

BAAP OF ALL FORMULA LISTS



FOR IIT JEE

MATRICES AND DETERMINANTS

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SL#	FORMULA
1	Second Order Determinant $\det A = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1$
2	Value of Determinant $\det A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}$
3	N-th Order Determinant $\det A = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1j} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2j} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{i1} & a_{i2} & \dots & a_{ij} & \dots & a_{in} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nj} & \dots & a_{nn} \end{vmatrix}$
4	Minor The minor M_{ij} associated with the element a_{ij} of n-th order matrix A is the (n-1)-th order determinant derived from the matrix A by deletion of its i-th row and j-th column.
5	Cofactor $C_{ij} = (-1)^{i+j}M_{ij}$
6	Laplace Expansion of n-th Order Determinant Laplace expansion by elements of the i-th row $\det A = \sum_{j=1}^n a_{ij}C_{ij}, i = 1, 2, \dots, n$ Laplace expansion by elements of the j-th column $\det A = \sum_{i=1}^n a_{ij}C_{ij}, j = 1, 2, \dots, n$
7	The value of a determinant remains unchanged if rows are changed to columns and columns to rows.

$$\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

8

If two rows (or two columns) are interchanged, the sign of the determinant is changed.

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = - \begin{vmatrix} a_2 & b_2 \\ a_1 & b_1 \end{vmatrix}$$

9

If two rows (or two columns) are identical, the value of the determinant is zero.

$$\begin{vmatrix} a_1 & a_1 \\ a_2 & a_2 \end{vmatrix} = 0$$

10

If the elements of any row (or column) are multiplied by a common factor, the determinant is multiplied by that factor.

$$\begin{vmatrix} ka_1 & kb_1 \\ a_2 & b_2 \end{vmatrix} = k \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

11

If the elements of any row (or column) are increased (or decreased) by equal multiples of the corresponding elements of any other row (or column), the value of the determinant is unchanged.

$$\begin{vmatrix} a_1 + kb_1 & b_1 \\ a_2 + kb_2 & b_2 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

12

An $m \times n$ matrix A is a rectangular array of elements (numbers or functions) with m rows and n columns.

$$A = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

13

Square matrix is a matrix of order $n \times n$

14

A square matrix $[a_{ij}]$ is symmetric if $a_{ij} = a_{ji}$, i.e. it is symmetric about the leading diagonal.

15

A square matrix $[a_{ij}]$ is skew-symmetric if $a_{ij} = -a_{ji}$

16

Diagonal matrix is a square matrix with all elements zero except those on the leading diagonal.

17

Unit matrix is a diagonal matrix in which the elements on the leading diagonal are all unity. The unit matrix is denoted by I

18

A null matrix is one whose elements are all zero

19

Two matrices A and B are equal if, and only if, they are both of the same shape $m \times n$ and corresponding elements are equal.

20

Two matrices A and B can be added (or subtracted) of, and only if, they have the same shape $m \times n$.
If

$$A = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix},$$

$$B = [b_{ij}] = \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{m1} & b_{m2} & \dots & b_{mn} \end{bmatrix},$$

then,

$$A + B = \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} & \dots & a_{1n} + b_{1n} \\ a_{21} + b_{21} & a_{22} + b_{22} & \dots & a_{2n} + b_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} + b_{m1} & a_{m2} + b_{m2} & \dots & a_{mn} + b_{mn} \end{bmatrix}$$

21

If k is a scalar, and $A = [a_{ij}]$ is a matrix, then

$$kA = [ka_{ij}] = \begin{bmatrix} ka_{11} & ka_{12} & \dots & ka_{1n} \\ ka_{21} & ka_{22} & \dots & ka_{2n} \\ \vdots & \vdots & & \vdots \\ ka_{m1} & ka_{m2} & \dots & ka_{mn} \end{bmatrix}$$

22

Multiplication of Two Matrices

Two matrices can be multiplied together only when the number of columns in the first is equal to the number of rows in the second.

If

$$A = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix},$$

$$B = [b_{ij}] = \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1k} \\ b_{21} & b_{22} & \dots & b_{2k} \\ \vdots & \vdots & & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nk} \end{bmatrix},$$

$$\text{then } AB = C = \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1k} \\ c_{21} & c_{22} & \dots & c_{2k} \\ \vdots & \vdots & & \vdots \\ b_{m1} & c_{m2} & \dots & c_{mk} \end{bmatrix},$$

where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \dots + a_{in}b_{nj} = \sum_{\lambda=1}^n a_{i\lambda}b_{\lambda j}$

($i = 1, 2, \dots, m; j = 1, 2, \dots, k$).

Thus if $A = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$, $B = [b_i] = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$,

$$\text{then, } AB = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} a_{11}b_1 & a_{12}b_2 & a_{13}b_3 \\ a_{21}b_1 & a_{22}b_2 & a_{23}b_3 \end{bmatrix}$$

23

Transpose of a Matrix

If the rows and columns of matrix are interchanged, then the new matrix is called the transpose of the original matrix. If A is the original matrix, its transpose is denoted A^T or \tilde{A}



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The matrix A is orthogonal if $AA^T = I$

25

If the matrix product AB is defined, then

$$(AB)^T = B^T A^T$$

26

Adjoint of Matrix

If A is a square $n \times n$ matrix, its adjoint, denoted by $\text{adj } A$, is the transpose of the matrix of cofactors C_{ij} of A : $\text{adj } A = [C_{ij}]^T$.

27

Trace of a Matrix

If A is a square $n \times n$ matrix, its trace, denoted by $\text{tr } A$, is defined to be the sum of the terms on the leading diagonal: $\text{tr } A = a_{11} + a_{22} + \dots + a_{nn}$.

28

Inverse of a Matrix

If A is a square $n \times n$ matrix with a nonsingular determinant $\det A$, then its inverse A^{-1} is given by $A^{-1} = \frac{\text{adj } A}{\det A}$

29

If the matrix product AB is defined, then

$$(AB)^{-1} = B^{-1}A^{-1}$$

30

If A is a square $n \times n$ matrix, the eigenvectors X satisfy the equation $AX = \lambda X$, while the eigenvalues λ satisfy the characteristic equation $|A - \lambda I| = 0$

31

$$\begin{cases} a_1x + b_1y = d_1 \\ a_2x + b_2y = d_2 \end{cases}$$

$$x = \frac{D_x}{D}, y = \frac{D_y}{D} \text{ (Cramer's rule),}$$

where

$$D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1,$$

$$D_x = \begin{vmatrix} d_1 & b_1 \\ d_2 & b_2 \end{vmatrix} = d_1b_2 - d_2b_1,$$

$$D_y = \begin{vmatrix} a_1 & d_1 \\ a_2 & d_2 \end{vmatrix} = a_1d_2 - a_2d_1$$

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32	<p>If $D \neq 0$, then the system has a single solution:</p> $x = \frac{D_x}{D}, y = \frac{D_y}{D}.$ <p>If $D = 0$ and $D_x \neq 0$ (or $D_y \neq 0$), then the system has no solution.</p> <p>If $D = D_x = D_y = 0$, then the system has infinitely many solutions.</p>
33	$\begin{cases} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \\ a_3x + b_3y + c_3z = d_3 \end{cases}$ $x = \frac{D_x}{D}, y = \frac{D_y}{D}, z = \frac{D_z}{D} \text{ (Cramer's rule),}$ <p>where</p> $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}, D_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix},$ $D_y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}, D_z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}.$
34	<p>If $D \neq 0$, then the system has a single solution:</p> $x = \frac{D_x}{D}, y = \frac{D_y}{D}, z = \frac{D_z}{D}.$ <p>If $D = 0$ and $D_x \neq 0$ (or $D_y \neq 0$ or $D_z \neq 0$), then the system has no solution.</p> <p>If $D = D_x = D_y = D_z = 0$, then the system has infinitely many solutions.</p>
35	<p>Matrix Form of a System of n Linear Equations in n Unknowns</p> <p>The set of linear equations</p> $\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \\ a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n \end{cases}$ <p>can be written in matrix form</p> $\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix} \cdot \begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix},$ <p>i.e. $A \cdot X = B$,</p>

$$\text{where, } A = \begin{pmatrix} a_{11} & a_{12} & \vdots & a_{1n} \\ a_{21} & a_{22} & \vdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}, X = \begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{pmatrix}, B = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}.$$

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Solution of a Set of Linear Equations $n \times n$

$$X = A^{-1} \cdot B,$$

where A^{-1} is the inverse of A .

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