sin\theta}{p\cos\theta + q\sin\theta}$ 

10. In a triangle  $ABC, \angle c = 60^0 and \angle A = 75^0$  . If D is a point on AC

such that the area of the  $BCD, the \angle ABD$ 



11. In a scalene triangle ABC, D is a point on the side AB such that  $CD^2 = AD \cdot DB$ ,  $\sin A \cdot \sin B = \sin^2\left(\frac{C}{2}\right)$  then prove that CD is internal bisector of  $\angle C$ 

Watch Video Solution

12. In a triangle ABC,  $\angle A=60^\circ\,\,{
m and}\,\,b\!:\!c=ig(\sqrt{3}+1ig)\!:\!2$ , then find the

value of  $(\angle B - \angle C)$ 

**13.** If the median AD of triangle ABC makes an angle  $\frac{\pi}{4}$  with the side BC, then find the value of  $|\cot B - \cot C|$ .

14. The base of a triangle is divided into three equal parts. If  $t_1, t_2, t_3$  are the tangents of the angles subtended by these parts at the opposite vertex, prove that  $\left(\frac{1}{t_1} + \frac{1}{t_2}\right)\left(\frac{1}{t_2} + \frac{1}{t_3}\right) = 4\left(1 + \frac{1}{t22}\right)$ .

Watch Video Solution

15. In any 
$$\Delta ABC$$
 , prove that  $(a-b)^2 \cos^2\left(rac{C}{2}
ight) + (a+b)^2 \sin^2\left(rac{C}{2}
ight) = c^2.$ 

Watch Video Solution

16. In  $ABC, = ext{if} (a+b+c)(a-b+c) = 3ac$ , then find  $\angle B$ .

17. If 
$$a = \sqrt{3}, b = rac{1}{2} \left( \sqrt{6} + \sqrt{2} \right)$$
, and  $c = \sqrt{2}$ , then find  $\angle A$ 

## Watch Video Solution

18. The sides of a triangle are  $x^2 + x + 1, 2x + 1$  and  $x^2 - 1$ . Prove

that the greatest angle is  $120^{\circ}$ 

Watch Video Solution

**19.** If the angles A,B,C of a triangle are in A.P. and sides a,b,c, are in G.P.,

then prove that  $a^2, b^2, c^2$  are in A.P.



20. Let a, bandc be the three sides of a triangle, then prove that the equation  $b^2x^2 + (b^2 = c^2 - \alpha^2)x + c^2 = 0$  has imaginary roots.

#### Watch Video Solution

21. Let  $a\leq b\leq c$  be the lengths of the sides of a triangle. If  $a^2+b^2< c^2, then provet \widehat{\angle} Cisobtuse.$ 

Watch Video Solution



**23.** If in a triangle 
$$ABC, \angle C = 60^{\circ}$$
, then prove that  
 $\frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c}$ .  
**Watch Video Solution**

**24.** In a triangle, if the angles A, B, andC are in A.P. show that

$$2rac{\cos 1}{2}(A-C) = rac{a+c}{\sqrt{a^2-ac+c^2}}$$

Watch Video Solution

25. If a = 9, b = 4andc = 8 then find the distance between the middle

point of BC and the foot of the perpendicular form  $A_{\cdot}$ 



26. Three parallel chords of a circle have lengths 2,3,4 units and subtend

angles lpha, eta, lpha+eta at the centre, respectively `(alpha



27. In a cyclic quadrilateral PQRS, PQ= 2 units, QR= 5 units, RS=3 units and

 $\angle PQR = 60^{0}, ext{ then what is the measure of SP?}$ 

Watch Video Solution

**28.** For any triangle ABC, prove that  $a(b\cos C - \mathrm{o}sB) = b^2 - c^2$ 

Watch Video Solution

**29.** If in a triangle a  $\frac{\cos^2 C}{2} + \frac{\cos^2 A}{2} = \frac{3b}{2}$ , then find the relation

between the sides of the triangle.

Watch Video Solution

**30.** Prove that  $(b+c) {
m cos}\, A + (c+a) {
m cos}\, B + (a+b) {
m cos}\, C = 2s {
m cos}$ 





length of the altitude.

**39.** In equilateral triangle ABC with interior point D, if the perpendicular distances from D to the sides of 4,5, and 6, respectively, are given, then find the area of ABC.



**40.** If area of a triangle is 2 sq. units, then find the value of the product of the arithmetic mean of the lengths of the sides of a triangle and harmonic mean of the lengths of the altitudes of the triangle.



**41.** A triangle has sides 6,7, and 8. The line through its incenter parallel to the shortest side is drawn to meet the other two sides at P and Q. Then find the length of the segment PQ.

**42.** Each side of triangle ABC is divided into three equal parts. Find the ratio of the area of hexagon PQRSTU to the area of the triangle ABC.

C	Watch	Video	Solution

**43.** The two adjacent sides of a cyclic quadrilateral are 2and5 and the angle between them is  $60^{0}$ . If the area of the quadrilateral is  $4\sqrt{3}$ , find the remaining two sides.

Watch Video Solution

**44.** In triangle ABC, a:b:c = 4:5:6. The ratio of the radius of the

circumcircle to that of the incircle is\_\_\_\_.



**45.** Given a triangle ABC with sides a=7, b=8 and c=5. Find the value of expression  $(\sin A + \sin B + \sin C)\left(\frac{\cot A}{2} + \frac{\cot B}{2} + \frac{\cot C}{2}\right)$ 

Watch Video Solution

**46.** If  $b = 3, c = 4, and B = \frac{\pi}{3}$ , then find the number of triangles that can be constructed.

Watch Video Solution

**47.** If  $A = 30^{0}, a = 7, andb = 8$  in ABC, then find the number of

triangles that can be constructed.

## Watch Video Solution

48. If in triangle ABC,  $ig(a=ig(1+\sqrt{3}ig)cm,b=2cm,and ot c=60^0$  , then

find the other two angles and the third side.

**49.** In ABC, sidesb, c and angle B are given such that a has two values

 $a_1 and a_{2^{\cdot}}$  Then prove that  $|a_1-a_2|=2\sqrt{b^2-c^2\sin^2 B}$ 

Watch Video Solution

50. In ABC, a, candA are given and  $b_1$ ,  $b_2$  are two values of the third

side b such that  $b_2=2b_1$  . Then prove that  $\sin A=\sqrt{rac{9a^2-c^2}{8c^2}}$ 

Watch Video Solution

**51.** O is the circumcenter of  $ABCandR_1, R_2, R_3$  are respectively, the

radii of the circumcircles of the triangle OBC, OCA and OAB. Prove that

$$rac{a}{R_1}+rac{b}{R_2}+rac{c}{R_3}=rac{abc}{R_3}$$

52. In ABC,  $C = 60^0 and B = 45^0$ . Line joining vertex A of triangle and its circumcenter (O) meets the side  $BC \in D$  Find the ratio BD:DCFind the ratio AO:OD



54. Find the lengths of chords of the circumcircle of triangle ABC, made

by its altitudes\_\_\_\_\_



**55.** Let ABC be a triangle with  $\angle B = 90^0$ . Let AD be the bisector of  $\angle A$  with D on BC. Suppose AC=6cm and the area of the triangle ADC is  $10cm^2$ . Find the length of BD.

**56.** If the distances of the vertices of a triangle =ABC from the points of contacts of the incercle with sides are  $\alpha$ ,  $\beta and\gamma$  then prove that  $r^2 = \frac{\alpha\beta\gamma}{\alpha + \beta + \gamma}$ 

Watch Video Solution

**57.** If x, yandz are the distances of incenter from the vertices of the triangle ABC, respectively, then prove that  $\frac{abc}{xyz} = \cot\left(\frac{A}{2}\right)\cot\left(\frac{B}{2}\right)\cot\left(\frac{C}{2}\right)$ 

58. Prove that  $\cos A + \cos B + \cos C = 1 + rac{r}{R}$ 

Watch Video Solution

59. Prove that 
$$rac{{
m a}\,{
m c}\,{
m o}\,{
m s}A+b\,{
m cos}\,B+{
m o}sC}{a+b+c}=rac{r}{R}$$

Watch Video Solution

**60.** Incircle of ABC touches the sides BC, CA and AB at D, E and F, respectively. Let  $r_1$  be the radius of incircle of BDF. Then prove that  $r_1 = \frac{1}{2} \frac{(s-b)\sin B}{\left(1+\sin\left(\frac{B}{2}\right)\right)}$ Watch Video Solution

**61.** In an acute angled triangle ABC, a semicircle with radius  $r_a$  is constructed with its base on BC and tangent to the other two sides.

 $r_b and r_c$  are defined similarly. If r is the radius of the incircle of triangle

ABC then prove that 
$$\displaystyle rac{2}{r} = \displaystyle rac{1}{r_a} + \displaystyle rac{1}{r_b} + \displaystyle rac{1}{r_c}$$

Watch Video Solution

**62.** Let the incircle with center I of ABC touch sides BC, CA and AB at D, E, F, respectively. Let a circle is drawn touching ID, IF and incircle of ABChaving radius  $r_2$ . similarly  $r_1andr_3$  are defined. Prove that  $\frac{r_1}{r-r_1}\frac{\dot{r_2}}{r-r_2}\frac{\dot{r_3}}{r-r_3} = \frac{a+b+c}{8R}$ 

Watch Video Solution

**63.** In ABC, the bisector of the angle A meets the side BC at D and the

circumscribed circle at E. Prove that  $DE = rac{a^2rac{\sec A}{2}}{2(b+c)}$ 

**64.** Let I be the incetre of  $\triangle$ ABC having inradius r. Al, BI and Ci intersect incircle at D, E and F respectively. Prove that area of  $\triangle DEF$  is  $\frac{r^2}{2} \left( \cos \cdot \frac{A}{2} + \cos \cdot \frac{B}{2} + \cos \cdot \frac{C}{2} \right)$ 

Watch Video Solution

**65.** In *ABC*, the three bisectors of the angle A, B and C are extended to intersect the circumcircle at D,E and F respectively. Prove that  $AD\frac{\cos A}{2} + BE\frac{\cos B}{2} + CF\frac{\cos C}{2} = 2R(\sin A + \sin B + \sin C)$ Watch Video Solution

**66.** Given a right triangle with  $\angle A = 90^{\circ}$ . Let M be the mid-point of BC. If the radii of the triangle ABM and ACM are  $r_1$  and  $r_2$  then find the range of  $\frac{r_1}{r_2}$ .

67. Prove that the distance between the circumcenter and the incenter of

triangle ABC is  $\sqrt{R^2-2Rr}$ 

Watch Video Solution

**68.** Prove that  $a\cos A + b\cos B + \mathrm{o}sC \leq s$ .



69. If  $\Delta$  is the area of a triangle with side lengths a, b, c, then show that as  $\Delta \leq \frac{1}{4}\sqrt{(a+b+c)abc}$  Also, show that the equality occurs in the above inequality if and only if a = b = c.

#### Watch Video Solution

**70.** If in  $\triangle ABC$ , the distance of the vertices from the orthocenter are x,y,

and z then prove that 
$$rac{a}{x}+rac{b}{y}+rac{c}{z}=rac{abc}{xyz}$$

71. ABC is an acute angled triangle with circumcenter O and orthocentre

H. If AO=AH, then find the angle A.

Watch Video Solution

**72.** In a acute angled triangle ABC, proint D, E and F are the feet of the perpendiculars from A,B and C onto BC, AC and AB, respectively. H is orthocentre. If  $\sin A = \frac{3}{5} and BC = 39$ , then find the length of AH

Watch Video Solution

73. Prove that the distance between the circumcenter and the orthocentre of triangle ABC is  $R\sqrt{1-8\cos A\cos B\cos C}$ 

74. Let ABC be an acute angled triangle whose orthocentre is at H. If altitude from A is produced to meet the circumcircle of triangle ABC at D, then prove  $HD = 4R \cos B \cos C$ 

**75.** In *ABC*, let *L*, *M*, *N* be the feet of the altitudes. The prove that  $\sin(\angle MLN) + \sin(\angle LMN) + \sin(\angle MNL) = 4\sin A \sin B \sin C$ 

Watch Video Solution

**76.** The lengths of the medians through acute angles of a right-angled

triangle are 3 and 4. Find the area of the triangle.



**77.** Two medians drawn from acute angles of a right angled triangles intersect at an angle of  $\pi/6$ . If the length of the hypotenuse of the triangle is 3 units, then find the area of the triangle.



78. Prove that 
$$r_1+r_2+r_3-r=4R$$

Watch Video Solution

**79.** If in a triangle  $r_1 = r_2 + r_3 + r$ , prove that the triangle is right

angled.

Watch Video Solution

80. Prove that 
$$rac{r_{1+r_2}}{1}=2R$$



Watch Video Solution	
82 If the distance between incenter and one of the excenter	of an

equilateral triangle is 4 units, then find the inradius of the triangle.

Watch Video Solution

83. If  $I_1, I_2, I_3$  are the centers of escribed circles of  $\Delta ABC$ , show that the area of  $\Delta I_1 I_2 I_3$  is (abc)/(2r)`



**84.** Prove that the sum of the radii of the radii of the circles, which are, respectively, inscribed and circumscribed about a polygon of n sides,



Watch Video Solution

**85.** If the area of the circle is  $A_1$  and the area of the regular pentagon inscribed in the circle is  $A_2$ , then find the ratio  $\frac{A_1}{A_2}$ .

Watch Video Solution

**86.** Prove that the area of a regular polygon hawing 2n sides, inscribed in a circle, is the geometric mean of the areas of the inscribed and circumscribed polygons of n sides.

Watch Video Solution

**Concept Application Exercise 51** 

**1.** Find the value of 
$$rac{a^2+b^2+c^2}{R^2}$$
 in any right-angled triangle.



5. In triangle ABC, if  $\cos^2 A + \cos^2 B - \cos^2 C = 1$ , then identify the type

of the triangle



6. Prove that  $b^2\cos 2A - a^2\cos 2B = b^2 - a^2$ 

Watch Video Solution

7. In any triangle 
$$ABC$$
 , prove that following :  

$$\frac{c}{a+b} = \frac{1 - \tan\left(\frac{A}{2}\right) \tan\left(\frac{B}{2}\right)}{1 + \tan\left(\frac{A}{2}\right) \tan\left(\frac{B}{2}\right)}$$

Watch Video Solution



9. In a triangle ABC, prove that 
$$\displaystyle rac{b+c}{a} \leq \cos ec. \; \displaystyle rac{A}{2}$$

# Watch Video Solution

10. In any triangle 
$$ABC$$
 , prove that:  $rac{1+\cos(A-B)\cos C}{1+\cos(A-C)\cos B}=rac{a^2+b^2}{a^2+c^2}$ 

**Watch Video Solution** 

**11.** In a triangle ABC, if a, b, c are in A.P. and 
$$\frac{b}{c}\sin 2C + \frac{c}{b}\sin 2B + \frac{b}{a}\sin 2A + \frac{a}{b}\sin 2B = 2$$
, then find the value of sin B

### Watch Video Solution

12. Prove that  $a \cos A + b \cos B + c \cos C = 4R \sin A \sin B \sin C$ .

**Concept Application Exercise 5 2** 

1. If the sides of a triangle are a, b and  $\sqrt{a^2 + ab + b^2}$ , then find the greatest angle

Watch Video Solution

2. If the segments joining the points A(a,b)and B(c,d) subtends an angle heta at the origin, prove that :  $heta=rac{ac+bd}{(a^2+b^2)(c^2+d^2)}$ 

Watch Video Solution

**3.** The sides of a triangle are 3x + 4y, 4x + 3y and 5x + 5y units, where

x > 0, y > 0. The triangle is

4. In  $\triangle ABC$ , angle A is  $120^{\circ}$ , BC + CA = 20, and AB + BC = 21

Find the length of the side BC



5. In 
$$\Delta ABC, AB = 1, BC = 1, \text{ and } AC = 1/\sqrt{2}.$$
 In

 $\Delta MNP, MN = 1, NP = 1, ext{ and } \angle MNP = 2 \angle ABC.$  Find the side

MP

Watch Video Solution

6. If in a triangle  $ABC, \, rac{bc}{2\cos A} = b^2 + c^2 - 2bc\cos A$  then prove that

the triangle must be isosceles.


2. Prove that 
$$rac{\cos C + \cos A}{c+a} + rac{\cos B}{b} = rac{1}{b}$$

3. Prove that  

$$a(b^2 + c^2)\cos A + b(c^2 + a^2)\cos B + c(a^2 + b^2)\cos C = 3abc$$
  
  
Watch Video Solution  
Concept Application Exercise 5 4  
1. In a triangle  $ABC$  if  $b + c = 3a$  then find the value of  
 $cot\left(\frac{B}{2}\right)cot\left(\frac{C}{2}\right)$   
Watch Video Solution  
2. Prove that  $bc cos^2$ .  $\frac{A}{2} + ca cos^2$ .  $\frac{B}{2} + ab cos^2$ .  $\frac{C}{2} = s^2$   
Watch Video Solution

**3.** If in 
$$\triangle ABC$$
,  $\tan \frac{A}{2} = \frac{5}{6}$  and  $\tan \frac{C}{2} = \frac{2}{5}$ , then prove that a, b,

and c are in A.P.

Watch Video Solution

**4.** Prove that 
$$(b+c-a)\left(\cot. rac{B}{2}+\cot. rac{C}{2}
ight)=2a\cot. rac{A}{2}$$

Watch Video Solution

5. If 
$$\sin^2\left(\frac{A}{2}\right)$$
,  $\sin^2\left(\frac{B}{2}\right)$ , and  $\sin^2\left(\frac{C}{2}\right)$  are in  $H.P.$ , then prove

that the sides of triangle are in H. P.

Watch Video Solution

Concept Application Exercise 5 5

1. If 
$$c^2=a^2+b^2$$
, then prove that  $4s(s-a)(s-b)(s-c)=a^2b^2$ 



the opposite sides are 2, 2 and 3. If the area of  $\Delta ABC$  is  $\Delta$ , then find



(a)  $b\sin A=a, A<\pi/2$ 



2. If in  $\Delta ABC$ , b = 3cm, c = 4cm and the length of the perpendicular from A to the side BC is 2 cm, then how many such triangle are possible ?

Watch Video Solution

**3.** In a triangle 
$$ABC$$
,  $\frac{a}{b} = \frac{2}{3}$  and  $\sec^2 A = \frac{8}{5}$ . Find the number of

triangle satisfying these conditions

**4.** In a triangle, the lengths of the two larger sides are 10 and 9, respectively. If the angles are in A.P, then the length of the third side can be (a)  $5 - \sqrt{6}$  (b)  $3\sqrt{3}$  (c) 5 (d)  $5 + \sqrt{6}$ 



5. If a, b and A are given in a triangle and  $c_1, c_2$  are possible values of the third side, then prove that  $c_1^2+c_2^2-2c_1c_2\cos 2A=4a^2\cos^2 A$ 

Watch Video Solution

**6.** In  $\triangle ABC$ , a, b and A are given and  $c_1, c_2$  are two values of the third side c. Prove that the sum of the area of two triangles with sides a, b,  $c_1$  and  $a, bc_2$  is  $\frac{1}{2}b^2 \sin 2A$ 

Watch Video Solution

**Concept Application Exercise 5 7** 

1. Let f, g and h be the lengths of the perpendiculars from the circumcenter of  $\triangle ABC$  on the sides a, b, and c, respectively. Prove that  $\frac{a}{f} + \frac{b}{g} + \frac{c}{h} = \frac{1}{4} \frac{abc}{fgh}$ Watch Video Solution

2. If AD, BE and CF are the altitudes of  $\Delta ABC$  whose vertex A is (-4,5). The coordinates of points E and F are (4,1) and (-1,-4), respectively. Equation of BC is

**D** Watch Video Solution

**3.** If the sides of triangle are in the ratio 3:5:7, then prove that the minimum distance of the circumcentre from the side of triangle is half the circmradius



**4.** If circumradius of triangle ABC is 4 cm, then prove that sum of perpendicular distances from circumcentre to the sides of triangle cannot exceed 6 cm

Concept Application Exercise 5 8

1. If the incircle of the triangle ABC passes through its circumcenter, then

find the value of  $4\sin. \frac{A}{2}\sin. \frac{B}{2}\sin. \frac{C}{2}$ 

Watch Video Solution

2. In  $\Delta ABC, a = 10, A = rac{2\pi}{3}$ , and circle through B and C passes

through the incenter. Find the radius of this circle

**3.** Let ABC be a triangle with  $\angle BAC = 2\pi/3$  and AB = x such that (AB) (AC) = 1. If x varies, then find the longest possible length of the angle bisector AD

**4.** If the incircle of the  $\triangle ABC$  touches its sides at L, M and N as shown in the figure and if x, y, z be the circumradii of the triangles MIN, NILand LIM respectively, where I is the incentre, then the product xyz is equal to:

(A) 
$$Rr^2$$
 (B) $rR^2$   
(C)  $\frac{1}{2}Rr^2$  (D)  $\frac{1}{2}rR^2$ 

Watch Video Solution

5. In a triangle ABC, CD is the bisector of the angle C. If  $\cos\left(\frac{C}{2}\right)$  has the value  $\frac{1}{3}$  and l(CD) = 6, then  $\left(\frac{1}{a} + \frac{1}{b}\right)$  has the value equal to -

**6.** In  $\Delta ABC$ ,  $\angle A = \frac{\pi}{3}$  and its incircle of unit radius. Find the radius of the circle touching the sides AB, AC internally and the incircle of  $\Delta ABC$  externally is x, then the value of x is



**Concept Application Exercise 5 9** 

1. Line joining vertex A of triangle ABC and orthocenter (H) meets the side

BC in D. Then prove that

(a)  $BD: DC = \tan C: \tan B$ 

(b)  $AH: HD = (\tan B + \tan C): \tan A$ 

Watch Video Solution

**2.** In a triangle ABC,  $\angle A = 30^2, BC = 2 + \sqrt{5}$ , then find the distance of

the vertex A from the orthocenter

Watch Video Solution

**3.** If the perimeter of the triangle formed by feet of altitudes of the triangle ABC is equal to four times the circumradius of  $\Delta ABC$ , then identify the type of  $\Delta ABC$ 



4. AD, BE and CF are the medians of triangle ABC whose centroid is G. If

the points A, F, G and E are concyclic, then prove that  $2a^2=b^2+c^2$ 

5. Consider an acute angled  $\triangle ABC$ . Let AD, BE and CF be the altitudes drawn from the vertice to the opposite sides. Prove that :  $\frac{EF}{a} + \frac{FD}{b} + \frac{DE}{@} = \frac{R+r}{R}$ . Watch Video Solution

**Concept Application Exercise 5 10** 

1. In  $\Delta ABC$ , if  $r_1 < r_2 < r_3$ , then find the order of lengths of the sides

Watch Video Solution

**2.** The exradii  $r_1, r_2, \text{ and } r_3 \text{ of } \Delta ABC$  are in H.P. show that its sides a,

b, and c are in A.P.

3. If in 
$$\triangle ABC$$
,  $(a-b)(s-c)=(b-c)(s-a)$ , prove that  $r_1,r_2,r_3$  are in A.P.

4. Prove that 
$$2R \cos A = 2R + r - r_1$$
  
Watch Video Solution

5. If the lengths of the perpendiculars from the vertices of a triangle ABC

on the opposite sides are  $p_1, p_2, p_3$  then prove that  $rac{1}{p_1} + rac{1}{p_2} + rac{1}{p_3} = rac{1}{r} = rac{1}{r_1} + rac{1}{r_2} + rac{1}{r_3}.$ 

Watch Video Solution

Watch Video Solution

6. Prove that  $r_1r_2 + r_2r_3 + r_3r_1 = rac{1}{4}(a+b+c)^2$ 



1. Regular pentagons are inscribed in two circles of radius 5and 2 units

respectively. The ratio of their areas is

2. Let A be a point inside a regular polygon of 10 sides. Let  $p_1, p_2, \ldots, p_{10}$ be the distances of A from the sides of the polygon. If each side is of length 2 units, then find the value of  $p_1 + p_2 + \ldots + p_{10}$ 



4. If  $I_n$  is the area of n-sided regular polygon inscribed in a circle of

unit radius and  $O_n$  be the area of the polygon circumscribing the given

circle, prove that  $I_n = rac{O_n}{2} \Biggl( \sqrt{1 + \left( rac{2I_n}{n} 
ight)^2} \Biggr)$ 



1. ln 
$$\Delta ABC$$
,  $\frac{\sin A(a - b \cos C)}{\sin C(c - b \cos A)} =$   
A. -2  
B. -1  
C. 0  
D. 1

## Answer: D

**Watch Video Solution** 

2. If in a triangle ABC,  
$$\frac{1+\cos A}{a} + \frac{1+\cos B}{b} + \frac{1+\cos C}{c} = \frac{k^2(1+\cos A)(1+\cos B)(1+\cos B)}{abc}$$

, then k is equal to

A. 
$$\frac{1}{2\sqrt{2}R}$$

B. 2R

C. 
$$\frac{1}{R}$$

## D. none of these

## Answer: B

## Watch Video Solution

**3.** In triangle 
$$ABC$$
,  $2ac\sin\left(\frac{1}{2}(A-B+C)\right)$  is equal to  $a^2 + b^2 - c^2$   
(b)  $c^2 + a^2 - b^2 b^2 - c^2 - a^2$  (d)  $c^2 - a^2 - b^2$ 

A. 
$$a^2 + b^2 - c^2$$
  
B.  $c^2 + a^2 - b^2$   
C.  $b^2 - c^2 - a^2$   
D.  $c^2 - a^2 - b^2$ 

## Answer: B

**4.** If the angles of a triangle are in the ratio 4:1:1, then the ratio of the

longest side to the perimeter is

A. 
$$\sqrt{3}$$
:  $(2 + \sqrt{3})$   
B. 1: 6  
C. 1:  $2 + \sqrt{3}$ 

D. 2:3

## Answer: A

Watch Video Solution

5. Which of the following pieces of data does NOT uniquely determine an acute-angled triangle ABC(R being the radius of the circumcircle)? (a)  $a, \sin A, \sin B$  (b) a, b, c(c)a ,sinB ,R(d)a ,sinA ,R`

A.  $a, \sin A, \sin B$ 

B.a, b, c

 $C.a, \sin B, R$ 

 $D.a, \sin A, R$ 

Answer: D

Watch Video Solution

**6.** The sides of a triangle are in the ratio  $1: \sqrt{3}: 2$ . Then the angles are in

the ratio

A. 1:3:5

B. 2:3:4

C.3:2:1

D. 1:2:3

Answer: D

7. In ABC, a = 5, b = 12,  $c = 90^{0} and D$  is a point on AB so that  $\angle BCD = 45^{0}$ . Then which of the following is not true? (a)  $CD = \frac{60\sqrt{2}}{17}$ (b)  $BD = \frac{65}{17}$  (c)  $AD = \frac{60\sqrt{2}}{17}$  (d) none of these A.  $CD = \frac{60\sqrt{2}}{17}$ B.  $BD = \frac{65}{17}$ C.  $AD = \frac{60\sqrt{2}}{17}$ 

D. none of these

#### Answer: C

Watch Video Solution

8. In 
$$\Delta ABC$$
,  $(a+b+c)(b+c-a)=kbc$  if

A. k < 0

 $\mathsf{B}.\,k>0$ 

 $\mathsf{C.0} < k < 4$ 

 $\mathsf{D.}\,k<4$ 

Answer: C



**9.** Let D be the middle point of the side BC of a triangle ABC. If the triangle ADC is equilateral, then  $a^2:b^2:c^2$  is equal to 1:4:3 (b) 4:1:3 (c) 4:3:1 (d) 3:4:1

A. 1:4:3

B. 4:1:3

C.4:3:1

D. 3:4:1

Answer: B

**10.** In a triangle ABC, the altitude from A is not less than BC and the altitude from B is not less than AC. (a) The triangle is right angled (b) isosceles obtuse angled (d) equilateral

A. right angled

B. isosceles

C. obtuse angled

D. equilateral

#### Answer: A

Watch Video Solution

11. In 
$$\triangle ABC$$
, if  $\frac{\sin A}{c \sin B} + \frac{\sin B}{c} + \frac{\sin C}{b} = \frac{c}{ab} + \frac{b}{ac} + \frac{a}{bc}$ , then the value of angle A is

A.  $120^{\,\circ}$ 

B.  $90^{\circ}$ 

C.  $60^{\circ}$ 

D.  $30^{\circ}$ 

Answer: B

**Watch Video Solution** 

12. If in  $\Delta ABC$ , sides a, b, c are in A.P. then

A.  $B > 60^\circ$ 

B.  $B < 60^{\circ}$ 

C.  $B \leq 60^{\circ}$ 

D. B = |A - C|

Answer: C

13.	In	а	$\Delta A$	BC,	AD	is	the	altitude	fror	m A.	Given
b >	$c, \angle c$	$\mathcal{C} = \mathcal{C}$	$23^{\circ}$	and	AD =	$\overline{(b^2)}$	$\frac{abc}{-c^2)}$ ,	find $\angle B$ .			
1	A. $83^\circ$										
I	B. $97^{\circ}$										
(	C. 113	0									
[	D. 127	0									

#### Answer: C

# Watch Video Solution

14. If the sides a, b, c of a triangle ABC form successive terms of G.P. with common ratio r(>1) then which of the following is correct? (a)  $A > \frac{\pi}{3}$ ' $(b)B \ge \pi/3$ ' $(c)C < \pi/3$ ' $(d)A < B < \pi/3$ `

A.  $A>\pi/3$ 

B.  $B \geq \pi/3$ 

C.  $C < \pi/3$ 

D.  $A < B < \pi/3$ 

Answer: D

**Watch Video Solution** 

15. In triangle ABC,  $b^2 \sin 2C + c^2 \sin 2B = 2bc$  where b = 20, c = 21, then inradius = A. 4 B. 6

C. 8

D. 9

Answer: B

16. In a ABC, if  $AB=x, BC=x+1, \angle C=rac{\pi}{3}$  , then the least

integer value of x is 6 (b) 7 (c) 8 (d) none of these

A. 6

B. 7

C. 8

D. none of these

Answer: B

Watch Video Solution

**17.** If one side of a triangle is double the other, and the angles on opposite sides differ by  $60^0$ , then the triangle is equilateral (b) obtuse angled (c) right angled (d) acute angled

A. equilateral

B. obtus angled

C. right angled

D. acute angled

Answer: C

Watch Video Solution

**18.** If the hypotenuse of a right-angled triangle is four times the length of the perpendicular drawn from the opposite vertex to it, then the difference of the two acute angles will be  $60^{0}$  (b)  $15^{0}$  (c)  $75^{0}$  (d)  $30^{0}$ 

A.  $60^{\,\circ}$ 

B.  $15^{\circ}$ 

C.  $75^{\circ}$ 

D.  $30^{\,\circ}$ 

Answer: A

19. If P is a point on the altitude AD of the triangle ABC such the  

$$\angle CBP = \frac{B}{3}, \text{ then AP is equal to } 2a \frac{\sin C}{3} \text{ (b) } 2b \frac{\sin C}{3} \text{ (c) } 2c \frac{\sin B}{3} \text{ (d)}$$

$$2c \frac{\sin C}{3}$$
A.  $2a \sin \frac{C}{3}$ 
B.  $2b \sin \frac{C}{3}$ 
C.  $2c \sin \frac{B}{3}$ 
D.  $2c \sin \frac{C}{3}$ 

## Answer: C

**20.** With usual notations, in triangle 
$$ABC$$
,  $a \cos(B - C) + b \cos(C - A) + c\cos(A - B)$  is equal to  $abc/R^2$   
(b)  $\frac{abc}{4R^2} \frac{4abc}{R^2}$  (d)  $\frac{abc}{2R^2}$   
A.  $\frac{abc}{R^2}$ 

B. 
$$\frac{abc}{4R^2}$$
  
C.  $\frac{4abc}{R^2}$   
D.  $\frac{abc}{2R^2}$ 

#### Answer: A

Watch Video Solution

**21.** If in  $\Delta ABC, 8R^2 = a^2 + b^2 + c^2$ , then the triangle ABC is

A. right angled

B. isosceles

C. equilateral

D. none of these

Answer: A

**22.** Let ABC be a triangle with  $\angle A = 45^{\circ}$ . Let P be a point on side BC with PB=3 and PC=5. If O is circumcenter of triangle ABC, then length OP is  $\sqrt{18}$  (b)  $\sqrt{17}$  (c)  $\sqrt{19}$  (d)  $\sqrt{15}$ 

A.  $\sqrt{18}$ 

 $\mathsf{B.}\,\sqrt{17}$ 

C.  $\sqrt{19}$ 

D.  $\sqrt{15}$ 

Answer: B



23. In any triangle 
$$ABC, \frac{a^2+b^2+c^2}{R^2}$$
 has the maximum value of 3 (b) 6

(c) 9 (d) none of these

A. 3

B. 6

C. 9

D. none of these

Answer: C

Watch Video Solution

**24.** In triangle ABC,  $R(b+c) = a\sqrt{bc}$ , where R is the circumradius of the

triangle. Then the triangle is

A. isosceles but not right

B. right but not isosceles

C. right isosceles

D. equilateral

Answer: C

<b>25.</b> In <i>ABC</i> , (c) $\frac{5}{-}$ (d) $\frac{5}{-}$	if $b^2 + c^2 = 2a^2$ ,	then v	alue of	$\frac{\cot B}{\cot B}$ +	$\frac{A}{-\cot C}$	is	$\frac{1}{2}$	(b)	$\frac{3}{2}$
A. $\frac{1}{-}$									
$\frac{2}{B.\frac{3}{2}}$									
C. $\frac{5}{2}$									
D. $\frac{5}{3}$									

#### Answer: A

Watch Video Solution

**26.** If  $\sin \theta$  and  $-\cos \theta$  are the roots of the equation  $ax^2 - bx - c = 0$ , where a, b, and c are the sides of a triangle ABC, then  $\cos B$  is equal to

A. 
$$1 - \frac{c}{2a}$$
  
B.  $1 - \frac{c}{a}$   
C.  $1 + \frac{c}{2a}$ 

$$\mathsf{D.1} + \frac{c}{3a}$$

Answer: C

# Watch Video Solution

27. If D is the mid-point of the side BC of triangle ABC and AD is perpendicular to AC, then  $3b^2=a^2-c$  (b)  $3a^2=b^23c^2$   $b^2=a^2-c^2$  (d)  $a^2+b^2=5c^2$ 

- A.  $3b^2 = a^2 c^2$ B.  $3a^2 = b^2 - 3c^2$
- $\mathsf{C}.\,b^2=a^2-c^2$
- $\mathsf{D}.\,a^2+b^2=5c^2$

#### Answer: A

**28.** In a triangle ABC, if  $\cot A : \cot B : \cot C = 30 : 19 : 6$  then the sides a, b, c are

A. in A.P.

B. in G.P.

C. in H.P.

D. none of these

## Answer: A

Watch Video Solution

29. In  $\triangle ABC$ , P is an interior point such that  $\angle PAB = 10^\circ, \angle PBA = 20^\circ, \angle PCA = 30^\circ, \angle PAC = 40^\circ$  then  $\triangle ABC$  is

A. isosceles

B. right angled

C. equilateral

D. obtuse angled

Answer: A

**Watch Video Solution** 

**30.** In  $\Delta ABC$ , if AB = c is fixed, and  $\cos A + \cos B + 2\cos C = 2$  then the

locus of vertex C is

A. ellipse

B. hyperbola

C. circle

D. parabola

Answer: A
**31.** If in ABC,  $A = \frac{\pi}{7}$ ,  $B = \frac{2\pi}{7}$ ,  $C = \frac{4\pi}{7}$  then  $a^2 + b^2 + c^2$  must be  $R^2$ (b)  $3R^2$  (c)  $4R^2$  (d)  $7R^2$ 

A.  $R^2$ 

 $\mathsf{B.}\, 3R^2$ 

 $\mathsf{C}.\,4R^2$ 

D.  $7R^2$ 

## Answer: D

32. In 
$$\Delta ABC$$
,  $\cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2}$  is equal to  
A.  $\frac{\Delta}{r^2}$   
B.  $\frac{(a+b+c)^2}{abc} 2R$   
C.  $\frac{\Delta}{r}$   
D.  $\frac{\Delta}{Rr}$ 

# Answer: A



**33.** In 
$$ABC$$
,  $\left(\cot\left(\frac{A}{2}\right) + \cot\left(\frac{B}{2}\right)\right) \left(a\sin^2\left(\frac{B}{2}\right) + b\sin^2\left(\frac{A}{2}\right)\right) =$   
(a)  $\cot C$  (b)  $\cot C$  (c)  $\cot\left(\frac{C}{2}\right)$  (d)  $\cot\left(\frac{C}{2}\right)$ 

A.  $\cot C$ 

 $\operatorname{B.} c \cot C$ 

C. cot. 
$$\frac{C}{2}$$
  
D.  $c$  cot.  $\frac{C}{2}$ 

## Answer: D

**O** Watch Video Solution

34. In a right-angled isosceles triangle, the ratio of the circumradius and

inradius is

A. 
$$2(\sqrt{2}+1):1$$
  
B.  $(\sqrt{2}+1):1$   
C.  $2:1$   
D.  $\sqrt{2}:1$ 

### Answer: B

Watch Video Solution

**35.** In a  $\triangle$ ABC, a semicircle is inscribed, whose diameter lies on the side c. Then the radius of the semicircle is (Where  $\triangle$  is the area of the triangle ABC)

36. In 
$$\Delta ABC, A = \frac{2\pi}{3}, b - c = 3\sqrt{3}cm$$
 and area of  $\Delta ABC = \frac{9\sqrt{3}}{2}cm^2$ , then BC =

A.  $6\sqrt{3}cm$ 

B. 9 cm

C. 18 cm

D. 27 cm

Answer: B

Watch Video Solution

**37.** In triangle ABC, let  $\angle C = \pi/2$ . If r is the inradius and R is circumradius of the triangle, then 2(r+R) is equal to

A. a + b

 $\mathsf{B}.\, b+c$ 

 $\mathsf{C.}\,c+a$ 

 $\mathsf{D}.\,a+b+c$ 

Answer: A

**38.** In the given figure, AB is the diameter of the circle, centered at O. If

 $\angle COA = 60^{\circ}, AB = 2r, Ac = d \hspace{0.1 cm} ext{and} \hspace{0.1 cm} CD = l, \hspace{0.1 cm} ext{then} \hspace{0.1 cm} ext{l is equal to}$ 



A.  $d\sqrt{3}$ 

B.  $d/\sqrt{3}$ 

C. 3d

D.  $\sqrt{3}d/2$ 

Answer: A

**39.** In triangle ABC, if PQ, R divides sides BC, AC, and AB, respectively, in the ratio  $k: 1 (\in \text{ or } der)$ . If the ratio  $\left(\frac{arEAPQR}{areaABC}\right)$  IS  $\frac{1}{3}$ , thenk is equal to  $\frac{1}{3}$  (b) 2 (c) 3 (d) none of these

A. 1/3

B. 2

C. 3

D. none of these

## Answer: B



**40.** If the angles of a triangle are  $30^\circ~{
m and}~45^\circ$ , and the included side is

 $\left(\sqrt{3}+1
ight)$  cm, then

A. 
$$\frac{\sqrt{3}+1}{2}$$
 sq. units  
B.  $(\sqrt{3}+1)$  sq. units  
C.  $2(\sqrt{3}-1)$  sq. units  
D.  $\frac{2\sqrt{3}-1}{2}$  sq. units

### Answer: A



**41.** In triangle ABC, base BC and area of triangle are fixed. The locus of the centroid of triangle ABC is a straight line that is parallel to side BC right bisector of side BC perpendicular to BC inclined at an angle  $\sin^{-1}\left(\frac{\sqrt{BC}}{BC}\right)$  to side BC

A. parallel to side BC

B. right bisector of side BC

C. prependicular to BC

D. inclined at an angle  $\sin^{-1} ig( \sqrt{\Delta} \, / \, BC ig)$  to side BC

## Answer: A



**42.** Let the area of triangle ABC be  $\frac{\left(\sqrt{3}-1\right)}{2}, b=2$ , and  $c=\left(\sqrt{3}-1\right)$ , and  $\angle A$  be acute. The measure of the angle C is  $15^0$  (b)  $30^0$  (c)  $60^0$  (d)  $75^0$ 

A.  $15^{\,\circ}$ 

B.  $30^{\circ}$ 

C.  $60^{\circ}$ 

D.  $75^{\circ}$ 

Answer: A

**43.** In  $\Delta ABC, \Delta = 6, abc = 60, r = 1$ . Then the value of  $rac{1}{a} + rac{1}{b} + rac{1}{c}$  is

# nearly

 $\mathsf{A.}\,0.5$ 

B. 0.6

C. 0.4

D. 0.8

### Answer: D



**44.** Triangle ABC is isosceles with AB = AC and BC = 65cm. P is a point on BC such that the perpendicular distances from P to AB and AC are 24cm and 36cm, respectively. The area of triangle ABC (in sq cm is)

A. 1254

B. 1950

C. 2535

D. 5070

Answer: C

Watch Video Solution

45. In an equilateral triangle, the inradius, circumradius, and one of the

exradii are in the ratio

A. 2:4:5

B. 1:2:3

C. 1: 2: 4

D. 2:4:3

Answer: B

**46.** In triangle ABC, if  $\cos A + \cos B + \cos C = \frac{7}{4}$ , then  $\frac{R}{r}$  is equal to

A. 
$$\frac{3}{4}$$
  
B.  $\frac{4}{3}$   
C.  $\frac{2}{3}$   
D.  $\frac{3}{2}$ 

#### Answer: B



**47.** If two sides of a triangle are roots of the equation  $x^2 - 7x + 8 = 0$ and the angle between these sides is  $60^\circ$  then the product of inradius and circumradius of the triangle is

A. 
$$\frac{8}{7}$$
  
B.  $\frac{5}{3}$ 

$$\mathsf{C}.\,\frac{5\sqrt{2}}{3}$$

D. 8

## Answer: B



**48.** Given 
$$b=2, c=\sqrt{3}, ot A=30^\circ$$
 , then inradius of  $\Delta ABC$  is

A. 
$$\frac{\sqrt{3}-1}{2}$$
  
B.  $\frac{\sqrt{3}+1}{2}$   
C.  $\frac{\sqrt{3}-1}{4}$ 

D. none of these

## Answer: A

**49.** In triangle ABC, if  $A - B = 120^2$  and R = 8r, where R and r have their usual meaning, then cos C equals

A. 3/4

B. 2/3

C.5/6

D. 7/8

## Answer: D

Watch Video Solution

**50.** ABC is an equilateral triangle of side 4cm. If R, r and h are the circumradius, inradius, and altitude, respectively, then  $\frac{R+r}{h}$  is equal to

A. 4

B. 2

C. 1

## Answer: C

# Watch Video Solution

**51.** A circle is inscribed in a triangle ABC touching the side AB at D such that AD = 5, BD = 3, if  $\angle A = 60^{0}$  then length BC equals. 9 (b) $\frac{120}{13}$ (c) 13(d) 12

## A. 9

B.  $\frac{120}{13}$ 

C. 13

D. 12

# Answer: C

**52.** The rational number which equals the number 2. 357 with recurring decimal is  $\frac{2355}{1001}$  b.  $\frac{2379}{997}$  c.  $\frac{2355}{999}$  d. none of these A.  $\frac{25}{9}$ B.  $\frac{25}{3}$ C.  $\frac{25}{18}$ D.  $\frac{10}{3}$ Answer: B

**53.** Let AD be a median of the  $\Delta ABC$ . If AE and AF are medians of the triangle ABD and ADC, respectively, and BD=a/2 AD =  $m_1, AE = m_2, AF = m_3$ , then  $a^2/8$  is equal to

A.  $m_2^2+m_3^2-2m_1^2$ B.  $m_1^2+m_2^2-2m_3^2$ 

$${\sf C}.\,m_1^2+m_3^2-2m_2^2$$

D. none of these

Answer: A

Watch Video Solution

**54.** For a triangle ABC,  $R = \frac{5}{2}$  and r = 1. Let D, E and F be the feet of

the perpendiculars from incentre I to BC, CA and AB, respectively. Then

the value of  ${(IA)(IB)(IC)\over (ID)(IE)(IF)}$  is equal to \_\_\_\_\_

A. 
$$\frac{5}{2}$$
  
B.  $\frac{5}{4}$   
C.  $\frac{1}{10}$   
D.  $\frac{1}{5}$ 

## Answer: C

**55.** In triangle ABC,  $\angle A = 60^{\circ}$ ,  $\angle B = 40^{\circ}$ ,  $and \angle C = 80^{\circ}$ . If P is the center of the circumcircle of triangle ABC with radius unity, then the radius of the circumcircle of triangle BPC is (a)1 (b)  $\sqrt{3}$  (c) 2 (d)  $\sqrt{3}$  2

A. 1

 $\mathsf{B.}\,\sqrt{3}$ 

C. 2

D.  $\sqrt{3}/2$ 

## Answer: A

Watch Video Solution

56. If H is the othrocenter of an acute angled triangle ABC whose circumcircle is  $x^2 + y^2 = 16$ , then circumdiameter of the triangle HBC is 1 (b) 2 (c) 4 (d) 8

D		2
D	•	2

C. 4

D. 8

## Answer: D

Watch Video Solution

57. In triangle ABC, the line joining the circumcenter and incenter is parallel to side AC, then  $\cos A + \cos C$  is equal to

A.  $\frac{1}{2}$ 

B. 1

C.  $\sqrt{3}$ 

D. 2

### Answer: B

**58.** In triangle ABC, line joining the circumcenter and orthocenter is parallel to side AC, then the value of tan A tan C is equal to

A.  $\sqrt{3}$ 

B. 3

C.  $3\sqrt{3}$ 

D. none of these

Answer: B

Watch Video Solution

**59.** In triangle ABC,  $\angle C = \frac{2\pi}{3}$  and CD is the internal angle bisector of  $\angle C$ , meeting the side ABatD. If Length CD is 1, the H.M. of aandb is equal to: 1 (b) 2 (c) 3 (d) 4

В	2

C. 3

D. 4

### Answer: B

Watch Video Solution

**60.** In the given figure  $\Delta ABC$  is equilateral on side AB produced. We choose a point such that A lies between P and B. We now denote 'a' as the length of sides of  $\Delta ABC$ ,  $r_1$  as the radius of incircle  $\Delta PAC$  and  $r_2$  as the ex-radius of  $\Delta PBC$  with respect to side BC. Then  $r_1 + r_2$  is equal to

A. (a) 
$$\frac{1}{2}$$
  
B. (b)  $\frac{3}{2}a$   
C. (c)  $\frac{\sqrt{3}}{2}a$   
D. (d)  $a\sqrt{2}$ 

-

# Answer: C

# Watch Video Solution

**61.** A variable triangle ABC is circumscribed about a fixed circle of unit radius. Side BC always touches the circle at D and has fixed direction. If B and C vary in such a way that (BD) (CD)=2, then locus of vertex A will be a straight line. parallel to side BC perpendicular to side BC making an angle  $\left(\frac{\pi}{6}\right)$  with BC making an angle  $\sin^{-1}\left(\frac{2}{3}\right)$  with BC

A. parallel to side BC

B. perpendicular to side BC

C. making an angle  $(\pi/6)$  with BC

D. making an angle  $\sin^{-1}(2/3)$  with BC

### Answer: A

**62.** In  $\triangle ABC$ , if a = 10 and  $b \cot B + c \cot C = 2(r+R)$  then the

maximum area of  $\Delta ABC$  will be

A. 50

B.  $\sqrt{50}$ 

C. 25

D. 5

### Answer: C

Watch Video Solution

**63.** Let C be incircle of  $\triangle ABC$ . If the tangents of lengths  $t_1, t_2$  and  $t_3$  are drawn inside the given triangle parallel to side a,b, and c, respectively, then  $\frac{t_1}{a} + \frac{t_2}{b} + \frac{t_3}{c}$  is equal to

A. 0

B. 1

C. 2

D. 3

### Answer: B

Watch Video Solution

**64.** A park is in the form of a rectangle 120mx100m. At the centre of the park there is a circular lawn. The area of park excluding lawn is  $8700m^2$ . Find the radius of the circular lawn.  $\left(Use\pi\frac{22}{7}\right)$ 



D. none of these

## Answer: C

**65.** In triangle ABC, if  $r_1=2r_2=3r_3$ , then  $a\!:\!b$  is equal to

A. 
$$\frac{5}{4}$$
  
B.  $\frac{4}{5}$   
C.  $\frac{7}{4}$   
D.  $\frac{4}{7}$ 

## Answer: A

**Watch Video Solution** 

**66.** If in a triangle, 
$$\left(1 - \frac{r_1}{r_2}\right)\left(1 - \frac{r_1}{r_3}\right) = 2$$
, then the triangle is

A. right angled

B. isosceles

C. equilateral

D. none of these

## Answer: A



67. If in a triangle 
$$rac{r}{r_1}=rac{r_2}{r_3}$$
 , then

- A.  $A=90^{\circ}$
- B.  $B=90^{\circ}$
- C.  $C=90^{\circ}$
- D. none of these

### Answer: C



**68.** In  $\triangle ABC$ , I is the incentre, Area of  $\triangle IBC$ ,  $\triangle IAC$  and  $\triangle IAB$  are, respectively,  $\Delta_1$ ,  $\Delta_2$  and  $\Delta_3$ . If the values of  $\Delta_1$ ,  $\Delta_2$  and  $\Delta_3$  are in A.P., then the altitudes of the  $\triangle ABC$  are in

A. A.P.

B. G.P.

C. H.P.

D. none of these

Answer: C

Watch Video Solution

**69.** In an acute angled triangle ABC,  $r+r_1=r_2+r_3$  and  $\angle B>rac{\pi}{3}$ , then

A. b+2c<2a<2b+2c

 $\mathsf{B}.\,b+4<4a<2b+4c$ 

 ${\sf C}.\,b+4c<4a<4b+4c$ 

 $\mathsf{D}.\,b+3c<3a<3b+3c$ 

Answer: D

70. If in triangle 
$$ABC$$
,  $\sum \frac{\sin A}{2} = \frac{6}{5}and \sum II_1 = 9$  (where  $I_1, I_2andI_3$  are excenters and  $I$  is incenter, then circumradius  $R$  is equal to  $\frac{15}{8}$  (b)  $\frac{15}{4}$  (c)  $\frac{15}{2}$  (d)  $\frac{4}{12}$   
A.  $\frac{15}{8}$ 

$$\mathsf{B.}\,\frac{15}{4}$$

C. 
$$\frac{15}{2}$$

D. 
$$\frac{4}{12}$$

# Answer: A



**71.** The radii  $r_1$ ,  $r_2$ ,  $r_3$  of the escribed circles of the triangle ABC are in H.P. If the area of the triangle is  $24cm^2$  and its perimeter is 24 cm, then the length of its largest side is A. 10

B. 9

C. 8

D. none of these

Answer: A

Watch Video Solution

**72.** In *ABC* with usual notations, if  $r = 1, r_1 = 7$  and R = 3, the (a)ABC is equilateral (b) acute angled which is not equilateral (c) obtuse angled (d) right angled

A. equilateral

B. acute angled which is not equilateral

C. obtuse angled

D. right angled

## Answer: D



73. Which of the following expresses the circumference of a circle inscribed in a sector OAB with radius RandAB = 2a?  $2\pi \frac{Ra}{R+a}$  (b)  $\frac{2\pi R^2}{a} 2\pi (r-a)^2$  (d)  $2\pi \frac{R}{R-a}$ A.  $2\pi \frac{Ra}{R+a}$ B.  $\frac{2\pi R^2}{a}$ C.  $2\pi (R-a)^2$ D.  $2\pi \frac{R}{R-a}$ 

Answer: A

74. In ABC, the median AD divides  $\angle BAC$  such that  $\angle BAD: \angle CAD = 2:1$ . Then  $\cos\left(\frac{A}{3}\right)$  is equal to  $\frac{\sin B}{2\sin C}$  (b)  $\frac{\sin C}{2\sin B}$  $\frac{2\sin B}{\sin C}$  (d) noneofthese A.  $\frac{\sin B}{2\sin C}$ 

D. none of these

B.  $\frac{\sin C}{2\sin B}$ 

C.  $\frac{2\sin B}{\sin C}$ 

#### Answer: A

**Watch Video Solution** 

**75.** The area of the circle and the area of a regular polygon of n sides and the perimeter of polygon equal to that of the circle are in the ratio of

$$\tan\left(\frac{\pi}{n}\right):\frac{\pi}{n}$$
 (b)  $\cos\left(\frac{\pi}{n}\right):\frac{\pi}{n}\frac{\sin\pi}{n}:\frac{\pi}{n}$  (d)  $\cot\left(\frac{\pi}{n}\right):\frac{\pi}{n}$ 

A. 
$$\tan\left(\frac{\pi}{n}\right): \frac{\pi}{n}$$

B. 
$$\cos\left(\frac{\pi}{n}\right): \frac{\pi}{n}$$
  
C.  $\sin \cdot \frac{\pi}{n}: \frac{\pi}{n}$   
D.  $\cot\left(\frac{\pi}{n}\right): \frac{\pi}{n}$ 

### Answer: A

Watch Video Solution

**76.** The ratio of the area of a regular polygon of n sides inscribed in a circle to that of the polygon of same number of sides circumscribing the same is 3:4. Then the value of n is 6 (b) 4 (c) 8 (d) 12

A. 6

B. 4

C. 8

D. 12

## Answer: A



77. In any triangle, the minimum value of  $r_1r_2r_3\,/\,r^3$  is equal to

A. 1

B. 9

C. 27

D. none of these

### Answer: C

Watch Video Solution

**78.** If  $R_1$  is the circumradius of the pedal triangle of a given triangle ABC, and  $R_2$  is the circumradius of the pedal triangle of the pedal triangle formed, and so on  $R_3$ ,  $R_4$ ..., then the value of  $\sum_{i=1}^{\infty} R_i$ , where R (circumradius) of  $\Delta ABC$  is 5 is

A. 8	
B. 10	
C. 12	

### Answer: B

D. 15

Watch Video Solution

**79.** A sector *OABO* of central angle  $\theta$  is constructed in a circle with centre *O* and of radius 6. The radius of the circle that is circumscribed about the triangle *OAB*, is  $6\frac{\cos\theta}{2}$  (b)  $6\frac{\sec\theta}{2} 3\frac{\sec\theta}{2}$  (d)  $3\left(\frac{\cos\theta}{2}+2\right)$ 

A.  $6 \cos \frac{\theta}{2}$ B.  $6 \sec \frac{\theta}{2}$ C.  $3 \sec \frac{\theta}{2}$ D.  $3\left(\cos \frac{\theta}{2} + 2\right)$ 

# Answer: C

# Watch Video Solution

**80.** There is a point P inside an equilateral  $\Delta ABC$  of side a whose distances from vertices A, B and C are 3, 4 and 5, respectively. Rotate the triangle and P through  $60^{\circ}$  about C. Let A go to A' and P to P'. Then the area of  $\Delta PAP'$  (in sq. units) is

A. 8

B. 12

C. 16

D. 6

Answer: D

1. The sides of ABC satisfy the equation  $2a^2 + 4b^2 + c^2 = 4ab + 2ac$ Then a) the triangle is isosceles b) the triangle is obtuse c)  $B = \cos^{-1}\left(\frac{7}{8}\right)$  d)  $A = \cos^{-1}\left(\frac{1}{4}\right)$ 

A. the triangle is isosceles

B. the triangle is obtuse

C. 
$$B = \cos^{-1}(7/8)$$

D. 
$$A = \cos^{-1}(1/4)$$

### Answer: A::C::D

Watch Video Solution

**2.** If sides of triangle ABC are a, b, and c such that 2b = a + c, then

A. 
$$\displaystyle rac{b}{c} > \displaystyle rac{2}{3}$$

$$\begin{array}{l} \mathsf{B}.\,\frac{b}{c}>\frac{1}{3}\\ \mathsf{C}.\,\frac{b}{c}<2\\ \mathsf{D}.\,\frac{b}{c}<\frac{3}{2} \end{array}$$

### Answer: A::C

Watch Video Solution

3. If the sines of the angle A and B of a triangle ABC satisfy the equation

 $c^2x^2-c(a+b)x+ab=0$ , then the triangle

A. is acute angled

B. is right angled

C. is obtus angled

D. satisfies the equation  $\sin A + \cos A = rac{(a+b)}{c}$ 

### Answer: B::D
4. There exist a triangle ABC satisfying

A. 
$$\tan A + \tan B + \tan C = 0$$
  
B.  $\frac{\sin A}{2} = \frac{\sin B}{3} = \frac{\sin C}{7}$   
C.  $(a+b)^2 = c^2 + ab$  and  $\sqrt{2}(\sin A + \cos A) = \sqrt{3}$   
D.  $\sin A + \sin B = \frac{\sqrt{3}+1}{2}$ ,  $\cos A \cos B = \frac{\sqrt{3}}{4} = \sin A \sin B$ 

# Answer: C::D

Watch Video Solution

5. In triangle, ABC if  $2a^2b^2+2b^2c^2=a^4+b^4+c^4$ , then angle B is equal

# to

A.  $45^{\,\circ}$ 

B.  $135^{\,\circ}$ 

C.  $120^{\circ}$ 

Answer: A::B



**6.** If in triangle ABC, a, c and angle A are given and  $c \sin A < a < c$ , then (

 $b_1 \hspace{0.1 cm} ext{and} \hspace{0.1 cm} b_2 ext{ are values of b)}$ 

- A.  $b_1 + b_2 = 2c \cos A$
- $\mathsf{B}.\,b_1+b_2=c\cos A$
- C.  $b_1b_2=c^2-a^2$
- D.  $b_1b_2=c^2+a^2$

### Answer: A::C

7. If area of  $\Delta ABC(\Delta)$  and angle C are given and if c opposite to given

angle is minimum, then

A. 
$$a = \sqrt{rac{2\Delta}{\sin C}}$$
  
B.  $b = \sqrt{rac{2\Delta}{\sin C}}$   
C.  $a = rac{4\Delta}{\sin C}$   
D.  $b = rac{4\Delta}{\sin^2 C}$ 

# Answer: A::B

8. If 
$$\Delta$$
 represents the area of acute angled triangle ABC  
 $\sqrt{a^2b^2 - 4\Delta^2} + \sqrt{b^2c^2 - 4\Delta^2} + \sqrt{c^2a^2 - 4\Delta^2} =$   
A.  $a^2 + b^2 + c^2$   
B.  $\frac{a^2 + b^2 + c^2}{2}$   
C.  $ab\cos C + bc\cos A + ca\cos B$ 

 $\mathsf{D}.\,ab\sin C+bc\sin A+ca\sin B$ 

## Answer: B::C



**9.** Sides of  $\Delta ABC$  are in A.P. If  $a < \min\{b, c\}$ , then  $\cos$  A may be equal to

A. 
$$\frac{4b - 3c}{2b}$$
  
B. 
$$\frac{3c - 4b}{2c}$$
  
C. 
$$\frac{4c - 3b}{2b}$$
  
D. 
$$\frac{4c - 3b}{2c}$$

Answer: A::D

10. If the angles of a triangle are  $30^\circ~{
m and}~45^\circ$ , and the included side is  $\left(\sqrt{3}+1
ight)$  cm, then

A. area of the triangle is  $\displaystyle rac{1}{-}ig(\sqrt{3}+1ig)$  sq. units

B. area of the triangle is  $rac{1}{2}ig(\sqrt{3}-1ig)$  sq. units

C. ratio of greater side to smaller side is  $\frac{\sqrt{3}+1}{\sqrt{2}}$ D. ratio of greater side to smaller side is  $\frac{1}{4\sqrt{3}}$ 

## Answer: A::C

**Watch Video Solution** 

11. Lengths of the tangents from A,B and C to the incircle are in A.P., then

A.  $r_1, r_2r_3$  are in H.P

B.  $r_1, r_2, r_3$  are in AP

C. a, b, c are in A.P

 $\mathsf{D.}\cos A = \frac{4c-3b}{2c}$ 

# Answer: A::C::D



12. CF is the internal bisector of angle C of ABC , then CF is equal to

(a) 
$$\frac{2ab}{a+b}\cos\left(\frac{C}{2}\right)$$
 (b)  $\frac{a+b}{2ab}\frac{\cos C}{2}$  (c)  $\frac{a\sin B}{\sin\left(B+\frac{C}{2}\right)}$  (d) none of these



D. none of these

# Answer: A::C



**13.** The incircle of  $\Delta ABC$  touches side BC at D. The difference between

BD and CD (R is circumradius of  $\Delta ABC$ ) is

A. 
$$\left| 4R \sin \frac{A}{2} \sin \frac{B-C}{2} \right|$$
  
B.  $\left| 4R \cos \frac{A}{2} \sin \frac{B-C}{2} \right|$   
C.  $\left| b-c \right|$ 

$$\mathsf{D}. \left| \frac{b-c}{2} \right|$$

# Answer: A::C



14. A circle of radius 4 cm is inscribed in  $\Delta ABC$ , which touches side BC at

D. If BD = 6 cm, DC = 8 cm then

A. the triangle is necessarily acute angled

 $\mathsf{B.}\tan.\,\frac{A}{2}=\frac{4}{7}$ 

C. perimeter of the triangle ABC is 42 cm

D. area of  $\triangle ABC$  is  $84cm^2$ 

Answer: A::B::C::D



15. If H is the orthocentre of triangle ABC, R = circumradius and P = AH + BH + CH, then

A. P = 2(R+r)

B. max. of P is 3R

C. min. of P is 3R

 $\mathsf{D}.\, P = 2(R-r)$ 

Answer: A::B

**16.** Let ABC be an isosceles triangle with base BC. If r is the radius of the circle inscribed in  $\Delta ABC$  and  $r_1$  is the radius of the circle ecribed opposite to the angle A, then the product  $r_1r$  can be equal to (where R is the radius of the circumcircle of  $\Delta ABC$ )

A.  $R^2 \sin^2 A$ B.  $R^2 \sin^2 2B$ C.  $\frac{1}{2}a^2$ D.  $\frac{a^2}{4}$ 

Answer: A::B::D

Watch Video Solution

17. If inside a big circle exactly  $n(n \le 3)$  small circles, each of radius r, can be drawn in such a way that each small circle touches the big circle and also touches both its adjacent small circles, then the radius of big

circle is 
$$r\left(1 + \cos ec \frac{\pi}{n}\right)$$
 (b)  $\left(\frac{1 + \frac{\tan \pi}{n}}{\frac{\cos \pi}{\pi}}\right)$   $r\left[1 + \cos ec \frac{2\pi}{n}\right]$  (d)  

$$\frac{r\left[s \in \frac{\pi}{2n} + \frac{\cos(2\pi)}{n}\right]^2}{\frac{\sin \pi}{n}}$$
A.  $r\left(1 + \cos ec. \frac{\pi}{n}\right)$ 
B.  $\left(\frac{1 + \tan \pi/n}{\cos \pi/n}\right)$ 
C.  $r\left[1 + \cos ec. \frac{2\pi}{n}\right]$ 

D. 
$$\frac{r\left[\sin.\frac{\pi}{2n} + \cos.\frac{2\pi}{n}\right]}{\sin\pi/n}$$

### Answer: A::D

**18.** The area of a regular polygon of n sides is (where r is inradius, R is circumradius, and a is side of the triangle)  $\frac{nR^2}{2}\sin\left(\frac{2\pi}{n}\right)$  (b)  $nr^2 \tan\left(\frac{\pi}{n}\right) \frac{na^2}{4} \frac{\cot \pi}{n}$  (d)  $nR^2 \tan\left(\frac{\pi}{n}\right)$ 

A. 
$$\frac{nR^2}{2}\sin\left(\frac{2\pi}{n}\right)$$

B. 
$$nr^2 \tan\left(\frac{\pi}{n}\right)$$
  
C.  $\frac{na^2}{4}$ cot.  $\frac{\pi}{n}$   
D.  $nR^2 \tan\left(\frac{\pi}{n}\right)$ 

### Answer: A::B::C

Watch Video Solution

**19.** In acute angled triangle ABC, AD is the altitude. Circle drawn with AD as its diameter cuts ABandACatPandQ, respectively. Length of PQ is equal to /(2R) (b)  $\frac{abc}{4R^2} 2R \sin A \sin B \sin C$  (d)  $\Delta/R$ 

A. 
$$\frac{\Delta}{2R}$$
  
B.  $\frac{abc}{4R^2}$ 

 $\mathsf{C.}\,2R\sin A\sin B\sin C$ 

D. 
$$\frac{\Delta}{R}$$

### Answer: C::D



**20.** If A is the area and 2s is the sum of the sides of a triangle, then

$$egin{aligned} A&\leq rac{s^2}{4} ext{ (b) } A&\leq rac{s^2}{3\sqrt{3}} \ 2R\sin A\sin B\sin C ext{ (d) } noneof these \ egin{aligned} \mathsf{A}.\,A&\leq rac{s^2}{4} \ \mathsf{B}.\,A&\leq rac{s^2}{3\sqrt{3}} \ \mathsf{C}.\,A&<rac{s^2}{\sqrt{3}} \end{aligned}$$

D. none of these

### Answer: A::B

Watch Video Solution

**21.** In *ABC*, internal angle bisector of  $\angle A$  meets side *BC* in *D*.  $DE \perp AD$  meets *AC* at *E* and *AB* at *F*. Then (a) *AE* is in *H*. *P*. of *b* and *c* (b)  $AD = \frac{2bc}{b+c} \frac{\cos A}{2}$  (c)  $EF = \frac{4bc}{b+c} \frac{\sin A}{2}$  (d) *AEF* is isosceles A. AE in H.M of b and c

B. 
$$AD = rac{2bc}{b+c} \cos{\cdot} rac{A}{2}$$
  
C.  $EF = rac{4bc}{b+c} \sin{\cdot} rac{A}{2}$ 

D.  $\Delta AEF$  is isosceles

### Answer: A::B::C::D

Watch Video Solution

**22.** In a triangle ABC, AB = 5, BC = 7, AC = 6. A point P is in the plane such that it is at distance '2' units from AB and 3 units form AC then its distance from BC

A. is  $\frac{12\sqrt{6}-28}{7}$  when P is inside the triagle B. may be  $\frac{12\sqrt{6}-8}{7}$  when P is outside the triangle C. may be  $\frac{12\sqrt{6}+14}{7}$  when P is inside the triangle D. may be  $\frac{12\sqrt{6}+14}{7}$  when P is outside the triangle

# Answer: A::B::C



**23.** The base *BC* of *ABC* is fixed and the vertex *A* moves, satisfying the condition  $\cot\left(\frac{B}{2}\right) + \cot\left(\frac{C}{2}\right) = 2\cot\left(\frac{A}{2}\right)$ , then (a) b + c = a (b) b + c = 2a (c) vertex *A* moves along a straight line (d) Vertex *A* moves along an ellipse

A. b + c = a

 $\mathrm{B.}\,b+c=2a$ 

C. vertex A moves along a straight line

D. vertex A moves along an ellipse

### Answer: B::D

24. If D,E and F are the middle points of the sides BC,CA and AB of the  $\Delta ABC$ , then AD+BE+CF is

A. centroid of the triangle DEF is the same as that of ABC

B. orthocenter of the triangle DEF is the circumcentre of ABC

C. orthocenter of the triangle DEF is the incenter of ABC

D. centroid of the triangle DEF is not the same as that of ABC

## Answer: A::B

Watch Video Solution

Linked Comprehension Type

1. Given that  $\Delta=6, r_1=3, r_3=6$ 

Circumradius R is equal to

B. 3.5

C. 1.5

D. none of these

# Answer: A

**Watch Video Solution** 

**2.** Given that  $\Delta=6, r_1=3, r_3=6$ 

# Inradius is equal to

A. 2

B. 1

C. 1.5

D. 2.5

### Answer: B

3. Given that  $\Delta=6, r_1=2, r_2=3, r_3=6$  Difference between the greatest and the least angles is

A. 
$$\cos^{-1} \cdot \frac{4}{5}$$
  
B.  $\tan^{-1} \cdot \frac{3}{4}$   
C.  $\cos^{-1} \cdot \frac{3}{5}$ 

D. none of these

# Answer: C

**Watch Video Solution** 

**4.** Let a = 6, b = 3 and 
$$\cos(A - B) = \frac{4}{5}$$

Area (in sq. units) of the triangle is equal to

A. 9

B. 12

C. 11

D. 10

Answer: A

Watch Video Solution

5. Let a = 6, b = 3 and 
$$\cos(A-B) = rac{4}{5}$$

Angle C is equal to

A. 
$$\frac{3\pi}{4}$$
  
B.  $\frac{\pi}{4}$   
C.  $\frac{\pi}{2}$ 

D. none of these

# Answer: C

6. Let a = 6, b = 3 and 
$$\cos(A - B) = \frac{4}{5}$$

Value of  $\sin A$  is equal to

A. 
$$\frac{1}{2\sqrt{5}}$$
  
B. 
$$\frac{1}{\sqrt{3}}$$
  
C. 
$$\frac{1}{\sqrt{5}}$$
  
D. 
$$\frac{2}{\sqrt{5}}$$

### Answer: D

# > Watch Video Solution

7. Let ABC be an acute angled triangle with orthocenter H.D, E, and F are the feet of perpendicular from A,B, and C, respectively, on opposite sides. Also, let R be the circumradius of  $\Delta ABC$ . Given AH. BH. CH = 3 and  $(AH)^2 + (BH)^2 + (CH)^2 = 7$ Then answer the following

Value of 
$$rac{\cos A. \cos B. \cos C}{\cos^2 A + \cos^2 B + \cos^2 C}$$
 is

A. 
$$\frac{3}{14R}$$
  
B. 
$$\frac{3}{7R}$$
  
C. 
$$\frac{7}{3R}$$
  
D. 
$$\frac{14}{3R}$$

### Answer: A

# Watch Video Solution

**8.** Let ABC be an acute angled triangle with orthocenter H.D, E, and F are the feet of perpendicular from A,B, and C, respectively, on opposite sides. Also, let R be the circumradius of  $\Delta ABC$ . Given AH. BH. CH = 3 and  $(AH)^2 + (BH)^2 + (CH)^2 = 7$ 

Then answer the following

Value of R is

A. 1

 $\mathsf{B}.\,\frac{3}{2}$ 

$$\mathsf{C}.\,\frac{5}{2}$$

D. none

### Answer: B

Watch Video Solution

**9.** Let ABC be an acute angled triangle with orthocenter H.D, E, and F are the feet of perpendicular from A,B, and C, respectively, on opposite sides. Also, let R be the circumradius of  $\Delta ABC$ . Given AH. BH. CH = 3 and  $(AH)^2 + (BH)^2 + (CH)^2 = 7$ Then answer the following

Value of HD. HE. HF is

A. 
$$\frac{9}{64R^3}$$
  
B.  $\frac{9}{8R^3}$   
C.  $\frac{8}{9R^3}$   
D.  $\frac{64}{9R^3}$ 

# Answer: B



10. Let 
$$O$$
 be a point inside a triangle  $ABC$  such that  
 $\angle OAB = \angle OBC = \angle OCA = \omega$ , then show that:  
 $\cot \omega = \cot A + \cot B + \cot C$   
 $\cos ec^2 \omega = \cos ec^2 A + \cos ec^2 B + \cos ec^2 C$   
A.  $\tan^2 \theta$   
B.  $\cot^2 \theta$   
C.  $\tan \theta$   
D.  $\cot \theta$ 

## Answer: D



$$\left(\frac{\sqrt{3}}{2}\right)$$

Watch Video Solution

12. Let O be a point inside  $\Delta ABC$  such that

$$\angle AOB = \angle BOC = \angle COA = \theta$$

Area of  $\Delta ABC$  is equal to

$$\begin{aligned} &\mathsf{A}. \left(\frac{a^2+b^2+c^2}{4}\right)\!\tan\theta\\ &\mathsf{B}. \left(\frac{a^2+b^2+c^2}{4}\right)\!\cot\theta\\ &\mathsf{C}. \left(\frac{a^2+b^2+c^2}{2}\right)\!\tan\theta\\ &\mathsf{D}. \left(\frac{a^2+b^2+c^2}{2}\right)\!\cot\theta \end{aligned}$$

## Answer: A

**View Text Solution** 

**13.** Given an isoceles triangle with equal side of length b and angle

 $lpha < \pi/4$ , then

the circumradius R is given by

A. 
$$\frac{1}{2}b\cos ec\alpha$$

B.  $b\cos ec\alpha$ 

 $\mathsf{C}.\,2b$ 

D. none of these

# Answer: A

Watch Video Solution

14. Given an isoceles triangle with equal side of length b and angle  $lpha < \pi/4$ , then

the inradius r is given by

A. 
$$rac{b\sin 2lpha}{2(1-\coslpha)}$$

B. 
$$\frac{b\sin 2\alpha}{2(1+\cos\alpha)}$$
C. 
$$\frac{b\sin\alpha}{2}$$
D. 
$$\frac{b\sin\alpha}{2(1+\sin\alpha)}$$

## Answer: B

**Watch Video Solution** 

**15.** Given an isoceles triangle with equal side of length b and angle

 $lpha < \pi/4$  , then

the distance between circumcenter O and incenter I is

A. 
$$\left| \frac{b \cos(3\alpha/2)}{2 \sin \alpha \cos(\alpha/2)} \right|$$
  
B. 
$$\left| \frac{b \cos 3\alpha}{\sin 2\alpha} \right|$$
  
C. 
$$\left| \frac{b \cos 3\alpha}{\cos \alpha \sin(\alpha/2)} \right|$$
  
D. 
$$\left| \frac{b}{\sin \alpha \cos \alpha/2} \right|$$

## Answer: A



16. Incircle of  $\Delta ABC$  touches the sides BC, AC and AB at D, E and F, respectively. Then answer the following question

 $\angle DEF$  is equal to

A. 
$$rac{\pi-B}{2}$$

 $\mathrm{B.}\,\pi-2B$ 

 $\mathsf{C}.A - C$ 

D. none of these

# Answer: A

**Watch Video Solution** 

17. Incircle of  $\Delta ABC$  touches the sides BC, AC and AB at D, E and F, respectively. Then answer the following question

 $\angle DEF$  is equal to

A.  $2r^2\sin(2A)\sin(2B)\sin(2C)$ 

 $\mathsf{B.} \, 2r^2 \cos. \, \frac{A}{2} \mathrm{cos.} \, \frac{B}{2} \mathrm{cos.} \, \frac{C}{2}$ 

C.  $2r^2\sin(A-B)\sin(B-C)\sin(C-A)$ 

D. none of these

### Answer: B

**Watch Video Solution** 

**18.** Incircle of  $\Delta ABC$  touches the sides BC, AC and AB at D, E and F, respectively. Then answer the following question

The length of side EF is

A. 
$$r \sin \frac{A}{2}$$
  
B.  $2r \sin \frac{A}{2}$   
C.  $r \cos \frac{A}{2}$   
D.  $2r \cos \frac{A}{2}$ 

## Answer: D



**19.** Bisectors of angles A, B and C of a triangle ABC intersect its circumcircle at D, E and F respectively. Prove that the angles of the triangle DEF are  $90^{\circ} - \frac{1}{2}A$ ,  $90^{\circ} - \frac{1}{2}B$  and  $90^{\circ} - \frac{1}{2}C$ 



### Answer: A

**20.** Internal bisectors of  $\Delta ABC$  meet the circumcircle at point D, E, and F

Area of  $\Delta DEF$  is

A. 
$$2R^2 \cos^2\left(\frac{A}{2}\right) \cos^2\left(\frac{B}{2}\right) \cos^2\left(\frac{C}{2}\right)$$
  
B.  $2R^2 \sin\left(\frac{A}{2}\right) \sin\left(\frac{B}{2}\right) \sin\left(\frac{C}{2}\right)$   
C.  $2R^2 \sin^2\left(\frac{A}{2}\right) \sin^2\left(\frac{B}{2} \sin^2\left(\frac{C}{2}\right)$   
D.  $2R^2 \cos\left(\frac{A}{2}\right) \cos\left(\frac{B}{2}\right) \cos\left(\frac{C}{2}\right)$ 

## Answer: D



**21.** Internal bisectors of  $\Delta ABC$  meet the circumcircle at point D, E, and F

Area of  $\Delta DEF$  is

- A.  $\geq 1$
- B.  $\leq 1$
- $\mathsf{C.}\ \geq 1/2$

D.  $\leq 1/2$ 

### Answer: B

# Watch Video Solution

22. The area of any cyclic quadrilateral ABCD is given by  $A^2 = (s-a)(s-b)(s-c)(s-d),$  where 2s = a + b + + c + d, a, b, c and d are the sides of the quadrilateral Now consider a cyclic quadrilateral ABCD of area 1 sq. unit and answer the following question

The minium perimeter of the quadrilateral is

A. 4

B. 2

C. 1

D. none of these

Answer: A

23. The area of any cyclic quadrilateral ABCD is given by  $A^2 = (s-a)(s-b)(s-c)(s-d),$  where 2s = a + b + + c + d, a, b, c and d are the sides of the quadrilateral Now consider a cyclic quadrilateral ABCD of area 1 sq. unit and answer the following question

The minimum value of the sum of the lenghts of diagonals is

A. 
$$2\sqrt{2}$$

B. 2

C.  $\sqrt{2}$ 

D. none of these

### Answer: A

24. The area of any cyclic quadrilateral ABCD is given by  $A^2 = (s-a)(s-b)(s-c)(s-d),$  where 2s = a + b + + c + d, a, b, c and d are the sides of the quadrilateral Now consider a cyclic quadrilateral ABCD of area 1 sq. unit and answer the following question

When the perimeter is minimum, the quadrilateral is necessarily

A. a square

B. a rectangle but not a square

C. a rhombus but not a square

D. none of these

## Answer: A



25. In  $\Delta ABC, R, r, r_1, r_2, r_3$  denote the circumradius, inradius, the exradii opposite to the vertices A,B, C respectively. Given that

 $r_1:r_2:r_3=1:2:3$ 

The sides of the triangle are in the ratio

A. 1:2:3

B. 3: 5: 7

C. 1:5:9

D. 5:8:9

### Answer: D

Watch Video Solution

26. In  $\Delta ABC, R, r, r_1, r_2, r_3$  denote the circumradius, inradius, the exradii opposite to the vertices A,B, C respectively. Given that  $r_1:r_2:r_3=1:2:3$ 

The value of R:r is

A. 5:2

B.5:4

C.5:3

 $\mathsf{D}.\,3\!:\!2$ 

Answer: A

**Watch Video Solution** 

27. In  $\Delta ABC, R, r, r_1, r_2, r_3$  denote the circumradius, inradius, the exradii opposite to the vertices A,B, C respectively. Given that  $r_1:r_2:r_3=1:2:3$ 

The greatest angle of the triangle is given by

A. 
$$\cos^{-1}\left(\frac{1}{30}\right)$$
  
B.  $\cos^{-1}\left(\frac{1}{3}\right)$   
C.  $\cos^{-1}\left(\frac{1}{10}\right)$   
D.  $\cos^{-1}\left(\frac{1}{5}\right)$ 

### Answer: C



**28.** In  $\triangle ABC$ , P, Q, R are the feet of angle bisectors from the vertices to their opposite sides as shown in the figure.  $\triangle PQR$  is constructed

If  $\angle BAC = 120^{\circ}$  , then measusred of  $\angle RPQ$  will be

A.  $60^{\circ}$ 

 $\text{B.}\,90^{\,\circ}$ 

C.  $120^{\circ}$ 

D.  $150^{\circ}$ 

# Answer: B

Watch Video Solution

**29.** In riangle ABC, P, Q, R are the feet of angle bisectors from the vertices

to their opposite sides as shown in the figure. riangle PQR is constructed



If AB = 7 units, BC = 8 units, AC = 5 units, then the side PQ will be

A. 
$$\frac{\sqrt{28}}{3}$$
 units  
B.  $\frac{\sqrt{88}}{3}$  units  
C.  $\frac{\sqrt{78}}{3}$  units  
D.  $\frac{\sqrt{84}}{3}$  units

## Answer: D



30. Let G be the centroid of triangle ABC and the circumcircle of triangle

AGC touches the side AB at A
If BC = 6, AC = 8, then the length of side AB is equal to

A. 
$$\frac{1}{2}$$
  
B.  $\frac{2}{\sqrt{3}}$ 

C.  $5\sqrt{2}$ 

D. none of these

### Answer: C

Watch Video Solution

**31.** Let G be the centroid of triangle ABC and the circumcircle of triangle AGC touches the side AB at A If  $\angle GAC = \frac{\pi}{3}$  and a = 3b, then sin C is equal to

A. 
$$\frac{3}{4}$$
  
B.  $\frac{1}{2}$   
C.  $\frac{2}{\sqrt{3}}$ 

D. none of these

### Answer: B



**32.** Let G be the centroid of triangle ABC and the circumcircle of triangle AGC touches the side AB at A If AC = 1, then the length of the median of triangle ABC through the vertex

A is equal to



### Answer: A

33. The inradius in a right angled triangle with integer sides is r

If r = 4, the greatest perimeter (in units) is

A. 96

B. 90

C. 60

D. 48

## Answer: B

Watch Video Solution

34. The inradius in a right angled triangle with integer sides is r

If r = 5, the greatest area (in sq. units) is

A. 150

B. 210

C. 330

D. 450

Answer: C



Matrix Match Type

1. show that 
$$an^{-1}igg(rac{1}{2}igg)+ an^{-1}igg(rac{2}{11}igg)= an^{-1}igg(rac{3}{4}igg)$$

Watch Video Solution

2. find the value of 
$$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

show

$$an^{-1}x+ an^{-1}igg(rac{2x}{1-x^2}igg)= an^{-1}igg(rac{3x-x^3}{1-3x^2}igg), |x|<rac{1}{\sqrt{3}}$$

Watch Video Solution

3.

4. Show that 
$$\frac{\sin^{-1}3}{5} - \frac{\sin^{-1}8}{17} = \frac{\cos^{-1}(84)}{85}$$
.  
Watch Video Solution

5. simplify 
$$\cos^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right), \frac{\pi}{4} < x < \frac{5\pi}{4}$$

Watch Video Solution

6. In a triangle ABC, a = 7, b = 8, c = 9, BD is the median and BE the altitude from the vertex B. Match the following lists

that

a. BD = p. 2b. BE = q. 7c.  $ED = r. \sqrt{45}$ d. AE = s. 6A.  $\begin{array}{c} a & b & c & d \\ p & r & q & q \\ B. & a & b & c & d \\ r & q & s & p \\ C. \begin{array}{c} a & b & c & d \\ q & r & p & s \\ p & s & p & q \end{array}$ 

### Answer: C

Watch Video Solution

Numerical Value Type

1. Suppose  $\alpha$ ,  $\beta$ ,  $\gamma and\delta$  are the interior angles of regular pentagon, hexagon, decagon, and dodecagon, respectively, then the value of  $|\cos \alpha \sec \beta \cos \gamma \cos ec\delta|$  is \_\_\_\_\_

2. Let ABCDEFGHIJKL be a regular dodecagon. Then the value of  $\frac{AB}{AF} + \frac{AF}{AB}$  is equal to \_\_\_\_

## Watch Video Solution

3. In a  $\Delta ABC, b=12$  units, c = 5 units and  $\Delta=30$ sq. units. If d is the

distance between vertex A and incentre of the triangle then the value of

 $d^2$  is \_\_\_\_\_

Watch Video Solution

**4.** In  $\Delta ABC$ , if  $r=1,\,R=3,\,\,\,{
m and}\,\,s=5$ , then the value of  $a^2+b^2+c^2$ 

is \_\_\_\_



## Watch Video Solution

**6.** In  $\Delta AEX$ , T is the midpoint of XE and P is the midpoint of ET. If  $\Delta APE$  is equilateral of side length equal to unity, then the vaue of  $(AX)^2$  is



7. In  $\Delta ABC$ , the incircle touches the sides BC, CA and AB, respectively, at D, E,and F. If the radius of the incircle is 4 units and BD, CE, and AF are consecutive integers, then

A. Sides are also consecutive integers

B. Perimeter of the triangle is 42 units

C. Area of triangle is 84 sq. units

D. Diameter of circumcircle is 65 units

Answer: 21

**Watch Video Solution** 

8. The altitudes from the angular points A,B, and C on the opposite sides

BC, CA and AB of  $\Delta ABC$  are 210, 195 and 182 respectively. Then the value

of a is \_\_\_\_

Watch Video Solution

**9.** In  $\triangle ABC$ , If  $\angle C = 3 \angle A, BC = 27$ , and AB = 48. Then the value of

AC is \_\_\_\_\_

10. The area of a right triangle is 6864 sq. units. If the ratio of its legs is

143:24, then the value of r is \_\_\_\_\_

Watch Video Solution

11. In  $\Delta ABC$ , if  $\cos A + \sin A - \frac{2}{\cos B + \sin B} = 0$ , then the value of  $\left(\frac{a+b}{c}\right)^4$  is

**Watch Video Solution** 

12. In  $\triangle ABC$ ,  $\angle C = 2\angle A$ , and AC = 2BC, then the value of  $\frac{a^2 + b^2c^2}{R^2}$  (where R is circumradius of triangle) is \_\_\_\_\_

## Watch Video Solution

**13.** In  $\triangle ABC$ , if  $b(b+c) = a^2$  and  $c(c+a) = b^2$ , then

 $|\cos A. \cos B. \cos C|$  is\_\_\_\_\_



17. In triangle	ABC,
-----------------	------

 $\sin A \sin B + \sin B \sin C + \sin C \sin A = 9/4$  and a = 2, then the

value of  $\sqrt{3}\Delta$ , where  $\Delta$  is the area of triangle, is \_\_\_\_\_

## Watch Video Solution

**18.** In a  $\triangle ABC$ , AB = 52, BC = 56, CA = 60. Let D be the foot of the altitude from A and E be the intersection of the internal angle bisector of  $\angle BAC$  with BC. Find the length DE.

# Watch Video Solution 19. Point D,E are taken on the side BC of an acute angled triangle ABC,, such that BD = DE = EC. If $\angle BAD = x, \angle DAE = y$ and $\angle EAC = z$ then the value of

 $rac{\sin(x+y)\sin(y+z)}{\sin x\sin z}$  is \_\_\_\_\_

**20.** For a triangle ABC,  $R = \frac{5}{2}$  and r = 1. Let D, E and F be the feet of the perpendiculars from incentre I to BC, CA and AB, respectively. Then the value of  $\frac{(IA)(IB)(IC)}{(ID)(IE)(IF)}$  is equal to \_\_\_\_\_ Watch Video Solution

**21.** Circumradius of  $\Delta ABC$  is 3 cm and its area is  $6cm^2$ . If DEF is the triangle formed by feet of the perpendicular drawn from A,B and C on the sides BC, CA and AB, respectively, then the perimeter of  $\Delta DEF$  (in cm) is

Watch Video Solution

**22.** The distance of incentre of the right-angled triangle ABC (right angled at A) from B and C are  $\sqrt{10}$  and  $\sqrt{5}$ , respectively. The perimeter of the triangle is \_\_\_\_\_

**1.** For a regular polygon, let r and R be the radii of the inscribed and the circumscribed circles. A false statement among the following is There is a regular polygon with  $\frac{r}{R} = \frac{1}{\sqrt{2}}$  (17) There is a regular polygon with  $\frac{r}{R} = \frac{2}{3}$  (30) There is a regular polygon with  $\frac{r}{R} = \frac{1}{2}$  (47) There is a regular polygon with  $\frac{r}{R} = \frac{1}{2}$  (60)

A. There is a regular polygon with  $\frac{r}{R} = \frac{\sqrt{3}}{2}$ B. There is a regular polygon with  $\frac{r}{R} = \frac{1}{2}$ C. There is a regular polygon with  $\frac{r}{R} = \frac{1}{\sqrt{2}}$ D. There is a regular polygon with  $\frac{r}{R} = \frac{2}{3}$ 

### Answer: D

2. ABCD is a trapezium such that AB and CD are parallel and  $BC \perp CD$ . If  $\angle ADB = \theta, BC = p$  and CD = q, then AB is equal to

A. 
$$\left(\frac{p^2 + q^2 \sin \theta}{p \cos \theta + q \sin \theta}\right)$$
  
B. 
$$\frac{\left(p^2 + q^2\right) \cos \theta}{p \cos \theta + q \sin t h e t}$$
  
C. 
$$\frac{p^2 + q^2}{p^2 \cos \theta + q^2 \sin \theta}$$
  
D. 
$$\frac{\left(p^2 + q^2\right) \sin \theta}{\left(p \cos \theta + q \sin \theta\right)}$$

### Answer: A

Watch Video Solution

Archives Jee Advanced

1. Let ABC be a triangle such that  $\angle ACB = \frac{\pi}{6}$  and let a, b and c denote the lengths of the side opposite to A, B, and C respectively. The value(s) of x for which  $a = x^2 + x + 1, b = x^2 - 1$ , and c = 2x + 1 is(are)  $-(2 + \sqrt{3})$  (b)  $1 + \sqrt{3}$  (c) $2 + \sqrt{3}$  (d)  $4\sqrt{3}$ 

A. 
$$-\left(2+\sqrt{3}
ight)$$
  
B.  $1+\sqrt{3}$   
C.  $2+\sqrt{3}$   
D.  $4\sqrt{3}$ 

### Answer: B



2. If the angles A, B and C of a triangle are in an arithmetic progression and if a, b and c denote the lengths of the sides opposite to A, B and C respectively, then the value of the expression  $\frac{a}{c}\sin 2C + \frac{c}{a}\sin 2A$  is

A. 
$$\frac{1}{2}$$
  
B.  $\frac{\sqrt{3}}{2}$   
C. 1  
D.  $\sqrt{3}$ 

### Answer: D



**3.** Let PQR be a triangle of area  $\Delta$  with a = 2, b = 7/2, and c = 5/2, where a, b and c are the lengths of the sides of the triangle opposite to the angles at P, Q and R, respectively. Then  $\frac{2\sin P - \sin 2P}{2\sin P + \sin 2P}$  equals



Answer: C

Watch Video Solution

Archives Multiple Correct Answer Type

1. In a triangle ABC with fixed base BC, the vertex A moves such that  $\cos B + \cos C = 4 \sin^2 A/2$ 

If a, b and c denote the lengths of the sides of the triangle opposite to the angles A,B and C respectively, then

A. b+c=4a

 $\mathsf{B}.\, b+c=2a$ 

C. locus of point A is an ellipse

D. locus of point A is a pair of straight lines

### Answer: B::C

Watch Video Solution

**2.** In a triangle PQR, P is the largest angle and  $\cos P = 1/3$ . Further the incircle of the triangle touches the sides PQ. QR and PR at N, L and M, respectively, such that the length of PN, QL, and RM are consecutive even integers. Then possible length (s) of the side(s) of the triangle is (are)

A. 16

B. 18

C. 24

D. 22

### Answer: B::D

Watch Video Solution

**3.** In a triangle XYZ, let x, y, z be the lengths of sides opposite to the angles X, Y, Z, respectively, and 2s = x + y + z. If  $\frac{s - x}{4} = \frac{s - y}{3} = \frac{s - z}{2}$  of incircle of the triangle XYZ is  $\frac{8\pi}{3}$ 

A. (a) area of the triangle XYZ is  $6\sqrt{6}$ 

B. (b) the radius of circumcircle of the triangle XYZ is  $\frac{35}{6}\sqrt{6}$ 

C. (c) sin. 
$$\frac{X}{2}$$
sin.  $\frac{Y}{2}$ sin.  $\frac{Z}{2} = \frac{4}{35}$   
D. (d) sin<sup>2</sup>  $\left(\frac{X+Y}{2}\right) = \frac{3}{5}$ 



**4.** In a triangle PQR, let  $\angle PQR = 30^{\circ}$  and the sides PQ and QR have lengths  $10\sqrt{3}$  and 10, respectively. Then, which of the following statement(s) is (are) TRUE ?

A.  $\angle QPR = 45^{\circ}$ 

B. The area of the triangle PQR is  $25\sqrt{3}$  and  $\angle QRP = 120^{\circ}$ 

C. The radius of the incircle of the triangle PQR is  $10\sqrt{3}-15$ 

D. The area of the circumcircle of the triangle PQR is  $100\pi$ 

### Answer: B::C::D

Watch Video Solution

Archives Matrix Match Type

1. solve equation  $2 an^{-1}(\cos x) = an^{-1}(2 \cos ecx)$ 



**Archives Numerical Value Type** 

1. Let ABCandABC' be two non-congruent triangles with sides  $AB = 4, AC = AC' = 2\sqrt{2}$  and angle  $B = 30^0$ . The absolute value of the difference between the areas of these triangles is

Watch Video Solution

**2.** Two parallel chords of a circle of radius 2 are at a distance  $\sqrt{3} + 1$  apart. If the chord subtend angles  $\frac{\pi}{k}$  and  $\frac{\pi}{2k}$  at the center, where k > 0, then the value of [k] is \_\_\_\_\_

**3.** Consider a triangle ABC and let a, b and c denote the lengths of the sides opposite to vertices A, B and C, respectivelu. Suppose a = 6, b = 10 and the triangle is  $15\sqrt{3}$ . If  $\angle ACB$  is obtus and if r denotes than radius of the incircle of the triangle, then the value of  $r^2$  is \_\_\_\_\_