MATHS

BOOKS - DEEPTI MATHS (TELUGU ENGLISH)

DE MOIVRE'S THEOREM

SOLVED EXAMPLES

1. If A, B, C are angles of a triangle such that $x = cisA, y = cisB, z = cisC$, then find the value of xyz.

A. -1

B. 0

C. 1

D. 2

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2. The equation whose roots are the n th powers of the roots of the equation $x^2 - 2x\cos\theta + 1 = 0$ is

A. $x^2 - 2x\cos n\theta + 1 = 0$

B. $x^2 + 2x\cos n\theta + 1 = 0$

C. $x^2 - 2x\cos n\theta - 1 = 0$

D. $x^2 + 2x\cos n\theta - 1 = 0$

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3. $\left[\sqrt{2}(\cos 56^\circ 15 + i\sin 56^\circ 15')\right]^8$

A. 1

B. i

C. 16

D. 16i

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4. The value of

$$1+\sum_{k=0}^{14}\left\{(\cos)\frac{(2k+1)\pi}{15}+(i\sin)\frac{(2k+1)\pi}{15}\right\}$$ is

A. 0

B. -1

C. 1

D. i

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5. If ω is a complex cube root of unity, then $\sin\left[\left(\omega^{10}+\omega^{23}\right)\pi-\frac{\pi}{4}\right]$

A. $1/\sqrt{2}$

B. $\frac{1}{2}$

C. 1

D. $\sqrt{3}/2$

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6. If $(\omega \neq 1)$ is a cube root of unity , then

`{:(1 , 1 + i+ omega^(2) , omega^(2)) , (1-i, -1 , omega^(2) -1) , (-i , -1 + omega - i, -1):}|

A. -1

B. -2

C. 0

D. 2

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7. The number of real roots of the equation $z^3 + iz - 1 = 0$ is

A. 0

B. 1

C. 2

D. 3

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8. If $x^2 - 2x\cos\theta + 1 = 0$, then $x^{2n} - 2x^n \cos n\theta + 1$

A. -1

B. 0

C. 1

D. 2

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9. If $z=\sqrt{\frac{1-i}{1+i}}$ then Arg z =

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. $\frac{3\pi}{2}$

D. $\frac{3\pi}{4}$

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10. If the roots of $(z-1)^n = i(z+1)^n$ are plotted on the argand plane, they are

A. onx=0

B. Concyclic

C. on a parabola

D. on y=0

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EXERCISE 1

1. $(\cos 3\alpha + i \sin 3\alpha)^5 (\cos 4\alpha - i \sin 4\alpha)^6$

A. $\cos 9\alpha - i \sin 9\alpha$

B. $\cos 9\alpha + i \sin 9\alpha$

C. $\cos 8\alpha - i \sin 8\alpha$

D. $\cos 8\alpha + i \sin 8\alpha$

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2. $$\frac{(\cos\theta + i\sin\theta)^5(\cos 3\theta - i\sin 3\theta)^6}{(\cos 2\theta + i\sin 2\theta)^3(\cos 4\theta - i\sin 4\theta)^5}$$

A. 1

B. $cis\theta$

C. i

D. -1

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3. $$\frac{(\cos 2\theta + i\sin 2\theta)^3(\cos 3\theta - i\sin 3\theta)^4}{(\cos 3\theta + i\sin 3\theta)^2(\cos 4\theta + i\sin 4\theta)^{-3}}$$

A. 1

B. $cis\theta$

C. i

D. -1

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4. $\dfrac{(\cos\alpha + i\sin\alpha)^5}{(\sin\beta + i\cos\beta)^3} =$

A. $\sin(5\alpha + 3\beta) + i\cos(5\alpha + 3\beta)$

B. $-\sin(5\alpha + 3\beta) + i\cos(5\alpha + 3\beta)$

C. $\sin(3\alpha + 5\beta) + i\cos(3\alpha + 5\beta)$

D. $-\sin(3\alpha + 5\beta) + i\cos(3\alpha + 5\beta)$

Answer: B

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5. $\dfrac{1 - \cos\theta + i\sin\theta}{1 + \cos\theta - i\sin\theta}$

A. $i\tan\frac{\theta}{2}$

B. $-i\tan\frac{\theta}{2}$

C. $\tan\frac{\theta}{2}$

D. $-\tan\frac{\theta}{2}$

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6. $\left(\frac{1+\cos\theta+i\sin\theta}{1+\cos\theta-i\sin\theta}\right)^n$

A. $cisn\theta$

B. $-cisn\theta$

C. $cis\theta$

D. $-cis\theta$

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7. $(\sin x + i \cos x)^n$

A. $cisn\left(\frac{\pi}{2} + x\right)$

B. $cisn\left(\frac{\pi}{2} - x\right)$

C. $cisn\left(\frac{\pi}{3} + x\right)$

D. $cisn\left(\frac{\pi}{3} - x\right)$

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8. $(1 + \sin\theta + i \cos\theta)^n$

A. $2^n . \cos^n\left[\frac{\pi}{3} - \frac{\theta}{2}\right] . cis\left[n\frac{\pi}{3} - \frac{\theta}{2}\right]$

B. $2^n . \cos^n\left[\frac{\pi}{4} - \frac{\theta}{2}\right] . cis\left[n\frac{\pi}{4} - \frac{\theta}{2}\right]$

C. $2^n . \cos^n\left[\frac{\pi}{5} - \frac{\theta}{2}\right] . cis\left[n\left(\frac{\pi}{5} - \frac{\theta}{2}\right)\right]$

D. none

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9. $(1+\cos\theta - i\sin\theta)^n$

A. $2^n\cos^n\left(\frac{\theta}{2}\right)cis\frac{n\theta}{2}$

B. $2^n\cos^n\left(\frac{\theta}{2}\right)cis\left(-\frac{n\theta}{2}\right)$

C. $2^n\sin^n\left(\frac{\theta}{2}\right)cis\frac{n\theta}{2}$

D. $2^n\sin^n\left(\frac{\theta}{2}\right)-cis\frac{n\theta}{9}$

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10. $(1-\cos\theta + i\sin\theta)^6$

A. $2^6\sin^6\frac{\theta}{2}(\cos 3\theta + i\sin 3\theta)$

B. $2^6\sin^6\frac{\theta}{2}(-\cos 3\theta + i\sin 3\theta)$

C. $2^6\sin^6\frac{\theta}{2}(\cot 3\theta + i\sin 3\theta)$

D. $2^6 \sin^6 \frac{\theta}{2}(-\cot 3\theta + i \sin 3\theta)$

Answer: B

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11. $(1+i)^4$

A. 4

B. -4

C. 0

D. i

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12. $(1+i\sqrt{3})^9 =$

A. -2^{6}

B. -2^{8}

C. -2^{9}

D. -2^{12}

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13. $(1-i)^{5}$

A. $2^{5/2}\sin\left(\frac{-5\pi}{4}\right)$

B. $2^{5/2}cis\left(\frac{-5\pi}{4}\right)$

C. $2^{5/2}\cot\left(\frac{-5\pi}{4}\right)$

D. $2^{5/2}\tan\left(\frac{-5\pi}{4}\right)$

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14. $\frac{\cos 75^{\circ} + i\sin 75^{\circ}}{\cos 30^{\circ} + i\sin 30^{\circ}}$

A. $\frac{1}{\sqrt{2}}(1+i)$

B. $\frac{1}{\sqrt{2}}(1-i)$

C. $\frac{1}{\sqrt{3}}(1+i)$

D. $\frac{1}{\sqrt{3}}(1-i)$

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15. $\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^{6}$

A. 0

B. 1

C. -1

D. 2

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16. If $\left(\frac{1+\cos\theta+i\sin\theta}{\sin\theta+i+i\cos\theta}\right)^4=\cos n\theta+i\sin n\theta$ then n=

A. 4

B. 8

C. 6

D. 2

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17. $\left(\frac{1+\cos\frac{\pi}{8}-i\sin\frac{\pi}{8}}{1+\cos\frac{\pi}{8}+i\sin\frac{\pi}{8}}\right)^8$

A. 1

B. -1

C. 2

D. $1/2$

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18. $\left[\dfrac{1+\sin(\pi/8)+i\cos(\pi/8)}{1+\sin(\pi/8)-i\cos\pi/8}\right]^{-8/3}$

A. $1+i$

B. 1-i

C. 1

D. -1

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19. $\left(\dfrac{\sin\pi/8+i\cos\pi/8}{\sin\pi/8-i\cos\pi/8}\right)^{8}$

A. -1

B. 0

C. 1

D. 2i

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20. $\left(\frac{1+\sin\theta+i\cos\theta}{1+\sin\theta-i\cos\theta}\right)^n$

A. $\cos\left(\frac{n\pi}{2}-n\theta\right)+i\sin\left(\frac{n\pi}{2}-n\theta\right)$

B. $\cos\left(\frac{n\pi}{2}-n\theta\right)+i\sin\left(\frac{n\pi}{2}+n\theta\right)$

C. $\cos\left(\frac{n\pi}{2}+n\theta\right)+i\sin\left(\frac{n\pi}{2}-n\theta\right)$

D. none

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21. If n is a positive integer, show that

$$(1+i)^n + (1-i)^n = 2^{\frac{n+2}{2}} \cos\left(\frac{n\pi}{4}\right).$$

A. $2^{(n/2)+1} . \cos \frac{n\pi}{3}$

B. $2^{(n/2)+1} . \cos \frac{n\pi}{4}$

C. $2^{(n/2)+1} . \cos \frac{n\pi}{5}$

D. $2^{(n/2)+1} . \cos \frac{n\pi}{5}$

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22. $(1+i)^5 + (1-i)^5$

A. -8

B. 81

C. 8

D. 32

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23. $\left(\sqrt{3}+i\right)^7+\left(\sqrt{3}-i\right)^7$

A. $128\sqrt{3}$

B. $256\sqrt{3}$

C. $-128\sqrt{3}$

D. $-256\sqrt{3}$

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24. If n is a positive integer and $(1+i)^{2n}+(1-i)^{2n}=k\cos(n\pi/2)$ then the value of k is

A. 2^n

B. 2^{n-1}

C. 2^{n+1}

D. none

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25. If n is a positive niteger, then $(\sqrt{3}+i)^n+(\sqrt{3}-i)^n$

A. $2^{(n+1)}\cos\left(\frac{n\pi}{2}\right)$

B. $2^{(n+1)}\cos\left(\frac{n\pi}{3}\right)$

C. $2^{(n+1)}\cos\left(\frac{n\pi}{5}\right)$

D. $2^{(n+1)}\cos\left(\frac{n\pi}{6}\right)$

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26. If n is a positive integer and $(1+i\sqrt{3})^n+(1-i\sqrt{3})^n=2^{n+1}\cos\theta$, then the value of θ is

A. $n\pi/3$

B. $n\pi/2$

C. $n\pi/4$

D. none

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27. If n is an integer which leaves remainder one when divided by three, then $(1+\sqrt{3}i)^n+(1-\sqrt{3}i)^n$

A. -2^{n+1}

B. 2^{n+1}

C. $-(-2)^n$

D. -2^n

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28. $\left(\frac{-1+i\sqrt{3}}{2}\right)^{3n}+\left(\frac{-1-i\sqrt{3}}{2}\right)^{3n}$

A. 0

B. 1

C. 2

D. 3

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29. $(1+\cos\theta+i\sin\theta)^n+(1+\cos\theta-i\sin\theta)^n$

A. $2^{n+1}\cos^n(\theta/2).\cos(n\theta/2)$

B. $2^{n}.\cos^{n}(\theta/2).\sin(n\theta/2)$

C. $2^{n+1}\sin^{n}(\theta/2).\cos(n\theta/2)$

D. none

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30. $(\cos\theta + i\sin\theta)^{10} + (\cos\theta - i\sin\theta)^{10}$

A. $\cos 10\theta$

B. $i\sin 10\theta$

C. $2\cos 10\theta$

D. $2i\sin 10\theta$

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31. If n is a positive integer, then $(a+ib)^{m/n}+(a-ib)^{m/n}$

A. $2\left(a^2+b^2\right)^{m/2n}\cos\left[\frac{m}{n}\tan^{-1}\frac{b}{a}\right]$

B. $2\left(a^2+b^2\right)^{m/2n}\sin\left[\frac{m}{n}\tan^{-1}\frac{b}{a}\right]$

C. $2\left(a^2+b^2\right)^{m/2n}\cot\left[\frac{m}{n}\tan^{-1}\frac{b}{a}\right]$

D. none

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32.

$[(\cos\alpha-\cos\beta)+i(\sin\alpha-\sin\beta)]^n+[(\cos\alpha-\cos\beta)-i(\sin\alpha-\sin\beta)]^n$

A. $2^{n+1}.\sin^n\left(\frac{\alpha-\beta}{2}\right)\cos\left[n\left(\frac{\pi}{2}+\frac{\alpha+\beta}{2}\right)\right]$

B. $2^{n-1}.\sin^n\left(\frac{\alpha-\beta}{2}\right)\cos\left[n\left(\frac{\pi}{2}+\frac{\alpha+\beta}{2}\right)\right]$

C. $2^{n+1}.\sin^n\left(\frac{\alpha+\beta}{2}\right)\cos\left[n\left(\frac{\pi}{2}+\frac{\alpha-\beta}{2}\right)\right]$

D. $2^{n-1}.\sin^n\left(\frac{\alpha+\beta}{2}\right)\cos\left[n\left(\frac{\pi}{2}+\frac{\alpha-\beta}{2}\right)\right]$

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33. If $\left(\sqrt{3}+i\right)^{100}=2^{99}(a+ib)$ then b=

A. 1

B. $\sqrt{3}$

C. $\sqrt{2}$

D. 2

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34. A value of n such that $\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^{n}=1$ is

A. 12

B. 3

C. 2

D. 1

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35. If n is a multiple of 3, then $\left(\frac{-1+i\sqrt{3}}{2}\right)^{n}+\left(\frac{-1-i\sqrt{3}}{2}\right)^{n}$

A. 1

B. 2

C. 3

D. 4

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36. $4+5\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^{334}+3\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^{365}$

A. $1-i\sqrt{3}$

B. $-1+i\sqrt{3}$

C. $i\sqrt{3}$

D. $-i\sqrt{3}$

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37. $\left(\frac{-1+i\sqrt{3}}{2}\right)^{6}+\left(\frac{-1-i\sqrt{3}}{2}\right)^{6}$

A. 2

B. -2

C. 1

D. 0

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38. If n is an integer and $z = -1 + i\sqrt{3}, \quad \text{then} \quad z^{2n} + 2^n . z^n + 2^{2n}$

A. 0

B. 2

C. 3

D. 4

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39. If $\theta = \pi / 6$ then the 10th term of the series $1 + (\cos\theta + i\sin\theta) + (\cos\theta + i\sin\theta)^2 + \ldots\ldots$ is

A. i

B. -1

C. 1

D. $-i$

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40. If $x = cis\theta$, then find the value of $\left[x^6 + \frac{1}{x^6}\right]$.

A. $2\cos 6\theta$

B. $2\sin 6\theta$

C. $-2\cos 6\theta$

D. $-2\sin 6\theta$

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41. If $x + \frac{1}{x} = 2\cos\theta$ then $x^{10} + \frac{1}{x^{10}}$

A. $2^{10}\cos 10\theta$

B. $2\cos 10\theta$

C. $2^{10}\cos^{10}\theta$

D. $2\cos^{10}\theta$

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42. If $z^2+z+1=0$ where z is a complex number, then the value of

$$\left(z+\frac{1}{z}\right)^2+\left(z^2+\frac{1}{z^2}\right)^2+\left(z^3+\frac{1}{z^2}\right)^2+\ldots\ldots\ldots+\left(z^6+\frac{1}{z^6}\right)^2$$

A. 6

B. 12

C. 18

D. 54

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43. If $z = \cos\theta + i\sin\theta$, then $\dfrac{z^{2n}-1}{z^{2n}+1}$

A. $\cos n\theta$

B. $\sin n\theta$

C. $-i\sin\theta$

D. $i\tan n\theta$

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44. If $x = \cos\alpha + i\sin\alpha, y = \cos\beta + i\sin\beta, \text{ then } \dfrac{x}{y} + \dfrac{y}{x}$

A. $2\sin(\alpha+\beta)$

B. $2\sin(\alpha-\beta)$

C. $2\cos(\alpha+\beta)$

D. $2\cos(\alpha-\beta)$

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45. If $x = \cos\alpha + i\sin\alpha, y = \cos\beta + i\sin\beta, \quad \text{then } \dfrac{x}{y} - \dfrac{y}{x}$

A. $2i\sin(\alpha+\beta)$

B. $2i\sin(\alpha-\beta)$

C. $2i\cos(\alpha+\beta)$

D. $2i\cos(\alpha-\beta)$

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46. If $x = \cos\alpha + i\sin\alpha, y = \cos\beta + i\sin\beta, \quad \text{then } \dfrac{x-y}{x+y}$

A. $i.\cos\left(\dfrac{\alpha-\beta}{2}\right)$

B. $i.\sin\left(\dfrac{\alpha-\beta}{2}\right)$

C. $i.\cot\left(\dfrac{\alpha-\beta}{2}\right)$

D. $i.\tan\left(\frac{\alpha-\beta}{2}\right)$

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47. If $x = \cos\alpha + i\sin\alpha, y = \cos\beta + i\sin\beta, \text{ then } x^m . y^n + \frac{1}{x^m . y^n}$

A. $2\cos(m\alpha + n\beta)$

B. $2\cos(m\alpha - n\beta)$

C. $2\sin(m\alpha + n\beta)$

D. $2\sin(m\alpha - n\beta)$

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48. If $p = \cos 2\alpha + i\sin 2\alpha, q = \cos 2\beta + i\sin 2\beta \text{ then } \sqrt{p/q} - \sqrt{q/p}$

A. $2i\sin(\alpha - \beta)$

B. $2\sin(\alpha-\beta)$

C. $2i\cos(\alpha-\beta)$

D. $2\cos(\alpha-\beta)$

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49. If $x+\frac{1}{x}=2\cos\alpha, y+\frac{1}{y}=2\cos\beta,$ then $xy+\frac{1}{xy}=$

A. $\cos(\alpha\pm\beta)$

B. $2\cos(\alpha\pm\beta)$

C. $2i\sin(\alpha\pm\beta)$

D. $\sin(\alpha\pm\beta)$

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50. If α and β are the roots of the equation $x^2 - x + 1 = 0$ then $\alpha^{2009} + \beta^{2009}$

A. -2

B. -1

C. 1

D. 2

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51. If α and β are the roots of $x^2 - 2x + 4 = 0$ then $\alpha^n + \beta^n$

A. $2^n \cos(n\pi/2)$

B. $2^{n+1} \cos(n\pi/3)$

C. $2^{n-1} \sin(n\pi/3)$

D. none

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52. If α and β are the roots of $x^2 - 2x + 4 = 0$ then $\alpha^n - \beta^n$

A. $2^{n+1}. i \cos(n\pi/3)$

B. $2^{n+1}. i \sin(n\pi/3)$

C. $2^{n-1}. i \cos(n\pi/3)$

D. $2^{n-1}. i \sin(n\pi/3)$

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53. If α, β are roots of the equation $x^2 - 4x + 8 = 0$ then for any $n \in N, \alpha^{2n} + \beta^{2n}$

A. $2^{2n+1} \cos \frac{n\pi}{2}$

B. $2^{3n} \cos \frac{n\pi}{2}$

C. $2^{3n+1} \cos \frac{n\pi}{2}$

D. $2^{3n} \cos \frac{n\pi}{4}$

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54. If α and β are the roots of $x^2 - 2x + 4 = 0$, then the value of $\alpha^6 + \beta^6$ is :

A. 32

B. 64

C. 128

D. 256

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55. If x satisfies the equation $x^2 - 2x\cos\theta + 1 = 0$, then the value of $x^n + 1/x^n$ is

A. $2^n \cos n\theta$

B. $2^n \cos^n \theta$

C. $2\cos n\theta$

D. $2\cos^n \theta$

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56. If $a = \cos 4\pi/3 + i\sin 4\pi/3 \text{ then } (1+a)^{3n}$

A. -1

B. 0

C. 1

D. $(-1)^n$

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57. If $\cos\alpha + \cos\beta = 0 = \sin\alpha + \sin\beta \text{ then } \cos 2\alpha + \cos 2\beta$

A. $-2\sin(\alpha + \beta)$

B. $-2\cos(\alpha + \beta)$

C. $2\sin(\alpha + \beta)$

D. $2\cos(\alpha + \beta)$

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58. If

$\cos\alpha + \cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma \text{ then } \cos 3\alpha + \cos 3\beta +$

A. $\cos(\alpha + \beta + \gamma)$

B. $2\cos(\alpha + \beta + \gamma)$

C. $3\cos(\alpha+\beta+\gamma)$

D. none

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59. If

$\cos\alpha+\cos\beta+\cos\gamma=0=\sin\alpha+\sin\beta+\sin\gamma$ then $\sin 3\alpha+\sin 3\beta+$

A. $3\sin(\alpha+\beta+\gamma)$

B. $2\sin(\alpha+\beta+\gamma)$

C. $\sin(\alpha+\beta+\gamma)$

D. none

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60. If $\cos\alpha+\cos\beta+\cos\gamma=0=\sin\alpha+\sin\beta+\sin\gamma$ then $\sin^2\alpha+\sin^2\beta+\sin^2\gamma=$

A. 2

B. 1

C. $1/2$

D. 0

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61. If $\cos\alpha+\cos\beta+\cos\gamma=0=\sin\alpha+\sin\beta+\sin\gamma$ then $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=$

A. 0

B. $1/2$

C. $3/2$

D. 2

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62. If $\cos\alpha + \cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$ then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma =$

A. 0

B. $1/2$

C. $3/2$

D. 2

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63. If $\cos\alpha + \cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$ then $\cos(2\alpha - \beta - \gamma) + \cos(2\beta - \gamma - \alpha) + \cos(2\gamma - \alpha - \beta) =$

A. 0

B. 1

C. 2

D. 3

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64. If $\cos\alpha + \cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$ then $\sin(2\alpha - \beta - \gamma) + \sin(2\beta - \gamma - \alpha) + \sin(2\gamma - \alpha - \beta) =$

A. 0

B. 1

C. 2

D. 3

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65. If $\cos\alpha + coa\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$ then $\cos(\alpha+\beta) + \cos(\beta+\gamma) + \cos(\gamma+\alpha) =$

A. 0

B. 1

C. $\cos 2(\alpha+\beta+\gamma)$

D. $3\cos(\alpha+\beta+\gamma)$

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66. If $a = cis\alpha, b = cis\beta, c = cis\gamma$, and $\frac{a}{b} + \frac{b}{c} + \frac{c}{a} - 1 = 0$ then $\cos(\alpha-\beta) + \cos(\beta-\gamma)+\cos(\gamma-\alpha)$=

A. 0

B. 1

C. -1

D. 2

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67. Let A and B denote the statements

$A: \cos\alpha + \cos\beta + \cos\gamma = 0$ $B: \sin\alpha + \sin\beta + \sin\gamma = 0$

If $\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha + \beta) = -\frac{3}{2}$, then :

A. A is false and B is true

B. both A and B are true

C. both A and B are false

D. A is true and B is false

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68. The value of $(i)^{i}$ is

A. ω

B. $-\omega^2$

C. $\pi/3$

D. none

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69. $\log(\log i)$

A. $\log \pi/2$

B. $i\pi/2$

C. $i\pi/2 + \log \pi/2$

D. none

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70. $\sqrt{i} + \sqrt{-i} =$

A. $i\sqrt{2}$

B. $1/(i\sqrt{2})$

C. 0

D. $-\sqrt{1}$

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71. One value of $(1+i)^{1/2}$ is $2^{1/4}e^{i\pi/8}$.The other value is

A. $\sqrt{2}e^{-i\pi/8}$

B. $\sqrt{2}e^{i5\pi/8}$

C. $\sqrt{2}e^{i5\pi/8}$

D. $\sqrt{2}e^{i9\pi/8}$

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72. The modulus of $\sqrt{2}i - \sqrt{-2i}$ is

A. 2

B. $\sqrt{2}$

C. 0

D. $2\sqrt{2}$

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73. $\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)^{1/2} + \left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)^{11/2}$

A. -1

B. 1

C. 0

D. 2

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74. If n is a multiple of 3 then $\omega^n + \omega^{2n} =$

A. 0

B. 1

C. -1

D. 2

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75. $(1 + \omega - \omega^2)(1 - \omega + \omega^2)$

A. 0

B. 1

C. 2

D. 4

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76. $\left(2+\omega^2+\omega^4\right)^5$

A. 0

B. 1

C. 2

D. 4

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77. $\left(3+5\omega+3\omega^2\right)^6$

A. 42

B. 48

C. 52

D. 64

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78. $(1-\omega)\left(1-\omega^2\right)\left(1-\omega^4\right)\left(1-\omega^5\right)\left(1-\omega^7\right)\left(1-\omega^8\right)$

A. 25

B. 36

C. 27

D. 30

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79. $(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^5)..........2n$ factors =

A. 0

B. 1

C. -1

D. none

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80. $(1+\omega)(1+\omega^2)(1+\omega^3)(1+\omega^4)(1+\omega^5)......(1+\omega^{3n})$

A. 2^{3n}

B. 2^{2n}

C. 2^n

D. none

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81. $\left(1-\omega+\omega^2\right)^7+\left(1+\omega-\omega^2\right)^7$

A. 2^5

B. 2^7

C. 2^9

D. 2^{10}

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82. $\left(1-\omega+\omega^2\right)^6+\left(1-\omega^2+\omega\right)^6$

A. 0

B. 6

C. 64

D. 128

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83. If ω is a complex cube root of unity, then $225 + (3\omega + 8\omega^2)^2 + (3\omega^2 + 8\omega)^2$

A. 72

B. 192

C. 200

D. 248

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84. If ω is a complex cube root of unity, then $(x+1)(x+\omega)(x-\omega-1)$

A. x^3-1

B. x^3+1

C. x^3+2

D. x^3-2

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85. $(a-b)\left(a\omega-b\omega^2\right)\left(a\omega^2-b\omega\right)$

A. a^2-b^2

B. a^2+b^2

C. a^3-b^3

D. a^3+b^3

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86. $(a+2b)^2 + (a\omega + 2b\omega^2)^2 + (a\omega^2 + 2b\omega)^2$

A. 8ab

B. 9ab

C. 11 ab

D. 12 ab

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87. If $1, \omega, \omega^2$ are the cube roots of unity , then $(a+b)^3 + (a\omega + b\omega^2)^3 + (a\omega^2 + b\omega)^3 =$

A. $a^2 + b^2$

B. $3(a^3 + b^3)$

C. $a^3 - b^3$

D. $a^3 + b^3 + 3ab$

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88. If $x = a + b, y = a\omega + b\omega^2, z = a\omega^2 + b\omega$ then xyz

A. $a^2 + b^2$

B. $a^2 - b^2$

C. $a^3 + b^3$

D. $a^3 - b^3$

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89. If a, b are real and $x = a + b, y = a\omega^2 + b\omega, z = a\omega + b\omega^2$ then $x^2 + y^2 + z^2$

A. 6ab

B. 12ab

C. $8a^2b^2$

D. $4a^2b^2$

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90. $\left(1-\omega+\omega^2\right)\left(1-\omega^2+\omega^4\right)\left(1-\omega^4+\omega^8\right)\left(1-\omega^8+\omega^{16}\right)$

A. 4

B. 8

C. 12

D. 16

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91. If $1, \omega, \omega^2$ are the cube roots of unity, then prove that

$$(x+y+z)\left(x+y\omega+z\omega^2\right)\left(x+y\omega^2+z\omega\right)=x^3+y^3+z^3-3xyz$$

A. $x^3+y^3+z^3+3xyz$

B. $x^3-y^3-z^3-3xyz$

C. $x^2+y^3+z^3$

D. none

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92. $\left(1-\omega+\omega^2\right)\left(1-\omega^2+\omega^4\right)\left(1-\omega^4+\omega^8\right)$ .... to 2n factors

A. 2

B. 2^{2n}

C. 2n

D. none

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93. If ω is a complex cube root of unity then $\omega^{\left(\frac{1}{3}+\frac{2}{9}+\frac{4}{27}+\ldots.\infty\right)}+\omega^{\left(\frac{1}{2}+\frac{3}{8}+\frac{9}{32}+\ldots\infty\right)}=$

A. 1

B. -1

C. ω

D. i

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94. $\dfrac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}$

A. ω

B. ω^2

C. $a^2 + b^2$

D. 0

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95. $\frac{1}{1+2\omega} - \frac{1}{1+\omega} + \frac{1}{2+\omega}$

A. ω

B. ω^2

C. $a^2 + b^2$

D. 0

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96. If ω is a complex cube root of unity, then $\sin\left[\left(\omega^{10}+\omega^{23}\right)\pi-\frac{\pi}{4}\right]$

A. $\frac{1}{\sqrt{2}}$

B. $\frac{1}{2}$

C. 1

D. $\frac{\sqrt{3}}{2}$

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97. If $1, \omega, \omega^2$ are the cube roots of unity and $\alpha=\omega+2\omega^2-3, \text{then} \alpha^3+12\alpha^2+48\alpha+3$ equals

A. -63

B. -62

C. -61

D. -60

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98. If α, β are the roots of 1 + x + x^(2)= 0, then the value of $\alpha^4 + \beta^4 + \alpha^{-4}\beta^{-4}$ is

A. 0

B. 1

C. -1

D. -2

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99. If $x^2 + x + 1 = 0$, then the value of $\left(x + \frac{1}{x}\right)^2 + \left(x^2 + \frac{1}{x^2}\right)^2 + \ldots + \left(x^{27} + \frac{1}{x^{27}}\right)^2$ is

A. 27

B. 72

C. 45

D. 54

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100. If the cube roots of unity are $1, \omega, \omega^2$ then the roots of the equation $(x-1)^3+8=0$ are

A. $-1, \ -1+2\omega, \ -1-2\omega^2$

B. $-1, \ -1, \ -1$

C. $-1, 1-2\omega, 1-2\omega^2$

D. $-1, 1+2\omega, 1+2\omega^2$

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101. If α, β are the non-real cube roots of 2, then $\alpha^6 + \beta^6 =$

A. 8

B. 4

C. 2

D. 1

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102. If α is a non-real root of $x^6 - 1 = 0$ then $\dfrac{\alpha^5 + \alpha^3 + \alpha + 1}{\alpha^2 + 1}$

A. α^2

B. 0

C. $-\alpha^2$

D. α

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103. If α is a non real root of the equation $x^6 - 1 = 0$ then $\frac{\alpha^2 + \alpha^3 + \alpha^4 + \alpha^5}{\alpha + 1}$

A. α

B. 1

C. 0

D. -1

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104. If $1, \alpha_1, \alpha_2, \ldots.. \alpha_{n-1}$ are the n^{th} roots of unity then $(1 - \alpha_1)(1 - \alpha_2)\ldots..(1 - \alpha_{n-1}) =$

A. n-1

B. n

C. -1

D. 1

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105. If $1, \alpha_1, \alpha_2, \ldots\ldots \alpha_{n-1}$ are the n^{th} roots of unity and n is an odd natural number then

$(1+\alpha_1)(1+\alpha_2)\ldots\ldots(1+\alpha_{n-1}) =$

A. 1

B. -1

C. 0

D. none

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106. $1, \alpha_1, \alpha_2, \alpha_3 \ldots . . \alpha_{n-1}$ are the n^{th} roots of unity and n is an even natural number ,then $(1+\alpha_1)(1+\alpha_2)(1+\alpha_3) \ldots . (1+\alpha_{n-1}) =$

A. 1

B. 0

C. -1

D. none

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107. If $1, z_1 z_2, \ldots \ldots z_{n-1}$ are the $n^t h$ roots of unity, then $(1-z_1)(1-z_2) \ldots . . (1-z_{n-1}) = .$

A. 0

B. n-1

C. n

D. 1

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108. If α is an n^{th} root of unity then prove that $1+2\alpha+3\alpha^2+4\alpha^3+\ldots.+n\alpha^{n-1}=-\frac{n}{1-\alpha}$

A. $\frac{n}{1-\alpha}$

B. $\frac{-n}{1-\alpha}$

C. $\frac{-n}{(1-\alpha)^2}$

D. none

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109. If $1, \alpha, \alpha^2, \ldots\ldots, \alpha^{n-1}$ are the n^{th} roots of unity, then the value of $(3-\alpha)(3-\alpha^2)\ldots\ldots(3-\alpha^{n-1})$ is

A. n

B. 0

C. $(3^n - 1)/2$

D. $(3^n + 1)/2$

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110. Statement-I :

If $x_n = \cos(\pi/2^n) + i\sin(\pi/2^n)$ then $x_1 . x_2 . x_3 . \dots . = -1$

Statement-II : $x_n = \cos(\pi/4^n) + i\sin(\pi/4^n)$ then $x_1 . x_2 . x_3 . \dots . \infty = -1$

A. $x_1 + x_2 + m - 1$

B. $x_1 + x_2 + x_3 m = 0$

C. $x_1 x_2 x_3 m - 1$

D. $x_1 - x_2 + x_3 m = -1$

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111. If $X_n = \cos\frac{\pi}{2^n} + \text{isin}\frac{\pi}{2^n}$, then $x_1, x_2, x_3.... \infty$

A. -1

B. 1

C. $1/\sqrt{2}$

D. $i/\sqrt{2}$

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112. If $x_n = \cos\left(\frac{\pi}{4^n}\right) + i\sin\left(\frac{\pi}{4^n}\right)$, then $x_1, x_2, x_3.... \infty$ is equal to

A. $\frac{1+i\sqrt{3}}{2}$

B. $\frac{-1+i\sqrt{3}}{2}$

C. $\frac{1-i\sqrt{3}}{2}$

D. $\frac{-1-i\sqrt{3}}{2}$

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113. $\sum_{k=1}^{6}\left[\sin\left(\frac{2k\pi}{7}\right)-i\cos\left(\frac{2k\pi}{7}\right)\right]=$

A. -1

B. 0

C. $-i$

D. i

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114. If $z_k=\cos\left(\frac{k\pi}{10}\right)+i\sin\left(\frac{k\pi}{10}\right)$, then $z_1z_2z_3z_4$ is equal to

A. -1

B. 1

C. -2

D. 2

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115. The cube roots of 1+I are

A. $(i-1)/\sqrt[3]{2}, \{(i-1)/\sqrt[3]{2}\}\omega, \{(i-1)/\sqrt[3]{2}\}\omega^2$

B. $(i-1)/\sqrt{2}, \{(i-1)/\sqrt{2}\}\omega, \{(i-1)/\sqrt[3]{2}\}\omega^3$

C. $(i-1)/\sqrt{3}, \{(i-1)/\sqrt{3}\}\omega, \{(i-1)/\sqrt{3}\}\omega^2$

D. none

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116. The values of $\left(1-\sqrt{3}i\right)^{1/3}$ are

A. $2^{1/5} cis\left[\frac{2k\pi + \pi/3}{3}\right], k = 0, 1, 2$

B. $2^{1/5} cis\left[\frac{2k\pi - \pi/6}{3}\right], k = 0, 1, 2$

C. $2^{1/3} cis\left[\frac{2k\pi - \pi/3}{3}\right], k = 0, 1, 2$

D. none

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117. Find all the values of $\left(\sqrt{3} + i\right)^{\frac{1}{4}}$.

A. $2^{1/4} cis\left[\frac{2k\pi + \pi/6}{4}\right], k = 0, 1, 2, 3$

B. $2^{1/4} cis\left[\frac{2k\pi + \pi/3}{4}\right], k = 0, 1, 2, 3$

C. $2^{1/4} cis\left[\frac{2k\pi - \pi/6}{4}\right], k = 0, 1, 2, 3$

D. none

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118. The product of the four values of $\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)^{3/4}$ is

A. -1

B. 1

C. i

D. i

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119. The product of the values of $\left[\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right]^{3/4}$ is

A. -1

B. 1

C. i

D. i

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120. The product of the distinct $(2n)^{th}$ roots $1 + i\sqrt{3}$ is

A. 0

B. $-1 - i\sqrt{3}$

C. $1 + i\sqrt{3}$

D. $-1 + i\sqrt{3}$

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121. If $x^6 + 1 = 0$, then x =

A. $cis\frac{(3k+1)\pi}{6}, k = 0, 1, 2, 3, 4, 5$

B. $cis\frac{(2k-1)\pi}{6}, k = 0, 1, 2, 3, 4, 5$

C. $cis\frac{(3k+1)\pi}{6}, k = 0, 1, 2, 3, 4, 5$

D. $cis\frac{(3k-1)\pi}{6}, k=0, 1, 2, 3, 4, 5$

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122. The number of common roots of 15th and of 25th roots of unity are

A. 1

B.

C. 5

D. 6

Answer: A

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123. The common roots of the equation

$z^3+2z^2+2z+1=0, z^{2014}+z^{2015}+1=0$

A. ω, ω^2

B. $1, \omega, \omega^2$

C. $-1, \omega, \omega^2$

D. $-\omega, \ -\omega^2$

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124. p,q,r are distinct cube roots of non-zero complex number z. Let a, b, c be complex numbers satisfying $ap+bq+cr \neq 0$. Then find the value of $\frac{(aq+br+cp)(ar+bp+cq)}{(ap+bq+cr)^2}$.

A. 1

B. -1

C. i

D. -1

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EXERCISE 2 SET-1

1. $1, \alpha_1, \alpha_2, \alpha_3 \ldots . . \alpha_{n-1}$ are the n^{th} roots of unity and n is an even natural number ,then $(1+\alpha_1)(1+\alpha_2)(1+\alpha_3) \ldots . (1+\alpha_{n-1}) =$

A. only I is true

B. only II is true

C. both I and II are true

D. neither I nor II true

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2. Statement-I :

If $x_n = \cos(\pi / 2^n) + i \sin(\pi / 2^n)$ then $x_1 . x_2 . x_3 \ldots . = -1$

Statement-II : $x_n = \cos(\pi / 4^n) + i\sin(\pi / 4^n)$ then $x_1 . x_2 . x_3 . \dots . \infty = -1$

A. only I is true

B. only II is true

C. both I and II are true

D. neither I nor II true

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3. $I: \sum_{k=1}^{6}\left[\sin\left(\frac{2k\pi}{7}\right) - i\cos\left(\frac{2k\pi}{7}\right)\right] = i$

$II: \sum_{r=1}^{8}\left[\sin\left(\frac{2\pi r}{9}\right) + i\cos\left(\frac{2\pi r}{9}\right)\right] = i$

Which of the above statements are true

A. only I is true

B. only II is true

C. both I and II are true

D. neither I nor II true

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4. Statement-I : If $e^{i\theta} = \cos\theta + i\sin\theta$ then for the $\Delta ABC e^{iA} e^{iB} e^{iC} = -1$

Statement-II : If $\left(\sqrt{3}+1\right)^{100} = 2^{99}(a+ib)$ then $b = 2\sqrt{3}$

A. only I is true

B. only II is true

C. both I and II are true

D. neither I nor II true

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5. I : Value of $i^i = e^{-\frac{\pi}{2}}$

II : $z = i \log(2 + \sqrt{3})$ then sinz=2

which of above statement is true

A. only I is true

B. only II is true

C. both I and II are true

D. neither I nor II true

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EXERCISE 2 SET-2

1. The ascending order of the values of

$z_1 = (1+i)^4,\ z_2 = (\sqrt{3}+i)^{12},\ z_3 = (1+i\sqrt{3})^9 + (1-i\sqrt{3})^9$

A. z_1, z_2, z_3

B. z_2, z_3, z_1

C. z_3, z_1, z_2

D. z_1, z_3, z_2

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2. The ascending order of the values of

$z_1(1+i)^4+(1-i)^4, z_2=\left(\sqrt{3}+i\right)^{12}+\left(\sqrt{3}-i\right)^{12}, z_3=\left(1+i\sqrt{3}\right)^9+($

A. z_1, z_2, z_3

B. z_2, z_3, z_1

C. z_3, z_1, z_2

D. z_1, z_3, z_2

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3. The descending order of the values of

$$z_1 = \left(\frac{\sqrt{3}}{2}i + \frac{1}{2}\right)^6, z_2 = \left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)^4, z_3 = \left(\frac{\sqrt{3}}{4} + \frac{i}{4}\right)^6$$

A. z_1, z_2, z_3

B. z_2, z_3, z_1

C. z_3, z_1, z_2

D. z_1, z_3, z_2

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EXERCISE 2 SET-3

1. Match the following.

I) $(i)^i =$	a) $i\pi/2$
II) $\log_e i =$	b) $i\pi/2 + \log \pi/2$
III) $\log(\log i) =$	c) $\sqrt{2}$
IV $\sqrt{i} + \sqrt{-i} =$	$e^{-\pi/2}$

A. d, a, c, b

B. b, c, a, d

C. d, a, b, c

D. d, c, b, a

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EXERCISE 2 SET-4

1. A : $(1+i)^6 + (1-i)^6 = 0$

R : If n is a positive integer then $(1+i)^n + (1-i)^n = 2^{(n/2)+1}. \cos \frac{n\pi}{4}$

A. Both A and R are true and R is the correct explanation of A

B. Both A and B are true but R is not correct explanation of A

C. A is true but R is false

D. A is false but R is true

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2. A : If $A+B+C=\pi$ and $cisA=x, cisB=y, cisC=z$, then xyz=-1

R: $(cis\alpha)(cis\beta)=cis(\alpha+\beta)$

A. Both A and R are true and R is the correct explanation of A

B. Both A and B are true but R is not correct explanation of A

C. A is true but R is false

D. A is false but R is true

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3. Assertion (A) : If

$\alpha=\cos\left(\frac{2\pi}{7}\right)+i\sin\left(\frac{2\pi}{7}\right), p=\alpha+\alpha^2+\alpha^4, q=\alpha^3+\alpha^5+\alpha^6$

then the equation whose roots are p and q is $x^2+x+2=0$

Reason (R) : If α is a roots of $z^7=1$ then $1+\alpha+\alpha^2+\ldots...+\alpha^6=0$

A. Both A and R are true and R is the correct explanation of A

B. Both A and B are true but R is not correct explanation of A

C. A is true but R is false

D. A is false but R is true

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