# JEE MAINS MATHS SOLUTIONS

## YEAR 2012



| Ques No. | Question |
|---|---|
| 1 | **JEE MAINS MATHS SOLUTIONS - 2012**<br><br>no.of solutions of the equation<br>$e^{\sin x} - e^{-\sin x} - 4$<br>$= 0$<br><br>Watch Free Video Solution on Doubtnut |
| 2 | **JEE MAINS MATHS SOLUTIONS - 2012**<br><br>Let $\hat{a}$ and $\hat{b}$ be two unit vectors. If the vectors<br>$\vec{c} = \hat{a} + 2\hat{b} and \vec{d}$<br>$= 5\hat{a} - 4\hat{b}$<br>are perpendicular to each other, then the angle between $\hat{a}$ and $\hat{b}$ is (1) $\frac{\pi}{6}$ (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{4}$<br><br>Watch Free Video Solution on Doubtnut |
| 3 | **JEE MAINS MATHS SOLUTIONS - 2012**<br><br>A spherical balloon is filled with 4500p cubic meters of helium gas. If a leak in the balloon causes the gas to escape at the rate of $72\pi$ cubic meters per minute, then the rate (in meters per minute) at which the radius of the balloon decreases 49 minutes after the leakage began is (1) $\frac{9}{7}$ (2) $\frac{7}{9}$ (3) $\frac{2}{9}$ (4) $\frac{9}{2}$<br><br>Watch Free Video Solution on Doubtnut |

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Statement 1: The sum of the series $1 + (1 + 2 + 4) + (4 + 6 + 9) + (9 + 12 + 16) + \ldots\ldots + (361 + 380 + 400) is 8000$. Statement 2: $\sum_{k=1}^{n} \left(k^3 - (k-1)^3\right) = n^3$ for any natural number n. (1) Statement 1 is false, statement 2 is true (2) Statement 1 is true, statement 2 is true; statement 2 is a correct explanation for statement 1 (3) Statement 1 is true, statement 2 is true; statement 2 is not a correct explanation for statement 1 (4) Statement 1 is true, statement 2 is false



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If the integral $\int \frac{5\tan x}{\tan x - 2} dx = x + a\ln|\sin x - 2\cos x| + k$ then a is equal to (1) 1 (2) 2 (3) 1 (4) 2



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Statement 1: An equation of a common tangent to the parabola $y^2 = 16\sqrt{3}x$ and the ellipse $2x^2 + y^2 = 4is = 2x + 2\sqrt{3}$

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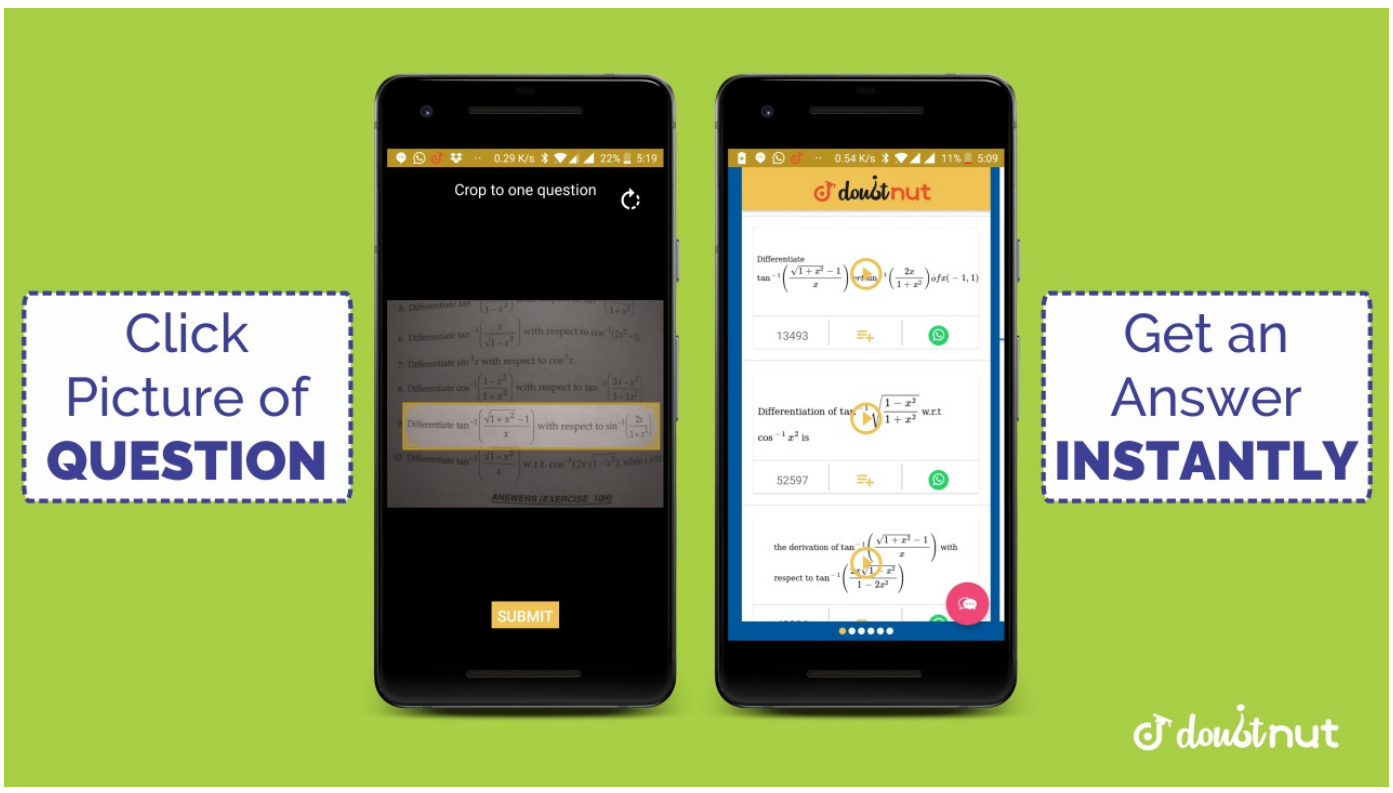
| 6 | . Statement 2: If the line $y = mx + \frac{4\sqrt{3}}{m}$, $(m \neq 0)$ is a common tangent to the parabola $y^2 = 16\sqrt{3}x$ and the ellipse $2x^2 + y^2 = 4$, then m satisfies $m^4 + 2m^2 = 24$. (1) Statement 1 is false, statement 2 is true (2) Statement 1 is true, statement 2 is true; statement 2 is a correct explanation for statement 1 (3) Statement 1 is true, statement 2 is true; statement 2 is not a correct explanation for statement 1 (4) Statement 1 is true, statement 2 is false<br><br>Watch Free Video Solution on Doubtnut |
| 7 | **JEE MAINS MATHS SOLUTIONS - 2012**<br><br>Let $A = (100210321)$ If $u_1$ and $u_2$ are column matrices such that $Au_1 = (100)$ and $Au_2 = (010)$, then $u_1 + u_2$ is equal to (1) $(-110)$ (2) $(-11-1)$ (3) $(-1-10)$ (4) $(1-1-1)$<br><br>Watch Free Video Solution on Doubtnut<br><br>Watch Free Video Solution on Doubtnut |
|  |  |
| 8 | **JEE MAINS MATHS SOLUTIONS - 2012**<br><br>If n is a positive integer, then $\left(\sqrt{3}+1\right)^{2n} - \left(\sqrt{3}-1\right)^{2n}$ is (1) an irrational number (2) an odd positive integer (3) an even positive integer (4) a rational number other than positive integers<br><br>Watch Free Video Solution on Doubtnut |
| 9 | **JEE MAINS MATHS SOLUTIONS - 2012**<br><br>If 100 times the $100^{th}$ term of an AP with non zero common difference equals the 50 times its $50^{th}$ term, then the $150^{th}$ term of this AP is (1) $150$ (2) 150 times its $50^{th}$ term (3) 150 (4) zero<br><br>Watch Free Video Solution on Doubtnut |

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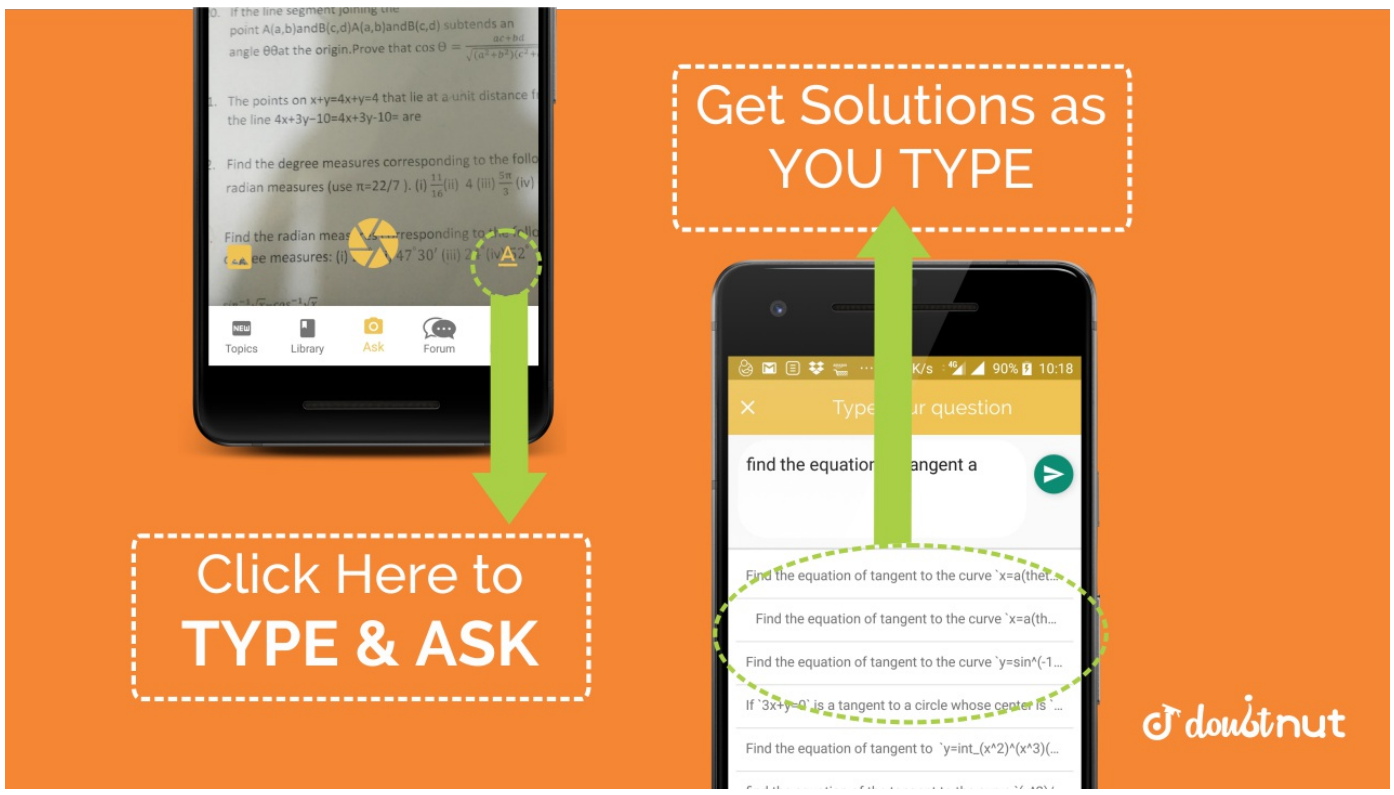
| 10 | **JEE MAINS MATHS SOLUTIONS - 2012**<br><br>In a $\Delta PQR$ , if<br>$3s \in P + 4\cos Q$<br>$= 6$<br>and<br>$4s \in Q + 3\cos P$<br>$= 1$<br>, then the angle R is equal to (1) $\frac{5\pi}{6}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{4}$ (4) $\frac{3\pi}{4}$<br><br>Watch Free Video Solution on Doubtnut |
| 11 | **JEE MAINS MATHS SOLUTIONS - 2012**<br><br>An equation of a plane parallel to the plane $x2y + 2z5 = 0$ and at a unit distance from the origin is (1) $x2y + 2z3 = 0$ (2) $x2y + 2z + 1 = 0$ (3) $x2y + 2z1 = 0$ (4) $x2y + 2z + 5 = 0$<br><br>Watch Free Video Solution on Doubtnut |
| |  |
| 12 | **JEE MAINS MATHS SOLUTIONS - 2012**<br><br>If the line $2x + y = k$ passes through the point which divides the line segment joining the points (1, 1) and (2, 4) in the ratio 3 : 2, then k equals (1) $\frac{29}{5}$ (2) 5 (3) 6 (4) $\frac{11}{5}$<br><br>Watch Free Video Solution on Doubtnut |
| | **JEE MAINS MATHS SOLUTIONS - 2012**<br><br>Let $x_1, x_2, \ldots\ldots, x_n$ be n observations, and let $\bar{x}$ be their arithematic mean and $\sigma^2$ be their variance. Statement 1: Variance of<br>$2x_1, 2x_2, \ldots\ldots,$<br>$2x_n is 4\sigma^2$<br>. Statement 2: Arithmetic mean of |

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$2x_1, 2x_2, \ldots\ldots,$
$2x_n is 4x$
. (1) Statement 1 is false, statement 2 is true (2) Statement 1 is true, statement 2 is true; statement 2 is a correct explanation for statement 1 (3) Statement 1 is true, statement 2 is true; statement 2 is not a correct explanation for statement 1 (4) Statement 1 is true, statement 2 is false

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The population p(t) at time t of a certain mouse species satisfies the differential equation
$\frac{dp(t)}{dt} = 0.5p(t)$
$- 450$
If $p(0) = 850$ , then the time at which the population becomes zero is (1) 2 ln 18 (2) ln 9 (3) $\frac{1}{2}$ ln 18 (4) ln 18

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Let a, b R be such that the function f given by
$f(x) = \ln|x| + bx^2$
$+ ax, x \neq 0$
has extreme values at $x = 1$ and $x = 2$ . Statement 1: f has local maximum at $x = 1$ and at $x = 2$ . Statement 2: $a = \frac{1}{2} \text{and} b = \frac{-1}{4}$ (1) Statement 1 is false, statement 2 is true (2) Statement 1 is true, statement 2 is true; statement 2 is a correct explanation for statement 1 (3) Statement 1 is true, statement 2 is true; statement 2 is not a correct explanation for statement 1 (4) Statement 1 is true, statement 2 is false

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The area bounded between the parabolas $x^2 = \frac{y}{4} \text{and} x^2 = 9y$ and the straight line

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| 16 | $y = 2$ is (1) $20\sqrt{2}$ (2) $\frac{10\sqrt{2}}{3}$ (3) $\frac{20\sqrt{2}}{3}$ (4) $10\sqrt{2}$<br><br>Watch Free Video Solution on Doubtnut |
| 17 | **JEE MAINS MATHS SOLUTIONS - 2012**<br><br>Assuming the balls to be identical except for difference in colours, the number of ways in which one or more balls can be selected from 10 white, 9 green and 7 black balls is<br>(1) 880 (2) 629 (3) 630 (4) 879<br><br>Watch Free Video Solution on Doubtnut |
| 18 | **JEE MAINS MATHS SOLUTIONS - 2012**<br><br>If $f: R\overrightarrow{R}$ is a function defined by<br>$f(x)$<br>$= [x]\cos\left(\frac{2x-1}{2}\right)\pi$<br><br>where [x] denotes the greatest integer function, then f is (1) continuous for every real x (2) discontinuous only at $x = 0$ (3) discontinuous only at non-zero integral values of x (4) continuous only at $x = 0$<br><br>Watch Free Video Solution on Doubtnut |
| 19 | **JEE MAINS MATHS SOLUTIONS - 2012**<br><br>If the lines<br>$\frac{x-1}{2} = \frac{y+1}{3}$<br>$= \frac{z-1}{4}$<br>and<br>$\frac{x-3}{1} = \frac{y-k}{2}$<br>$= \frac{z}{1}$<br>intersect, then k is equal to (1) $-1$ (2) $\frac{2}{9}$ (3) $\frac{9}{2}$ (4) 0<br><br>Watch Free Video Solution on Doubtnut |
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| 20 | **JEE MAINS MATHS SOLUTIONS - 2012**<br><br>Three numbers are chosen at random without replacement from {1, 2, 3, ..... 8}. The probability that their minimum is 3, given that their maximum is 6, is (1) $\frac{3}{8}$ (2) $\frac{1}{5}$ (3) $\frac{1}{4}$ (4) $\frac{2}{5}$<br><br>Watch Free Video Solution on Doubtnut |
| 21 | **JEE MAINS MATHS SOLUTIONS - 2012**<br><br>If $z \neq 1$ and $\frac{z^2}{z-1}$ is real, then the point represented by the complex number z lies (1) either on the real axis or on a circle passing through the origin (2) on a circle with centre at the origin (3) either on the real axis or on a circle not passing through the origin (4) on the imaginary axis<br><br>Watch Free Video Solution on Doubtnut |
| 22 | **JEE MAINS MATHS SOLUTIONS - 2012**<br><br>Let P and Q be $3 \times 3$ matrices with $P \neq Q$ . If $P^3 = Q^3 and P^2Q = Q^2P$ , then determinant of $\left(P^2 + Q^2\right)$ is equal to (1) 2(2) 1 (3)0 (4) 1<br><br>Watch Free Video Solution on Doubtnut |
| 23 | **JEE MAINS MATHS SOLUTIONS - 2012**<br><br>If $g(x) = \int 0x \cos 4t \mathrm{dt}$ , then $g(x+\pi)$ equals: (1) $\left(\frac{g(x)}{g(\pi)}\right.$ (2) $g(x) + g(\pi)$ (3) $g(x) - g(\pi)$ (4) $g(x)g(\pi)$<br><br>Watch Free Video Solution on Doubtnut |

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The length of the diameter of the circle which touches the x-axis at the point (1, 0) and passes through the point (2, 3) is (1) $\frac{10}{3}$ (2) $\frac{3}{5}$ (3) $\frac{6}{5}$ (4) $\frac{5}{3}$

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Let $X = \{1, 2, 3, 4, 5\}$ . The number of different ordered pairs (Y, Z) that can be formed such that $Y \subseteq X$, $Z \subseteq X$ and $Y \cap Z$ is empty, is (1) $5^2$ (2) $3^5$ (3) $2^5$ (4) $5^3$

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An ellipse is drawn by taking a diameter of the circle $(x1)^2 + y^2 = 1$ as its semiminor axis and a diameter of the circle $x^2 + (y2)^2 = 4$ as its semi-major axis. If the centre of the ellipse is the origin and its axes are the coordinate axes, then the equation of the ellipse is (1) $4x^2 + y^2 = 4$ (2) $x^2 + 4y^2 = 8$ (3) $4x^2 + y^2 = 8$ (4) $x^2 + 4y^2 = 16$

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Consider the function
$f(x) = |x2| + |x5|,$
$x \in R$
. Statement 1: $f'(4) = 0$ Statement 2: f is continuous in [2, 5], differentiable in (2, 5) and f(2) = f(5). (1) Statement 1 is false, statement 2 is true (2) Statement 1 is true, statement 2 is true; statement 2 is a correct explanation for statement 1 (3) Statement 1 is true, statement 2 is true; statement 2 is not a correct explanation for statement 1 (4) Statement 1 is true, statement 2 is false

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A line is drawn through the point (1, 2) to meet the coordinate axes at P and Q such that it forms a triangle OPQ, where O is the origin. If the area of the triangle OPQ is least, then the slope of the line PQ is (1) $-\frac{1}{4}$ (2) $-4$ (3) $-2$ (4) $-\frac{1}{2}$

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Let ABCD be a parallelogram such that
$\overrightarrow{A}B = \overrightarrow{q}, \overrightarrow{A}D$
$= \overrightarrow{p} and \angle BAD$
be an acute angle. If $\overrightarrow{r}$ is the vector that coincides with the altitude directed from the vertex B to the side AD, then $\overrightarrow{r}$ is given by (1)
$\overrightarrow{r} = 3\overrightarrow{q}$
$- \frac{3\left(\overrightarrow{p}\dot{\overrightarrow{q}}\right)}{\left(\overrightarrow{p}\dot{\overrightarrow{p}}\right)}\overrightarrow{p}$
(2)
$\overrightarrow{r} = -\overrightarrow{q}$
$+ \left(\frac{\overrightarrow{p}\dot{\overrightarrow{q}}}{\overrightarrow{p}\dot{\overrightarrow{p}}}\right)\overrightarrow{p}$
(3)
$\overrightarrow{r} = \overrightarrow{q}$
$+ \left(\frac{\overrightarrow{p}\dot{\overrightarrow{q}}}{\overrightarrow{p}\dot{\overrightarrow{p}}}\right)\overrightarrow{p}$
(4)

$$\vec{r} = -3\vec{q} + \frac{3\left(\vec{p}\,\dot{\vec{q}}\right)}{\left(\vec{p}\,\dot{\vec{p}}\right)}\vec{p}$$