JEE ADVANCED MATHS SOLUTIONS

YEAR 2010

Ques No.	Question
1	**JEE ADVANCED MATHS SOLUTIONS - 2010 || Paper 1** Let $p and q$ be real numbers such that $p \neq 0, p^3 \neq q, and p^3 \neq -q.$ If $\alpha and \beta$ are nonzero complex numbers satisfying $\alpha + \beta = -p and \alpha^2 + \beta^2 = q$, then a quadratic equation having $\alpha / \beta and \beta / \alpha$ as its roots is $(p^3 + q)x^2 - (p^3 + 2q)x + (p^3 + q) = 0$ $(p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$ $(p^3 + q)x^2 - (5p^3 - 2q)x + (p^3 - q) = 0$ $(p^3 + q)x^2 - (5p^3 + 2q)x + (p^3 + q) = 0$ Watch Free Video Solution on Doubtnut
2	**JEE ADVANCED MATHS SOLUTIONS - 2010 || Paper 1** Let f, g and h be real-valued functions defined on the interval $[0, 1]$ by $f(x) = e^{x^2} + e^{-x^2}$, $g(x) = xe^{x^2} + e^{-x^2}$ and $h(x) = x^2e^{x^2} + e^{-x^2}$. if a, b and c denote respectively, the absolute maximum of f, g and h on $[0, 1]$ then Watch Free Video Solution on Doubtnut
3	**JEE ADVANCED MATHS SOLUTIONS - 2010 || Paper 1** If the angle $A, B and C$ of a triangle are in an arithmetic propression and if $a, b and c$ denote the lengths of the sides opposite to $A, B and C$ respectively, then the value of

	the expression $\frac{a}{c}\sin 2C+\frac{c}{a}\sin 2A$ is $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) 1 (d) $\sqrt{3}$ Watch Free Video Solution on Doubtnut
4	**JEE ADVANCED MATHS SOLUTIONS - 2010 || Paper 1** Equation of the plane containing the straight line $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{2}=\frac{y}{4}=\frac{z}{2}$ and $\frac{x}{4}=\frac{y}{2}=\frac{z}{3}$ is Watch Free Video Solution on Doubtnut
5	**JEE ADVANCED MATHS SOLUTIONS - 2010 || Paper 1** Let ω be a complex cube root unity with $\omega \neq 1.$ A fair die is thrown three times. If $r_1, r_2 and r_3$ are the numbers obtained on the die, then the probability that $\omega^{r1}+\omega^{r2}+\omega^{r3}=0$ is $1/18$ b. $1/9$ c. $2/9$ d. $1/36$ Watch Free Video Solution on Doubtnut
6	**JEE ADVANCED MATHS SOLUTIONS - 2010 || Paper 1** Let P, Q, R and S be the points on the plane with position vectors $-2i-j, 4i, 3i$ $+3jand-3j+2j,$ respectively. The quadrilateral $PQRS$ must be a Parallelogram, which is neither a rhombus nor a rectangle Square Rectangle, but not a square Rhombus, but not a square Watch Free Video Solution on Doubtnut
	JEE ADVANCED MATHS SOLUTIONS - 2010 || Paper 1 The number of 3×3 matrices A whose entries are either 0 or 1 and for which the

7	system $A[xyz] = [100]$ has exactly two distinct solution is a. 0 b. $2^9 - 1$ c. 168 d. 2 Watch Free Video Solution on Doubtnut
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8	**JEE ADVANCED MATHS SOLUTIONS - 2010 || Paper 1** The value of $\int_0^1 \frac{x^4(1-x)^4}{1+x^2}dx$ is/are (a)$\frac{22}{7} - \pi$ (b) $\frac{2}{105}$ (c)0 (d) $\frac{71}{15} - \frac{3\pi}{2}$ Watch Free Video Solution on Doubtnut
9	**JEE ADVANCED MATHS SOLUTIONS - 2010 || Paper 1** Let f be a real-valued function defined on interval $(0, \infty)$,by $f(x) = \ln x +$ $\int_0^x \sqrt{1+\sin t}.\,dt$. Then which of the following statement(s) is (are) true? A. f''(x) exists for all $\in (0, \infty)$. B. f'(x) exists for all x $\in (0, \infty)$ and f' is continuous on $(0, \infty)$, but not differentiable on $(0, \infty)$. C. there exists $\alpha > 1$ such that $\lvert f'(x)\rvert < \lvert f(x)\rvert$ for all x $\in (\alpha, \infty)$. D. there exists $\beta > 1$ such that $\lvert f(x)\rvert + \lvert f'(x)\rvert \le \beta$ for all x $\in (0, \infty)$. Watch Free Video Solution on Doubtnut
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10	Let A and B be two distinct points on the parabola $y^2 = 4x$. If the axis of the parabola touches a circle of radius r having AB as its diameter, then the slope of the line joining A and B can be Watch Free Video Solution on Doubtnut
11	**JEE ADVANCED MATHS SOLUTIONS - 2010 || Paper 1** Let ABC be a triangle such that $\angle ACB = \frac{\pi}{6}$ and let a, b and c denote the lengths of the sides opposite to A, B and C respectively. The value(s) of x for which $a = x^2 + x + 1, b$ $= x^2 - 1$ and $c = 2x + 1$ Watch Free Video Solution on Doubtnut
12	**JEE ADVANCED MATHS SOLUTIONS - 2010 || Paper 1** Let p be an odd prime number and T_p, be the following set of 2×2 matrices $T_p = \left\{ A = \begin{bmatrix} a & b \\ c & a \end{bmatrix}\right.$ $: a, b, c \in \{0, 1, 2, \ldots$ $\left. \ldots\ldots p - 1\} \right\}$ The number of A in T_p, such that A is either symmetric or skew-symmetric or both, and det (A) divisible by p is Watch Free Video Solution on Doubtnut
13	**JEE ADVANCED MATHS SOLUTIONS - 2010 || Paper 1** Let p be an odd prime number and T_p, be the following set of 2×2 matrices

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$\left. \dots\dots p-1\} \right\}$
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16	**JEE ADVANCED MATHS SOLUTIONS - 2010 || Paper 1** The circle $x^2 + y^2 - 8x = 0$ and hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ I intersect at the points A and B. Equation of a common tangent with positive slope to the circle as well as to the hyperbola is Watch Free Video Solution on Doubtnut
17	**JEE ADVANCED MATHS SOLUTIONS - 2010 || Paper 1** The circle $x^2 + y^2 - 8x = 0$ and hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ intersect at points A and B. Then Equation of the circle with A B as its diameter is Watch Free Video Solution on Doubtnut
18	**JEE ADVANCED MATHS SOLUTIONS - 2010 || Paper 1** Let f be a real-valued differentiable function on R (the set of all real numbers) such that $f(1) = 1.$ If the $y - \in tercept$ of the tangent at any point $P(x, y)$ on the curve $y = f(x)$ is equal to the cube of the abscissa of $P,$ then the value of $f(-3)$ is equal to_________ Watch Free Video Solution on Doubtnut
19	**JEE ADVANCED MATHS SOLUTIONS - 2010 || Paper 1** The number of values of θ in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ such that $\theta \neq n\frac{\pi}{5}$ for $n \in N$ and $\tan\theta = \cot 5\theta$ as well as $\sin 2\theta = \cos 4\theta$ is Watch Free Video Solution on Doubtnut

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The maximum value of the expression $\frac{1}{\sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta}$ is

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If $a and b$ are vectors in space given by $\vec{a} = \frac{\hat{i} - 2\hat{j}}{\sqrt{5}} and\, \vec{b} = \frac{2\hat{i} + \hat{j} + 3\hat{k}}{\sqrt{14}}$, then find the value of $\left(2\vec{a} + \vec{b}\right)\left(\vec{a} \times \vec{b}\right) \times \left(\vec{a} - 2\vec{b}\right)$.

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The line $2x + y = 1$ is tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If this line passes through the point of intersection of the nearest directrix and the x-axis, then the eccentricity of the hyperbola is

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If the distance between the plane $Ax - 2y + z = d.$ and the plane containing the lies

$$\frac{x+1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \text{ and } \frac{x-2}{3} = \frac{4-3}{4} = \frac{z-4}{5}$$

is $\sqrt{6}$, then $|d|$ is

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For any real number x, $let[x]$ denote the largest integer less than or equal to x, $Let f$ be a real-valued function defined on the interval $[-10, 10]$ be

$$f(x) = \{x - [x], \quad \text{if } [x] is odd 1 + [x] - x, \quad \text{if } [x] is even$$

Then the value of

$$\frac{\pi^2}{10} \int_{-1}^{10} f(x) \cos \pi x dx is \_\_$$

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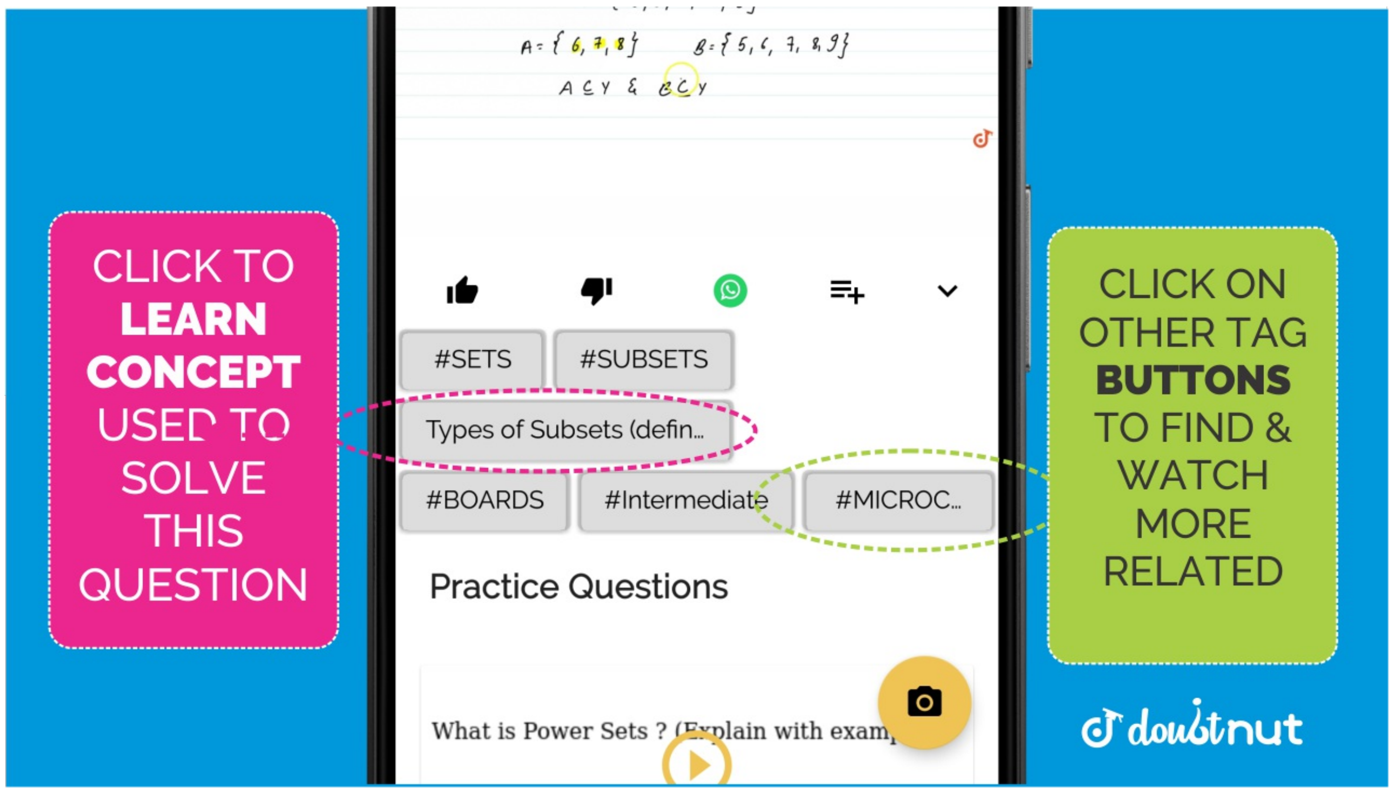
JEE ADVANCED MATHS SOLUTIONS - 2010 || Paper 1

Let ω be the complex number

25	$\cos\left(\frac{2\pi}{3}\right)$ $+ i\sin\left(\frac{2\pi}{3}\right)$. Then the number of distinct complex cos numbers z satisfying Δ $= \begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix}$ $= 0$ is Watch Free Video Solution on Doubtnut
26	**JEE ADVANCED MATHS SOLUTIONS - 2010 || Paper 1** Let $S_k, k = 1, 2, , 100,$ denotes thesum of the infinite geometric series whose first term s $\frac{k-1}{k!}$ and the common ratio is $\frac{1}{k}$, then the value of $\frac{100^2}{100!} +$ $\sum_{k=1}^{100} \left(k^2 - 3k + 1\right) S_k$ is ________. Watch Free Video Solution on Doubtnut
27	**JEE ADVANCED MATHS SOLUTIONS - 2010 || Paper 2** Let f be a real-valued function defined on the inverval $(-1, 1)$ such that $e^{-x} f(x) = 2 +$ $\int_0^x \sqrt{t^4 + 1} dt,$ for all, $x \in (-1,$ $1) and let f^{-1}$ be the inverse function of f. Then $\left(f^{-1}\right)'(2)$ is equal to 1 (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{e}$ Watch Free Video Solution on Doubtnut

28	**JEE ADVANCED MATHS SOLUTIONS - 2010 || Paper 2** If $P(A) = 0.4, P(B) = 0.48$ and $P(A \cap B) = 0.16$, then find th e value of each of the following: Watch Free Video Solution on Doubtnut
29	**JEE ADVANCED MATHS SOLUTIONS - 2010 || Paper 2** There are n AM's between 1 & 31 such that $7thmean:(n-1)thmean = 5:9$, then find the value of n. Watch Free Video Solution on Doubtnut
30	**JEE ADVANCED MATHS SOLUTIONS - 2010 || Paper 2** A signal which can be green or red with probability $\frac{4}{5}$ and $\frac{1}{5}$ respectively, is received by station A and then and 3 transmitted to station B. The probability of each station receiving the signal correctly is $\frac{3}{4}$ If the signal received at station B is green, then the probability that the original signal was green is Watch Free Video Solution on Doubtnut
31	**JEE ADVANCED MATHS SOLUTIONS - 2010 || Paper 2** Let $S = \{1, 2, 3, 4\}$. The total number of unordered pairs of disjoint subsets of S is equal a.25 b. 34 c. 42 d. 41 Watch Free Video Solution on Doubtnut

	
32	**JEE ADVANCED MATHS SOLUTIONS - 2010 || Paper 2** For $r = 0, 1, ..., 10$, let $A_r, B_r,$ and C_r denote, respectively, the coefficient of x^r in the expansions of $(1+x)^{10}$, $(+x)^{20}$ and $(1$ $+x)^{30}$ .Then $\sum_{r=1}^{10} A_r(B_{10}B_r$ $- C_{10}A_r)$ is equal to Watch Free Video Solution on Doubtnut
33	**JEE ADVANCED MATHS SOLUTIONS - 2010 || Paper 2** Two adjacent sides of a parallelogram $ABCD$ are given by $\overrightarrow{AB} = 2\hat{i} + 10\hat{j}$ $+ 11\hat{k} and \overrightarrow{AD} =$ $-\hat{i} + 2\hat{j} + 2\hat{k}$. The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD'. If AD' makes a right angle with the side AB, then the cosine of the angel α is given by $\frac{8}{9}$ b. $\frac{\sqrt{17}}{9}$ c. $\frac{1}{9}$ d. $\frac{4\sqrt{5}}{9}$ Watch Free Video Solution on Doubtnut
	JEE ADVANCED MATHS SOLUTIONS - 2010 || Paper 2 Let K be a positive real number and

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A
$= [2k$
$- 12\sqrt{k}2\sqrt{k}2\sqrt{k}1$
$- 2k - 2\sqrt{k}2k$
$- 1] and B$
$= [02k - 1\sqrt{k}1$
$- 2k02 - \sqrt{k}$
$- 2\sqrt{k}0]$
. If det
$(adj A) + \det(adj B)$
$= 10^6, then [k]$
is equal to. [Note: $adj M$ denotes the adjoint of a square matix M and $[k]$ denotes the largest integer less than or equal to K].

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Let f be a function defined on R (the set of all real numbers) such that
$f'(x) = 2010(x$
$- 2009)(x$
$- 2010)^2(x$
$- 2011)^3(x$
$- 2012)^4,$
for all $x \in R$. If g is a function defined on R with values in the interval $(0, \infty)$ such that $f(x) = \ln(g(x)),$ for all $x \in R,$ then the number of point is R at which g has a local maximum is ___

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Let a1,a2,a3 ...... a11 be real numbers satisfying

36	$a_1 = 15, 27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for $k = 3, 4, \ldots .11$ If $\frac{a1^2 + a2^2 \ldots\ldots a11^2}{11} = 90$ then find the value of $\frac{a_1 + a_2 \ldots. + a_{11}}{11}$ Watch Free Video Solution on Doubtnut
37	**JEE ADVANCED MATHS SOLUTIONS - 2010 || Paper 2** Consider a triangle ABC and let $a, bandc$ denote the lengths of the sides opposite to vertices $A, B, andC$, respectively. Suppose $a = 6, b = 10,$ and the area of triangle is $15\sqrt{3}$. If $\angle ACB$ is obtuse and if r denotes the radius of the incircle of the triangle, then the value of r^2 is Watch Free Video Solution on Doubtnut
38	**JEE ADVANCED MATHS SOLUTIONS - 2010 || Paper 2** Two parallel chords of a circle of radius 2 are at a distance. $\sqrt{3+1}$ apart. If the chord subtend angles $\frac{\pi}{k}$ and $\frac{2\pi}{k}$ at the center, where $k > 0,$ then the value of [k] is Watch Free Video Solution on Doubtnut
39	**JEE ADVANCED MATHS SOLUTIONS - 2010 || Paper 2** Consider the polynomial f $(x) = 1 + 2x + 3x^2 + 4x^3$. Let s be the sum of all distinct real roots of $f(x)$and let $t = \|s\|$. Watch Free Video Solution on Doubtnut
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Consider the polynomial f $(x) = 1 + 2x + 3x^2 + 4x^3$. Let s be the sum of all distinct real roots of $f(x)$ and let $t = |s|$.

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JEE ADVANCED MATHS SOLUTIONS - 2010 || Paper 2

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The set of points z $|z - i|z|| = |z + i|z||$ is contained in or equal to

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