MATHS

BOOKS - NCERT MATHS (ENGLISH)

COMPLEX NUMBERS AND QUADRATIC EQUATIONS

Short Answer Type Questions

1. For a positive integer n , find the value of $(1-i)^n\left(1-\frac{1}{i}\right)^n$.

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2. Evaluate $\sum_{n=1}^{13}\left(i^n+i^{n+1}\right)$, where $n \in N$.

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3. If $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 = x + iy,\ f \in d\,(x, y)$

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4. $\frac{(1+i)^2}{2-i} = x + iy$, then find the value of x+y.

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5. If $\left(\frac{1-i}{1+i}\right)^{100} = a + ib,\ f \in d(a, b)$

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6. If a = $\cos\theta + \theta i \sin$, then find the value of $\frac{1+a}{1-a}$.

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7. $If(1+i)z=(1-i)\bar{z}$, then show that $z=-i\bar{z}$.

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8. If $z=x+iy$, then show that $zz+2(z+z)+a=0$, where $a\in R$, represents a circle.

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9. If the real part of $\frac{\bar{z}+2}{\bar{z}-1}$ is 4, then show that the locus of the point representing z in the complex plane is a circle.

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10. Show that the complex number z, satisfying are $\frac{z-1}{z+1}=\frac{\pi}{4}$ lies on a circle.

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11. Solve the equation $|z| = z + 1 + 2i$.

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Long Answer Type Questions

1. If $|z + 1| = z + 2(1 + i)$, find z.

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2. If $arg(z - 1) = arg(z + 3i)$, then find $(x - 1) : y$, where $z = x + iy$.

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3. Show that $\left|\frac{z-2}{z-3}\right| = 2$ represents a circle, find its centre and radius.

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4. If $\frac{z-1}{z+1}$ is a purely imaginary number $(z \neq -1)$, then find the value of $|z|$.

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5. $z_1 \text{the} z_2 \text{are two complex numbers such that} |z_1| = |z_2|$. "and" arg $(z_1) + arg(z_2 = \pi, \quad \text{then show that} \quad z_1 = -\bar{z}_2.$

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6. If $|z+1| = 1 (z_1 \neq -1)$ and $z_2 = \dfrac{z_1 - 1}{z_1 - 2}$, then show that the real part of z_2 is zero.

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7. If z_1, z_2 and z_3, z_4 are two pairs of conjugate complex numbers, the find the value of $arg\left(\dfrac{z_1}{z_4}\right) + arg(z_2 / z_3)$.

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8. If $|z_1| = |z_2| = \vdots = |z_n| = 1$, prove that $|z_1 + z_2 + z_3 + + z_n| = \dfrac{1}{z_1} + \dfrac{1}{z_2} + \dfrac{1}{z_3} + + \dfrac{1}{z_n}$.

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9. If the complex number Z_1 and $Z_2, arg(Z_1) - arg(Z_2) = 0$. then show that $|z_1 - z_2| = |z_1| - |z_2|$.

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10. Solve the system of equations $Re(z^2) = 0,\ |z| = 2$

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11. Find a complex number z satisfying the equation $z + \sqrt{2}|z + 1| + i = 0$.

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12. Convert the complex number $z = \frac{i - 1}{\frac{\cos \pi}{3} + i\frac{\sin \pi}{3}}$ in the polar form.

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13. If $zandw$ are two complex number such that $|zw| = 1andarg(z) - arg(w) = \frac{\pi}{2}$, then show that $zw = -i$.

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14. Fill in the blanks of the following .

(i) For any two complex numbers z_1, z_2 and any real numbers a, b, $|az_1 - bz_2|^2 + |bz_1 + az_2|^2 = \cdots$

(ii) The value of $\sqrt{-25} \times \sqrt{-9}is...$

(iii) The number $\frac{(1-i)^3}{1-i^3}$ is equal to ...

(iv) The sum of the series $i + i^2 + i^3 + \cdots$ upto 1000 terms us ...

(v) Multiplicative inverse of 1 + i is ...

(vi) If $z_1 \text{and} z_2$ are complex numbers such that $z_1 + z_2$ is a real number, then $z_1 = \cdots$

(vii) $arg(z) + arg\bar{z}$ where, $(\bar{z} \neq 0) is \ldots$

(viii) If

$|z+4| \leq 3$, then the greatest and least values of $|z+1| are \ldots$ and $\ldots$

(ix) If $\left|\frac{z-2}{z+2}\right| = \frac{\pi}{6}$, then the locus of z is ...

$(x) If |z| = 4 \text{and } \arg(z) = \frac{5\pi}{6}, then z = \cdots$

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True False

1. State ture or false for the following. (i) The order relation is defined on the set of complex numbers. (ii) Multiplication of a non-zero complex number by -i rotates the point about origin through a right angle in the anti-clockwise direction.(iiI) For any complex number z, the minimum value of $|z| + |z-1|$ is 1. (iv) The locus represent by $|z-1| = |z-i|$ is a line perpendicular to the join of the points $(1, 0)$ and $(0, 1)$. (v) If z is a complex number such that

$z \neq 0$ and $Re(z) = 0$, $then Im(z^2) = 0$. $(vi) The \in equality$|z-4|lt |z-2|$represents the region given by$xgt3. (vii) Let $z_1 and z_2$ be two complex numbers such that $|z_1 + z_2| = |z_1 + z_2|$,then arg $(z_1 - z_2) = 0$. 2 is not a complex number.

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2. Match the statements of column A and Column B.

	Column A		Column B
(i)	The polar form of $i + \sqrt{3}$ is	(a)	Perpendicular bisector of segment joining $(-2, 0)$ and $(2, 0)$.
(ii)	The amplitude of $-1 + \sqrt{-3}$ is	(b)	On or outside the circle having centre at $(0, -4)$ and radius 3.
(iii)	It $\|z + 2\| = \|z - 2\|$, then locus of z is	(c)	$\frac{2\pi}{3}$
(iv)	It $\|z + 2i\| = \|z - 2i\|$, then locus of z is	(d)	Perpendiculor bisectar of segment joining $(0, -2)$ and $(0, 2)$.
(v)	Region represented by	(e)	$2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$
(vi)	Region represented by $\|z + 4\| \leq 3$ is	(f)	On or inside the circle having centre $(-4, 0)$ and radius 3 units.
(vii)	Conjugate of $\frac{1+2i}{1-i}$ lies in	(g)	First quadrant
(viii)	Reciprocal of $1 - i$ lies in	(h)	Third quadrant

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3. What is the conjugate of $\frac{2-i}{(1-2i)^2}$?

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4. If $|z_1| = |z_2|$, is it necesssary that $z_1 = z_2$.

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5. If $\frac{(a^2+1)^2}{2a-i} = x + iy$, then when is the value of $x^2 + y^2$?

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6. Find the value of z, if $|z| = 4$ and $arg(z) = \frac{5\pi}{6}$.

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7. Find the value of $\left|(1+i)\frac{(2+i)}{(3+i)}\right|$

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8. Find the principal argument of $\left(1+i\sqrt{3}\right)^2$.

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9. where does z lie , if $\left|\frac{z-5i}{z+5i}\right| = 1$?

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Objective Type Questions

1. sinx + i cos 2x and cos x - i sin2x are conjugate to each other for

(A) x=nπ (B) x=(n+1/2)π/2 (C) x=0 (D) no value of x

A. $x = n\pi$

B. $x = \left(\mathrm{n} + \frac{1}{2}\right)\frac{\pi}{2}$

C. x=0

D. No value of x

Answer: D

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2. The real value of α for which the expression $\frac{1 - i\sin\alpha}{1 + 2i\sin\alpha}$ is purely real is

A. $(n+1)\frac{\pi}{2}$

B. $(2n+1)\frac{\pi}{2}$

C. $n\pi$

D. None of these

Answer: C

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3. If z = x + iy lies in III quadrant, then $\frac{\bar{z}}{z}$ also lies in III quadrant If:

A. $x > y > 0$

B. $x < y < 0$

C. $y < x < 0$

D. $y > x > 0$

Answer: B

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4. The value of $(z+3)(\bar{z}+3)$ is equivlent to (A) |z+3|^(2) (B) |z-3| (C) z^2+3 (D) none of these

A. $|z+3|^2$

B. $|z-3|$

C. z^2+3

D. None of these

Answer: A

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5. If $\left(\frac{1+i}{1-i}\right)^x = 1$, then (A) x=2n+1 (B) x=4n (C) x=2n (D) x=4n+1, n $\in$ N.

A. $x = 2n+1$

B. $x = 4n$

C. $x = 2n$

D. $x = 4n+1$

Answer: B

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6. A real value of x satisfies the equation $\frac{3-4ix}{3+4ix} = \alpha - i\beta(\alpha, \beta \in R)$, if $\alpha^2 + \beta^2 =$

A. $x = 2n + 1$

B. $x = 4n$

C. $x = 2n$

D. $x = 4n + 1$

Answer: A

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7. Which of the following is correct for any tow complex numbers $z_1 and z_2$? $|z_1 z_2| = |z_1||z_2|$ (b) $arg(z_1 z_2) = arg(z_1) arg(z_2)$ (c) $|z_1 + z_2| = |z_1| + |z_2|$ (d) $|z_1 + z_2| \geq |z_1| + |z_2|$

A. $|z_1 z_2| = |z_1||z_2|$

B. $\arg(z_1 z_2) = arg(z_1 . Arg(z_2)$

C. $|z_1 + z_2| = |z_1| + |z_2|$

D. $|z_1 + z_2 \geq |z_1| - |z_2|$

Answer: A

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8. The point represented by the complex number $(2 - i)$ is rotated about origin through an angle $\frac{\pi}{2}$ in the clockwise direction, the new position of point is (A) 1+2i (B) -1-2i (C) 2+i (D) -1+2i

A. $1 + 2i$

B. $-1 - 2i$

C. $2 + i$

D. $-1 + 2i$

Answer: B

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9. Let $x, y \in R$. Then $x + iy$ is a non real complex number if

A. x = 0

B. y = 0

C. $x \neq 0$

D. $y \neq 0$

Answer: D

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10. If $a + ib = c + id$, then

A. $a^2 + c^2 = 0$

B. $b^2 + c^2 = 0$

C. $b^2 + d^2 = 0$

D. $a^2 + b^2 = c^2 + d^2$

Answer: D

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11. The complex number which satisfies the condition $\left|\frac{i+z}{i-z}\right| = 1$ lies on a. $circle\ x^2 + y^2 = 1$ b. $the\ x - a\xi s$ c. $the\ y - a\xi s$ d. $the\ l \in e\ x + y = 1$

A. Circle $x^2 + y^2 = 1$

B. the X-axis

C. the Y-axis

D. the line $x + y = 1$

Answer: B

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12. If z is a complex number, then

A. $\left|z^2\right| > |z|$

B. $\left|z^2\right| = \left|z^2\right|$

C. $\left|z^2\right| < |z|^2$

D. $\left|z^2\right| \geq |z|^2$

Answer: B

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13. $|z_1 + z_2| = |z_1| + |z_2|$ is possible, if

A. $z_2 = \bar{z}_1$

B. $z_2 = \dfrac{1}{z_1}$

C. $\arg(z_1) = arg(z_2)$

D. $|z_1| + |z_2|$

Answer: C

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14. The real value of θ for which the expression $\dfrac{1 + i\cos\theta}{1 - 2i\cos\theta}$ is real number is

A. $n\pi + \dfrac{\pi}{4}$

B. $n\pi + (-1)^2 \frac{\pi}{4}$

C. $2n\pi \pm \frac{\pi}{2}$

D. None of these

Answer: C

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15. the vaule of arg (x) when $x < 0$ is (a) 0 (b) Ï€Â /2 (c) Ï€Â (d) none of these

A. 0

B. $\frac{\pi}{2}$

C. π

D. None of these

Answer: C

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16. If $f(z) = \frac{7 - z}{1 - z^2}$, where $z = 1 + 2i$, then $|f(z)|$ is

A. $\frac{|z|}{2}$

B. $|z|$

C. $2|z|$

D. None of these

Answer: A

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