BAAP OF ALL FORMULA LISTS

FOR IIT JEE

DIFFERENTIAL EQUATIONS

SL#	FORMULA
1	Linear Equation $\frac{dy}{dx} + p(x)y = q(x)$. The general solutin is $y = \frac{\int u(x)q(x)dx + C}{u(x)}$, where $u(x) = \exp\left(\int p(x)dx\right)$.
2	Separable Equations $\frac{dy}{dx} = f(x, y) = g(x)h(y)$ The general solution is given by $\int \frac{dy}{h(y)} = \int g(x)dx + C$, or $H(y) = G(x) + C$
3	Homogeneous Equations The differential equation $\frac{dy}{dx} = f(x, y)$ is homogeneous, if the function $f(x, y)$ is homogeneous, that is $f(tx, ty) = f(x, y)$ The substitution $z = \frac{y}{x}(theny = zx)$ leads to the separable equation $x\frac{dz}{dx} + z = f(1, z)$.
4	Bernoulli Equation $\frac{dy}{dx} + p(x)y = q(x)y^n$. The substitution $z = y^{1-n}$ leads to the linear equation $\frac{dz}{dx} + (1-n)p(x)z = (1-n)q(x)$.
5	Riccati Equation $\frac{dy}{dx} = p(x) + q(x)y + r(x)y^2$ If a particular solution y_1 is known, then the general solution can be obtained with the help of substitutin $z = \frac{1}{y - y_1}$ which leads to the first order linear equation $\frac{dz}{dx} = -[q(x) + 2y_1(x)]z - r(x)$.
6	Exact and Nonexact Equations The equation $M(x, y)dx + N(x, y)dy = 0$ is called exact if $\frac{\partial M}{\partial Y} = \frac{\partial N}{\partial x}$, nd nonexact other wise. The general solution is $\int M(x, y)dx + \int N(x, y)dy = C$.
7	Radioactive Decay $\frac{dy}{dx} = -ky$, where y(t) is the amount of radioactive element at time t,k is the rate of decay. The solution is $y(t) = y_0e^{-kt}, where y_0 = y(0)$ is the initial amount.
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	Newton\'s Law of cooling $\frac{dT}{dt} = -k(T-S)$, where T(t) is the temperature of an object at time t,S is the temperature of the surrounding environement k, is a possitive constant.The Solution is $T(t) = S + (T_0 - S)e^{-kt}, where T_0 = T(0)$ is the initial temperature of the object at time t=0.
9	Population Dynamics (Logistic Model) $\frac{dP}{dt} = kP\left(1 - \frac{P}{M}\right)$, where P(t) is population at time t,k is a positive costant, M is a limiting size for the population . The solution of the differential equation is $P(t) = \frac{MP_0}{P_0 + (M - P_0)e^{-kt}}, where P_0 = P(0)$ is the initial population at time t=0.
10	Homogeneous Linear Equations with Constant Coefficients $y'' + py' + qy = 0$ The characteristic equatin is $\lambda^2 + p\lambda + q = 0$ If $\lambda_1 \text{ and } \lambda_2$ are distinct real roots of the characteristc equation then the general solution is $y = C_1 e^{\lambda_1} + C_2(e^{\lambda_2}, where C_1 \text{ and } C_2$ integration constants. If $\lambda_1 = \lambda_2 = -\frac{p}{2}$ then the general solution is $y = (C_1 + C_2 x)e^{-\frac{p}{2}x}. If \lambda_1 \text{ and } \lambda_2$ are complex numbers. $\lambda_1 = \alpha + \beta i, \lambda_2 = \alpha - \beta i, where \alpha = -\frac{p}{2}, \beta = \frac{\sqrt{4q - p^2}}{2}$, then the general solutin is $y = e^{\alpha x}(C_1 \cos\beta x + C_2 \sin\beta x)$.
11	Inhomogeneous Linear Equations with Constant Coefficietns $y'' + py' + qy = f(x)$. The general solution is given by $y = y_p + y_h, where y_p$ is a particular solution o fthe inhomogeneous equation and y_h is the general solution of the associated homogeneous equation. If the right side has the form $f(x) = e^{\alpha x}(P_1(x)\cos\beta x + P_1(x)\sin\beta x)$, then the particular solution y_p is given by $y_p = x^k e^{\alpha x}(R_1(x)\cos\beta x + R_2(x)\sin\beta x)$, where the polynomials $R_1(x)$ and $R_2(x)$ have to be found by using the method of undetermined coefficiets. If $\alpha + \beta i$ is a simle root, then $k = 1$, If $\alpha + \beta i$ is a double root, then $k = 2$.
12	Differential Equatins with y missing $y'' = f(x, y'). Set u = y'$. then the new equation satisfied by v is $u' = f(x, u)$ where is a first order differential equation.
13	Differential Equations with x missing $y'' = f(y, y')$. Since $y'' = \frac{du}{dx} = \frac{du}{dy}\frac{dy}{dx} = u\frac{du}{dy}$, we have $u\frac{du}{dy} = f(y, u)$, which is a first order differential equation.
14	Free Undamped Vibrations The motion of a Mass on a Spring is described by the equation $m\ddot{y} + ky = 0$, where m is the mass of the object, k is the stiffness of the spring, y is displacement of the mass from equilibrium. The general solution is $y = A\cos(\omega_0 t - \delta)$, where is the amplitude of the displacement, $om\eta_0$ is the fundamenta frequency, the perod is $T = \frac{2\pi}{\omega_0}, \delta$ is phase angle o the displacement. This is an example of simple harmonic motion.
15	Free Damped Vibrations $m\ddot{y} + \gamma\dot{y} + ky = 0$, where gamma is the damping coefficient. There are 3 cases for the general solution: Case 1. $\gamma^2 > 4km$(overdamped) $y(t) = Ae^{\lambda_1 t} + Be^{\lambda_2 t}$, where $\lambda_1 = \frac{-\gamma - \sqrt{\gamma^2 - 4km}}{2m}$, $\lambda_2 = \frac{-\gamma + \sqrt{\gamma^2 - 4km}}{2m}$. Case 2. $\gamma^2 = 4km$(critically damped)y(t)=(A+Bt)e^(lambdat), $where$lambda=- gamma/(2m) $. Case 3.$ gamma^2lt4km$(underdamed)$ $y(t) = e^{-\frac{\gamma}{2m}}A\cos(\omega t - \delta)$, where, $\omega = \sqrt{4km - \gamma^2}$

16	simple Pendulum $\frac{d^2\theta}{dt^2}+\frac{g}{L}\theta=0,$ where theta is the angular displacement, L is the pendulum length, g is the acceleration of gravity. The general solution for small angles theta is $\theta(t)=\theta_{\max}\sin\left(\sqrt{\frac{g}{L}}t,\right.$ the period is $T=2\pi\sqrt{\frac{L}{g}}$.
17	RLC circuit $L\frac{d^2I}{dt^2}+R\frac{dI}{dt}+\frac{1}{C}I=V'(t)=\omega E_0\cos(\omega t)$, where I is the current is an RLC circuit with an ac voltage source $V(t)=E_0\sin(\omega t)$. The general solution is $I(t)=C_1e^{r_1t}+C_2e^{r_2t}+A\sin(\omega t-\varphi)$, where $r_{1,2}=\frac{-R\pm\sqrt{R^2-\frac{4L}{C}}}{2L}$, $A=\left.\frac{\omega E_0}{\sqrt{L\omega^2-\frac{1}{C}^{-2}+R^2\omega^2}}\right)$, $\varphi=\tan^{-1}\left(\frac{L\omega}{R}-\frac{1}{RC\omega}\right)$, C_1, C_2 are constants depending on intial conditions.
18	The Lalace Equation $\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2}=0$ applies to potential energy function $u(x,y)$ for a conservative force field in the xy-plane. Partial differential equations of this type are called elliptic.
19	The Heat Equation $\frac{\partial^2u}{\partial x^2+\frac{\partial^2u}{\partial y^2}=\frac{\partial u}{\partial t}}$ applies to the temperature distribution$u(x,y)$ in the xy-plane whenheat is allowed to flow from warm areas to cool ones. The equations of this type are called parabolic.
20	The Wave Equatin $\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2}=\frac{\partial^2u}{\partial t^2}$ applies to the displacement $u(x,y)$ of vibrating membrances and other wave functions. The equations of this type are caled hyperbolic.