

MATHS

BOOKS - CENGAGE MATHS (ENGLISH)

SET THEORY AND REAL NUMBER SYSTEM

Illustration

1. State which of the following statement are true and which ones are false . Justify your answer .

(i) $21 \in \{x \mid x ext{ has exactly four positive factors }\}$

(ii) $64 \in \{y|$ the sun of the all the positve factors of y is 2y }

(iii)
$$2 \in ig\{x \mid x^4 - 3x^3 + 4x^2 - 5x + 6 = 0ig\}$$

(iv) 23562 $\in \{y \mid y ext{ is divisible by 9}\}$

2. Are the following pairs of sets equal ? (i) A={x | x is prime factor of 6 } B= {x |x is a solution of $x^2 - 5x + 6 = 0$ }*Mlbtr* (ii) A={x|y is a letter in the word REPLACED}, B= { y is a letter in the word PARCELED } (iii) A= { x| x is a natural number x > 1} B= {x|x is natural number $x \ge 1$ }



A= set ofnaturai numbers which are multiples of 2

B= set ofnaturai numbers which are multiples of 3

C= set ofnaturai numbers which are multiples of 5

Then find the following set

 $(i)A\cup B(ii)B\cup C(iii)A-B(iv)B-C(v)A\cap C(vi)A\cap B\cap C(vii)(A\cup C(vii)A))$



8. Show that $A \cap B = A \cap C$ need not impl B=C





```
10. Show that A \subset B then C- B \subset C - A
```

11. Assume that P(A) = P(B). Show that A = B

12. If sets A= (-3,2] and B= (-1, 5] then find the following sets :

A. $A\cap B$

 $\mathsf{B.}\, A \cup B$

C. A-B

D. B- A

Answer:

Watch Video Solution

13. In a group of 500 people, 350 speak Hindi and 300 speak English. It is

given that each person speaks at least one language.

(i) How many people can speak both Hindi and English?

(ii) How many people can speak Hindi only?

(iii) How many people can speak English only?

14. In a group of 50 students, the number of students studying French, English, Sanskrit were found to be as follows French = 17, English = 13, Sanskrit = 15 French and English = 09, English and Sanskrit = 4, French and Sanskrit = 5, English , French and Sanskrit = 3. Find the number of students who study

- (i) only French, (ii) only English.
- (iii) only Sanskrit. , (iv) English and Sanskrit but not French.
- (v) French and Sanskrit but not English.
- (vi) French and English but not Sanskrit.
- (vii) atleast one of the three languages.
- (viii) none of the three languages.

Watch Video Solution

15. Solve the following inequlaities ,Write the solution in the from of intervals (i) $3x \ge 18$ (ii) 2x + 17 < 3

$$(iii)7-4x>~-17$$

$$\mathsf{(iv)}\frac{x}{7}+3 \leq \ -2$$

16. Solve the following inequlaities

 $egin{aligned} &7x+15 \geq 9-4x \ &-5 \leq rac{2-3x}{4} \leq 9 \ &5x-6 \leq 4 \ ext{ and } \ 7-3x \geq 2x \end{aligned}$

Watch Video Solution

17. Abhinav obtained 65 and 80 marks in first two unit test .Find the minimum marks he should get in the third test of have an average of at least 70 marks.

18. Solve
$$x>\sqrt{(z-5)}-\sqrt{9-z}>1, x\in Z$$
 .

19. Solve
$$\sqrt{x-2} \geq -1$$
.

Watch Video Solution

20. Solve
$$\sqrt{x-1} > \sqrt{3-x}$$
 .



21. Solve
$$x+\sqrt{x} \geq \sqrt{x}-3$$

22. Find tha values of x^2 for the given values of x. (i) x < 2

- (ii) x > -1
- (iii) $x \geq 2$
- (iv) x < -1

Watch Video Solution

23. Find all the possible values of the expression $\sqrt{x^2-4}$.

Watch Video Solution

24. Solve
$$\left(x^2-4
ight)\sqrt{x^2-1}<0.$$

Watch Video Solution

25. Find all the possible values of $\frac{1}{x}$ for x > 3.



29. Solve
$$(x-1)(x-2)(1-2x) > 0$$

30. Solve
$$\displaystyle rac{2}{x} > 3$$



31. Solve
$$\displaystyle rac{x-2}{x+2} > \displaystyle rac{2x-3}{4x-1}$$

32. Solve
$$x > \sqrt{1-x}$$

Watch Video Solution

33. Solve
$$rac{2}{x^2-x+1} - rac{1}{x+1} - rac{2x-1}{x^3+1} \geq 0.$$

34. Sovle
$$x(x+2)^2(x-1)^5(2x-3)(x-3)^4 \geq 0$$

35. Solve
$$x(2^x-1)(3^x-9)^5(x-3) < 0$$
.



36. Solve
$$\left(x^2 - x - 1
ight)\left(x^2 - x - 7
ight) < -5$$

37. Solve the following :

- (i) |x| = 5
- (ii) $x^2 |x| 2 = 0$

38. Find the value of x for which following expressions are defined:



42. Solve the following: (i)|x-2|=1 (ii) $2|x+1|^2-|x+1|=3$



45. Solve
$$1 - x = \sqrt{x^2 - 2x + 1}$$
.

46. Solve |3x - 2| = x.



49. For $x \in R, ext{ find all possible values of (i)} |x-3|-2$ (ii) 4-|2x+3|

50. Find all possible values of

$$\sqrt{|x|-2}$$

.

Watch Video Solution

51. Solve
$$|x - 3| + |x - 2| = 1$$
.

Watch Video Solution

52. Solve
$$x^2 - 4|x| + 3 < 0$$
.

Watch Video Solution

53. Solve 0 < |x| < 2

54. Solve |3x-2| < 4



55. Solve
$$1 \leq |x-2| \leq 3$$

Watch Video Solution

56. Solve
$$0 < |x-3| \le 5$$

Watch Video Solution

57. Solve
$$||x-1|-2| < 5$$

Watch Video Solution

58. Solve $|x-3| \geq 2$

59. Solve :
$$||x| - 3| > 1$$
.

60. Solve
$$\left|rac{x-3}{x+1}
ight|\leq 1$$

Watch Video Solution

61. Solve
$$\left|1+rac{3}{x}
ight|>2$$



62. Solve
$$|x| + |x - 2| = 2$$
.

63. Solve |2x-3|+|x-1|=|x-2|.



64. Solve
$$|x^2 + x - 4| = |x^2 - 4| + |x|$$
 .

Watch Video Solution

65. If $|\sin x + \cos x| = |\sin x| + |\cos x|(\sin x, \cos x \neq 0)$, then in which

quadrant does x lie?

Watch Video Solution

66. Is $|tanx + \cot x| < |tanx| + |\cot x|$ true for any x? If it is true, then

find the values of x.



67. Solve
$$\left|rac{x+1}{x}
ight|+|x+1|=rac{{(x+1)}^2}{|x|}$$

68. Solve
$$\left|x^2-2x
ight|+\left|x-4
ight|>\left|x^2-3x+4
ight|.$$

Watch Video Solution

69. Solve
$$|2^x - 1| + |4 - 2^x| < 3$$

Watch Video Solution

Solved Exp

1. Find the total number of integer n such that $2 \le n \le 2000$ and H.C.F.

of n and 36 is 1.





4. Suppose A_1, A_2, \ldots, A_{30} are thirty sets each having 5 elements and B_1B_2, \ldots, B_n are n sets each having 3 elements ,Let $\bigcup_{i=1}^{30} A_1 = \bigcup_{j=1}^n B_j = s$

and each element of S belongs to exactly 10 of the A_1 and exactly 9 of the

value of n.

5. Let a>2 be a constant. If there are just 18 positive integers satisfying the inequality $(x-a)(x-2a)(x-a^2)<0$, then the value of a is

Watch Video Solution

6. Find the set of all possible real value of a such that the inequality

$$(x-(a-1))ig(x-ig(a^2+2ig)ig) < 0$$
 holds for all $x\in(-1,3)$.

Watch Video Solution

7. Find all possible values of
$$rac{x^2+1}{x^2-2}$$
 .

8.
$$Solve{\left(rac{1}{3}
ight)^{rac{|x+2|}{2-|x|}}}>9$$





10. Solve
$$x + 1 | + |2x - 3| = 4$$
.

11. Solve
$$\displaystyle rac{x}{x+2} \leq \displaystyle rac{1}{|x|}$$

Watch Video Solution

Concept Application Exercise 11

1. Let $A = \{1, 2, \{3, 4\}, 5\}$. Which of the following statements are incorrect and why?



2. If x = {1,2,3, 10 } and a represents any elements of X then write the follwing sets containing all the elements satisfing the given conditions $a \in Xbuta^2 \in X$

 $a \in Xbuta \, / \, 2 \in X$

a is factor of 24





10. If A= [-4 , 1) and B =[0,3), then find $A\cap B$.

11. In a survey conduced on 800 students of a school , 250 students were found to like tea and 300 like coffee , 150 like both tea and coffee .Find how many students like neither tea nor coffee ?

Watch Video Solution

12. Out of 100 students, 15 passed in English, 12 passed in Mathmatics, 8

in Science, 6 in English and Mathematics, 7 in Mathematics and Science , 4

in English and Science, 4 in all the three. Find how many passed

(i) in English and Mathematics but not in Science.

(ii) in Mathematics and Science but not in English.

(iii) in Mathematics only.

(iv) in more than one subject only.

1. Find the values of x which satisfy the inequality -3 < 2x - 1 < 19 .

Watch	Video	Sol	ution
, materi	1400		acioni

2. The longest side of a triangle is three times the shortest side and the third side is 2 cm shorter than the longest side. If the perimeter of the triangle is at least 61 cm, find the minimum length of the shortest side.

Watch Video Solution

3. Find the values of 1/x for $2 \le x \le 5$.

4. Find the values of
$$\frac{1}{x}$$
 for $-5 \le x \le -1$.



5. Find all possible values (range) of the quadratic expression:

$$1+6x-x^2$$
, when $x\in [\,-3,2].$

Watch Video Solution

6. Find all possible values of expressions $\displaystyle rac{2+x^2}{4-x^2}$

Watch Video Solution

7. Solve
$$rac{\sqrt{x-1}}{x-2} < 0$$

Watch Video Solution

8. Solve
$$\sqrt{x-2} \leq 3$$

1. Solve
$$\displaystyle rac{x(3-4x)(x+1)}{2x-5} < 0$$

2. Solve
$$rac{(2x+3)(4-3x)^3(x-4)}{{(x-2)}^2x^5} \leq 0$$

3. Solve
$$rac{(x-3)(x+5)(x-7)}{|x-4|(x+6)} \leq 0$$

Watch Video Solution

4. Solve
$$rac{5x+1}{\left(x+1
ight)^2} < 1$$

5. Solve
$$\displaystyle rac{x+2}{x^2+1} > \displaystyle rac{1}{2}$$

6. The solution of the inequation $4^{-x+0.5}-7.2^{-x} < 4, x \in R$ is



Watch Video Solution

7. Solve
$$rac{x^4}{\left(x-2
ight)^4}>0$$

Watch Video Solution

8. Solve
$$\displaystyle rac{6x^2-5x-3}{x^2-2x+6} \leq 4$$

9. Solve
$$rac{(x+2)ig(x^2-2x+1ig)}{-4+3x-x^2}\geq 0$$

10. solve
$$\sqrt{x+2} \geq x$$

11. Solve:
$$\sqrt{2+x-x^2} > x-4$$

12. The number of integral values of
$$x$$
 satisfying $\sqrt{(-x^2+10x-16)} < x-2$ is Watch Video Solution

13. Find all the possible values of $f(x) = rac{1-x^2}{x^2+3}$



Concept Application Exercise 14

1. Which is the following is always true ? (a) If a < b, then $a^2 < b^2$

(b) If $a < b ext{then} \ \ rac{1}{a} > rac{1}{b}$

(c) If $a < b, ext{ then } |a| < |b|$

Watch Video Solution

2. Which of the following equations has maximum number of real roots ?

A.
$$x^2-|x|-2=0$$

B.
$$x^2 - 2|x| + 3 = 0$$

C.
$$x^2-3|x|+2=0$$

D.
$$x^2+3|x|+2=0$$

Answer: c



3. Find the number of solution of the system of equation x+2y=6 and |x-

3|=y

Watch Video Solution

4. Find the values of x for which the following function is defined:

$$f(x) = \sqrt{rac{1}{|x-2| - (x-2)}}$$

5. Find all values of f(x) for which f(x) $= x + \sqrt{x^2}$



6. Solve
$$\left|rac{x+2}{x-1}
ight|=2$$

Watch Video Solution

Watch Video Solution

8. Solve
$$|2^x - 1| + |2^x + 1| = 2$$

9. Solve
$$\left|x^2+4x+3
ight|=x+1$$



2. Solve
$$||x-2|-3|<5$$

3. Which of the following is / are true ?

If |x+y|=|x|+|y| then points (x,y) lie in 1st or 3rd quadrant or any of the x-

axis of y axis

If |x + y| < |x| + |y| then points (x,y) lie in 2nd or 4th quadrant.

(c) If |x-y|=|x|+|y| then points (x,y) lie in 2nd or 4th quadrant.



6. Solve
$$\left|x^2+x-6
ight|<6$$



7. Solve
$$|x|+|rac{4-x^2}{x}|=|rac{4}{x}|$$

8. Solve
$$\displaystyle rac{1}{|x|-3} < \displaystyle rac{1}{2}$$

9. Solve
$$|x-\pi|+\left|x^2-\pi^2
ight|\leq 0$$

10. Solve the inequation:
$$\left|1-\left(rac{|x|}{1+|x|}
ight)
ight|\geq rac{1}{2}$$

11. Solve
$$\displaystyle rac{|x-1|}{x+2} > 1$$



12. Solve
$$\left|x^2-1
ight|+\left|x^2-4
ight|>3$$

Exercises

1. Let F_1 be the set of parallelograms, F_2 the set of rectangle , F_3 the set of rhombuses, F_4 the set of squares and F_5 the set of trapeziums in a plane. Then, F_1 may be equal to

A. $F_2\cap F_3$

B. $F_3 \cap F_4$

 $\mathsf{C}.\,F_2\cup F_5$

D. $F_2 \cup F_3 \cup F_4 \cup F_1$

Answer: D

Watch Video Solution

2. If n (A) =3 ,n(B)=6 and $A\subseteq B$.Then the number of elements in $A\cup B$ is

equal to

A. 3

B. 9

C. 6

D. Non of these

Answer: C

3. If set A and B are defined as

$$A = igg\{(x,y) \mid y = rac{1}{x}, 0
eq x \in Rigg\}, B = \{(x,y) \mid y = -x, x \in R, \}.$$

Then

A. $A \cap B = A$ B. $A \cup B = B$ C. $A \cup B = \phi$ D. $A \cup B - A$

Answer: C

Watch Video Solution

4. Two finite sets have m and n(m>n) elements .The number of subes of the first set is 112 more than that of the second set. The value of mn is B. 28

C. 32

D. 36

Answer: B

Watch Video Solution

5. Let A and B be two non empty subsets of set X such that A is not a subset of B, then:

A. A is a subset of complement of B

B. B is a subset of A

C. A and B are disjoint sets

D. A and complement of B are non-disjoint sets

Answer: D

6. If a N = $\{ax : x \in N\}$ then the set $4N \cap 6N$ is

A. 8N

B. 10 N

C. 12 N

D. Non of these

Answer: C

Watch Video Solution

7. The set $(A \cap B')' \cup (B \cap C)$ equals a). $A' \cup B \cup C$ b). $A' \cup B$ c). $A' \cup C$ d). $A' \cap B$ A. $A' \cup B \cup C$

 $\mathsf{B.}\,A\,'\cup B$

 $\mathsf{C}.\,A\,'\,\cup\,C$

 $\mathsf{D}.\,A\,'\,\cap\,B$

Answer: B



8. For sets $(A \cup B) \cup (A \cap B)$ equals

A. A'

B. B'

C. A

D. None of these

Answer: D



9. Let U be the universal set and $A \cup B \cup C = U$ Then $[(A - B) \cup (B - C) \cup (C - A)]$ equals A. $A \cup B \cup C$ B. $A \cap B \cap C$

 $\mathsf{C}.\, A \cup (B \cap C)$

 $\mathsf{D}.\,A\cap (B\cup C)$

Answer: B

10. The shaded region in the given figure is



A. $A \cap (B \cup C)$

B. $A \cup (B \cap C)$

 $\mathsf{C}.\,A\cap (B-C)$

D. $A - (B \cup C)$

Answer: D

11. Which is the simplified representation of $(A' \cap B' \cap C) \cup (B \cap C) \cup (A \cap C)$ where A,B and C are subsets of set X (a).A (b). B (c).C (d). $X \cap (A \cup B \cup C)$

A. A

B. B

C. C

 $\mathsf{D}.\,X\cap (A\cup B\cup C)$

Answer: C



12. In a statistical investigation of 1,003 families of Calcutta, it was found that 63 families had neither a radio nor a T.V, 794 families had a radio and 187 had a T.V. The number of families in that group having both a radio and a T.V is (a) 36 (b)41 (c) 32 (d) None of these

B. 41

C. 32

D. None of these

Answer: B

Watch Video Solution

13. A survey shows that 63 % of the pepole watch a news channel whereas

, 76 % watch an entertiament channel at a particular time If X% of the pepole watch both types of channels , then

A. x= 35

 $\mathrm{B.}\,x\geq 63$

C. $39 \leq x \leq 63$

D. x=39

Answer: C



14. In a town of 10000 families, it was found that 40% families buy newspaper A, 20% families buy newspaper B, 10 % families buy newspaper C, 5% families buy A and B, 3% buy B and C and 4 % buy a and C. If 2% families buy all the three newspaper. Find

(i) the number of families which buy newspaper A only.

(ii) the number of families which buy none of A , B and C.

A. 3100

B. 3300

C. 2900

D. 1400

Answer: B

15. Complete solution set of inequlaity $rac{(x+2)(x+3)}{(x-2)(x-3)} \leq 1$

A.
$$(-\infty,0)$$

B.
$$(-\infty,0]\cup(2,3)$$

C. [2,3]

D.
$$(-\infty,2)\cup(3,\infty)$$

Answer: B

Watch Video Solution

16. The number of intergal values of x if $5x-1 < \left(x+1
ight)^2 < 7x-3$ is

A. 0

B. 1

C. 2

D. 3

Answer: B



17. If set A
$$= \left\{ x \mid \frac{x^2(x-5)(2x-1)}{(5x+1)(x+2)} < 0 \right\}$$
 and
Set $B = \left\{ x \mid \frac{3x+1}{6x^3+x^2-x} > 0 \right\}$ then $A \cap B$ does not contain

B. (5,11)

$$\mathsf{C}.\left(\,-\frac{3}{2},\frac{-1}{2}\right)$$

D. None of these

Answer: B



18. Number of intergers satisfying the inequality

 $x^4-29x^2+100\leq 0$ is

A. 2		
B. 4		
C. 6		
D. 8		

Answer: D

Watch Video Solution

19. If n > 0 and exactly 15 integers satisfying (x+6)(x-4) (x-5) $(2x - n) \le 0$ then least possible value of n is

A. a. 10

B. b. 12

C. c. 14

D. d. 16

Answer: D

20. The solution of the inequality $\left(rac{x+7}{x-5}+rac{3x+1}{2}\ge 0 ext{ is }
ight.$

A.
$$[1,3] \cup (5,\infty)$$

- $\mathsf{B}.\,(1,3)\cup(5,\infty)$
- $\mathsf{C}.\,(\,-\infty,1)\cup(5,\infty)$

D. None of these

Answer: A



21. The complete solution set of inequality

$$\frac{(x-5)^{1005}(x+8)^{1008}(x-1)}{x^{1006}(x-2)^3(x-3)^5(x-6)(x+9)^{1010}} \le 0$$
A. $(-\infty, -9) \cup (-8, 0) \cup (0, 1) \cup (2, 3) \cup [5, 6]$
B. $(-\infty, -9) \cup (-9, 0) \cup (0, 1) \cup (2, 3) \cup [5, 6]$

C.
$$(\,-\infty,\,-9)\cup(\,-9,0)\cup(0,1]\cup(2,3)\cup[5,6)$$

D.
$$(\,-\infty,0)\cup(0,1]\cup(2,3)\cup[5,6)]$$

Answer: C

Watch Video Solution



Answer: C

23. Number of intergral roots of $|x-1| \left| x^2 - 2
ight| = 2$ is

A. 0 B. 1 C. 2

D. 3

Answer: D

Watch Video Solution

24. The solution set of the inequlity
$$rac{|x-2|-x|}{x} < 2$$
 is

- A. (0,1)
- B. [0,2]

$$\mathsf{C}.\,(\,-\infty,0)\cup(1,\infty)$$

D. None of these

Answer: C



$$\overline{|x+2||x+3|} <$$

A. 5	
B. 6	
C. 7	
D. 8	

Answer: A

Watch Video Solution

27. If $-4 \leq x < 2$ then ||x+2|-3 lies in the inerval

A. (1,3]

B. [1,3]

C. [0,3]

D. $[0,\infty)$

Answer: C

28. Complete set of values of x satisfying inequality ||x - 1| - 5| < 2x - 5 is A. $(5/2, \infty)$ B. $(11/3, \infty)$ C. $(-1, \infty)$ D. $(-\infty, 1/3)$

Answer: B

29. If
$$\left|x^2-2x+2
ight|-\left|2x^2-5x+2
ight|=\left|x^2-3x
ight|$$
 then the set of values of x is

A.
$$(-\infty,0]\cup[3,00)$$

B. $\left[0,rac{1}{2}
ight]\cup[2,3]$

C.
$$(-\infty,0]\cup\left[rac{1}{2},2
ight]\cup[3,\infty)$$

D. $[0,2]\cup[3,\infty)$

Answer: B

Watch Video Solution

30. The complete solution set of the equation

$$|x^2 - 5x + 6| + |x^2 + 12x + 27| = |17x + 21|$$
 is $a.x \in [-9, 3]$ b.
 $x \in [-3, 2) \cup (2, 3] c.x \in [-9, -3] \cup [2, 3] d.x \in (-2, 3)$
A. $x \in [-9, 3]$
B. $x \in [-3, 2) \cup (2, 3]$
C. $x \in [-9, -3] \cup [2, 3]$
D. $x \in (-2, 3)$

Answer: C

Archives

1.	If	A,B	and	С	are	three	sets	such	that
A ($A\cap B=A\cap C { m and} A\cup B=A\cup C$ then								
	A. A=B								
	B. A=C								
	C. B=C								
	D. $A\cap$	$B=\phi$							
Ans	Answer: C								