MATHS

BOOKS - CENGAGE MATHS (ENGLISH)

RELATIONS AND FUNCTIONS

ILLUSTRATION

1. If sets A= (-3,2] and B= (-1, 5] then find the following sets :

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2. Find the value of x^2 for the given values of x $x < 2$ (ii) $x > 1$ (iii) $x \geq 2$ (iv) $x < -1$

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3. Find all possible values of the following expressions :

$(i)\sqrt{x^2 - 4}(ii)\sqrt{9 - x^2}(iii)\sqrt{x^2 - 2x + 10}$

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4. Find the value of $1/x$ for the given values of x $x > 3$ (ii) $x < -2$ (iii) $x \in (-1, 3) - \{0\}$

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5. Find all the possible values of the following expressions: $\frac{1}{x^2 + 2}$ (ii) $\frac{1}{x^2 - 2x + 3}$ (iii) $\frac{1}{x^2 - x - 1}$

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6. Find the values of x for which expression $\sqrt{1 - \sqrt{1 - \sqrt{1 - x^2}}}$ is meaningful.

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7. Solve $x^2 - x - 2 > 0.$

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8. Solve $x^2 - x - 1 < 0.$

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9. Solve $(x - 1)(x - 2)(1 - 2x) > 0.$

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10. Solve $\frac{2}{x} < 3.$

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11. Solve $\frac{x-2}{x+2} > \frac{2x-3}{4x-1}\cdot$

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12. Solve $x > \sqrt{1-x}; x \geq 0$

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13. Solve $x(x+2)^2(x-1)^5(2x-3)(x-3)^4 \geq 0.$

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14. Solve $x\left(2^x-1\right)\left(3^x-9\right)^5(x-3) < 0.$

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15. Solve $\left(2^x-1\right)\left(3^x-9\right)(\sin x-\cos x)\left(5^x-1\right) < 0, \ -\pi/2 < x < 2\pi.$

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16. Find the value of x for which following expressions are defined: $\frac{1}{\sqrt{x - |x|}}$ (ii) $\frac{1}{\sqrt{x + |x|}}$

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17. For $2 < x < 4$ find the values of |x|.

(ii) For $-3 \leq x \leq -1$, find the values of |x|.

(iii) For $-3 \leq x < 1$, find the values of |x|

(iv) For $-5 < x < 7$ find the values of |x-2|

(v) For $1 \leq x \leq 5$ find fthe values of |2x -7|

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18. Solve the following :

(i) $|x - 2| = (x - 2)$ (ii) $|x + 3| = -x - 3$

(iii) $\left|x^2 - x\right| = x^2 - x$ (iv) $\left|x^2 - x - 2\right| = 2 + x - x^2$

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19. Prove that

$\sqrt{x^2 + 2x + 1} - \sqrt{x^2 - 2x + 1} = \{ - 2, x < \ - 12x, \ - 1 \leq x \leq 12, x > 1$

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20. For $x \in R$, find all possible values of (i)|x-3|-2(ii)4-|2x+3|`

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21. Find the possible values of $\sqrt{|x| - 2}$ (ii) $\sqrt{3 - |x - 1|}$ (iii) $\sqrt{4 - \sqrt{x^2}}$

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22. Solve $|x - 3| + |x - 2| = 1$.

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23. Solve: $\frac{|x+3|+x}{x+2} > 1$

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24. Solve $|3x - 2| \le \frac{1}{2}$.

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25. Solve $||x - 1| - 5| \ge 2$.

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26. Solve: $\frac{-1}{|x| - 2} \ge 1$.

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27. Solve $|x - 1| + |x - 2| \geq 4$

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28. Solve $|\sin x + \cos x| = |\sin x| + |\cos x|, x \in [0, 2\pi]$.

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29. Solve: $\left|- 2x^2 + 1 + e^x + \sin x\right| = \left|2x^2 - 1\right| + e^x + |\sin x|, x \in [0, 2\pi]$.

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30. Let $A = \{1, 2, 3, 4, 6\}$. Let R be the relation on A defined by $\{(a, b) : a, b \in A, a$is exactly divisible by b}.(i) Write R in roster form(ii) Find the domain of R(iii) Find the range of R.

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31. If $R = \left\{(x, y) : x, y \in W, x^2 + y^2 = 25\right\}$, then find the domain and range of R.

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32. If $R_1 = \{(x, y) \mid y = 2x + 7, \text{ where } x \in R \text{ and } -5 \leq x \leq 5\}$ is a relation. Then find the domain and Range of R_1.

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33. Show that the relation R in the set R of real numbers, defined as $R = \left\{(a, b) : a \leq b^2\right\}$ is neither reflexive nor symmetric nor transitive.

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34. Prove that the relation R in set A = {1, 2, 3, 4, 5} given by R = {(a,b): |a-b| is even} is an equivalence relation .

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35. Show that the relation R in the set A of points in a plane given by $R = \{(P, Q)$: distance of the point P from the origin is same as the distance of the point Q from the origin}, is an equivalence relation. Further, show that the set o

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36. Show that the relation R defined in the set A of all triangles as $R = \{(T_1, T_2) : T_1$ is similar to $T_2\}$, is equivalence relation.

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37. Given a non-empty set X, consider P(X) which is the set of all subsets of X. Define the relation R in P(X) as follows: For subsets A, B in P(X), ARB if and only if A B. Is R an equivalence relation on P(X)? Justify you answer

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38. Examine each of the following relations given below and state in each case, giving reasons whether it is a function or not ?

(i) $R = \{(4, 1), (5, 1), (6, 7)\}$

(ii) $R = \{(2, 3), (2, 5), (3, 3), (6, 6)\}$

(iii) $R = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6), (6, 7)\}$

(iv) $R = \{(1, 1), (2, 1), (3, 1), (4, 1), (5, 1)\}$

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39. If A is set of different triangles in the plane and B is set of all positive real numbers. A relation R is defined from set A to set B such that every element of set A is associated with some number in set B which is measure of area of triangle. Is this relation as function?

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40. A relation R is defined from N to N as R $= \{(ab, a+b) : a, b \in N\}$. Is R a function from N to N ? Justify your answer.

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41. Set A has m distinct elements and set B has n distinct elements. Then how many different mappings from set A to set B can be formed?

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42. Write explicit functions of y defined by the following equations and also find the domains of definitions of the given implicit functions: $x + |y| = 2y$ (b) $e^y - e^{-y} = 2x$ $10^x + 10^y = 10$ (d) $x^2 - \sin^{-1}y = \dfrac{\pi}{2}$

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43. Find the domain and range of the following functions.

(i) $f(x) = \sqrt{2x - 3}$ (ii) $f(x) = \frac{1}{x - 2}$

(iii) $f(x) = x^2 + 3$ (iv) $f(x) = \frac{1}{x^2 + 2}$

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44. Find the domain and range function $f(x) = \frac{x^2 - 3x + 2}{x^2 - 4x + 3}$.

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45. Find the values of x for which the following functions are identical.

(i) $f(x) = x$ and $g(x) = \frac{1}{1/x}$

(ii) $f(x) = \frac{\sqrt{9 - x^2}}{\sqrt{x - 2}}$ and $g(x) = \sqrt{\frac{9 - x^2}{x - 2}}$

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46. ABCD is a square of side l. A line parallel to the diagonal BD at a distance 'x' from the vertex A cuts two adjacent sides. Express the area of the segment of the square with A at a vertex, as a function of x. Find this area at $x = 1/\sqrt{2}$ and at $x = 2$, when $l = 2$.

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47. The relation f is defined by $f(x) = \begin{cases} 3x + 2, & 0 \le x \le 2 \\ x^3, & 2 \le x \le 5 \end{cases}$.

The relation g is defined by $g(x) = \begin{cases} 3x + 2, & 0 \le x \le 1 \\ x^3, & 1 \le x \le 5 \end{cases}$

Show that f is a function and g is not a function

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48. If $f: [-3, 4] \to R, f(x) = 2x,$ and $g: [-2, 6] \to R, g(x) = x^2$. Then find function $(f + g)(x)$.

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49. If $f(x) = \begin{cases} x^3, & x < 1 \\ 2x - 1, & x \geq 1 \end{cases}$ and $g(x) = \begin{cases} 3x, & x \leq 2 \\ x^2, & x > 2 \end{cases}$ then find $(f - g)(x)$.

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50. Check the nature of the following function.

(i) $f(x) = \sin x, x \in R$ (ii) $f(x) = \sin x, x \in N$

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51. Check the nature of the function $f(x) = x^3 + x + 1, x \in R$ using analytical method and differentiation method.

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52. Let $f : R \rightarrow R$ where $f(x) = \dfrac{x^2 + 4x + 7}{x^2 + x + 1}$. Is $f(x)$ one one?

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53. Let $f: R \rightarrow R$ where $f(x) = \sin x$. Show that f is into. Also find the codomain if f is onto.

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54. Let $f: N \rightarrow Z$ be a function defined as $f(x) = x - 1000$. Show that f is an into function.

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55. If the function $f: R\vec{A}$ given by $f(x) = \dfrac{x^2}{x^2 + 1}$ is surjection, then find A.

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56. Let $A = \{x: -1 \leq x \leq 1\} = B$ be a function $f: A \rightarrow B$. Then find the nature of each of the following functions.

(i) $f(x) = |x|$ (ii) $f(x) = x|x|$

(iii) $f(x) = x^3$ (iv) $f(x) = \sin\frac{\pi x}{2}$

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57. If $f: R \to R$ be defined by $f(x) = 2x + \sin x$ for $x \in R$, then check the nature of the function.

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58. If $f: [0, \infty) \to [0, 1)$, and $f(x) = \frac{x}{1 + x}$ then check the nature of the function.

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59. If the functions $f(x)$ and $g(x)$ are defined on $R \to R$ such that $f(x) = \{0, x \in$ retional and $x, x \in$ irrational ; $g(x) = \{0, x \in$ irratinal and $x, x \in$ rational then $(f - g)(x)$ is

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60. Show that $f: R \rightarrow R$ defined by $f(x) = (x - 1)(x - 2)(x - 3)$ is surjective but not injective.

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61. If $f: R \rightarrow R$ be a function such that $f(x) = \{x|x| - 4; x \in Q, x|x| - \sqrt{3}; x \notin Q$ then f(x) is

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62. If $f: R \rightarrow R$ be a function such that $f(x) = x^3 + x^2 + 3x + \sin x$, then discuss the nature of the function.

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63. If function f(x) is defined from set A to B, such that $n(A) = 3$ and $n(B) = 5$. Then find the number of one-one functions and number of onto functions that can be formed.

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64. Find the range of $f(x) = x^2 - x - 3$.

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65. Find the domain and range of the following

(i) $f(x) = \sqrt{x^2 - 3x + 2}$ (ii) $f(x) = \sqrt{x^2 - 4x + 6}$

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66. Find the range of $f(x) = \dfrac{x^2 - x + 1}{x^2 + x + 1}$

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67. Find the complete set of values of a such that $\frac{x^2 - x}{1 - ax}$ attains all real values.

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68. Find the domain of the function $f(x) = \frac{1}{1 + 2\sin x}$

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69. Find domain for $f(x) = \sqrt{\cos(\sin x)}$

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70. Find the range of $f(x) = \sin^2 x - \sin x + 1$.

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71. Find the range of $f(x) = \frac{1}{2\cos x - 1}$

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72. Find the value of x for which function are identical. $f(x) = \cos x and g(x) = \frac{1}{\sqrt{1 + \tan^2 x}}$

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73. Find the range of the function $f(x) = 3\sin\left(\sqrt{\frac{\pi^2}{16} - x^2}\right)$.

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74. if:$f(x) = \frac{\sin x}{\sqrt{1 + \tan^2 x}} - \frac{\cos x}{\sqrt{1 + \cot^2 x}}$, then find the range of $f(x)$

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75. Find the range of $f(x) = |\sin x| + |\cos x|, x \in R$.

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76. Find the range of $f(\theta) = 5\cos\theta + 3\cos\left(\theta + \frac{\pi}{3}\right) + 3$

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77. Solve $\sin x > \frac{1}{2}$ or find the domain of $f(x) = \frac{1}{\sqrt{1 + 2\sin x}}$

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78. Find the number of solutions of $\sin x = \frac{x}{10}$

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79. Find the number of solutions of the equation $\sin x = x^2 + x + 1$.

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80. Find the domain and range of the function $f(x) = \sin^{-1}\frac{x^2}{2}$

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81. Domain of the function f(x)=$\sin^{-1}\left(1 + 3x + 2x^2\right)$

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82. Find the values of x for which the following pair of functions are identical.

(i) $f(x) = \tan^{-1}x + \cot^{-1}x$ and $g(x) = \sin^{-1}x + \cos^{-1}x$

(ii) $f(x) = \cos\left(\cos^{-1}x\right)$ and $g(x) = \cos^{-1}(\cos x)$

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83. Find the domain and range of the function $f(x) = \sin^{-1}\left(\left(1 + e^x\right)^{-1}\right)$.

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84. Find the domain for $f(x) = \sin^{-1}\left(\frac{1 + x^2}{2x}\right)$

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85. Find the range of $f(x) = \sin^{-1}x + \tan^{-1}x + \cos^{-1}x$

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86. Find the domain of $f(x) = \sqrt{\cos^{-1}x - \sin^{-1}x}$

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87. Find the range of $\tan^{-1}\left(\frac{2x}{1+x^2}\right)$

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88. Find the range of $\cot^{-1}\left(2x - x^2\right)$

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89. Find the range of $f(x) = \cos^{-1}\left(\frac{\sqrt{1+2x^2}}{1}\right)$

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90. Find the domain of $f(x) = \sqrt{\left(\frac{1-5^x}{7^{-1}-7}\right)}$

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91. Find the domain of $f(x) = \sqrt{(0.625)^{4-3x} - (1.6)^{x(x+8)}}$

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92. Is the pair of the functions $e^{\sqrt{\log_e x}}$ and $\sqrt{x}$ identical ?

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93. Find the domain and range of following functions

(i) $f(x) = \log_e(\sin x)$

(ii) $f(x) = \log_3\left(5 - 4x - x^2\right)$

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94. Range of the function : $f(x) = \log_2\left(\frac{\pi + 2\sin^{-1}\left(\frac{3-x}{7}\right)}{\pi}\right)$

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95. Find the domain of $f(x) = (\log)_{10}(\log)_2(\log)_{\frac{2}{\pi}}\left(tan^{-1}x\right)^{-1}$

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96. Find the domain and range of $f(x) = \sqrt{(\log)_3\{\cos(\sin x)\}}$

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97. Find the domain of $f(x) = \sin^{-1}\left\{(\log)_9\left(\frac{x^2}{4}\right)\right\}$

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98. Find the domain of function $f(x) = (\log)_4\left[(\log)_5\left\{(\log)_3\left(18x - x^2 - 77\right\}\right]\right.$

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99. Let $x = \in \left(0, \frac{\pi}{2}\right)$. Then find the domain of the function $f(x) = \frac{1}{-(\log)_{\sin x}\tan x}$

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100. Find the domain of $f(x) = \sqrt{(\log)_{0.4}\left(\frac{x-1}{x+5}\right)}$

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101. Find the range of $f(x) = \log_e x - \frac{\left(\log_e x\right)^2}{\left|\log_e x\right|}$.

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102. If $f(x) = \sin(\log)_e \left\{ \frac{\sqrt{4 - x^2}}{1 - x} \right\}$, then the domain of $f(x)$ is ____ and its range is _________.

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103. Find the domain of the function $f(x) = \frac{1}{[x]^2 - 7[x] - 8}$, where [.] represents the greatest integer function.

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104. Find the domain of $f(x) = \sqrt{([x] - 1)} + \sqrt{(4 - [x])}$ (where [ ] represents the greatest integer function).

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105. Find the domain and range of $f(x) = \sin^{-1}[x] where []$ represents the greatest function).

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106. Solve $\left[\cot^{-1}x\right] + \left[\cos^{-1}x\right] = 0$, where $[.]$ denotes the greatest integer function

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107. Write the piecewise definition of the function:

$f(x) = \left[\sqrt{x}\right]$,

where [.] denotes the greatest integer function.

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108. The range of $f(x) = [\sin x \mid [\cos x[\tan x[\sec x]]]], x \in \left(0, \frac{\pi}{4}\right)$, *where* [.] denotes the greatest integer function less than or equal to x, is (0,1) (b) - $\{1, 0, 1\}$ $\{1\}$ (d) none of these

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109. The range of

$$f(x) = [1 + \sin x] + \left[2 + s \in \frac{2}{x}\right] + \left[3 + s \in \frac{x}{3}\right] + + \left[n + s \in \frac{x}{n}\right] \forall x \in [0, \pi]$$

, where [.] denotes the greatest integer function, is,

$\left\{\frac{n + n - 2^2}{2}, \frac{n(n+1)}{2}\right\}$ $\left\{\frac{n(n+1)}{2}\right\}$ $\left\{\frac{n^2 + n - 2^{\square}}{2}, \frac{n(n+1)}{2} \frac{n^2 + n + 2}{2}\right\}$ $\left[\frac{n(n+1)}{2}, \frac{n^2 + n + 2}{2}\right]$

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110. Solve $x^2 - 4 - [x] = 0$ (where [] denotes the greatest integer function).

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111. Find the domain and range of $f(f) = \log\{x\}, where\{\}$ represents the fractional part function).

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112. Find the domain and range of $f(x) = \sin^{-1}(x - [x])$, where [.] represents the greatest integer function.

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113. Write the function $f(x) = \{\sin x\}$ where {.} denotes the fractional part function) in piecewise definition.

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114. Solve $2[x] = x + \{x\}, where[]and\{\}$ denote the greatest integer function and the fractional part function, respectively.

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115. Find the range of $f(x) = \frac{x - [x]}{1 - [x] + x'}, where[]$ represents the greatest integer function.

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116. The domain of the function $f(x) = \frac{1}{\sqrt{\{\sin x\} + \{\sin(\pi + x)\}}}$ where $\{.\}$ denotes the fractional part, is (a) $[0, \pi]$ (b) $(2n + 1)\frac{\pi}{2}, n \in Z\ (0, \pi)$ (d) none of these

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117. Solve : $[x]^2 = x + 2\{x\}$, where [.] and {.} denote the greatest integer and the fractional part functions, respectively.

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118. Solve the system of equation in $x, yandz$ satisfying the following equations: $x + [y] + \{z\} = 3.1$ $\{x\} + y + [z] = 4.3$ $[x] + \{y\} + z = 5.4$ (where [] denotes the greatest integer function and {} denotes the fractional part function.)

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119. If $f(x) = [x], 0 \leq \{x\} < 0.5$ and $f(x) = [x] + 1, 0.5 < \{x\} < 1$ then prove that f (x) = -f(-x) (where[.] and{.} represent the greatest integer function and the fractional part function, respectively).

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120. Verify that $xsgnx = |x|; |x|sgnx = x; x(sgnx)(sgnx) = x.$

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121. For the following functions write the piecewise definition and draw the graph

(i) $f(x) = \text{ sgn } \left(\log_e x\right)$ (ii) $f(x) = \text{ sgn } (\sin x)$

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122. Find the range of the following

(i) $f(x) = \text{sgn}\left(x^2\right)$ (ii) $f(x) = \text{sgn}\left(x^2 - 2x + 3\right)$

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123. If $f: \vec{R}\vec{R}R$ are two given functions, then prove that

$2m \in$ if $\dot{(x)} - g(x), 0 = f(x) - |g(x) - f(x)|$

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124. Draw the graph of the function $f(x) = \max . \{\sin x, \cos 2x\}, x \in [0, 2\pi]$. Write the equivalent definition of f(x) and find the range of function.

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125. Which of the following function is (are) even, odd, or neither? (a). $f(x) = x^2 \sin x$ (b). $f(x) = \log\left(\frac{1-x}{1+x}\right)$ (c). $f(x) = \log\left(x + \sqrt{1+x^2}\right)$ (d). $f(x) = \frac{e^x + e^{-x}}{2}$

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126. If $f(x) = \left(h_1(x) - h_1(-x)\right)\left(h_2(x) - h_2(-x)\right)\dots\left(h_{2n+1}(x) - h_{2n+1}(-x)\right)$ and $f(200) = 0$, then prove that $f(x)$ is many one function.

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127. Find whether the given function is even or odd: $f(x) = \left\{ x\frac{\sin x + \tan x}{\left[\frac{x}{\pi}\right] - \frac{1}{2}};\right.$

whether [] denotes the greatest integer function.

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128. if`f(x)={x^3+x^2,for0lt=xlt=2x+2,for2

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129. Prove that period of function $f(x) = \sin x, x \in R$ is 2π.

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130. Verify that the period of function $f(x) = \sin^{10} x$ is π.

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131. Prove that function $f(x) = \cos\sqrt{x}$ is non-periodic.

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132. Find the period of the following functions

(i) $f(x) = |\sin 3x|$

(ii) $f(x) = 2\operatorname{cosec}(5x - 6) + 7$

(iii) $f(x) = x - [x - 2.6]$, where [.] represents the greatest integer function.

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133. The fundamental period of the function $f(x) = 4\cos^4\left(\frac{x - \pi}{4\pi^2}\right) - 2\cos\left(\frac{x - \pi}{2\pi^2}\right)$ is equal to :

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134. Find the period of the following.

(i) $f(x) = \frac{2^x}{2^{[x]}}$, where [.] represents the greatest integer function.

(ii) $f(x) = e^{\sin x}$

(iii) $f(x) = \sin^{-1}(\sin 3x)$

(iv) $f(x) = \sqrt{\sin x}$

(v) $f(x) = \tan\left(\frac{\pi}{2}[x]\right)$, where [.] represents greatest integer function.

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135. Period of $f(x) = \sin((\cos x) + x)$ is

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136. In each of the following cases find the period of the function if it is periodic.

(i) $f(x) = \sin\frac{\pi x}{\sqrt{2}} + \cos\frac{\pi x}{\sqrt{3}}$ (ii) $f(x) = \sin\frac{\pi x}{\sqrt{3}} + \cos\frac{\pi x}{2\sqrt{3}}$

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137. Find the period of

(i) $f(x) = \sin\pi x + \{x/3\}$, where {.} represents the fractional part.

(ii) $f(x) = |\sin 7x| - \cos^4\frac{3x}{4} + \tan\frac{2x}{3}$

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138. Find the period $f(x) = \sin x + \{x\}$, where {x} is the fractional part of $\dot{x}$

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139. If $f(x) = \sin x + \cos ax$ is a periodic function, show that a is a rational number

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140. Find the period of the following function

(i) $f(x) = |\sin x| + |\cos x|$

(ii) $f(x) = \cos(\cos x) + \cos(\sin x)$

(iii) $f(x) = \dfrac{|\sin x + \cos x|}{|\sin x| + |\cos x|}$

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141. about to only mathematics

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142. Let $f : \{2, 3, 4, 5\} \to \{3, 4, 5, 9\}$ and $g : \{3, 4, 5, 9\} \to \{7, 11, 15\}$ be functions defined as $f(2) = 3, f(3) = 4, f(4) = f(5) = 5, g(3) = g(4) = 7,$ and $g(5) = g(9) = 11.$ Find g

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143. Let f(x) and g(x) be bijective functions where $f:\{a, b, c, d\} \to \{1, 2, 3, 4\}$ and $g:\{3, 4, 5, 6\} \to \{w, x, y, z\}$, respectively. Then, find the number of elements in the range set of $g(f(x))$.

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144. Suppose that $g(x) = 1 + \sqrt{x}$ and $f(g(x)) = 3 + 2\sqrt{x} + x$. Then find the function $f(x)$.

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145. The function $f(x)$ is defined in $[0, 1]$. Find the domain of $f(\tan x)$.

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146. $f(x) = \{x + 1, x < 0, x^2, x \geq 0$ and $g(x) = \{x^3, x < 1, 2x - 1, x \geq 1$ Then find $f(g(x))$ and find its domain and range.

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147. If $f(x) = -1 + |x - 1|, \ -1 \le x \le 3$ and $g(x) = 2 - |x + 1|, \ -2 \le x \le 2,$ then find $fog(x)$ and $gof(x)$.

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148. Two functions are defined as under : $f(x) = \begin{cases} x + 1 & x \le 1 \\ 2x + 1 & 1 < x \le 2 \end{cases}$ and $g(x) = \begin{cases} x^2 & -1 \le x \le 2 \\ x + 2 & 2 \le x \le 3 \end{cases}$ Find fog and gof

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149. Suppose $f: A \rightarrow B$ and $B \rightarrow C$.

(i) Prove that if f is onto and g is not one-one, then gof is not one-to-one

(ii) Prove that if f is not and g is one-one, then gof is not onto.

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150. Let $f: A \rightarrow B$ and $g: B \rightarrow C$ be two functions. Then; if gof is onto then g is onto; if gof is one one then f is one-one and if gof is onto and g is one one then f is onto and if gof is one one and f is onto then g is one one.

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151. Which of the following functions has inverse function? $f: Z \vec{d} ef \in edbyf(x) = x + 2$ $f: Z \vec{d} ef \in edbyf(x) = 2x$ $f: Z \vec{d} ef \in edbyf(x) = x$ $f: Z \vec{d} ef \in edbyf(x) = |x|$

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152. Let $f: R \rightarrow [1, \infty), f(x) = x^2 - 4x + 5$. Then find the largest possible intervals for which $f^{-1}(x)$ is defined and find corresponding $f^{-1}(x)$.

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153. Let $A = R - \{3\}, B = R - \{1\}$, and let $f: A\vec{B}$ be defined by $f(x) = \frac{x - 2}{x - 3}$ is f invertible? Explain.

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154. Let $f: R \rightarrow R$ be defined by $f(x) = e^x - e^{-x}$. Prove that $f(x)$ is invertible. Also find the inverse function.

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155. Find the inverse of $f(x) = \{x, < 1x^2, 1 \le x \le 48\sqrt{x}, x > 4$

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156. Find the inverse of the function $f: [-1, 1] \rightarrow [-1, 1], f(x) = x^2 \times sgn(x)$.

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157. Find the inverse of the function

$$f:\left[-\frac{\pi}{2}-\tan^{-1}\frac{3}{2}, \frac{\pi}{2}-\tan^{-1}\frac{3}{4}\right] \to [-1, 1],$$

$f(x) = 3\cos x + 4\sin x + 7.$

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158. If $f(x) = 3x - 2 and (gof)^{-1}(x) = x - 2,$ then find the function $g(x)$

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159. Let $f(x) = x + f(x-1)$ for $\forall x \in R$. If $f(0) = 1, find\ f(100).$

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160. The function f(x) is defined for all real x. If $f(a+b) = f(ab) \forall a$ and b and $f\left(-\frac{1}{2}\right) = -\frac{1}{2}$ then find the value of

$f(1005)$.

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161. Let a function $f(x) satii e x + f(2x) + f(2 - x) + f(1 + x) = \forall x \in R^{\cdot}$ Then find the value of $f(0)^{\cdot}$

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162. Let f be a function satisfying of $x^{\cdot}$ Then $f(xy) = \frac{f(x)}{y}$ for all positive real numbers x and $y^{\cdot}$ If $f(30) = 20$, then find the value of $f(40)^{\cdot}$

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163. If $f(x)$ is a polynomial function satisfying $f(x)f\left(\frac{1}{x}\right)^{\cdot} = f(x) + f\left(\frac{1}{x}\right)$ and $f(4) = 65, then f \in d f(6)^{\cdot}$

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164. Let $f(x) = \dfrac{9^x}{9^x + 3}$. Show $f(x) + f(1 - x) = 1$ and, hence, evaluate.

$f\left(\dfrac{1}{1996}\right) + f\left(\dfrac{2}{1996}\right) + f\left(\dfrac{3}{1996}\right) + \; + f\left(\dfrac{1995}{1996}\right)$

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165. Consider a real-valued function f(x) satisfying $2f(xy) = (f(x))^y + (f(y))^x \forall x, y \in Randf(1) = a, wherea \neq 1.$ Prove that

$(a - 1)\sum_{i=1}^{n} f(i) = a^{n+1} - a$

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166. Let f be a real-valued function such that $f(x) + 2f\left(\dfrac{2002}{x}\right) = 3x^{\cdot}$ Then find $f(x)^{\cdot}$

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167. If $f: R \rightarrow R$ is an odd function such that $f(1+x) = 1 + f(x)$and $x^2 f\left(\frac{1}{x}\right) = f(x), x \neq 0$ then find $f(x)$

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168. Let $f: R^+ \vec{R}$ be a function which satisfies $f(x)f(y) = f(xy) + 2\left(\frac{1}{x} + \frac{1}{y} + 1\right)$ for $x, y > 0$. Then find $f(x)$

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169. A continuous function $f(x)onR \rightarrow R$ satisfies the relation $f(x) + f(2x + y) + 5xy = f(3x - y) + 2x^2 + 1f$ or $\forall x, y \in R$ Then find $f(x)$

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170. If for all real values of $uandv, 2f(u)\cos v = (u + v) + f(u - v)$, prove that for all real values of x, $f(x) + f(- x) = 2a\cos x$ $\dot{}$ $f(\pi - x) + f(- x) = 0$ $f(\pi - x) + f(x) = 2b\sin x$ $\dot{}$ Deduce that $f(x) = a\cos x + b\sin x, wherea, b$ are arbitrary constants.

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171. Prove that $f(x)givenbyf(x + y) = f(x) + f(y)\,\forall x \in R$ is an odd function.

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172. If $f(x + y) = f(x)f(y)$ $\dot{}$ for all real $x, yandf(0) \neq 0$, then prove that the function $g(x) = \dfrac{f(x)}{1 + \{f(x)\}^2}$ is an even function.

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173. Let $f(x)$ be periodic and k be a positive real number such that $f(x+k)+f(x)=0f \text{ or } allx \in R^{\cdot}$ Prove that $f(x)$ is periodic with period $2k^{\cdot}$

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174. If $f(x)$ satisfies the relation $f(x)+f(x+4)=f(x+2)+f(x+6)$ for all$x$, then prove that $f(x)$ is periodic and find its period.

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175. An odd function is symmetric about the vertical line $x=a, (a>0), and$ if $\sum_{r=0}^{\infty}\left[f(1+4r]^{r}=8,\right.$ then find the value of $f(1)^{\cdot}$

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176. Check whether the function defined by $f(x+\lambda)=1+\sqrt{2f(x)-f^{2}(x)}$ $\forall x \in R$ is periodic or not. If yes, then find its period $(\lambda>0)$.

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177. Draw the graph of $y = \log_e(-x)$, $-\log_e x$, $y = \left|\log_e x\right|$, $y = \log_e|x|$ and $y = \left|\log_e|x|\right|$ transforming the graph of $y = \log_e x$.

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178. Draw the graph of $y = |||x| - 2| - 3|$ by transforming the graph of $y = |x|$

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179. Consider the function $f(x) = \begin{cases} 2 + x3, & x \leq 1 \\ -x^2 + 6, & x > 1 \end{cases}$

Then draw the graph of the functions $y = f(x)$, $y = f(|x|)$, $y = |f(x)|$ and $y = |f(|x|)|$.

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180. Sketch the curve $|y| = (x - 1)(x - 2)$

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Solved Examples

1. Find the range of $f(x) = \sqrt{\sin(\cos x)} + \sqrt{\cos(\sin x)}$.

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2. Let $g(x) = \sqrt{x - 2k}, \forall 2k \leq x < 2(k + 1)$, where $k \in$ integer. Check whether $g(x)$ is periodic or not.

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3. Let $f(x) = x^2 - 2x, x \in R, andg(x) = f(f(x) - 1) + f(5 - (x))^{.}$ Show that $g(w) \geq o \forall x \in R^{.}$

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4. If $fandg$ are two distinct linear functions defined on R such that they map $\{-1, 1]$ onto $[0, 2]$ and $h: R - \{-1, 0, 1\}\vec{R}$ defined by $h(x) = \frac{f(x)}{g(x)},$ then show that $\left|h(h(x)) + h\left(h\left(\frac{1}{x}\right)\right)\right| > 2.$

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5. Let $f(x) = (\log)_2(\log)_3(\log)_4(\log)_5\left(\sin x + a^2\right)^{.}$ Find the set of values of a for which the domain of $f(x) is R^{.}$

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6. If f is polynomial function satisfying $2 + f(x)f(y) = f(x) + f(y) + f(xy) \forall x, y \in R$ and if $f(2) = 5$, then find the value of $f(f(2))^{\cdot}$

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7. Let $f: X \rightarrow Y$ be a function defined by f(x) = a sin (x + $\frac{\pi}{4}$) + b cosx + c. If f is both one-one and onto, then find the set X and Y

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8. Let $f: R\vec{R}, f(x) = \frac{x - a}{(x - b)(x - c)}, b > \cdot$ If f is onto, then prove that $a \in (b, c)^{\cdot}$

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9. If p, q are positive integers, f is a function defined for positive numbers and attains only positive values such that $f(xf(y)) = x^p y^q$, then prove that $p^2 = q$.

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10. If $f : \overrightarrow{R0}, \infty isafunctionsuchthatf(x - 1) + f(x + 1) = \sqrt{3}f(x),$ then prove that $f(x)$ is periodic and find its period.

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11. If a, b are two fixed positive integers such that $f(a + x) = b + \left[b^3 + 1 - 3b^2 f(x) + 3b\{f(x)\}^2 - \{f(x)\}^3\right]^{\frac{1}{3}}$ for all real x, then prove that $f(x)$ is periodic and find its period.

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13. Consider the function $f(x) = \begin{cases} x - [x] - \frac{1}{2} & x \notin \\ 0 & x \in I \end{cases}$ where [.] denotes the fractional integral function and I is the set of integers. Then find $g(x)$ max . $\left[x^2, f(x), |x|\right\}$, $-2 \leq x \leq 2.$

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14. Let $f(x)$ be defined on $[-2, 2]$ and be given by

$f(x) = \begin{cases} -1, & -2 \leq x \leq 0 \\ x - 1, & 0 < x \leq 2 \end{cases}$ and $g(x) = f(|x|) + |f(x)|$.

Then find $g(x)$.

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15. If $f(x) = \dfrac{a^x}{a^x + \sqrt{a,}}, (a > 0)$, then find the value of $\sum_{r=1}^{2n1} 2f\left(\dfrac{r}{2n}\right)$

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CONCEPT APPLICATION EXERCISE 1.1

1. If sets $A = [-4, 1]$ and $B = [0, 3)$, then find the following sets:

(a) $A \cap B$ (b) $A \cup B$ (c) $A - B$

(d) $B - A$ (e) $(A \cup B)'$ (f) $(A \cap B)'$

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2. Find the value of x^2 for the following values of x:

(a) $[-5, -1]$ (b) $(3, 6)$ (c) $(-2, 3]$

(d) $(-3, \infty)$ (e) $(-\infty, 4)$

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3. Find the values of 1/x for the following values fo x:

(a) $(2, 5)$ (b) $[-5, -1)$ (c) $(3, \infty)$

(d) $(-\infty, -2]$ (e) $[-3, 4]$

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4. Find all possible values (range) of the following quadratic expressions when $x \in R$ and when $x \in [-3, 2]$

(a) $4x^2 + 28x + 41$

(b) $1 + 6x - x^2$

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5. Find all possible values of expressions $\dfrac{2 + x^2}{4 - x^2}$

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6. Solve $\dfrac{x(3 - 4x)(x + 1)}{2x - 5} < 0$

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7. Solve $\dfrac{(2x+3)(4-3x)^3(x-4)}{(x-2)^2x^5} \leq 0$

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8. Solve $\dfrac{5x+1}{(x+1)^2}$ - 1 < 0

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9. Find the number of integal values of x satisfying

$\sqrt{-x^2+10x-16} < x-2$

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10. Find all the possible values of $f(x) = \dfrac{1-x^2}{x^2+3}$

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11. Find the values of x for which the following function is defined: $f(x) = \sqrt{\frac{1}{|x - 2| - (x - 2)}}$

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12. Solve |4-|x-1||=3

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13. Find all values of f(x) for which f(x) $= x + \sqrt{x^2}$

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14. Solve the following :

(a) $1 \leq |x - 2| \leq 3$ (b) $0 < |x - 3| \leq 5$

(c) $|x - 2| + |2x - 3| = |x - 1|$ (d) $\left|\dfrac{x - 3}{x + 1}\right| \leq 1$

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15. Find all possible values of expression $\sqrt{1 - \sqrt{x^2 - 6x + 9}}$.

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CONCEPT APPLICATION EXERCISE 1.2

1. (a) If $n(A) = 6$ and $n(A \times B) = 42$ then find $n(B)$

(b) If some of the elements of $A \times B$ are $(x, p), (p, q), (r, s)$. Then find the minimum value of $n(A \times B)$.

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2. Let $A = \{1, 2, 3, \ldots, 14\}$. Define a relation on a set A by $R = \{(x, y): 3x - y = 0. \text{ where } x, y \in A\}$. Depict this relationship using an arrow diagram. Write down its domain, co-domain and range.

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3. Let $A = \{1, 2, 3\}$. Then, the number of equivalence relations containing (1, 2) is (a) 1 (b) 2 (c) 3 (d) 4

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4. Let a relation R_1 on the set R of real numbers be defined as $(a, b) \in R_{11} + ab > 0$ for all $a, b \in R$. Show that R_1 is reflexive and symmetric but not transitive.

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5. Let Z be the set of all integers and R be the relation on Z defined as $R = \{(a, b); a, b \in Z,$ and $(a - b)$ is divisible by $5.\}$. Prove that R is an equivalence relation.

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CONCEPT APPLICATION EXERCISE 1.3

1. Find the domain of the following functions

(a) $f(x) = \frac{1}{\sqrt{x - 2}}$ (b) $f(x) = \frac{1}{x^3 - x}$

(c) $f(x) = \sqrt[3]{x^2 - 2}$

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2. Find the range of the following functions.

(a) $f(x) = 5 - 7x$ (b) $f(x) = 5 - x^2$

(c) $f(x) = \frac{x^2}{x^2 + 1}$

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3. Find the domain and range of $f(x) = \frac{2 - 5x}{3x - 4}$.

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4. Find the domain and range of $f(x) = \sqrt{4 - 16x^2}$.

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5. Find the range fo the function $f(x) = \frac{x^2 - x - 6}{x - 3}$

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6. If the relation $f(x) = \begin{cases} 2x - 3, & x \le 2 \\ x^3 - a, & x \ge 2 \end{cases}$ is a function, then find the value of a.

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7. If the relation $f(x) = \begin{cases} 1, & x \in Q \\ 2, & x \notin Q \end{cases}$ where Q is set of rational numbers, then find the value $f(\pi) + f\left(\frac{22}{7}\right)$.

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8. Let $f(x) = \begin{cases} x^2 - 4x + 3, & x < 3 \\ x - 4, & x \geq 3 \end{cases}$ and

$g(x) = \begin{cases} x - 3, & x < 4 \\ x^2 + 2x + 2, & x \geq 4 \end{cases}$.

Describe the function f/g and find its domain.

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9. Which of the following functions is/are identical to $|x - 2|$?

(a) $f(x) = \sqrt{x^2 - 4x + 4}$ (b) $g(x) = |x| - |2|$

(c) $h(x) = \frac{|x - 2|^2}{|x - 2|}$ (d) $t(x) = \left|\frac{x^2 - x - 2}{x + 1}\right|$

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CONCEPT APPLICATION EXERCISE 1.4

1. Which of the following function from Z to itself are bijections? $f(x) = x^3$

(b) $f(x) = x + 2$ $f(x) = 2x + 1$ (d) $f(x) = x^2 + x$

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2. A function f from the set of natural numbers to the set of integers defined by $f(n) = \begin{cases} \frac{n-1}{2}, & \text{when } n \text{ is odd } -\frac{n}{2}, \text{ when } n \text{ is even} \end{cases}$ (a) neither one-one nor onto (b) one-one but not onto (c) onto but not one-one (d) one-one and onto both

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3. If $f: R\vec{R}$ is given by $f(x) = \frac{x^2 - 4}{x^2 + 1}$, identify the type of function.

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4. If $f: R\vec{S}$, defined by $f(x) = \sin x - \sqrt{3}\cos x + 1, ison \to$, then find the set $\dot{S}$

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5. Let $g: R0, \overrightarrow{\frac{\pi}{3}}$ be defined by $g(x) = \cos^{-1}\left(\frac{x^2 - k}{1 + x^2}\right)$. Then find the possible values of k for which g is a subjective function.

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6. Identify the type of the function $f: R \to R$,

$f(x) = e^{x^2} + \cos x.$

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7. Let a function $f: R \rightarrow R$ be defined by $f(x) = 2x + \cos x + \sin x$ for $x \in R$. Then find the nature of $f(x)$.

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8. If $f: R \rightarrow R$ given by $f(x) = x^3 + px^2 + qx + r$, is then find the condition for which $f(x)$ is one-one.

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CONCEPT APPLICATION EXERCISE 1.5

1. The entire graph of the equation $y = x^2 + kx - x + 9$ in strictly above the $x - a\xi s$ if and only if $k < 7$ (b) \`-5-5\` (d) none of these

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2. Find the range of $f(x) = \dfrac{x^2 + 34x - 71}{x^2 + 2x - 7}$

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3. Find the range of $f(x) = \sqrt{x - 1} + \sqrt{5 - x}$

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4. If $f(x) = \sqrt{x^2 + ax + 4}$ is defined for all x, then find the values of a

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5. Find the domain and range of $f(x) = \sqrt{3 - 2x - x^2}$

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CONCEPT APPLICATION EXERCISE 1.6

1. Find the domain of $f(x) = \sqrt{\sin x} + \sqrt{16 - x^2}$

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2. Solve (a) $\tan x < 2 \cos x \leq -\frac{1}{2}$

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3. Prove that the least positive value of x, satisfying $\tan x = x + 1$, lies in the interval $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

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4. Find the range of $f(x) = \sec\left(\frac{\pi}{4}\cos^2 x\right)$, where `-oo < x < oo

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5. If $x \in [1, 2]$, then find the range of $f(x) = \tan x$

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6. Find the range of $f(x) = \dfrac{1}{1 - 3\sqrt{1 - \sin^2 x}}$

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7. Find the range of $f(x) = \dfrac{2\sin^2 x + 2\sin x + 3}{\sin^2 x + \sin x + 1}$

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8. Draw the graph of $y = (\sin 2x)\sqrt{1 + \tan^2 x}$, find its domain and range.

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CONCEPT APPLICATION EXERCISE 1.7

1. Find the domain of the following following functions:

(a) $f(x) = \dfrac{\sin^{-1}}{x}$

(b) $f(x) = \sin^{-1}(|x - 1| - 2)$

(c) $f(x) = \cos^{-1}\left(1 + 3x + 2x^2\right)$

(d) $f(x) = \dfrac{\sin^{-1}(x - 3)}{\sqrt{9 - x^2}}$

(e) $f(x) = \cos^{-1}\left(\dfrac{6 - 3x}{4}\right) + \operatorname{cosec}^{-1}\left(\dfrac{x - 1}{2}\right)$

(f) $f(x) = \sqrt{\sec^{-1}\left(\dfrac{2 - |x|}{4}\right)}$

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2. Find the range of $f(x) = \tan^{-1}\sqrt{\left(x^2 - 2x + 2\right)}$

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3. Find the range of the function $f(x) = \cot^{-1}(\log)_{0.5}\left(x^4 - 2x^2 + 3\right)$

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4. The domain of definition of the function $f(x) = \sqrt{\sin^{-1}(2x) + \frac{\pi}{6}}$ for real-valued x is (a) $\left[-\frac{1}{4}, \frac{1}{2}\right]$ (b) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (c) $\left(-\frac{1}{2}, \frac{1}{9}\right)$ (d) $\left[-\frac{1}{4}, \frac{1}{4}\right]$

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5. Find the domain and range of

$f(x) = \sin^{-1}x + \tan^{-1}x + \sec^{-1}x.$

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CONCEPT APPLICATION EXERCISE 1.8

1. Find the domain of the function :

$f(x) = \sqrt{4^x + 8\left(\frac{2}{3}\right)^{(2x-2)} - 13 - 2^{2(x-1)}}$

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2. Find the domain of the function : $f(x) = \sin^{-1}\left((\log)_2 x\right)$

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3. Find the domain of the function : $f(x) = (\log)_{(x-4)}\left(x^2 - 11x + 24\right)$

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4. Find the domain of the function : $f(x) = \dfrac{3}{4 - x^2} + (\log)_{10}\left(x^3 - x\right)$

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5. Find the domain $f(x) = \sqrt{\dfrac{\log_{0.3}|x - 2|}{|x|}}$.

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6. Find the domain of the following functions :

$$f(x) = \sqrt{\log_{10}\left(\frac{\log_{10}x}{2\left(3 - \log_{10}x\right)}\right)}$$

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7. Find the domain of the function : $f(x) = \dfrac{1}{\sqrt{(\log)_{\frac{1}{2}}\left(x^2 - 7x + 13\right)}}$

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8. Find the range of $f(x) = (\log)_2\left(\dfrac{\sin x - \cos x + 3\sqrt{2}}{\sqrt{2}}\right)$

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9. Find the value of x in $[-\pi, \pi]$ for which

$f(x) = \sqrt{(\log)_2\left(4\sin^2 x - 2\sqrt{3}\sin x - 2\sin x + \sqrt{3} + 1\right)}$ is defined.

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CONCEPT APPLICATION EXERCISE 1.9

1. Solve $[x]^2 - 5[x] + 6 = 0.$

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2. In the questions, $[x] and \{x\}$ represent the greatest integer function and the fractional part function, respectively. If $y = 3[x] + 1 = 4[x - 1] - 10,$ then find the value of $[x + 2y]^{\cdot}$

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3. Find the domain of $f(x) = \dfrac{1}{\sqrt{x - [x]}}$ (b) $f(x) = \dfrac{1}{\log[x]}$ $f(x) = \log\{x\}$

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4. Find the domain of $f(x) = \dfrac{1}{\sqrt{|[|x| - 1]| - 5}}$

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5. Find the domain of $f(x) = \dfrac{\sqrt{(1 - \sin x)}}{(\log)_5\left(1 - 4x^2\right)} + \cos^{-1}(1 - \{x\})$.

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6. Find the range of $f(x) = \cos\left((\log)_e\{x\}\right)$.

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7. Find the domain and range of $f(x) = \cos^{-1}\sqrt{(\log)_{[x]}\left(\frac{|x|}{x}\right)}$

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8. Find the range of $f(x) = (\log)_{[x-1]}\sin x$

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9. Solve $9x - 2)[x] = \{x\} - 1,$ (where $[x] and \{x\}$ denote the greatest integer function less than or equal to x and the fractional part function, respectively).

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10. Find the domain of the function $f(x) = \dfrac{1}{8\{x\}^2 - 6\{x\} + 1}$.

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11. Write the equivalent definition and draw the graphs of the following functions. $f(x) = sgn\left((\log)_e|x|\right)$

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12. Consider the function: $f(x) = max1, |x - 1|, \text{ min } \{4, |3x - 1|\}\} \forall x \in R$.

Then find the value of $f(3)$

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13. Find the equivalent definition of

$f(x) = maxx^2, (- x)^2, 2x(1 - x)where0 \leq x \leq 1$

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CONCEPT APPLICATION EXERCISE 1.10

1. Identify the type of the functions: $f(x) = \{g(x) - g(- x)\}^3$

A. Odd

B. Even

C. Neither

D. Both

Answer: A

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2. Identify the type of the functions: $f(x) = \log\left(\frac{x^4 + x^2 + 1}{x^2 + x + 1}\right)$

A. Odd

B. Even

C. Neither

D. Both

Answer: c

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3. Identify the following functions : $f(x) = xg(x)g(-x) + \tan(\sin x)$

A. Odd

B. Even

C. Neither

D. Both

Answer: A

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4. Identify the following functions: $f(x) = \cos[x] + \left[\frac{\sin x}{2}\right]$ where [.] denotes the greatest integer function.

A. Odd

B. Even

C. Neither

D. Both

Answer: C

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5. Identify the given functions whether odd or even or neither:

$$f(x) = \begin{cases} x|x| & x \le -1 \\ [x+1] + [1-x] & -1 < x < 1 \\ -x|x| & x \ge 1 \end{cases}$$

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6. Let the function $f(x) = x^2 + x + \sin x - \cos x + \log(1 + |x|)$ be defined on the interval $[-1, 1]$ .Define functions $g(x)$ and $h(x) \in [-1, 0]$ satisfying $g(-x) = -f(x)$ and $h(-x) = f(x) \forall x \in [0, 1]$.

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CONCEPT APPLICATION EXERCISE 1.11

1. Which of the following functions is not periodic?

(a) $|\sin 3x| + \sin^2 x$ (b) $\cos\sqrt{x} + \cos^2 x$

(c) $\cos 4x + \tan^2 x$ (d) $\cos 2x + \sin x$

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2. Which of the following function/functions is/are periodic ?

(a) $sgn\left(e^{-x}\right)$ (b) $\sin x + |\sin x|$

(c) $\min(\sin x, |x|)$ (d) $\frac{x}{x}$

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3. Find the period of

(a) $\frac{|\sin 4x| + |\cos 4x|}{|\sin 4x - \cos 4x| + |\sin 4x + \cos 4x|}$

(b) $f(x) = \sin\frac{\pi x}{n!} - \cos\frac{\pi x}{(n+1)!}$

(c) $f(x) = \sin x + \tan\frac{x}{2} + \sin\frac{x}{2^2} + \tan\frac{x}{2^3} + \ldots + \sin\frac{x}{2^{n-1}} + \tan\frac{x}{2^n}$

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4. Match the column

Column I (Function)	Column II (Period)
p. $f(x) = \sin^3 x + \cos^4 x$	**a.** $\pi/2$
q. $f(x) = \cos^4 x + \sin^4 x$	**b.** π
r. $f(x) = \sin^3 x + \cos^3 x$	**c.** 2π
s. $f(x) = \cos^4 x - \sin^4 x$	

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5. Let $[x]$ denotes the greatest integer less than or equal to x . If the function $f(x) = \tan\left(\sqrt{[n]}x\right)$ has period $\frac{\pi}{3}$ then find the value of n

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6. about to only mathematics

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7. Find the fundamental period of $f(x) = \cos x \cos 2x \cos 3x$.

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CONCEPT APPLICATION EXERCISE 1.12

1. Let $f(x) = 1 + |x|, x < -1 [x], x \geq -1, where [\cdot]$ denotes the greatest integer function.Then $f\{f(-2.3)\}$ is equal to

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2. If $f(x) = \log\left[\frac{1+x}{1-x}\right]$, then prove that $f\left[\frac{2x}{1+x^2}\right] = 2f(x)$.

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3. Let $f(x) = \frac{\alpha x}{x+1}$ Then the value of α for which $f(f(x)) = x$ is

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4. If the domain of $y = f(x) is[-3, 2]$, then find the domain of $g(x) = f(|[x]|), wher[]$ denotes the greatest integer function.

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5. Let f be a function defined on [0,2]. The prove that the domain of function $g(x) i 9x^2 - 1$

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6. A function f has domain $[-1, 2]$ and range $[0, 1]$. Find the domain and range of the function g defined by $g(x) = 1 - f(x + 1)$

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7. Let $f(x) = \tan x and g(f(x)) = f\left(x - \frac{\pi}{4}\right)$, where $f(x) and g(x)$ are real valued functions. Prove that $f(g(x)) = \tan\left(\frac{x - 1}{x + 1}\right)$.

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8. Let $g(x) = 1 + x - [x]$ and $f(x) = \{ - 1, x < 00, x = 01, x > 0.$ Then for all $x, f(g(x))$ is equal to (where [.] represents the greatest integer function).

(a) x (b) 1 (c) $f(x)$ (d) $g(x)$

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9. $f(x) = \begin{cases} \log_e x, & 0 < x < 1 \\ x^2 - 1, & x \geq 1 \end{cases}$ and $g(x) = \begin{cases} x + 1, & x < 2 \\ x^2 - 1, & x \geq 2 \end{cases}$.

Then find $g(f(x))$.

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CONCEPT APPLICATION EXERCISE 1.13

1. The inverse of the function $f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} + 2$ is given by

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2. Find the inverse of the function: $f: R \rightarrow (-\infty, 1) given by\ f(x) = 1 - 2^{-x}$

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3. Find the inverse of the function: $f:(2, 3) \rightarrow (0, 1)$ defined by $f(x) = x - [x],$ where[.] represents the greatest integer function

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4. Find the inverse of the function: $f: Z \rightarrow Z$ defined by $f(x) = [x + 1],$ where [.] denotes the greatest integer function.

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5. Find the inverse of the function: $f(x) = \{x^3 - 1, , x < 2x^2 + 3, x \geq 2$

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6. Find the inverse of the function: `f : [− 1 , 1] → [− 1 , 1] is defined by f (x) = x | x |

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7. Find the inverse of the function: $f: [1, \infty) \rightarrow [1, \infty), where f(x) = 2^{x(x-2)}$

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CONCEPT APPLICATION EXERCISE 1.14

1. If $f(x+y+1) = \left\{\sqrt{f(x)} + \sqrt{f(y)}\right\}^2$ and $f(0) = 1 \forall x, y \in R, determ \in ef(n), n \in N$.

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2. Let $g(x)$ be a function such that $g(a+b) = g(a)g(b) \forall a, b \in R$. If zero is not an element in the range of g, then find the value of $g(x)g(-x)$.

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3. If $f(x+2a) = f(x-2a), thenprovethatf(x)isperiodi \cdot$

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4. If $f(x+f(y)) = f(x) + y \forall x, y \in Randf(0) = 1$, then find the value of $f(7)$.

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5. If $f: R^+ \vec{R}, f(x) + 3xf\left(\frac{1}{x}\right) = 2(x+1) then f(x)$ is

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6. Represent the following function: $f: f\left(x^2 + x + 3\right) + 2f\left(x^2 - 3x + 5\right) = 6x^2 - 10x + 17$ in the form of ax + b.

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7. Consider $f: R^+ \vec{R} such that f(3) = 1$ for $a \in R^+ and f(x) f(y) + f\left(\frac{3}{x}\right) f\left(\frac{3}{y}\right) = 2f(xy) \forall x, y \in R^+$. Then find $f(x)$.

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8. Determine all functions $f: R \vec{s} uch that f(x - f(y)) = f(f(y)) + xf(y) + f(x) - 1 \forall x, y \geq \in R$.

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9. Determine the function satisfying $f^2(x+y) = f^2(x) + f^2(y) \, \forall x, y \in R$.

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10. If $f: R\vec{R}$ is a function satisfying the property $f(2x+3) + f(2x+7) = 2 \, \forall x \in R$, then find the fundamental period of $f(x)$.

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11. If $f(x)$ is an even function and satisfies the relation $x^2 f(x) - 2f\left(\frac{1}{x}\right) = g(x)$, $where g(x)$ is an odd function, then find the value of $f(5)$.

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12. If $f(a - x) = f(a + x)$ and $f(b - x) = f(b + x)$ for all real x, where $a, b(a > b > 0)$ are constants, then prove that $f(x)$ is a periodic function.

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13. A real-valued functin $f(x)$ satisfies the functional equation $f(x - y) = f(x)f(y) - f(a - x)f(a + y)$, where a given constant and $f(0) = 1$. Then prove that $f(x)$ is symmetrical about point (a, 0).

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CONCEPT APPLICATION EXERCISE 1.15

1. Draw the graph of $y = \sin|x|$.

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2. Draw the graph of the function: $|f(x)| = \tan x$

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3. Draw the graph of the function: $f(x) = \left|x^2 - 3\left|x\right| + 2\right|$

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4. Draw the graph of the function: $f(x) = -|x-1|^{\frac{1}{2}}$

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5. Find the total number of solutions of $\sin\pi x = | \in |x \mid \mid$

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6. Draw the graph of the function: Solve $\left|\frac{x^2}{x-1}\right| \leq 1$ using the graphical method.

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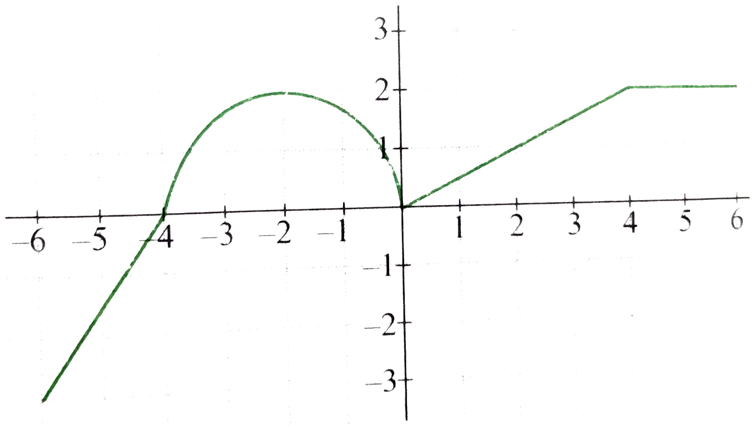
7. Given the graph of $f(x)$, draw the graph each one of the following functions :



(a) $y = f(x) + 3$ (b) $y = -f(x) + 2$

(c) $y = f(x + 1) - 2$ (d) $y = -f(x - 1)$

(e) $y = f(-x)$ (f) $y = f(|x|)$

(g) $y = f(1 - x)$

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8. Draw the graph and find the points of discontinuity for $f(x) = \left[x^2 - x - 1\right], x \in [-1, 2]$ ([.] represents the greatest integer function).

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Single Correct Answer Type

1. The relation $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\}$ on a set A={1, 2, 3} is

A. Reflexive but not symmetric

B. Reflexive but not transitive

C. Symmetric and transitive

D. Neither symmetric nor transitive

Answer: A

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2. Let $P = \left\{(x, y) \mid x^2 + y^2 = 1, x, y \in R\right\}$. Then, R, is

A. Reflexive

B. Symmetric

C. Transitive

D. Anti-symmetric

Answer: B

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3. Let R be an equivalence relation on a finite set A having n elements. Then the number of ordered pairs in R is

A. Less than n

B. Greater than or equal to n

C. Less than or equal to n

D. None of these

Answer: B

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4. A relation R on the set of complex numbers is defined by $z_1 R z_2$ if and only if $\frac{z_1 - z_2}{z_1 + z_2}$ is real Show that R is an equivalence relation.

A. R is reflexive

B. R is symmetric

C. R is transitive

D. R is not equivalence

Answer: D

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5. Which one of the following relations on R is an equivalence relation?

A. $aR_1b \Leftrightarrow |a| = |b|$

B. $aR_2b \Leftrightarrow a \geq b$

C. $aR_3b \Leftrightarrow a \text{ divides } b$

D. $aR_4b \Leftrightarrow a < b$

Answer: A

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6. Let R be the relation on the set R of all real numbers defined by aRb iff $|a-b| \leq 1$. Then, R is

A. Reflexive and symmetric

B. Symmetric only

C. Transitive only

D. None of these

Answer: A

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7. The function $f: N\vec{N}(N$ is the set of natural numbers) defined by $f(n) = 2n + 3is$ (a) surjective only (b) injective only (c) bijective (d) none of these

A. surjective only

B. injective only

C. bijective

D. none of these

Answer: B

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8. $f: N \rightarrow N$, where $f(x) = x - (-1)^x$, Then f is

A. one-one and into

B. many-one and into

C. one-one and onto

D. many-one and onto

Answer: C

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9. Let S be the set of all triangles and R^+ be the set of positive real numbers. Then the function $f: S \rightarrow R^+, f(\Delta) = area of \Delta, where \Delta \in S,$ is injective but not surjective. surjective but not injective injective as well as surjective neither injective nor surjective

A. injective but not surjective

B. surjective but not injective

C. injective as well as surjective

D. neither injective nor surjective

Answer: B

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10. The function $f:(-\infty, -1)0, e^5 \overrightarrow{} $ defined by $f(x) = e^x \wedge (3 - 3x + 2)$ is many one and onto many one and into one-one and onto one-one and into

A. many-one and onto

B. many-one and into

C. one-one and onto

D. one-one and into

Answer: D

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11. Let $f: N \rightarrow N$ be defined by $f(x) = x^2 + x + 1, x \in N$. Then $f(x)$ is

A. one-one and onto

B. many-one onto

C. one-one but not onto

D. none of these

Answer: C

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12. Let $X = \{a_1, a_2, , a_6\} and Y = \{b_1, b_2, b_3\}^{\cdot}$ The number of functions f from $x \rightarrow y$ such that it is onto and there are exactly three elements $\xi n X$ such that $f(x) = b_1$ is 75 (b) 90 (c) 100 (d) 120

A. 75

B. 90

C. 100

D. 120

Answer: D

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13. Which of the following functions is an injective (one-one) function in its respective domain? (A) $f(x) = 2x + \sin 3x$ (B) $x.[x]$, (where $[.]$ denotes the G.I.F) (C) $f(x) = \frac{2^x - 1}{4^x + 1}$ (D) $f(x) = \frac{2^x + 1}{4^x - 1}$

A. $f(x) = 2x + \sin 3x$

B. $f(x) = x \cdot [x]$, (where [.] denotes the G.I.F)

C. $f(x) = \frac{2^x - 1}{4^x + 1}$

D. $f(x) = \frac{2^x + 1}{4^x - 1}$

Answer: D

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14. Given the function $f(x) = \frac{a^x + a^{-x}}{2}(wherea > 2)\dot{T}henf(x + y) + f(x - y) =$ $2f(x)\dot{f}(y)$ (b) $f(x)\dot{f}(y)$ $\frac{f(x)}{f(y)}$ (d) none of these

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15. If $f(x) = \cos(\log x)$ then $f(x)f(y) - \frac{1}{2}\left[f\left(\frac{x}{y}\right) + f(xy)\right]$ has the value

A. -1

B. $1/2$

C. -2

D. 0

Answer: D

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16. The domain of the function $f(x) = \dfrac{1}{\sqrt{{}^{10}C_{x-1} - 3 \times {}^{10}C_{x}}}$ is

A. $\{9, 10, 11\}$

B. $\{9, 10, 12\}$

C. all natural numbers

D. $\{9, 10\}$

Answer: D

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17. The domain of the function $f(x) = \dfrac{\sin^{-1}(3 - x)}{\text{In}(|x| - 2)}$ is

A. $[2, 4]$

B. $(2, 3) \cup (3, 4]$

C. $[2, \infty)$

D. $(-\infty, -3) \cup [2, \infty)$

Answer: B

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18. The domain of $f9x\Big) = \dfrac{(\log)_2(x+3)}{x^2+3x+2}$ is $R - \{-1, 2\}$ (b) $(-2, \infty)$ $R - \{-1, -2, -3\}$ (d) $(-3, \infty) - (-1, -2\}$

A. $R - \{-1, -2\}$

B. $(-2, \infty)$

C. $R - \{-1, -2, -3\}$

D. $(-3, \infty) - \{-1, -2\}$

Answer: D

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19. The domain of the function $f(x) = \sqrt{x^2 - [x]^2}$, where $[x]$ is the greatest integer less than or equal to x, is (a) R (b) $[0, \infty]$ (c) $(-\infty, 0)$ (d) none of these

A. R

B. $[0, +\infty)$

C. $(-\infty, 0]$

D. none of these

Answer: D

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20. The domain of the function $f(x) = (\log)_{3+x}\left(x^2 - 1\right)$ is $(-3, -1) \cup (1, \infty)$ $(-3, -1) \cup (1, \infty)$ $(-3, -2) \cup (-2, -1) \cup (1, \infty)$ $(-3, -2) \cup (-2, -1) \cup (1, \infty)$

A. $(-3, -1) \cup (1, \infty)$

B. $[-3, -1) \cup [1, \infty)$

C. $(-3, -2) \cup (-2, -1) \cup (1, \infty)$

D. $(-3, -2) \cup (-2, -1) \cup (1, \infty)$

Answer: C

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21. Domain of the function, $f(x) = \left[\log_{10}\left(\frac{5x - x^2}{4}\right)\right]^{\frac{1}{2}}$ is

A. $-\infty < x < \infty$

B. $1 \leq x \leq 4$

C. $4 \leq x \leq 16$

D. $-1 \leq x \leq 1$

Answer: B

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22. The domain of $f(x) = \log|\log x|$ is

A. $(0, \infty)$

B. $(1, \infty)$

C. $(0, 1) \cup (1, \infty)$

D. $(-\infty, 1)$

Answer: C

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23. If $x^3 f(x) = \sqrt{1 + \cos 2x} + |f(x)|$, $\frac{-3\pi}{4} < x < \frac{-\pi}{2}$ and $f(x) = \frac{\alpha \cos x}{1 + x^3}$, then the value of α is (A)2 (B) $\sqrt{2}$ (C) $-\sqrt{2}$ (D) 1

A. 2

B. $-\sqrt{2}$

C. $\sqrt{2}$

D. 1

Answer: c

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24. The function $f(x) = \frac{\sec^{-1}x}{\sqrt{x - [x]},}$ where $[x]$ denotes the greatest integer less than or equal to x, is defined for all $x \in$ (a) R (b) $R - \{(-1, 1) \cup \{n|n \in Z\}\}$ (c) $R - (0, 1)$ (d) $R - \{n \mid n \in N\}$

A. R

B. $R - \{(-1, 1) \cup \{n \mid n \in Z\}\}$

C. $R^{+} - (0, 1)$

D. $R^{+} - \{n \mid n \in N\}$

Answer: B

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25. The domain of definition of the function $f(x)$ given by the equation $2^y = 2$ is `0

A. $0 < x \leq 1$

B. $0 \leq x \leq 1$

C. $-\infty < x \leq 0$

D. $-\infty < x < 1$

Answer: D

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26. The domain of $f(x) = \cos^{-1}\left(\frac{2 - |x|}{4}\right) + [l\log(3 - x)]^1$ is $[-2, 6]$ (b) $[-6, 2) \cup (2, 3)$ $[-6, 2]$ (d) $[-2, 2] \cup (2, 3)$

A. $[-2, 6]$

B. $[-6, 2) \cup (2, 3)$

C. $[-6, 2]$

D. $[-2, 2] \cup (2, 3)$

Answer: B

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27. The domain of the function $f(x) = \sqrt{\log\left(\frac{1}{|\sin x|}\right)}$

A. $R - \{-\pi, \pi\}$

B. $R - \{n\pi \mid n \in Z\}$

C. $R - \{2n\pi \mid n \in z\}$

D. $(-\infty, \infty)$

Answer: B

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28. Domain of definition of the function $f(x) = \log_2\left(-\log_{\frac{1}{2}}\left(1 + x^{-4}\right) - 1\right)$ is

A. $(0, 1)$

B. $(0, 1]$

C. $[1, \infty)$

D. $(1, \infty)$

Answer: A

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29. The number of real solutions of the $(\log)_{0.5}|x| = 2|x|$ is (a) 1 (b) 2 (c) 0 (d) none of these

A. 1

B. 2

C. 0

D. none of these

Answer: B

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30. Let $f: R0, \frac{\pi}{2}$ $\rightarrow$ be defined by $f(x) = \tan^{-1}\left(x^2 + x + a\right)^{\cdot}$ Then the set of values of a for which f is onto is $(0, \infty)$ (b) $[2, 1]$ (c) $\left[\frac{1}{4}, \infty\right]$ (d) none of these

A. $[0, \infty)$

B. $[2, 1]$

C. $\left[\frac{1}{4}, \infty\right)$

D. none of these

Answer: C

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31. The domain of the function $f(x)=\sqrt{1n_{(|x|-1)}\left(x^2+4x+4\right)}$ is $(-3,-1)\cup(1,2)$ $(-2,-1)\cup(2,\infty)$ $(-\infty,-3)\cup(-2,-1)\cup(2,\infty)$ none of these

A. $[-3,-1]\cup[1,2]$

B. $(-2,-1)\cup[2,\infty)$

C. $(-\infty,-3]\cup(-2,-1)\cup(2,\infty)$

D. None of these

Answer: C

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32. The domain of $f(x)=1n\left(ax^3+(a+b)x^2+(b+c)x+c\right)$, where $a>0, b^2-4ac=0, is(where[.]$ represetns greatest integer function).

$(-1,\infty)\sim\left(-\frac{b}{2a}\right)(1,\infty)\sim\left\{-\frac{b}{2a}\right\}(-1,1)\sim\left\{-\frac{b}{2a}\right\}$ *noneofthese*

A. $(-1,\infty)\sim\left\{-\frac{b}{2a}\right\}$

B. $(1,\infty)\sim\left\{-\frac{b}{2a}\right\}$

C. $(-1,1)\sim\left\{-\frac{b}{2a}\right\}$

D. None of these

Answer: A

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33. The domain of the function $f(x)=\frac{1}{\sqrt{4x-\left|x^2-10x+9\right|}}$ is (a) $\left(7-\sqrt{40},7+\sqrt{40}\right)$(b) $\left(0,7+\sqrt{40}\right)(c)\left(7-\sqrt{40},\infty\right)$ (d) none of these

A. $\left(7-\sqrt{40},7+\sqrt{40}\right)$

B. $\left(0,7+\sqrt{40}\right)$

C. $\left(7-\sqrt{40},\infty\right)$

D. none of these

Answer: D

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34. The domain of the function $f(x) = \frac{1}{\sqrt{|\cos x| + \cos x}}$ is

A. $[-2n\pi, 2n\pi], n \in Z$

B. $\left(2n\pi, 2n + 1\pi\right), n \in Z$

C. $\left(\frac{(4n+1)\pi}{2}, \frac{(4n+3)\pi}{2}\right), n \in Z$

D. $\left(\frac{(4n-1)\pi}{2}, \frac{(4n+1)\pi}{2}\right), n \in Z$

Answer: D

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35. $f(x) = \sqrt{x^{12} - x^9 + x^4 - x + 1}$

A. $[0, 1]$

B. $[1, \infty)$

C. $(-\infty, 1]$

D. R

Answer: D

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36. The domain of the function $f(x) = \sqrt{\sin x + \cos x} + \sqrt{7x - x^2 - 6}$ is

A. $[1, 6]$

B. $\left[1, \frac{3\pi}{4}\right] \cup \left[\frac{7\pi}{4}, 6\right]$

C. $[1, \pi] \cup \left[\frac{7\pi}{4}, 6\right]$

D. None of these

Answer: B

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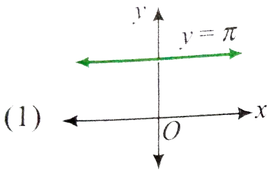
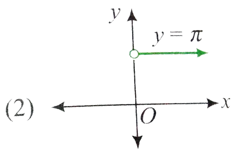
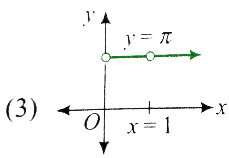
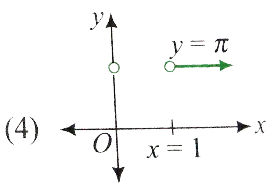
37. Which one of following best represents the graph of $y = x^{\log_x \pi}$

A. 

B. 

C. 

D. 

Answer: C

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38. If x is real, then the value of the expression $\frac{x^2 + 14x + 9}{x^2 + 2x + 3}$ lies between

A. [4, 5]

B. [-4, 5]

C. [-5, 4]

D. none of these

Answer: C

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39. The range of the function $f(x) = |x - 1| + |x - 2|, \ -1 \le x \le 3,$ is

A. [1, 3]

B. [1, 5]

C. [3, 5]

D. None of these

Answer: B

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40. The function $f: R \to R$ is defined by $f(x) = \cos^2 x + \sin^4 x$ for $x \in R$. Then the range of $f(x)$ is

A. $\left(\frac{3}{4}, 1\right]$

B. $\left[\frac{3}{4}, 1\right)$

C. $\left[\frac{3}{4}, 1\right]$

D. $\left(\frac{3}{4}, 1\right)$

Answer: C

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41. The range of $f9x\Big) = \Big[|s \in x| + \ | \cos x|]$ Where [.] denotes the greatest integer function, is {0} (b) {0,1} (c) {1} (d) none of these

A. {0}

B. {0, 1}

C. {1}

D. None of these

Answer: C

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42. The range of function $f(x) = {}^{7-x}P_{x-3} is$ (a) {1,2,3} (b) $\{1, 2, 3, 4, 5, 6\}$ $(c)\{1, 2, 3, 4\}$ (d) $\{1, 2, 3, 4, 5\}$

A. $\{1, 2, 3\}$

B. $\{1, 2, 3, 4, 5, 6\}$

C. $\{1, 2, 3, 4\}$

D. $\{1, 2, 3, 4, 5\}$

Answer: A

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43. The range of $f(x) = \sin^{-1}\left(\dfrac{x^2+1}{x^2+2}\right)$ is $(a)\left[0, \dfrac{\pi}{2}\right]$ (b) $\left(0, \dfrac{\pi}{6}\right)$ (c) $\left[\dfrac{\pi}{6}, \dfrac{\pi}{2}\right]$

(d) none of these

A. $[0, \pi/2]$

B. $(0, \pi/6)$

C. $[\pi/6, \pi/2)$

D. None of these

Answer: C

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44. The range of the function $f(x) = \dfrac{e^x - e^{|x|}}{e^x + e^{|x|}}$ is

A. $(-\infty, \infty)$

B. $[0, 1)$

C. $(-1, 0]$

D. $(-1, 1)$

Answer: C

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45. Domain (D) and range (R) of $f(x) = \sin^{-1}\left(\cos^{-1}[x]\right)$, where [.] denotes the greatest integer function, is $D \equiv x \in [1, 2], R \in \{0\}$ D $\equiv x \in 90, 1], R \equiv \{-1, 0, 1\}$ $\equiv x \in [-1, 1], R \equiv \left\{0, \sin^{-1}\left(\frac{\pi}{2}\right), \sin^{-1}(\pi)\right\}$ $\equiv x \in [-1, 1], R \equiv \left\{-\frac{\pi}{2}, 0, \frac{\pi}{2}\right\}$

A. $D \equiv x \in [1, 2), R \equiv \{0\}$

B. $D \equiv x \in [0, 1], R = \{-1, 0, 1\}$

C. $D \equiv x \in [-1, 1], R \equiv \left\{0, \sin^{-1}\left(\frac{\pi}{2}\right), \sin^{-1}(\pi)\right\}$

D. $D \equiv x \in [-1, 1], R \equiv \left\{-\frac{\pi}{2}, 0, \frac{\pi}{2}\right\}$

Answer: A

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46. The range of the function f defined by $f(x) = \left[\frac{1}{\sin\{x\}}\right]$ (where [.] and $\{.\}$, respectively, denote the greatest integer and the fractional part functions) is I, the set of integers N, the set of natural number W, the set of whole numbers {1,2,3,4,...}

A. I, the set of integers

B. N, the set of natural numbers

C. W, the set of whole numbers

D. $\{1, 2, 3, 4, \ldots\}$

Answer: D

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47. Range of function $f(x) = \cos(k\sin x)$ is [-1, 1], then the least positive integral value of k will be

A. 1

B. 2

C. 3

D. 4

Answer: D

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48. Let $f(x) = \sqrt{|x| - \mid x +}(where\{.\}$ denotes the fractional part of $(x)andX, Y$ are its domain and range, respectively). Then

$x \in \left(-\infty, \frac{1}{2}\right) and Y \in \left(\frac{1}{2}, \infty\right)$ $x \in \left(-\infty \in, \frac{1}{2}\right) \cup [0, \infty) and Y \in \left(\frac{1}{2}, \infty\right)$

$X \in \left(-\infty, -\frac{1}{2}\right) \cup [0, \infty) and Y \in \left(\frac{1}{2}, \infty\right)$

A. $x \in \left(-\infty, \frac{1}{2}\right]$ and $Y \in \left[\frac{1}{2}, \infty\right)$

B. $x \in \left(-\infty, -\frac{1}{2}\right] \cup [0, \infty)$ and $Y \in \left[\frac{1}{2}, \infty\right)$

C. $X \in \left(-\infty, -\frac{1}{2}\right] \cup [0, \infty)$ and $Y \in [0, \infty)$

D. None of these

Answer: C

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49. The range of $f(x) = \cos^{-1}\left(\frac{1 + x^2}{2x}\right) + \sqrt{2 - x^2}$ is (a) $\left\{0, 1 + \frac{\pi}{2}\right\}$ (b) $\{0, 1 + \pi)$ $\left\{1, 1 + \frac{\pi}{2}\right\}$ (d) $\{1, 1 + \pi\}$

A. $\left\{0, 1 + \frac{\pi}{2}\right\}$

B. $\{0, 1 + \pi\}$

C. $\left\{1, 1 + \frac{\pi}{2}\right\}$

D. $\{1, 1 + \pi\}$

Answer: D

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50. The range of the following function is $f(x) = \sqrt{(1 - \cos x)\sqrt{(1 - \cos x)\sqrt{(1 - \cos x)\sqrt{\infty}}}}$ (a)(0,1) (b) $\left(0, \frac{1}{2}\right)$ (c) $(0, 2)$ (d) $none of these$

A. [0, 1]

B. [0, 1/2]

C. [0, 2)

D. None of these

Answer: C

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51. The range of $f(x) = (x+1)(x+2)(x+3)(x+4)+5$ for $x \in [-6, 6]$ is [4, 5045] (b) [0, 5045] $[-20, 5045]$ (d) none of these

A. [4, 5045]

B. [0, 5045]

C. [-20, 5045]

D. None of these

Answer: A

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52. The range of $f(x) = \sec^{-1}\left((\log)_3 \tan x + (\log)_{\tan x} 3\right)$ is (a) $\left[\frac{\pi}{3}, \frac{\pi}{2}\right] \cup \left[\frac{\pi}{2}, \frac{2\pi}{3}\right]$ (b) $\left[0, \frac{\pi}{2}\right]$ (c) $\left(\frac{2\pi}{3}, \pi\right)$ (d) none of these

A. $\left[\frac{\pi}{3}, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{2\pi}{3}\right]$

B. $\left[0, \frac{\pi}{2}\right)$

C. $\left(\frac{2\pi}{3}, \pi\right]$

D. None of these

Answer: A

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53. The domain of definition of the function $f(x) = \{x\}^{\{x\}} + [x]^{[x]}$ is where $\{.\}$ represents fractional part and $[.]$ represent greatest integral function). $R - I$ (b) $R - [0, 1]$ $R - \{I \cup (0, 1)\}$ (d) $I \cup (0, 1)$

A. $R - I$

B. $R - \{0, 1)$

C. $R - \{I \cup (0, 1)\}$

D. $I \cup (0, 1)$

Answer: C

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54. 49. If $\left[x^2 - 2x + a\right] = 0$has no solution then

A. $-\infty < a < 1$

B. $2 \leq a < \infty$

C. $1 < a < 2$

D. $a \in R$

Answer: B

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55. If $[x]$ and $\{x\}$ represent the integral and fractional parts of x respectively, then the value of $\sum_{r=1}^{2000} \frac{\{x+r\}}{2000}$ is

A. x

B. [x]

C. {x}

D. x+2001

Answer: C

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56. If $f(x) = \frac{\sin([x]\pi)}{x^2 + x + 1}$, where [.] denotes the greatest integer function, then (a) f is one-one (b) f is not one-one and non-constant (c) f is a constant function (d) None of these

A. f is one-one

B. f is not one-one and non-constant

C. f is a constant function

D. None of these

Answer: C

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57. Let $f(x) = \left([a]^2 - 5[a] + 4\right)x^3 - \left(6\{a\}^2 - 5\{a\} + 1\right)x - (\tan x)x\,sgn\,x$ be an even function for all $x \in R$. Then the sum of all possible values of a is (where $[.] and \{.\}$ denote greatest integer function and fractional part function, respectively). $\frac{17}{6}$ (b) $\frac{53}{6}$ (c) $\frac{31}{3}$ (d) $\frac{35}{3}$

A. $\frac{17}{6}$

B. $\frac{53}{6}$

C. $\frac{31}{3}$

D. $\frac{35}{3}$

Answer: D

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58. The solution set for $[x]\{x\} = 1$ (where {x} and [x] are respectively, fractional part function and greatest integer function) is $R^{\pm}(0, 1)$ (b) $r^{\pm}\{1\}$ $\left\{m + \frac{1}{m}m \in I - \{0\}\right\}$ $\left\{m + \frac{1}{m}m \in I - \{1\}\right\}$

A. $R^{+} - (0, 1)$

B. $R^{+} - \{1\}$

C. $\left\{m + \frac{1}{m}/m \in I - \{0\}\right\}$

D. $\left\{m + \frac{1}{m}/m \in N - \{1\}\right\}$

Answer: D

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59. Let [x] represent the greatest integer less than or equal to x If $\left[\sqrt{n^2 + \lambda}\right] = \left[n^2 + 1\right] + 2$, where $\lambda, n \in N,$ then λ can assume $(2n + 4)d \Leftrightarrow erentvalus$ $(2n + 5)d \Leftrightarrow erentvalus$ $(2n + 3)d \Leftrightarrow erentvalus$ $(2n + 6)d \Leftrightarrow erentvalus$

A. $(2n+4)$ different values

B. $(2n+5)$ different values

C. $(2n+3)$ different values

D. $(2n+6)$ different values

Answer: B

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60. The number of roots of $x^2 - 2 = [\sin x]$, $where [.]$ stands for the greatest integer function is 0 (b) 1 (c) 2 (d) 3.

A. 0

B. 1

C. 2

D. 3

Answer: C

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61. The domain of $f(x) = \sin^{-1}\left[2x^2 - 3\right]$, *where*[.] denotes the greatest integer function, is $\left(-\sqrt{\frac{3}{2}}, \sqrt{\frac{3}{2}}\right)$ $\left(-\sqrt{\frac{3}{2}}, -1\right) \cup \left(-\sqrt{\frac{5}{2}}, \sqrt{\frac{5}{2}}\right)$ $\left(-\sqrt{\frac{5}{2}}, \sqrt{\frac{5}{2}}\right)$ $\left(-\sqrt{\frac{5}{2}}, -1\right) \cup \left(1, \sqrt{\frac{5}{2}}\right)$

A. $\left(-\sqrt{\frac{3}{2}}, \sqrt{\frac{3}{2}}\right)$

B. $\left(-\sqrt{\frac{3}{2}}, -1\right] \cup \left(-\sqrt{\frac{5}{2}}, \sqrt{\frac{5}{2}}\right)$

C. $\left(-\sqrt{\frac{5}{2}}, \sqrt{\frac{5}{2}}\right)$

D. $\left(-\sqrt{\frac{5}{2}}, -1\right] \cup \left[1, \sqrt{\frac{5}{2}}\right)$

Answer: D

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62. The domain of $f(x) = \sqrt{2\{x\}^2 - 3\{x\} + 1}$, where {.} denotes the fractional part in $[-1, 1]$ is $[-1, 1] - \left(\frac{1}{2, 1}\right)$ $\left[-1, -\frac{1}{2}\right] \cup \left[\frac{0, 1}{2}\right] \cup \{1\}$ $\left[-\frac{1, 1}{2}\right]$ (d) $\left[-\frac{1}{2}, 1\right]$

A. $[-1, 1] \sim \left(\frac{1}{2}, 1\right)$

B. $\left[-1, -\frac{1}{2}\right] \cup \left[0, \frac{1}{2}\right] \cup \{1\}$

C. $\left[-1, \frac{1}{2}\right]$

D. $\left[-\frac{1}{2}, 1\right]$

Answer: B

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63. The range of $\sin^{-1}\left[x^2 + \frac{1}{2}\right] + \cos^{-1}\left[x^2 - \frac{1}{2}\right]$, where [.] denotes the greatest integer function, is (a) $\left\{\frac{\pi}{2}, \pi\right\}$ (b) $\{\pi\}$ (c) $\left\{\frac{\pi}{2}\right\}$ (d) none of these

A. $\left\{\frac{\pi}{2}, \pi\right\}$

B. $\{\pi\}$

C. $\left\{\frac{\pi}{2}\right\}$

D. None of these

Answer: B

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64. Let $f(x) = e^{\left\{e^{|x| sgnx}\right\}} and g(x) = e^{\left[e^{|x| sgnx}\right]}, x \in R,$ where { } and [ ] denote the fractional and integral part functions, respectively. Also, $h(x) = \log(f(x)) + \log(g(x))$. Then for real $x, h(x)$ is

A. an odd function

B. an even function

C. neither an odd nor an even function

D. both odd and even function

Answer: A

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65. Number of solutions of the equation, $[y+[y]]=2\cos x$ is: (where $y=1/3)[\sin x+[\sin x+[\sin x]]]$ and $[]=$ greatest integer function) 0 (b) 1 (c) 2 (d) ∞

A. 4

B. 2

C. 3

D. 0

Answer: D

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66. The function $f(x) = \sin\left(\log\left(x + \sqrt{1 + x^2}\right)\right)$ is (a) even function (b) odd function (c) neither even nor odd (d) periodic function

A. even function

B. odd function

C. neither even nor odd

D. periodic function

Answer: B

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67. If $f(x) = x^m n, n \in N,$ is an even function, then m is even integer (b) odd integer any integer (d) $f(x)$ - $evenis\neg possible$

A. even integer

B. odd integer

C. any integer

D. $f(x)$-even is not possible

Answer: A

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68. If $f(x) = \left\{x^2 \sin\left(\frac{\pi x}{2}\right), |x| < 1; x|x|, |x| \geq 1\right.$ then $f(x)$ is

A. an even function

B. an odd function

C. a periodic function

D. None of these

Answer: B

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69. If the graph of the function $f(x) = \dfrac{a^x - 1}{x^n\left(a^x + 1\right)}$ is symmetrical about the $y - a\xi s, the \cap$ equals 2 (b) $\dfrac{2}{3}$ (c) $\dfrac{1}{4}$ (d) $\dfrac{1}{3}$

A. 2

B. $\dfrac{2}{3}$

C. $\dfrac{4}{3}$

D. $-\dfrac{1}{3}$

Answer: D

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70. If $f: R \rightarrow$ is an invertible function such that $f(x) and f^{-1}(x)$ are symmetric about the line $y = -x$, then $f(x) is odd$ $f(x) and f^{-1}(x)$ may not be symmetric about the line $y = x$ $f(x)$ may not be odd $none of these$

A. $f(x)$ is odd

B. $f(x)$ and $f^{-1}(x)$ may not be symmetric about the line $y = x$

C. $f(x)$ may not be odd

D. None of these

Answer: A

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71. If $f9x\big) = ax^7 + bx^3 + cx - 5, a, b, c$ are real constants, and $f(-7) = 7$, then the range of $f(7) + 17\cos\xi s\ [-34, 0]$ (b) $[0, 34]\ [-34, 34]$ (d) none of these

A. [-34, 0]

B. [0, 34]

C. [-34, 34]

D. None of these

Answer: A

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72. If $g:[-2,2]\vec{R}$, where $f(x)=x^3+\tan x+\left[\frac{x^2+1}{P}\right]$ is an odd function, then the value of parametric P, where [.] denotes the greatest integer function, is

A. $-5 < P < 5$

B. $P < 5$

C. $P > 5$

D. None of these

Answer: C

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73. Let $f: [-1, 10] \rightarrow R$, $where f(x) = \sin x + \left[\frac{x^2}{a}\right]$, be an odd function. Then the set of values of parameter a is/are $(-10, 10) \sim \{0\}$ (b) $(0, 10)$ $(100, \infty)$ (d) $(100, \infty)$

A. $(-10, 10) \sim \{0\}$

B. $(0, 10)$

C. $[100, \infty)$

D. $(100, \infty)$

Answer: D

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74. $f(x) = \dfrac{\cos x}{\left[\frac{2x}{\pi}\right] + \frac{1}{2}}$, where x is not an integral multiple of π and $[.]$ denotes the greatest integer function, is an odd function an even function neither odd nor even none of these

A. an odd function

B. an even function

C. neither odd nor even

D. None of these

Answer: A

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75. Let $f(x) = \begin{cases} \sin x + \cos x, & 0 < x < \frac{\pi}{2} \\ a, & x = \pi/2 \\ \tan^2 x + \operatorname{cosec} x, & \pi/2 < x < \pi \end{cases}$

Then its odd extension is

A. $\begin{cases} -\tan^2 x - \operatorname{cosec} x, & -\pi < x < -\frac{\pi}{2} \\ -a, & x = -\frac{\pi}{2} \\ -\sin x + \cos x, & -\frac{\pi}{2} < x < 0 \end{cases}$

B. $\begin{cases} -\tan^2 x + \text{cosec}x, & -\pi < x < -\frac{\pi}{2} \\ -a, & x = -\frac{\pi}{2} \\ \sin x - \cos x, & -\frac{\pi}{2} < x < 0 \end{cases}$

C. $\begin{cases} -\tan^2 x + \text{cosec}x, & -\pi < x < -\frac{\pi}{2} \\ a, & x = -\frac{\pi}{2} \\ \sin x - \cos x, & -\frac{\pi}{2} < x < 0 \end{cases}$

D. $\begin{cases} \tan^2 x + \text{cosec}x, & -\pi < x < -\frac{\pi}{2} \\ -a, & x = -\frac{\pi}{2} \\ \sin x + \cos x, & -\frac{\pi}{2} < x < 0 \end{cases}$

Answer: B

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76. The period of the function $\left|\sin^3\left(\frac{x}{2}\right)\right| + \left|\cos^5\left(\frac{x}{5}\right)\right|$ is

A. 2π

B. 10π

C. 8π

D. 5π

Answer: B

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77. If f is periodic, g is polynomial function and $f(g(x))$ is periodic and $g(2) = 3, g(4) = 7$ then $g(6)$ is

A. 13

B. 15

C. 11

D. None of these

Answer: C

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78. The period of function $2^{\{x\}} + \sin \pi x + 3^{\{x/2\}} + \cos \pi x$ (where {x} denotes the fractional part of x) is

A. 2

B. 1

C. 3

D. None of these

Answer: A

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79. The period of the function $f(x) = [6x + 7] + \cos \pi x - 6x$, where [.] denotes the greatest integer function is:

A. 3

B. 2π

C. 2

D. None of these

Answer: C

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80. If $f(x)$ and $g(x)$ are periodic functions with periods 7 and 11, respectively, then the period of $f(x) = f(x)g\left(\frac{x}{5}\right) - g(x)f\left(\frac{x}{3}\right)$ is

A. 177

B. 222

C. 433

D. 1155

Answer: D

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81. The period of the function $f(x) = c^{\left(\sin^2 x\right) + \sin^2\left(x + \frac{\pi}{3}\right) + \cos x \cos\left(x + \frac{\pi}{3}\right)}$ is (where c is constant)

A. 1

B. $\frac{\pi}{2}$

C. π

D. None of these

Answer: D

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82. Let $f(x) = \left\{(0.1)^{3[x]}\right\}$. (where [.] denotes greatest integer function and denotes fractional part). If $f(x+T) = f(x) \forall x \in 0$, where T is a fixed positive number then the least x value of T is

A. 2

B. 4

C. 6

D. None of these

Answer: B

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83. If the period of $\frac{\cos(\sin(nx))}{\tan\left(\frac{x}{n}\right)}, n \in N, is 6\pi$, then $n =$ (a)3 (b) 2 (c) 6 (d) 1

A. 3

B. 2

C. 6

D. 1

Answer: C

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84. The period of $f(x) = [x] + [2x] + [3x] + [4x] + [nx] - \frac{n(n+1)}{2}x$, where $n \in N$, is (where [.] represents greatest integer function) then which of the following is correct a. n (b) 1 (c) $\frac{1}{n}$ (d) none of these

A. n

B. 1

C. $\frac{1}{n}$

D. none of these

Answer: B

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85. If $f(x) = (-1)\left[\frac{2}{\pi}\right]$, $g(x) = |\sin x| - |\cos x|$, $and \varphi(x) = f(x)g(x)$ (where [.] denotes the greatest integer function), then the respective fundamental periods of $f(x), g(x), and \varphi(x)$ are π, π, π (b) $\pi, 2\pi, \pi$ $\pi, \pi, \frac{\pi}{2}$ (d) $\pi, \frac{\pi}{2}, \pi$

A. π, π, π

B. $\pi, 2\pi, \pi$

C. $\pi, \pi, \frac{\pi}{2}$

D. $\pi, \frac{\pi}{2}, \pi$

Answer: C

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86. If $f(x) = \frac{1}{x}$, $g(x) = \frac{1}{x^2}$, and $h(x) = x^2$, then

(A) $f(g(x)) = x^2, x \neq 0, h(g(x)) = \frac{1}{x^2}$

(B) $h(g(x)) = \frac{1}{x^2}, x \neq 0, fog(x) = x^2$

(C) $fog(x) = x^2, x \neq 0, h(g(x)) = (g(x))^2, x \neq 0$

(D) none of these

A. $fog(x) = x^2, x \neq 0, h(g(x)) = \frac{1}{x^2}$

B. $h(g(x)) = \frac{1}{x^2}, x \neq 0, fog(x) = x^2$

C. $fog(x) = x^2, x \neq 0, h(g(x)) = (g(x))^2, x \neq 0$

D. None of these

Answer: C

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87. If $f(x) = \begin{cases} x^2, & \text{for } x \geq 0 \\ x, & \text{for } x < 0 \end{cases}$, then fof(x) is given by

A. $x^2 \text{ for } x \geq 0, x \text{ for } x < 0$

B. $x^4 \text{ for } x \geq 0, x^2 \text{ for } x < 0$

C. $x^4 \text{ for } x \geq 0, -x^2 \text{ for } x < 0$

D. $x^4 \text{ for } x \geq 0, x \text{ for } x < 0$

Answer: D

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88. Let $f(x) = \sin x and g(x) = (\log)_e|x|$ If the ranges of the composition functions $fog and gof are R_1 and R_2$, respectively, then `R_1-{u :-1lt=u<1},R_2={v

:-oo

A. $R_1 = \{u: -1 \leq u < 1\}. R_2 = \{v: -\infty < v < 0\}$

B. $R_1 = \{u: -\infty < u < 0\}. R_2 = \{v: -\infty < v < 0\}$

C. $R_1 = \{u: -1 < u < 1\}. R_2 = \{v: -\infty < v < 0\}$

D. $R_1 = \{u: -1 \leq u \leq 1\}. R_2 = \{v: -\infty < v \leq 0\}$

Answer: D

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89. If $f(x) = \{x, \xi s rational 1 - x, \xi s irrational, then f(f(x))$ is $x \forall x \in R$ (b) $\{x, \xi s irrational 1 - x, \xi s rational$ $\{x, \xi s rational 1 - x, \xi s irrational$ (d) none of these

A. $x \forall x \in R$

B. $f(x) = \begin{cases} x, & x \text{ is irrational} \\ 1 - x, & x \text{ is rational} \end{cases}$

C. $f(x) = \begin{cases} x, & x \text{ is rational} \\ 1 - x, & x \text{ is irrational} \end{cases}$

D. None of these

Answer: A

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90. If f and g are one-one functions, then a. $f + g$ is one one b. fg is one one c. fog is one one d. $no \neq of these$

A. $f + g$ is one-one

B. fg is one-one

C. fog is one-one

D. None of these

Answer: C

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91. The domain of $f(x) is (0, 1)$. Then the domain of $\left(f\left(e^{x}\right) + f(1n|x|)\right.$ is a. $(-1, e)$ (b) $(1, e)$ (c) $(-e, -1)$ (d) $(-e, 1)$

A. $(-1, e)$

B. $(1, e)$

C. $(-e, -1)$

D. $(-e, 1)$

Answer: C

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92. Let $h(x) = |kx + 5|, thedoma \in off(x)be[-5, 7]$, the domain of $f(h(\times))be[-6, 1], andtheran \geq ofh(x)bethesameasthedoma \in off(x)$. Then the value of k is. 1 (b) 2 (c) 3 (d) 4

A. 1

B. 2

C. 3

D. 4

Answer: B

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93. If $f(x) = \sin x + \cos x$ and $g(x) = x^2 - 1$, then $g(f(x))$ is invertible in the domain .

A. $\left[0, \frac{\pi}{2}\right]$

B. $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

C. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

D. $[0, \pi]$

Answer: B

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94. If the function $f:[1, \infty) \rightarrow [1, \infty)$ is defined by $f(x) = 2^{x(x-1)}$, then $f^{-1}(x)$ is (A) $\left(\frac{1}{2}\right)^{x(x-1)}$ (B) $\frac{1}{2}\left(1 + \sqrt{1 + 4\log_2 x}\right)$ (C) $\frac{1}{2}\left(1 - \sqrt{1 + 4\log_2 x}\right)$ (D) not defined

A. $\left(\frac{1}{2}\right)^{x(x-1)}$

B. $\frac{1}{2}\left(1 + \sqrt{1 + 4\log_2 x}\right)$

C. $\frac{1}{2}\left(1 - \sqrt{1 + 4\log_2 x}\right)$

D. not defined

Answer: B

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95. Let $f(x) = (x+1)^2 - 1, x \geq -1$. Then the set $\left\{x : f(x) = f^{-1}(x)\right\}$ is $\left\{0, 1, \frac{-3+i\sqrt{3}}{2}, \frac{-3-i\sqrt{3}}{2}\right\}$ (b) $\{0, 1, -1$ $\{0, 1, 1\}$ (d) *empty*

A. $\left\{0, -1, \frac{-3+i\sqrt{3}}{2}, \frac{-3-i\sqrt{3}}{2}\right\}$

B. $\{0, 1, -1\}$

C. $\{0, -1\}$

D. empty

Answer: C

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96. if $f : [1, \infty) \rightarrow [2, \infty)$ is given by $f(x) = x + \frac{1}{x}$ then $f^{-1}(x)$ equals to : a) $\frac{x+\sqrt{x^2-4}}{2}$ b) $\frac{x}{1+x^2}$ c) $\frac{x-\sqrt{x^2-4}}{2}$ d) $1+\sqrt{x^2-4}$

A. $\dfrac{\left(x + \sqrt{x^2 - 4}\right)}{2}$

B. $\dfrac{x}{1 + x^2}$

C. $\dfrac{\left(x - \sqrt{x^2 - 4}\right)}{2}$

D. $1 + \sqrt{x^2 - 4}$

Answer: A

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97. Suppose $f(x) = (x + 1)^2 f$ or $x \geq -1$. If $g(x)$ is the function whose graph is the reflection of the graph of $f(x)$ with respect to the line $y = x$, then $g(x)$ equal. $a - \sqrt{x} - 1, x \geq 0$ (b) $\dfrac{1}{(x + 1)^2}, x \succ 1$ $\sqrt{x + 1}, x \geq -1$ (d) $\sqrt{x} - 1, x \geq 0$

A. $1 - \sqrt{x} - 1, x \geq 0$

B. $\dfrac{1}{(x + 1)^2}, x > -1$

C. $\sqrt{x+1}, x \geq -1$

D. $\sqrt{x}-1, x \geq 0$

Answer: D

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98. Let $f:\left[-\frac{\pi}{3}, \frac{2\pi}{3}\right] \overrightarrow{0}, 4$ be a function defined as $f(x)=\sqrt{3}\sin x-\cos x+2$.

Then $f^{-1}(x)$ is given by $\sin^{-1}\left(\frac{x-2}{2}\right)-\frac{\pi}{6}$ $\sin^{-1}\left(\frac{x-2}{2}\right)+\frac{\pi}{6}$ $\frac{2\pi}{3}+\cos^{-1}\left(\frac{x-2}{2}\right)$ (d) none of these

A. $\sin^{-1}\left(\frac{x-2}{2}\right)-\frac{\pi}{6}$

B. $\sin^{-1}\left(\frac{x-2}{2}\right)+\frac{\pi}{6}$

C. $\frac{2\pi}{3}+\cos^{-1}\left(\frac{x-2}{2}\right)$

D. None of these

Answer: B

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99. Which of the following functions is the inverse of itself? (a) $f(x) = \frac{1-x}{1+x}$ (b) $f(x) = 5^{\log x}$ (c) $f(x) = 2^{x(x-1)}$ (d) None of these

A. $f(x) = \frac{1-x}{1+x}$

B. $f(x) = 5^{\log x}$

C. $f(x) = 2^{x(x-1)}$

D. None of these

Answer: A

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100. If $g(x) = x^2 + x - 2 and \frac{1}{2} gof(x) = 2x^2 - 5x + 2,$ then which is not a possible $f(x)$? (a)2x-3(b)-2x+2(c)x-3` (d) None of these

A. $2x - 3$

B. $-2x+2$

C. $x-3$

D. None of these

Answer: C

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101. Let $f: X \to y f(x) = \sin x + \cos x + 2\sqrt{2}$ be invertible. Then which $X \to Y$ is not possible? $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right] \to \left[\sqrt{2}, 3\sqrt{2}\right]$ $\left[-\frac{3\pi}{4}, \frac{\pi}{4}\right] \to \left[\sqrt{2}, 3\sqrt{2}\right]$ $\left[-\frac{3\pi}{4}, \frac{3\pi}{4}\right] \to \left[\sqrt{2}, 3\sqrt{2}\right]$ none of these

A. $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right] \to \left[\sqrt{2}, 3\sqrt{2}\right]$

B. $\left[-\frac{3\pi}{4}, \frac{\pi}{4}\right] \to \left[\sqrt{2}, 3\sqrt{2}\right]$

C. $\left[-\frac{3\pi}{4}, \frac{3\pi}{4}\right] \to \left[\sqrt{2}, 3\sqrt{2}\right]$

D. None of these

Answer: C

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102. If $f(x)$ is an invertible function and $g(x) = 2f(x) + 5,$ then the value of $g^{-1}(x) is$ (a) $2f^{-1}(x) - 5$ (b) $\dfrac{1}{2f^{-1}(x) + 5}$ $\dfrac{1}{2}f^{-1}(x) + 5$ (d) $f^{-1}\left(\dfrac{x - 5}{2}\right)$

A. $2f^{-1}(x) - 5$

B. $\dfrac{1}{2f^{-1}(x) + 5}$

C. $\dfrac{1}{2}f^{-1}(x) + 5$

D. $f^{-1}\left(\dfrac{x - 5}{2}\right)$

Answer: D

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103. Discuss the differentiability of $f(x) = [x] + \sqrt{\{x\}}\big)$, where [.] and {.} denote the greatest integer function and fractional part repectively .

A. $[x] + \sqrt{\{x\}}$

B. $[x] + \{x\}^2$

C. $[x]^2 + \{x\}$

D. $\{x\} + \sqrt{\{x\}}$

Answer: B

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104. If f is a function such that $f(0) = 2, f(1) = 3, and f(x + 2) = 2f(x) - f(x + 1)$ for every real x, then $f(5)$ is 7 (b) 13 (c) 1 (d) 5

A. 7

B. 13

C. 1

D. 5

Answer: B

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105. A function $f(x)$ satisfies the functional equation $x^2f(x) + f(1 - x) = 2x - x^4$ for all real x. $f(x)$ must be

A. x^2

B. $1 - x^2$

C. $1 + x^2$

D. $x^2 + x + 1$

Answer: B

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106. If $f(x)$ is a polynomial satisfying $f(x)f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right) andf(3) = 28, thenf(4)$ is equal to 63 (b) 65 (c) 17 (d) none of these

A. 63

B. 65

C. 17

D. none of these

Answer: B

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107. If $f\left(2x + \frac{y}{8}, 2x - \frac{y}{8}\right) = xy$, then $f(m, n) = 0$ only when $m = n$ only when $m \neq n\ onlywhenm = -n$ (d) f or $allmandn$

A. only when $m = n$

B. only when $m \neq n$

C. only when $m = -n$

D. for all m and n

Answer: D

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108. Let $f: R \to R$ be a function such that $f(0) = 1$ and for any $x, y \in R, f(xy + 1) = f(x)f(y) - f(y) - x + 2$. Then f is

A. one-one and onto

B. one-one but not onto

C. many one but onto

D. many one and into

Answer: A

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109. If $f(x+y) = f(x) + f(y) - xy - 1 \forall x, y \in R and f(1) = 1,$ then the number of solution of $f(n) = n, n \in N,$ is 0 (b) 1 (c) 2 (d) more than 2

A. 0

B. 1

C. 2

D. more than 2

Answer: B

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110. The function f satisfies the functional equation $3f(x) + 2f\left(\frac{x+59}{x1}\right) = 10x + 30$ for all real $x \neq 1.$ The value of $f(7) is$ 8 (b) 4 (c) -8 (d) 11

A. 8

B. 4

C. -8

D. 11

Answer: B

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111. Let $f : R\vec{R}$ be a continuous and differentiable function such that $\left(f\left(x^2+1\right)\right)^{\sqrt{x}} = 5f \text{ or } \forall x \in (0, \infty)$, then the value of $\left(f\left(\frac{16+y^2}{y^2}\right)\right)^{\frac{4}{\sqrt{y}}} f \text{ or } eachy \in (0, \infty)$ is equal to 5 (b) 25 (c) 125 (d) 625

A. 5

B. 25

C. 125

D. 625

Answer: B

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112. Let $g(x) = f(x) - 1.$ If $f(x) + f(1 - x) = 2 \forall x \in R,$ then $g(x)$ is symmetrical about. (a)The origin (b) the line$x = \frac{1}{2}$ (c) the point (1,0) (d) the point $\left(\frac{1}{2}, 0\right)$

A. the orgin

B. the line$x = \frac{1}{2}$

C. the point (1, 0)

D. the point $\left(\frac{1}{2}, 0\right)$

Answer: D

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113. If $f(x + 1) + f(x - 1) = 2f(x) and f(0), \ = 0,$ then $f(n), n \in N,$ is $nf(1)$ (b) $\{f(1)\}^n$ 0 (d) none of these

A. $nf(1)$

B. $\{f(1)\}^n$

C. 0

D. none of these

Answer: A

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114. If $f\left(x+\frac{1}{2}\right)+f\left(x-\frac{1}{2}\right)=f(x)f \text{ or } all x \in R,$ then the period of $f(x)$ is 1 (b) 2 (c) 3 (d) 4

A. 1

B. 2

C. 3

D. 4

Answer: C

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115. If $af(x+1)+bf\left(\frac{1}{x+1}\right)=x, x \neq -1, a \neq b$, then $f(2)$ is equal to

A. $\dfrac{2a+b}{2\left(a^2-b^2\right)}$

B. $\dfrac{a}{a^2-b^2}$

C. $\dfrac{a+2b}{a^2-b^2}$

D. none of these

Answer: A

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116. If $f(3x+2)+f(3x+29)=0 \,\forall x \in R,$ then the period of $f(x)$ is

A. 7

B. 8

C. 10

D. none of these

Answer: D

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117. If the graph of $y = f(x)$ is symmetrical about the lines $x = 1 and x = 2$, then which of the following is true? $f(x+1) = f(x)$ (b) $f(x+3) = f(x)$ $f(x+2) = f(x)$ (d) None of these

A. $f(x+1) = f(x)$

B. $f(x+3) = f(x)$

C. $f(x+2) = f(x)$

D. none of these

Answer: C

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118. Find f(x) when it is given by

$$f(x) = \max\left\{x^3, x^2, \frac{1}{64}\right\}, \forall x \in [0, \infty).$$

A. $f(x) = \begin{cases} x^2, & 0 \leq x \leq 1 \\ x^3, & x > 1 \end{cases}$

B. $f(x) = \begin{cases} \frac{1}{64}, & 0 \leq x \leq \frac{1}{4} \\ x^2, & \frac{1}{4} < x \leq 1 \\ x^3, & x > 1 \end{cases}$

C. $f(x) = \begin{cases} \frac{1}{64}, & 0 \leq x \leq \frac{1}{8} \\ x^2, & \frac{1}{8} < x \leq 1 \\ x^3, & x > 1 \end{cases}$

D. $f(x) = \begin{cases} \frac{1}{64}, & 0 \leq x \leq \frac{1}{8} \\ x^3, & x > 1/8 \end{cases}$

Answer: C

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119. The equation $||x - 2| + a| = 4$ can have four distinct real solutions for x if a belongs to the interval

a) $(-\infty, -4)$

(b) $(-\infty, 0)$

c) $(4, \infty)$

(d) none of these

A. $(-\infty, -4)$

B. $(-\infty, 0]$

C. $[4, \infty)$

D. none of these

Answer: A

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120. Number of integral values of k for which the equation $4\cos^{-1}(-|x|) = k$ has exactly two solutions, is: (a) 4 (b) 5 (c) 6 (d) 7

A. 4

B. 5

C. 6

D. 7

Answer: C

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121. If $f(x)$ is a real-valued function defined as $f(x) = In(1 - \sin x)$, then the graph of $f(x)$ is

A. symmetric about the line $x = \pi$

B. symmetric about the y-axis

C. symmetric and the line $x = \frac{\pi}{2}$

D. symmetric about the origin

Answer: C

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122. Let $f(x) = x + 2|x + 1| + 2|x - 1|$. If $f(x) = k$ has exactly one real solution, then the value of k is

A. 3

B. 0

C. 1

D. 2

Answer: A

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123. The number of solutions of $2\cos x = |\sin x|, 0 \le x \le 4\pi,$ is (a) 0 (b) 2 (c) 4 (d) infinite

A. 0

B. 2

C. 4

D. infinite

Answer: C

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124. about to only mathematics

A. $f_4(x) = f_1(x)$ for all x

B. $f_1(x) = -f_3(-x)$ for all x

C. $f_2(-x) = f_4(x)$ for all x

D. $f_1(x) + f_3(x) = 0$ for all x

Answer: B

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125. If $\log_4\left(\frac{2f(x)}{1 - f(x)}\right) = x, \;\; \text{then} \;\; (f(2010) + f(-2009))$ is equal to

A. 0

B. -1

C. 1

D. 2

Answer: C

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Multiple Correct Answer Type

1. Let $f(x)=\sec^{-1}\left[1+\cos^2 x\right]$, where [.] denotes the greatest integer function. Then the

A. domain of f is R

B. domain of f is $[1, 2]$

C. domain of f is $[1, 2]$

D. range of f is $\left\{\sec^{-1}1, \sec^{-1}2\right\}$

Answer: A::B

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2. Let $f: R \to [-1, \infty]$ and $f(x)=\ln([|\sin 2x|+|\cos 2x|])$ (where[.] is greatest integer function), then -

A. $f(x)$ has range Z

B. Range of $f(x)$ is singleton set

C. $f(x)$ is invertible in $\left[0, \frac{\pi}{4}\right]$

D. $f(x)$ is into function

Answer: B::D

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3. If $f: R\vec{N} \cup \{0\}$, where f (area of triangle joining points $P(5, 0), Q(8, 4) and R(x, y)$ such that angle PRQ is a right angle $=$ number of triangles, then which of the following is true? $f(5) = 4$ (b) $f(7) = 0$ $f(6.25) = 2$ (d) `f(x)is into

A. $f(5) = 4$

B. $f(7) = 0$

C. $f(6.25) = 2$

D. $f(4.5) = 4$

Answer: A::B::C::D

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4. The domain of the function

$$f(x) = \log_e\left\{\log_{|\sin x|}\left(x^2 - 8x + 23\right) - \frac{3}{\log_2|\sin x|}\right\}$$

contains which of the following interval (s) ?

A. $(3, \pi)$

B. $\left(\pi, \frac{3\pi}{2}\right)$

C. $\left(\frac{3\pi}{2}, 5\right)$

D. None of these

Answer: A::B::C

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5. Let $f(x) = sgn\left(\cot^{-1}x\right) + \tan\left(\frac{\pi}{2}[x]\right)$, where $[x]$ is the greatest integer function less than or equal to x, then which of the following alternatives is/are true? $f(x)$ is many-one but not an even function. $f(x)$ is a periodic function. $f(x)$ is a bounded function. The graph of $f(x)$ remains above the x-axis.

A. $f(x)$ is many-one but not an even function.

B. $f(x)$ is a periodic function.

C. $f(x)$ is a bounded function.

D. The graph of $f(x)$ remains above the x-axis.

Answer: A::B::C::D

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6. $f(x) = \sqrt{1 - \sin^2 x} + \sqrt{1 + \tan^2 x}$ then

A. fundamental period of $f(x)$ is π

B. range of $f(x)$ is $[2, \infty)$

C. domain of $f(x)$ is R

D. $f(x) = 2$ has 3 solution in $[0, 2\pi]$

Answer: A::B::D

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7. If the following functions are defined from $[-1, 1] \to [-1, 1]$, select those which are not objective. $\sin\left(s \in^{-1} x\right)$ (b) $\frac{2}{\pi}\sin^{-1}(\sin x)$ $(sgn(x))1N\left(e^{x}\right)$ (d) $x^3(sgn(x))$

A. $\sin\left(\sin^{-1} x\right)$

B. $\frac{2}{\pi}\sin^{-1}(\sin x)$

C. $(sgn(x))In\left(e^{x}\right)$

D. $x^3(sgn(x))$

Answer: B::C::D

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8. Let $f(x) = \{x^2 - 4x + 3, x < 3x - 4, x \geq 3$ $and g(x) = \{x - 3, x < 4x^2 + 2x + 2, x \geq 4$ then which of the following is/are true? $(f + g)(3.5) = 0$ $f(g(3)) = 3$ $(fg)(2) = 1$ (d) $(f - g)(4) = 0$

A. $(f + g)(3.5) = 0$

B. $f(g(3)) = 3$

C. $(fg)(2) = 1$

D. $(f - g)(4) = 0$

Answer: A::B::C

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9. Let

$f(x) = \max(1 + s \in x, 1, 1 - \cos x), x \in [0, 2\pi], and g(x) = \max \{1, |x - 1|\}, x \in R$

Then $g(f(0)) = 1$ (b) $g(f(1)) = 1$ $f(f(1)) = 1$ (d) $f(g(0)) = 1 + \sin 1$

A. $g(f(0)) = 1$

B. $g(f(1)) = 1$

C. $f(f(1)) = 1$

D. $f(g(0)) = 1 + \sin 1$

Answer: A::B::D

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10. Consider the function $y = f(x)$ satisfying the condition $f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}(x \neq 0)$. Then the

A. domain of $f(x)$ is R

B. domain of f is $R - (-2, 2)$

C. range of $f(x)$ is $[-2, \infty)$

D. range of $f(x)$ is $[2, \infty)$

Answer: B::D

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11. Consider the real-valued function satisfying $2f(\sin x) + f(\cos x) = \dot{x}$ then the domain of $f(x) is R$ domain of $f(x) is[-1, 1]$ range of $f(x)$ is $\left[-\frac{2\pi}{3}, \frac{\pi}{3}\right]$ range of $f(x) is R$

A. domain of $f(x)$ is R

B. domain of $f(x)$ is $[-1, 1]$

C. range of $f(x)$ is $\left[-\frac{2\pi}{3}, \frac{\pi}{3}\right]$

D. range of $f(x)$ is R

Answer: B::C

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12. If $f: R^+ \vec{R}^+$ is a polynomial function satisfying the functional equation $f(f(x)) = 6x = f(x), thenf(17)$ is equal to 17 (b) 51 (c) 34 (d) -34

A. 17

B. -51

C. 34

D. -34

Answer: B::C

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13. $f(x) = x^2 - 2ax + a(a+1), f: [a, \infty)\overrightarrow{a, \infty}.$ If one of the solution of the equation $f(x) = f^{-1}(x) is 5049,$ then the other may be (a)5051 (b) 5048 (c) 5052 (d) 5050

A. 5051

B. 5048

C. 5052

D. 5050

Answer: B::D

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14. Which of the following function is/are periodic?

$(a) f(x) = \{1, \xi s rational 0, \xi s irrational$ `(b)f(x)={x-[x];2n

A. $f(x) = \begin{cases} 1, & x \text{ is rational} \\ 0, & x \text{ is irrational} \end{cases}$

B. $f(x) = \begin{cases} x - [x], & 2n \le x < 2n + 1 \\ \frac{1}{2}, & 2n + 1 \le x < 2n + 2 \end{cases}$

where [.] denotes the greatest integer function $n \in Z$

C. $f(x) = (-1)\left[\frac{2x}{\pi}\right]$, where [.] denotes the greatest integer function

D. $f(x) = x - [x + 3] + \tan\left(\frac{\pi x}{2}\right)$, where [.] denotes the greatest integer function, and a is a rational number

Answer: A::B::C::D

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15. Let $f(x)=\frac{3}{4}x+1, f^n(x)$ be defined as $f^2(x)=f(f(x))$, and for $n\geq 2, f^{n+1}(x)=f\left(f^n(x)\right)$. If $\lambda = \lim_{n\to\infty} f^n(x)$, then

A. λ is independent of x

B. λ is a linear polynomial in x

C. the line $y=\lambda$ has slope 0

D. the line $4y=\lambda$ touches the unit circle with center at the origin.

Answer: A::C::D

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16. If the fundamental period of function $f(x)=\sin x+\cos\left(\sqrt{4-a^2}\right)x$ is 4π, then the value of a is/are

A. $\frac{\sqrt{15}}{2}$

B. $-\frac{\sqrt{15}}{2}$

C. $\frac{\sqrt{7}}{2}$

D. $-\frac{\sqrt{7}}{2}$

Answer: A::B::C::D

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17. $f(x) = \sin^{-1}\left[e^x\right] + \sin^{-1}\left[e^{-x}\right]$ where [.] greatest integer function then

A. (a) domain of $f(x)$ is $\left(-\log_e 2, \log_e 2\right)$

B. (b) range of $f(x) = \{\pi\}$

C. (c) Range of $f(x)$ is $\left\{\frac{\pi}{2}, \pi\right\}$

D. (d) $f(x) = \cos^{-1} x$ has only one solution

Answer: A::C

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18. $[2x] - 2[x] = \lambda$ where [.] represents greatest integer function and {.} represents fractional part of a real number then

A. (a) $\lambda = 1 \forall x \in R$

B. (b) $\lambda = 0 \forall x \in R$

C. (c) $\lambda = 1 \forall \{x\} \geq \frac{1}{2}$

D. (d) $\lambda = 0 \forall \{x\} < \frac{1}{2}$

Answer: C::D

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19. The set of all values of x satisfying $\{x\} = x[\times]$ where $[\times]$ represents greatest integer function $\{\times\}$ represents fractional part of x

A. 0

B. $-\frac{1}{2}$

C. $-1 < x < 1$

D. Both A and B

Answer: D

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20. The function 'g' defined by $g(x) = \sin\left(\sin^{-1}\sqrt{\{x\}}\right) + \cos\left(\sin^{-1}\sqrt{\{x\}}\right) - 1$ (where {x} denotes the functional part function) is (1) an even function (2) a periodic function (3) an odd function (4) neither even nor odd

A. an even function

B. periodic function

C. odd function

D. Neither even nor odd

Answer: A::B

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21. If the function / satisfies the relation $f(x+y)+f(x-y)=2f(x), f(y)\,\forall x, y \in R$ and $f(0) \neq 0$, then

A. $f(x)$ is an even function

B. $f(x)$ is an odd function

C. If $f(2)=a, \ \text{then} \ f(-2)=a$

D. If $f(4)=b, \ \text{then} \ f(-4)=-b$

Answer: A::C

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22. Let $f(x)+f(y)=f\left(x\sqrt{1-y^2}+y\sqrt{1-x^2}\right)$[$f(x)$ is not identically zero]. Then $f\left(4x^3-3x\right)+3f(x)=0$ $f\left(4x^3-3x\right)=3f(x)$ $f\left(2x\sqrt{1-x^2}+2f(x)=0\right.$ $f\left(2x\sqrt{1-x^2}=2f(x)\right.$

A. $f\left(4x^3-3x\right)+3f(x)=0$

B. $f\left(4x^3 - 3x\right) = 3f(x)$

C. $f\left(2x\sqrt{1 - x^2}\right) + 2f(x) = 0$

D. $f\left(2x\sqrt{1 - x^2}\right) = 2f(x)$

Answer: A::D

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23. Let $f: R\vec{R}$ be a function defined by $f(x+1) = \dfrac{f(x) - 5}{f(x) - 3}\ \forall x \in R^{\cdot}$ Then which of the following statement(s) is/are ture? $f(2008) = f(2004)$ $f(2006) = f(2010)$ $f(2006) = f(2002)$ $f(2006) = f(2018$

A. $f(2008) = f(2004)$

B. $f(2006) = f(2010)$

C. $f(2006) = f(2002)$

D. $f(2006) = f(2018)$

Answer: A::B::C::D

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24. Let a function $f(x), x \neq 0$ be such that

$$f(x) + f\left(\frac{1}{x}\right) = f(x) \cdot f\left(\frac{1}{x}\right) \text{ then } f(x) \text{ can be}$$

A. $1 - x^{2013}$

B. $\sqrt{|x|} + 1$

C. $\frac{\pi}{2\tan^{-1}|x|}$

D. $\frac{2}{1 + k \text{ In } |x|}$

Answer: A::B::C::D

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25. Let f be a differential function such that $f(x) = f(2 - x)$ and $g(x) = f(1 + x)$ then

A. g(x) is an odd function

B. g(x) is an even function

C. Graph of $f(x)$ is symmetrical about the line $x = 1$

D. $f'(1) = 0$

Answer: B::C::D

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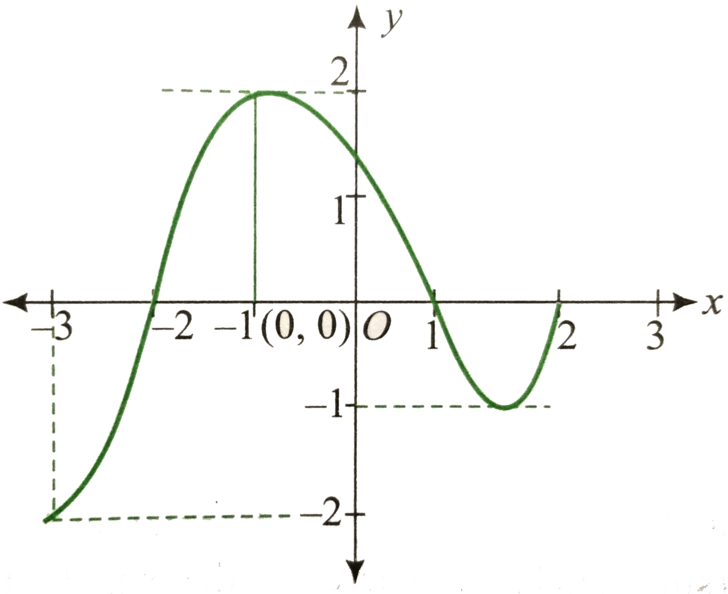
26. The figure illustrates the graph of the function $y = f(x)$ defined in [-3, 2].



Identify the correct statement(s)?

A. Range of $y = f(-|x|)$ is $[-2, 2]$

B. Domain of $y = f(|x|)$ is $[-2, 2]$

C. Domain of $y = f|x| + 1)$ is $[-1, 1]$

D. Range of $y = f(|x| + 1)$ is $[-1, 0]$

Answer: A::B::C::D

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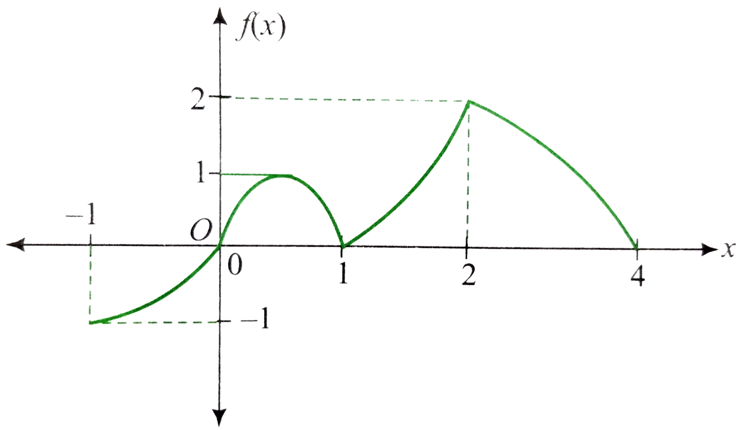
27. If graph of a function $f(x)$ which is defined in [-1, 4] is shown in the following figure then identify the correct statement(s).



A. domain of $f(|x| - 1)$ is $[-5, 5]$

B. range of $f(|x| + 1)$ is $[0, 2]$

C. range of $f(-|x|)$ is $[-1, 0]$

D. domain of $f(|x|)$ is $[-3, 3]$

Answer: A::B::C

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Linked Comprehension Type

1. Consider the functions

$$f(x) = \begin{cases} x+1, & x \le 1 \\ 2x+1, & 1 < x \le 2 \end{cases} \quad \text{and} \quad g(x) = \begin{cases} x^2, & -1 \le x < 2 \\ x+2, & 2 \le x \le 3 \end{cases}$$

The range of the function $f(g(x))$ is

A. $\left[0, \sqrt{2}\right]$

B. $[-1, 2]$

C. $\left[-1, \sqrt{2}\right]$

D. None of these

Answer: C

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2. Consider the functions

$$f(x) = \begin{cases} x+1, & x \le 1 \\ 2x+1, & 1 < x \le 2 \end{cases} \quad \text{and} \quad g(x) = \begin{cases} x^2, & -1 \le x < 2 \\ x+2, & 2 \le x \le 3 \end{cases}$$

The range of the function $f(g(x))$ is

A. $[1, 5]$

B. $[2, 3]$

C. $[1, 2] \cup [3, 5]$

D. None of these

Answer: C

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3. If the function $f(x) = \{x + 1$ if $x \le 1$, $2x + 1$ if $1 < x \le 2$ and $g(x) = \{x^2$ if $-1 \le x \le 2, x + 2 \quad \text{if} \quad 2 \le x \le 3$ then the number of roots of the equation $f(g(x)) = 2$

A. 1

B. 2

C. 4

D. None of these

Answer: B

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4. Consider the function $f(x)$ satisfying the identity

$$f(x) + f\left(\frac{x-1}{x}\right) = 1 + x \, \forall x \in R - \{0, 1\}, \text{ and } g(x) = 2f(x) - x + 1.$$

The domain of $y = \sqrt{g(x)}$ is

A. $\left(-\infty, \frac{1-\sqrt{5}}{2}\right] \cup \left[1, \frac{1+\sqrt{5}}{2}\right]$

B. $\left(-\infty, \frac{1-\sqrt{5}}{2}\right] \cup (0, 1) \cup \left[\frac{1+\sqrt{5}}{2}, \infty\right)$

C. $\left[\frac{-1-\sqrt{5}}{2}, 0\right] \cup \left[\frac{-1+\sqrt{5}}{2}, 1\right)$

D. None of these

Answer: B

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5. Consider the function f(x) satisfyig the identity $f(x) + f\left(\frac{x - 1}{x}\right) = 1 + x, \forall x \in R - \{0, 1\}$ and $g(x) = 2f(x) - x + 1$ Then range of y=g(x) is:

A. $(-\infty, 5]$

B. $[1, \infty)$

C. $(-\infty, 1) \cup [5, \infty)$

D. None of these

Answer: C

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6. Consider the function $f(x)$ satisfying the identity $f(x) + f\left(\frac{x - 1}{x}\right) = 1 + x \forall x \in R - \{0, 1\}, \text{ and } g(x) = 2f(x) - x + 1.$

The domain of $y = \sqrt{g(x)}$ is

A. 2

B. 1

C. 3

D. 0

Answer: D

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7. If $(f(x))^2 \cdot f\left(\frac{1-x}{1+x}\right) = 64x \,\forall\, \in D_f$ then

A. $4x^{2/3}\left(\frac{1+x}{1-x}\right)^{1/3}$

B. $x^{1/3}\left(\frac{1-x}{1+x}\right)^{1/3}$

C. $x^{1/3}\left(\frac{1-x}{1+x}\right)^{1/3}$

D. $x\left(\frac{1+x}{1-x}\right)^{1/3}$

Answer: A

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8. If $(f(x))^2 \times f\left(\frac{1-x}{1+x}\right) = 64x \, \forall x \in D_f,$ then

The domain of $f(x)$ is

A. $[0, \infty)$

B. $R - \{1\}$

C. $(-\infty, \infty)$

D. None of these

Answer: B

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9. If $(f(x))^2 \times f\left(\frac{1-x}{1+x}\right) = 64x \, \forall x \in D_f,$ then

The value of $f(9/7)$ is

A. $8(7/9)^{2/3}$

B. $4(9/7)^{1/3}$

C. $-8(9/7)^{2/3}$

D. None of these

Answer: C

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10. $f(x) = \begin{cases} x-1, & -1 \le x \le 0 \\ x^2, & 0 \le x \le 1 \end{cases}$ and $g(x) = \sin x$

Consider the functions $h_1(x) = f(|g(x)|)$ and $h_2(x) = |f(g(x))|$.

then

A. It is a periodic function with period π.

B. The range is [0, 1].

C. The domain is R.

D. None of these

Answer: D

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11. $f(x) = \begin{cases} x - 1, & -1 \leq x \leq 0 \\ x^2, & 0 \leq x \leq 1 \end{cases}$ and $g(x) = \sin x$

Consider the functions $h_1(x) = f(|g(x)|)$ and $h_2(x) = |f(g(x))|$.

Which of the following is not true about $h_2(x)$?

A. The domain is R

B. It is periodic with period 2π.

C. The range is [0, 1].

D. None of these

Answer: B

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12. $f(x) = \begin{cases} x - 1, & -1 \le x \le 0 \\ x^2, & 0 \le x \le 1 \end{cases}$ and $g(x) = \sin x$

Consider the functions $h_1(x) = f(|g(x)|)$ and $h_2(x) = |f(g(x))|$.

Which of the following is not true about $h_1(x)$?

A. Domain of $h_1(x)$ and $h_2(x)$ is $x \in [2n\pi, (2n + 1)\pi], n \in Z$.

B. Range of $h_1(x)$ and $h_2(x)$ is $[0, 1]$

C. Period of $h_1(x)$ and $h_2(x)$ is π

D. None of these

Answer: C

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13. If $a_0 = x, a_{n+1} = f\left(a_n\right)$, where $n = 0, 1, 2, \ldots,$ then answer thefollowing questions. If $f(x) = m\sqrt{a - x^m}, x < 0, m \le 2, m \in N$,then

A. $a_n = x, n = 2k + 1,$ where k is an integer

B. $a_n = f(x)$ if $n = 2k$, where k is an integer

C. The inverse of a_n exists for any value of n and m

D. None of these

Answer: D

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14. If $a_0 = x, a_{n+1} = f(a_n)$, where $n = 0, 1, 2, \ldots,$ then answer the following questions.

If $f(x) = \frac{1}{1 - x}$, then which of the following is not true?

A. $a_n = \frac{1}{1 - x}$ if $n = 3k + 1$

B. $a_n = \frac{x - 1}{x}$ if $n = 3k + 2$

C. $a_n = x$ if $n = 3k$

D. None of these

Answer: D

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15. If $a_0 = x, a_{n+1} = f(a_n)$, where $n = 0, 1, 2, \ldots,$ then answer the following questions.

If $f: R \to R$ is given by $f(x) = 3 + 4x$ and $a_n = A + Bx,$ then which of the following is not true?

A. $A + B + 1 = 2^{2n+1}$

B. $|A - B| = 1$

C. $\lim_{h \to \infty} \frac{A}{B} = -1$

D. None of these

Answer: C

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16. Let $f(x) = f_1(x) - 2f_2(x)$, where ,where $f_1(x) = \left\{ \begin{pmatrix} \min \left\{x^2, |x|\right\} & |x| \leq 1 \\ \max \left\{x^2, |x|\right\} & |x| \leq 1 \end{pmatrix} \right.$

and $f_2(x) = \left\{ \begin{pmatrix} \min \left\{x^2, |x|\right\} & |x| < 1 \\ \left\{x^2, |x|\right\} & |x| \leq 1 \end{pmatrix} \right.$ and let

$g(x) = \left\{ \begin{pmatrix} \min \{f(t): -3 \leq t \leq x, -3 \leq x \leq 0\} \\ \max \{f(t): 0 \leq t \leq x, 0 \leq x \leq 3\} \end{pmatrix} \right.$ for $-3 \leq x \leq -1$ the range of $g(x)$ is

A. $[-1, 3]$

B. $[-1, -15]$

C. $[-1, 9]$

D. None of these

Answer: A

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17. Let $f(x) = f_1(x) - 2f_2(x)$, where

$$\text{where } f(x) = \begin{cases} \min\left\{x^2, |x|\right\}, & |x| \le 1 \\ \max\left\{x^2, |x|\right\}, & |x| > 1 \end{cases}$$

$$\text{and } f_2(x) = \begin{cases} \min\left\{x^2, |x|\right\}, & |x| > 1 \\ \max\left\{x^2, |x|\right\}, & |x| \le 1 \end{cases}$$

$$\text{and let } g(x) = \begin{cases} \min\{f(t): -3 \le t \le x, \ -3 \le x < 0\} \\ \max\{f(t): 0 \le t \le x, \ 0 \le x \le 3\} \end{cases}$$

For $-3 \le x \le -1$, the range of g(x) is

A. $x^2 - 2x + 1$

B. $x^2 + 2x - 1$

C. $x^2 + 2x + 1$

D. $x^2 - 2x - 1$

Answer: B

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18. Let $f(x) = f_1(x) - 2f_2(x),$ where

where $f(x) = \begin{cases} \min\left\{x^2, |x|\right\}, & |x| \le 1 \\ \max\left\{x^2, |x|\right\}, & |x| > 1 \end{cases}$

and $f_2(x) = \begin{cases} \min\left\{x^2, |x|\right\}, & |x| > 1 \\ \max\left\{x^2, |x|\right\}, & |x| \le 1 \end{cases}$

and let $g(x) = \begin{cases} \min\{f(t): -3 \le t \le x, & -3 \le x < 0\} \\ \max\{f(t): 0 \le t \le x, & 0 \le x \le 3\} \end{cases}$

For $-3 \le x \le -1,$ the range of g(x) is

A. 1 point

B. 2 points

C. 3 points

D. None of these

Answer: A

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19. Let $f(x) = \begin{cases} 2x + a, & x \geq -1 \\ bx^2 + 3, & x < -1 \end{cases}$

and $g(x) = \begin{cases} x + 4, & 0 \leq x \leq 4 \\ -3x - 2, & -2 < x < 0 \end{cases}$

$g(f(x))$ is not defined if

A. $a \in (10, \infty), b \in (5, \infty)$

B. $a \in (4, 10), b \in (5, \infty)$

C. $a \in (10, \infty), b \in (0, 1)$

D. $a \in (4, 10), b \in (1, 5)$

Answer: A

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20. Let $f(x) = \begin{cases} 2x + a, & x \geq -1 \\ bx^2 + 3, & x < -1 \end{cases}$

and $g(x) = \begin{cases} x + 4, & 0 \leq x \leq 4 \\ -3x - 2, & -2 < x < 0 \end{cases}$

If the domain of $g(f(x))$ is $[-1, 4]$, then

A. $a = 1, b > 5$

B. $a = 2, b > 7$

C. $a = 2, b > 10$

D. $a = 0, b \in R$

Answer: D

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21. Let $f(x) = \begin{cases} 2x + a, & x \geq -1 \\ bx^2 + 3, & x < -1 \end{cases}$

and $g(x) = \begin{cases} x + 4, & 0 \leq x \leq 4 \\ -3x - 2, & -2 < x < 0 \end{cases}$

If $a = 2$ and $b = 3$, then the range of $g(f(x))$ is

A. $(-2, 8]$

B. $(0, 8]$

C. $[4, 8]$

D. $[-1, 8]$

Answer: C

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22. Let $f: R \to R$ is a function satisfying $f(2 - x) = f(2 + x)$ and $f(20 - x) = f(x), \forall x \in R$. On the basis of above information, answer the following questions If $f(0) = 5$, then minimum possible number of values of x satisfying $f(x) = 5$, for $x \in [10, 170]$ is

A. 21

B. 12

C. 11

D. 22

Answer: C

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23. Let $f : R \to R$ be a function satisfying $f(2 - x) = f(2 + x)$ and $f(20 - x) = f(x)\, \forall x \in R$. For this function f, answer the following.

The graph of $y = f(x)$ is not symmetrial about

A. symmetrical about $x = 2$

B. symmetrical about $x = 10$

C. symmetrical about $x = 8$

D. None of these

Answer: C

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24. Let $f : R \to R$ be a function satisfying $f(2 - x) = f(2 + x)$ and $f(20 - x) = f(x)\, \forall x \in R$. For this function f, answer the following.

If $f(2) \neq f(6)$, then the

A. (a) fundamental period of $f(x)$ is 1

B. (b) fundamental period of $f(x)$ may be 1

C. (c) period of $f(x)$ cannot be 1

D. (d) fundamental period of $f(x)$ is 8

Answer: C

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25. Consider two functions

$$f(x) = \begin{cases} [x], & -2 \le x \le -1 \\ |x| + 1, & -1 < x \le 2 \end{cases} \text{ and } g(x) = \begin{cases} [x], & -\pi \le x < 0 \\ \sin x, & 0 \le x \le \pi \end{cases}$$

where [.] denotes the greatest integer function.

The number of integral points in the range of $g(f(x))$ is

A. $[0, 2]$

B. $[-2, 0]$

C. $[-2, 2]$

D. $[-2, 2]$

Answer: C

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26. Consider two functions

$$f(x) = \begin{cases} [x], & -2 \le x \le -1 \\ |x| + 1, & -1 < x \le 2 \end{cases} \text{ and } g(x) = \begin{cases} [x], & -\pi \le x < 0 \\ \sin x, & 0 \le x \le \pi \end{cases}$$

where [.] denotes the greatest integer function.

The exhaustive domain of $g(f(x))$ is

A. $[\sin 3, \sin 1]$

B. $[\sin 3, 1] \cup \{-2, -1, 0\}$

C. $[\sin 3, 1] \cup \{-2, -1\}$

D. $[\sin 1, 1]$

Answer: C

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27. Consider two functions

$$f(x) = \begin{cases} [x], & -2 \le x \le -1 \\ |x| + 1, & -1 < x \le 2 \end{cases} \text{ and } g(x) = \begin{cases} [x], & -\pi \le x < 0 \\ \sin x, & 0 \le x \le \pi \end{cases}$$

where [.] denotes the greatest integer function.

The number of integral points in the range of $g(f(x))$ is

A. 2

B. 4

C. 3

D. 5

Answer: B

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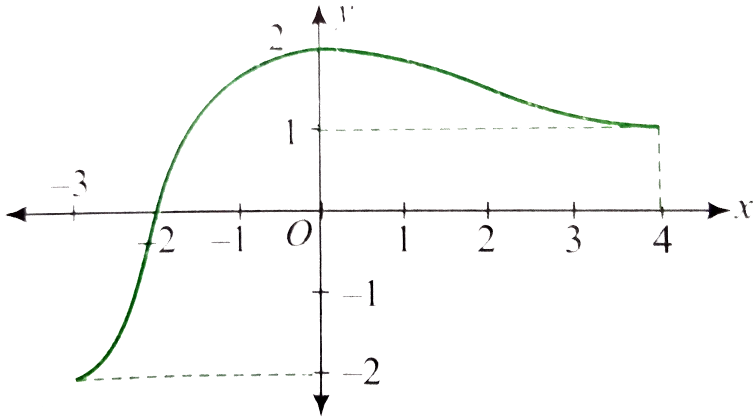
28. Consider a function f whose domain is [-3, 4] and range is [-2, 2] with following graph.



Domain and range of $g(x) = f(|x|)$ is [a, b] and [c, d] respectively, then $(b - a + c + d)$ is

A. 11

B. 10

C. 8

D. 7

Answer: A

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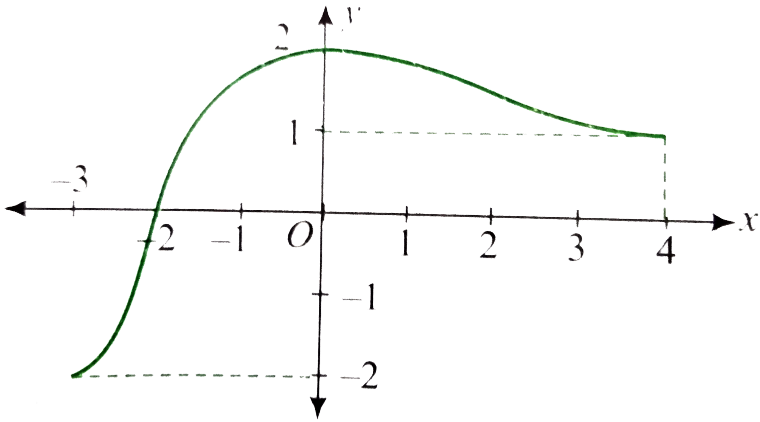
29. Consider a function f whose domain is [-3, 4] and range is [-2, 2] with following graph.



If $h(x) = \left|f(x) - \frac{3}{2}\right|$ has range [e, f] and n be number of real solutions of $h(x) = \frac{1}{4}$, then $(n + e + 2f)$ is

A. 8

B. 9

C. 10

D. 11

Answer: D

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30. Consider a differentiable $f: R \to R$ for which $f(1) = 2$ and $f(x + y) = 2^x f(y) + 4^y f(x) \forall x, y \in R$.

The value of f(4) is

A. 160

B. 240

C. 200

D. None of these

Answer: A

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31. Consider a differentiable function $f: R \to R$ for which $f(1) = 2$ and $f(x + y) = 2^x f(y) + 4^y f(x) \forall x, y \in R$.

The minimum value of f(x) is

A. 1

B. - $\frac{1}{2}$

C. - $\frac{1}{4}$

D. None of these

Answer: C

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32. Consider a differentiable $f: R \to R$ for which $f(1) = 2$ and $f(x + y) = 2^x f(y) + 4^y f(x) \forall x, y \in R.$

The number of solutions of $f(x) = 2$ is

A. 0

B. 1

C. 2

D. infinite

Answer: B

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Matrix Match Type

1. The function $f(x)$ is defined on the interval [0, 1]. Now, match the following lists:

Lisr I: Function	List II: Domain
a. $f(\tan x)$	p. $\left[2n\pi - \frac{\pi}{2}, 2n\pi + \frac{\pi}{2}\right], n \in Z$
b. $f(\sin x)$	q. $\left[2n\pi, 2n\pi + \frac{\pi}{6}\right] \cup \left[2n\pi + \frac{5\pi}{6}, (2n+1)\pi\right]$ $n \in Z$
c. $f(\cos x)$	r. $[2n\pi, (2n+1)\pi], n \in Z$
d. $f(2\sin x)$	s. $\left[n\pi, n\pi + \frac{\pi}{4}\right], n \in Z$

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2. Match the following lists:

List I: Function	List II: Type of function
a. $f(x) = \{(\operatorname{sgn} x)^{\operatorname{sgn} x}\}^n$; $x \neq 0$, n is an odd integer	**p.** odd function
b. $f(x) = \dfrac{x}{e^x - 1} + \dfrac{x}{2} + 1$	**q.** even function
c. $f(x) = \begin{cases} 0, & \text{if } x \text{ is rational} \\ 1, & \text{if } x \text{ is irrational} \end{cases}$	**r.** neither odd nor even function
d. $f(x) = \max\{\tan x, \cot x\}$	**s.** periodic

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3. Let $f(x) = \dfrac{x^2 - 6x + 5}{x^2 - 5x + 6}$. Then match the expressions/statements in List I with expression /statements in List II.

List I	List II
a. If $-1 < x < 1$, then $f(x)$ satisfies	**p.** $0 < f(x) < 1$
b. If $1 < x < 2$, then $f(x)$ satisfies	**q.** $f(x) < 0$
c. If $3 < x < 5$, then $f(x)$ satisfies	**r.** $f(x) > 0$
d. If $x > 5$, then $f(x)$ satisfies	**s.** $f(x) < 1$

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4. Match the following lists:

List I: Function	List II: Values of x for which both the functions in any option of List I are identical
a. $f(x) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, $g(x) = 2\tan^{-1}x$	**p.** $x \in \{-1, 1\}$
b. $f(x) = \sin^{-1}(\sin x)$ and $g(x) = \sin(\sin^{-1}x)$	**q.** $x \in [-1, 1]$
c. $f(x) = \log_{x^2} 25$ and $g(x) = \log_x 5$	**r.** $x \in (-1, 1)$
d. $f(x) = \sec^{-1}x + \operatorname{cosec}^{-1}x$, $g(x) = \sin^{-1}x + \cos^{-1}x$	**s.** $x \in (0, 1)$

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5. Match the following lists:

List I	List II
a. $f: R \to \left[\frac{3\pi}{4}, \pi\right)$ and $f(x) = \cot^{-1}(2x - x^2 - 2)$. Then $f(x)$ is	**p.** one-one
b. $f: R \to R$ and $f(x) = e^x \sin x$. Then $f(x)$ is	**q.** into
c. $f: R^+ \to [4, \infty]$ and $f(x) = 4 + 3x^2$. Then $f(x)$ is	**r.** many-one
d. $f: X \to X$ and $f(f(x)) = x \ \forall\ x \in X$. Then $f(x)$ is	**s.** onto

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6. Match the following lists:

List I: Function	List II: Fundamental Period
a. $f(x) = \cos(\lvert\sin x\rvert - \lvert\cos x\rvert)$	**p.** π
b. $f(x) = \cos(\tan x + \cot x) \times \cos(\tan x - \cot x)$	**q.** $\pi/2$
c. $f(x) = \sin^{-1}(\sin x) + e^{\tan x}$	**r.** 4π
d. $f(x) = \sin^3 x \sin 3x$	**s.** 2π

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7. { . } denotes the fractional part function and [.] denotes the greatest integer function. Now, match the following lists:

List I: Function	List II: Period
a. $f(x) = e^{\cos^4 \pi x + x - [x] + \cos^2 \pi x}$	**p.** 1/3
b. $f(x) = \cos 2\pi\{2x\} + \sin 2\pi\{2x\}$	**q.** 1/4
c. $f(x) = \sin 3\pi\{x\} + \tan \pi[x]$	**r.** 1/2
d. $f(x) = 3x - [3x + a] - b$, where $a, b \in R^+$	**s.** 1

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8. Match the following lists and then choose the correct code.

List I: Function	List II: Range
a. $f(x) = \log_3(5 + 4x - x^2)$	**p.** Function not defined
b. $f(x) = \log_3(x^2 - 4x - 5)$	**q.** $[0, \infty)$
c. $f(x) = \log_3(x^2 - 4x + 5)$	**r.** $(-\infty, 2]$
d. $f(x) = \log_3(4x - 5 - x^2)$	**s.** R

A. $\begin{matrix} a & b & c & d \\ p & r & s & q \end{matrix}$

B. $\begin{matrix} a & b & c & d \\ r & s & q & p \end{matrix}$

C. $\begin{matrix} a & b & c & d \\ r & q & s & p \end{matrix}$

D. $\begin{matrix} a & b & c & d \\ p & q & s & r \end{matrix}$

Answer: B

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9. Match the following lists and then choose the correct code.

List I: Equation	List II: Number of roots
a. $x^2 \tan x = 1, x \in [0, 2\pi]$	**p.** 5
b. $2^{\cos x} = \lvert\sin x\rvert, x \in [0, 2\pi]$	**q.** 2
c. If $f(x)$ is a polynomial of degree 5 with real coefficients such that $f(\lvert x \rvert) = 0$ has 8 real roots, then the number of roots of $f(x) = 0$	**r.** 3
d. $7^{\lvert x \rvert}(\lvert 5 - \lvert x \rvert\rvert) = 1$	**s.** 4

A. $\begin{matrix} a & b & c & d \\ q & s & r & p \end{matrix}$

B. $\begin{matrix} a & b & c & d \\ p & s & r & s \end{matrix}$

C. $\begin{matrix} a & b & c & d \\ q & s & p & s \end{matrix}$

D. $\begin{matrix} a & b & c & d \\ s & p & q & r \end{matrix}$

Answer: C

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Numerical value Type

1. Let f be a real-valued invertible function such that $f\left(\frac{2x-3}{x-2}\right)=5x-2, x \neq 2$. Then value of $f^{-1}(13)$ is_______

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2. Let $f(x)=3x^2-7x+c$, where c is a variable coefficient and $x>\frac{7}{6}$. Then the value of $[c]$ such that $f(x)$ touches $f^{-1}(x)$ is (where [.] represents greatest integer function)_______

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3. The number of points on the real line where the function $f(x) = \log_{|x^2-1|}|x - 3|$ is not defined is

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4. The number of integral values of x for which $\frac{\left(\frac{\pi}{2^{\tan \wedge ((-1)x-4)}}\right)(x - 4)(x - 10)}{x! - (x - 1)!} < 0 is\_\_\_\_$

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5. If $f(x) = \left\{x\cos x + (\log)_e\left(\frac{1 - x}{1 + x}\right)a; x = 0; x \neq 0\right.$ is odd, then $a\_\_\_\_,$

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6. The number of integers in the range of the function $f(x) = \mid 4\dfrac{\left(\sqrt{\cos x} - \sqrt{\sin x}\right)\left(\sqrt{\cos x} + \sqrt{\sin x}\right)}{(\cos x + \sin x)} \mid is\_\ \_\_\_$

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7. The number of integers in the domain of function, satisfying $f(x) + f\left(x^{-1}\right) = \dfrac{x^3 + 1}{x}, is\_\_\_$

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8. If a polynomial function $f(x)$ satisfies $f(f(f(x)) = 8x + 21,$ where $pandq$ are real numbers, then $p + q$ is equal to ______

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9. If $f(x)$ is an odd function, $f(1) = 3, f(x+2) = f(x) + f(2),$ then the value of $f(3)$ is________

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10. Let $f: R \to R$ be a continuous onto function satisfying $f(x) + f(-x) = 0 \forall x \in R.$ If $f(-3) = 2 \text{ and } f(5) = 4 \text{ in } [-5, 5],$ then the minimum number of roots of the equation $f(x) = 0$ is

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11. The set of all real values of x for which the funciton $f(x) = \sqrt{\sin x + \cos x} + \sqrt{7x - x^2 - 6}$ takes real values is

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12. Suppose that f is an even, periodic function with period 2, and $f(x) = x$ for all x in the interval $[0, 1]$. The values of $[10f(3.14)]$ is (where [.] represents the greatest integer function) _____

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13. If $f(x) = \sqrt{4 - x^2} + \sqrt{x^2 - 1}$, then the maximum value of $(f(x))^2$ is

____

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14. The function $f(x) = \dfrac{x + 1}{x^3 + 1}$ can be written as the sum of an even function $g(x)$ and an odd function $h(x)$. Then the value of $|g(0)|$ is__________

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15. If T is the period of the function $f(x) = [8x + 7] + |\tan 2\pi x + \cot 2\pi x| - 8x]$ (where [.] denotes the greatest integer function), then the value of $\frac{1}{T}$ is

____

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16. An even polynomial function $f(x)$ satisfies a relation $f(2x)\left(1 - f\left(\frac{1}{2x}\right)\right) + f\left(16x^2y\right) = f(-2) - f(4xy)\ \forall x, y \in R - \{0\}$ and $f(4) = -255, f(0) = 1.$ Then the value of $|(f(2) + 1)/2|$ is________

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17. If $f(x) = \sin^2 x + \sin^2\left(x + \frac{\pi}{3}\right) + \cos x \cos\left(x + \frac{\pi}{3}\right) and g\left(\frac{5}{4}\right) = 1,$ then $(gof)(x)$ is __________

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18. Let $E = \{1, 2, 3, 4, \}$ and $F = \{1, 2\}$. Then the number of onto functions from E to F, is _____.

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19. The function of f is continuous and has the property $f(f(x)) = 1 - x$.

Then the value of $f\left(\frac{1}{4}\right) + f\left(\frac{3}{4}\right)$ is

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20. A function f from integers to integers is defined as

$$f(n) = \begin{cases} n + 3, & n \in odd \\ n/2, & n \in even \end{cases}$$

Suppose $k \in odd$ and $f(f(f(k))) = 27$. Then the value of k is _______

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22. If $x = \frac{4}{9}$ satisfies the equation $(\log)_a\left(x^2 - x + 2\right) > (\log)_a\left(-x^2 + 2x + 3\right)$, then the sum of all possible distinct values of $[x]$ is (where[.] represetns the greatest integer function)

___

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23. If $4^x - 2^{x+2} + 5 + ||b - 1| - 3| - |\sin y|, x, y, b \in R,$ then the possible value of b is________

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24. If $f: N \rightarrow N,$ and $x_2 > x_1 \Rightarrow f\left(x_2\right) > f(x) \forall x_1, x_2 \in N$ and $f(f(n)) = 3n \forall n \in N,$ t

______.

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25. The number of integral values of a for which $f(x) = \log\left((\log)_{\frac{1}{3}}\left((\log)_7(\sin x + a)\right)\right)$ is defined for every real value of x is _______

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26. Let $f(x) = \sin^{23}x - \cos^{22}x and g(x) = 1 + \frac{1}{2}\tan^{-1}|x|$. Then the number of values of x in the interval $[-10\pi, 8\pi]$ satisfying the equation $f(x) = sgn(g(x))$ is ________

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27. Suppose that $f(x)$ is a function of the form $f(x) = \frac{ax^8 + bx^6 + cx^4 + dx^2 + 15x + 1}{x}, (x \neq 0)$. If $f(5) = 2$, then the value of $f(-5)$ is _______.

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28. If $f:(2, -\infty) \to [8, \infty)$ is a surjective function defined by $f(x) = x^2 - (p-2)x + 3p - 2, p \in R$ then sum of values of p is $m + \sqrt{n}$, where $m, n \in N$. Find the value of $\frac{n}{m}$.

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29. Period of the function

$f(x) = \sin\left(\frac{x}{2}\right)\cos\left[\frac{x}{2}\right] - \cos\left(\frac{x}{2}\right)\sin\left[\frac{x}{2}\right]$, where [.] denotes the greatest integer function, is ________.

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30. If the interval x satisfying the equation

$|x| + |-x| = \frac{\log_3(x-2)}{\left|\log_3(x-2)\right|}$ is (a, b), then $a + b =$ ______.

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31. Let $f(x)$ be a polynomial of degree 5 such that $f(|x|) = 0$ has 8 real distinct , Then number of real roots of $f(x) = 0$ is ______.

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Archives(single correct Answer Type)

1. For real x, let $f(x) = x^3 + 5x + 1$, then (1) f is oneone but not onto R (2) f is onto R but not oneone (3) f is oneone and onto R (4) f is neither oneone nor onto R

A. f is one-one but not onto R

B. f is onto R but not one-one

C. f is one-one and onto R

D. f is neither one-one nor onto R

Answer: C

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2. Let $f : [-1, \infty] \in [-1, \infty]$ be a function given $f(x) = (x+1)^2 - 1, x \geq -1$

Statement-1: The set $\left[x : f(x) = f^{-1}(x)\right] = \{0, 1\}$

Statement-2: f is a bijection.

A. Statement 1 is ture, statement 2 is true, statement 2 is a correct explanation for statement 1.

B. Statement 1 is ture, statement 2 is true, statement 2 is not a correct explanation for statement 1.

C. Statement 1 is ture, statement 2 is false.

D. Statement 1 is false, statement 2 is true.

Answer: C

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3. Consider the following relations: R = {(x, y) | x, y are real numbers and x = wy for some rational number w};

$$S = \left\{\left(\frac{m}{n}, \frac{p}{q}\right) \text{m , n , pandqa r ei n t e g e r ss u c ht h a tn ,q} \neq 0 \text{andq m} = \text{p n}\right\}$$

. Then (1) neither R nor S is an equivalence relation (2) S is an equivalence relation but R is not an equivalence relation (3) R and S both are equivalence relations (4) R is an equivalence relation but S is not an equivalence relation

A. R and S both are equivalence relations.

B. R is an equivalence relation but S is not an equivalence relation.

C. Neither R nor S is an equivalence relation.

D. S is an equivalence relation but R is not an equivalence relation.

Answer: D

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4. Let R be the set of real numbers.

Statement 1:$A = \{(x, y) \in R \times R : y - x$ is an integer$\}$ is an equivalence relation on R.

Statement 2: $B = \{x, y\} \in R \times R : x = \alpha y$ for some rational number $\alpha\}$ is an equivalence relation on R.

A. Statement 1 is false, statement 2 is true.

B. Statement 1 is ture, statement 2 is true, statement 2 is a correct explanation for statement 1.

C. Statement 1 is true, statement 2 is true, statement 2 is not a correct explanation for statement 1.

D. Statement 1 is false, statement 2 is false.

Answer: D

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5. The domain of the function $f(x) = \dfrac{1}{\sqrt{|x| - x}}$ is: (1) $(- \infty, \infty)$ (2) $(0, \infty$ (3) $(- \infty, 0)$ (4) $(- \infty, \infty)$-$\{0\}$

A. $(- \infty, \infty)\sim\{0\}$

B. $(- \infty, \infty)$

C. $(0, \infty)$

D. (-oo,0)`

Answer: D

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6. If $a \in R$ and the equation $-3(x - [x])^2 + 2(x - [x]) + a^2 = 0$ (where [x] denotes the greatest integer $\leq x$) has no integral solution, then all possible values of a lie in the interval: (1) (-2,-1) (2) $(\infty, - 2)$ ∪ $(2, \infty)$ (3) $(- 1, 0)$ ∪ $(0, 1)$ (4) (1,2)

A. $(- 1, 0)$ ∪ $(0, 1)$

B. $(1, 2)$

C. $(-2, -1)$

D. $(-\infty, -2) \cup (2, \infty)$

Answer: A

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7. If $f(x) + 2f\left(\frac{1}{x}\right) = 3x, x \neq 0,$ and $S = \{x \in R : f(x) = f(-x)\}$; then S: (1) is an empty set. (2) contains exactly one element. (3) contains exactly two elements. (4) contains more than two elements

A. contains exactly one element

B. contains exactly two elements

C. contains more than two elements

D. is an empty set

Answer: B

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8. The function $f: R \rightarrow \left[-\frac{1}{2}, \frac{1}{2}\right]$ defined as $f(x) = \frac{x}{1+x^2}$ is

A. neither injective nor surjective.

B. invertible.

C. injective but not surjective.

D. Surjective but not injective.

Answer: D

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9. Let $a, b, c \in R$. If $f(x) = ax^2 + bx + c$ is such that $a + B + c = 3$ and $f(x+y) = f(x) + f(y) + xy, \forall x, y \in R$, then $\sum_{n=1}^{10} f(n)$ is equal to

A. 255

B. 330

C. 165

D. 190

Answer: B

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Archives(Multiple Correct Answer Type)

1. Let $f:\left(-\frac{\pi}{2},\frac{\pi}{2}\right)\vec{R}$ be given by $f(x)=(\log(\sec x+\tan x))^3$ then $f(x)$ is an odd function $f(x)$ is a one-one function $f(x)$ is an onto function $f(x)$ is an even function

A. $f(x)$ is an odd function

B. $f(x)$ is a one-one function

C. $f(x)$ is an onto function

D. $f(x)$ is an event function

Answer: A::B::C

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2. Let $f(x) = \sin\left[\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right]$ for all $x \in \mathbb{R}$

A. Range of f is $\left[-\frac{1}{2}, \frac{1}{2}\right]$

B. Range of fog is $\left[-\frac{1}{2}, \frac{1}{2}\right]$

C. $\lim_{x \to 0} \frac{f(x)}{g(x)} = \frac{\pi}{6}$

D. There is an $x \in R$ such that $(gof)(x) = 1$

Answer: A::B::C

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Archives(Matrix Match Type)

1. Let

$E_1 = \left\{x \in R : x \neq 1 \text{ and } \frac{x}{x-1} > 0\right\}$ and $E_2 = \left\{x \in E_1 : \sin^{-1}\left(\log_e\left(\frac{x}{x-1}\right)\right)$ i

(Here, the inverse trigonometric function $\sin^{-1}x$ assumes values in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.)

Let $f : E_1 \to R$ be the function defined by $f(x) = \log_e\left(\frac{x}{x-1}\right)$ and $g : E_2 \to R$ be the function defined by $g(x) = \sin^{-1}\left(\log_e\left(\frac{x}{x-1}\right)\right)$.

List I	List II
a. The range of f is	p. $\left(-\infty, \frac{1}{1-e}\right] \cup \left[\frac{e}{e-1}, \infty\right)$
b. The range of g contains	q. $(0, 1)$
c. The domain of f contains	r. $[-1/2, 1/2]$
d. the domain of g is	s. $(-\infty, 0) \cup (0, \infty)$
	t. $\left(-\infty, \frac{e}{e-1}\right]$
	u. $(-\infty, 0) \cup \left(\frac{1}{2}, \frac{e}{e-1}\right]$

The correct option is

A. $a \to s, b \to q, c \to p, d \to p$

B. $a \to r, b \to r, c \to u, d \to t$

C. $a \to s, b \to q, c \to p, d \to u$

D. $a \to s, b \to r, c \to u, d \to t$

Answer: A

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