EXERCISE 5.1 - Question No. 1

Prove that the function $f(x) = 5x - 3$ is continuous at $x = 0$, at $x = -3$ and at $x = 5$.

EXERCISE 5.1 - Question No. 2

Examine the continuity of the function $f(x) = 2x^2 - 1$ at $x = 3$.

EXERCISE 5.1 - Question No. 3

Examine the following functions for continuity. (a) $f(x) = x - 5$ (b) $f(x) = \frac{1}{x-5}$ (c) $f(x) = \frac{x^2 - 25}{x + 5}$ (d) $f(x) = |x - 5|$

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EXERCISE 5.1 - Question No. 4

Prove that the function $f(x) = x^n$ is continuous at $x = n$, where n is a positive integer.

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EXERCISE 5.1 - Question No. 5

Is the function f defined by $f(x) = \{x, \quad \text{if} \quad x \leq 15, \quad \text{if} \quad x > 1$

continuous at $x = 0 ? At x = 1 ? At x = 2 ?$

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EXERCISE 5.1 - Question No. 6

Find all points of discontinuity of f, where f is defined by

$f(x) = \{2x + 3, \quad \text{if} \quad x \leq 22x - 3, \quad \text{if} \quad x > 2$

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EXERCISE 5.1 - Question No. 7

Find all points of discontinuity of f, where f is defined by

$f(x) = \{|x| + 3, \quad \text{if} \quad x \leq -3 - 2x, \quad \text{if} \quad -3 < x < 36x + 2 \quad \text{if}$

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EXERCISE 5.1 - Question No. 8

Find all points of discontinuity of f, where f is defined by

$$f(x) = \begin{cases} \frac{|x|}{x}, & \text{if } x \neq 00, \quad \text{if } x = 0 \end{cases}$$

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EXERCISE 5.1 - Question No. 9

Find all points of discontinuity of f, where f is defined by

$$f(x) = \begin{cases} \frac{x}{|x|}, & \text{if } x < 01, \quad \text{if } x \geq 0 \end{cases}$$

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EXERCISE 5.1 - Question No. 10

Find all points of discontinuity of f, where f is defined by

$f(x) = \{x + 1, \quad \text{if} \quad x \geq 1, x^2 + 1, \quad \text{if} \quad x \text{ and } < ; 1$

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EXERCISE 5.1 - Question No. 11

Find all points of discontinuity of f, where f is defined by

$f(x) = \{x^3 - 3, \quad \text{if} \quad x \leq 2x^2 + 1, \quad \text{if} \quad x < 2$

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EXERCISE 5.1 - Question No. 12

Find all points of discontinuity of f, where f is defined by

$f(x) = \{x^{10} - 1, \quad \text{if} \quad x \leq 1x^2, \quad \text{if} \quad x > 1$

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EXERCISE 5.1 - Question No. 13

Is the function defined by

$f(x) = \{x + 5, \quad \text{if} \quad x \leq 1x - 5, \quad \text{if} \quad x > 1$ a continuous

function?

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EXERCISE 5.1 - Question No. 14

Discuss the continuity of the function f, where f is defined by

$f(x) = \{3, \quad \text{if} \quad 0 \leq x \leq 14, \quad \text{if} \quad 1 < x < 35, \quad \text{if} \quad 3 \leq x \leq 10$

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EXERCISE 5.1 - Question No. 15

Discuss the continuity of the function f, where f is defined by

$f(x) = \{(2x, \quad \text{if} \quad x \text{ and } \quad < ; 1), (0, \quad \text{if} \quad 0 \leq x \leq 1), (4x \quad \text{if} \quad x \; a$

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EXERCISE 5.1 - Question No. 16

Discuss the continuity of the function f, where f is defined by

$f(x) = \{ -2, \quad \text{if} \quad x \leq \; -12x, \quad \text{if} \quad -1 < x \leq 12, \quad \text{if} \quad x > 1$

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EXERCISE 5.1 - Question No. 17

Find the relationship between a and b so that the function f defined by

$f(x) = \{ax + 1, \quad \text{if} \quad x \leq 3bx + 3 \quad \text{if} \quad x > 3$ is continuous at

$x = 3$.

EXERCISE 5.1 - Question No. 18

For what value of λ is the function defined by

$f(x) = \{\lambda(x^2 - 2x), \quad \text{if} \quad x \leq 04x + 1, \quad \text{if} \quad x > 0$ continuous at

$x = 0$? What about continuity at $x = 1$?

EXERCISE 5.1 - Question No. 19

Show that the function defined by $g(x) = x[x]$ is discontinuous at all integral points. Here [x] denotes the greatest integer less than or equal to x.

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EXERCISE 5.1 - Question No. 20

Is the function defined by $f(x) = x^2 - \sin x + 5$ continuous at

$x = \pi$?

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EXERCISE 5.1 - Question No. 21

Discuss the continuity of the following functions: (a)

$f(x) = s \in x + \cos x$ (b) $f(x) = s \in x \cos x$ (c)

$f(x) = s \in x \cos x$

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EXERCISE 5.1 - Question No. 22

Discuss the continuity of the cosine, cosecant, secant and cotangent functions.

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EXERCISE 5.1 - Question No. 23

Find all points of discontinuity of f, where

$$f(x) = \begin{cases} \frac{\sin x}{x}, & \text{if } x < 0 \\ x + 1, & \text{if } x \geq 0 \end{cases}$$

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EXERCISE 5.1 - Question No. 24

Determine if f defined by

$$f(x) = \begin{cases} x^2 \dfrac{\sin 1}{x}, & \text{if } x \neq 00, \quad \text{if } x = 0 \end{cases}$$

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EXERCISE 5.1 - Question No. 25

Examine the continuity of f, where f is defined by

$$f(x) = \{\sin x - \cos x, \quad \text{if } x \neq 0 - 1, \quad \text{if } x = 0$$

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EXERCISE 5.1 - Question No. 26

Find the values of k so that the function f is continuous at the indicated point in $f(x) = \begin{cases} \left(\dfrac{k \cos x}{\pi - 2x}, \quad \text{if } x \neq \dfrac{\pi}{2} \right), \left(3 \quad \text{if } x = \dfrac{\pi}{2} \right) \end{cases}$ at $x = \dfrac{\pi}{2}$

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EXERCISE 5.1 - Question No. 27

Find the values of k so that the function f is continuous at the indicated point in $f(x) = \{kx^2, \quad \text{if} \quad x \leq 2, 3 \quad \text{if} \quad x \text{ and } > ; 2 \text{ at } x = 2 .$

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EXERCISE 5.1 - Question No. 28

Find the values of k so that the function f is continuous at the indicated point in $f(x) = \{kx + 1, \quad \text{if} \quad x \leq \pi \cos x, \quad \text{if} \quad x \text{ and } > ; \pi \text{ at } x = \pi$

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EXERCISE 5.1 - Question No. 29

Find the values of k so that the function f is continuous at the indicated point in $f(x) = \{kx + 1, \quad \text{if} \quad x \leq 53x - 5, \quad \text{if} \quad x > 5 \text{ at } x = 5$

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EXERCISE 5.1 - Question No. 30

Find the values of a and b such that the function defined by $f(x) = \{(5, \quad \text{if} \quad x \leq 2), (ax + b, \quad \text{if} \quad 2 < x < 10), (21, \quad \text{if} \quad x \geq$ is a continuous function.

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EXERCISE 5.1 - Question No. 31

Show that the function defined by $f(x) = \cos\left(x^2\right)$ is a continuous function.

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EXERCISE 5.1 - Question No. 32

Show that the function defined by $f(x) = |\cos x|$ is a continuous function.

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EXERCISE 5.1 - Question No. 33

Examine that sin $|x|$ is a continuous function.

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EXERCISE 5.1 - Question No. 34

Find all the points of discontinuity of f defined by

$f(x) = |x| - |x+1|$.

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EXERCISE 5.2 - Question No. 1

Differentiate the functions with respect to x $\sin\left(x^2+5\right)$

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EXERCISE 5.2 - Question No. 2

Differentiate the functions with respect to x $\cos(\sin x)$

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EXERCISE 5.2 - Question No. 3

Differentiate the functions with respect to x $\sin(ax + b)$

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EXERCISE 5.2 - Question No. 4

Differentiate the functions with respect to x $\sec\left(\tan\left(\sqrt{x}\right)\right)$

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EXERCISE 5.2 - Question No. 5

Differentiate the functions with respect to x $\left(\frac{\sin(ax + b)}{\cos(cx + d)}\right.$

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EXERCISE 5.2 - Question No. 6

Differentiate the functions with respect to x $\cos x^3 \dot{\sin}^2 (x^5)$

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EXERCISE 5.2 - Question No. 7

Differentiate the functions with respect to x $2\sqrt{\cot(x^2)}$

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EXERCISE 5.2 - Question No. 8

Differentiate the functions with respect to x $\cos(\sqrt{x})$

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EXERCISE 5.2 - Question No. 9

Prove that the function f given by $f(x) = |x - 1|, x \in R$ is not differentiable at $x = 1$

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EXERCISE 5.2 - Question No. 10

Prove that the greatest integer function defined by

$f(x) = [x], 0 < x < 3$ is not differentiable at $x = 1 and x = 2$.

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EXERCISE 5.3 - Question No. 1

Find $\frac{dy}{dx}$ in the following: $2x + 3y = s \in x$

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EXERCISE 5.3 - Question No. 2

Find $\frac{dy}{dx}$ in the following: $2x + 3y = \sin y$

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EXERCISE 5.3 - Question No. 3

Find $\frac{dy}{dx}$ in the following: $ax + by^2 = \cos y$

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EXERCISE 5.3 - Question No. 4

Find $\frac{dy}{dx}$ in the following: $xy + y^2 = tanx + y$

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EXERCISE 5.3 - Question No. 5

Find $\frac{dy}{dx}$ in the following: $x^2 + xy + y^2 = 100$

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EXERCISE 5.3 - Question No. 6

Find $\frac{dy}{dx}$ in the following: (a)$x^3 + x^2y + xy^2 + y^3 = 81$ (b)

$xy + y^2 = \tan x + y$ (c) $x^2 + xy + y^2 = 100$

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EXERCISE 5.3 - Question No. 7

Find $\frac{dy}{dx}$ in the following: $\sin^2 y + \cos xy = \pi$

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EXERCISE 5.3 - Question No. 8

Find $\frac{dy}{dx}$ in the following: $\sin^2 x + \cos^2 y = 1$

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EXERCISE 5.3 - Question No. 9

Find $\frac{dy}{dx}$ in the following: $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

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EXERCISE 5.3 - Question No. 10

Find $\frac{dy}{dx}$ in the following:

$$y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right), \; -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$$

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EXERCISE 5.3 - Question No. 11

Find $\frac{dy}{dx}$ in the following: $y = \cos^{-1}\left(\frac{1 - x^2}{1 + x^2}\right), 0 < x < 1$

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EXERCISE 5.3 - Question No. 12

Find $\frac{dy}{dx}$ in the following: $y = \sin^{-1}\left(\frac{1 - x^2}{1 + x^2}\right), 0 < x < 1$

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EXERCISE 5.3 - Question No. 13

Find $\frac{dy}{dx}$ in the following: $y = \cos^{-1}\left(\frac{2x}{1+x^2}\right), \ -1 < x < 1$

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EXERCISE 5.3 - Question No. 14

Find $\frac{dy}{dx}$ in the following:

$y = \sin^{-1}\left(2x\sqrt{1-x^2}\right), \ -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$

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EXERCISE 5.3 - Question No. 15

Find $\frac{dy}{dx}$ in the following: $y = \sec^{-1}\left(\frac{1}{2x^2-1}\right)$

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EXERCISE 5.4 - Question No. 1

Differentiate the following w.r.t. x: $\frac{e^x}{\sin x}$

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EXERCISE 5.4 - Question No. 2

Differentiate the following w.r.t. x: $e^{\sin - 1x}$

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EXERCISE 5.4 - Question No. 3

Differentiate the following w.r.t. x: e^{x^3}

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EXERCISE 5.4 - Question No. 4

Differentiate the following w.r.t. x: $\sin\left(\tan^{-1} e^{-x}\right)$

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EXERCISE 5.4 - Question No. 5

Differentiate the following w.r.t. x: $\log(\cos e^{x})$

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EXERCISE 5.4 - Question No. 6

Differentiate the following w.r.t. x: $e^{x} + e^{x} \hat{} 2 + \ldots . + e^{x} \hat{} 5$

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EXERCISE 5.4 - Question No. 7

Differentiate the following w.r.t. x: $\sqrt{e^{\sqrt{x}}}, x > 0$

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EXERCISE 5.4 - Question No. 8

Differentiate the following w.r.t. x: $\log(\log x), x > 1$

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EXERCISE 5.4 - Question No. 9

Differentiate the following w.r.t. x: $\frac{\cos x}{\log x}, x > 0$

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EXERCISE 5.4 - Question No. 10

Differentiate the following w.r.t. x: $\cos(\log x + e^x), x < 0$

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EXERCISE 5.5 - Question No. 1

Differentiate the functions given w.r.t. x: $\cos x \cos 2x \cos 3x$

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EXERCISE 5.5 - Question No. 2

Differentiate the functions given w.r.t. x: $\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$

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EXERCISE 5.5 - Question No. 3

Differentiate the functions given w.r.t. x: $(\log x)^{\cos x}$

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EXERCISE 5.5 - Question No. 4

Differentiate the functions given w.r.t. x: $x^x - 2^{\sin x}$

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EXERCISE 5.5 - Question No. 5

Differentiate the functions given w.r.t. x: $(x+3)^2 x + \dot{4}^3 x + \dot{5}^4$

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EXERCISE 5.5 - Question No. 6

Differentiate the functions given w.r.t. x: $\left(x+\frac{1}{x}\right)^{x}+x^{\left(1+\frac{1}{x}\right)}$

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EXERCISE 5.5 - Question No. 7

Differentiate the following w.r.t. x: $(\log x)^{x}+x^{\log x}$

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EXERCISE 5.5 - Question No. 8

Differentiate the following w.r.t. x: $(\sin x)^{x}+\sin^{-1}\sqrt{x}$

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EXERCISE 5.5 - Question No. 9

Differentiate the following w.r.t. x: $x^{\sin x} + (\sin x)^{\cos x}$

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EXERCISE 5.5 - Question No. 10

Differentiate the following w.r.t. x: $x^{x \cos x} + \frac{x^2 + 1}{x^2 - 1}$

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EXERCISE 5.5 - Question No. 11

Differentiate the following w.r.t. x: $(x \cos x)^x + (x \sin x)^{\frac{1}{x}}$

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EXERCISE 5.5 - Question No. 12

Find $\frac{dy}{dx}$ of the functions given $x^y + y^x = 1$

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EXERCISE 5.5 - Question No. 13

Find $\frac{dy}{dx}$ of the functions given $y^x = x^y$

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EXERCISE 5.5 - Question No. 14

Find $\frac{dy}{dx}$ of the functions given $(\cos x)^y = (\cos y)^x$

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EXERCISE 5.5 - Question No. 15

Find $\frac{dy}{dx}$ of the functions given $xy = e^{(x-y)}$

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EXERCISE 5.5 - Question No. 16

Find the derivative of the function given by

$f(x) = (1+x)(1+x^2)(1+x^4)(1+x^8)$ and hence find $f'(1)$.

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EXERCISE 5.5 - Question No. 17

Differentiate $\left(x^2-5x+8\right)\left(x^3+7x+9\right)$ in three ways mentioned below: (i) by using product rule (ii) by expanding the product to obtain a single polynomial. (iii) by logarithmic differentiation. Do they all give the same answer?

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EXERCISE 5.5 - Question No. 18

If u, v and w are functions of x, then show that

$\frac{d}{dx}(uvw)=\frac{du}{dx}vw+u\frac{dv}{dx}w+uv\frac{dw}{dx}$ in two ways - first by repeated application of product rule, second by logarithmic differentiation.

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EXERCISE 5.6 - Question No. 1

If x and y are connected parametrically by the equations given,

without eliminating the parameter, Find $\frac{dy}{dx}$. $x = 2at^2, y = at^4$

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EXERCISE 5.6 - Question No. 2

If x and y are connected parametrically by the equations given,

without eliminating the parameter, Find $\frac{dy}{dx}$. $x = a\cos\theta, y = b\cos\theta$

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EXERCISE 5.6 - Question No. 3

If x and y are connected parametrically by the equations given,

without eliminating the parameter, Find $\frac{dy}{dx}$. $x = \sin t, y = \cos 2t$

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EXERCISE 5.6 - Question No. 4

If x and y are connected parametrically by the equations given,

without eliminating the parameter, Find $\frac{dy}{dx}$. $x = 4t, y = \frac{4}{t}$

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EXERCISE 5.6 - Question No. 5

If x and y are connected parametrically by the equations given,

without eliminating the parameter, Find $\frac{dy}{dx}$.

$x = \cos\theta - \cos 2\theta, y = \sin\theta - \sin 2\theta$

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EXERCISE 5.6 - Question No. 6

If x and y are connected parametrically by the equations given,

without eliminating the parameter, Find $\frac{dy}{dx}$.

$x = a(\theta - \sin\theta), y = a(1 + \cos\theta)$

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EXERCISE 5.6 - Question No. 7

If x and y are connected parametrically by the equations given,

without eliminating the parameter, Find $\frac{dy}{dx}$.

$x = \frac{\sin^3 t}{\sqrt{\cos 2t}}, y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$

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EXERCISE 5.6 - Question No. 8

If x and y are connected parametrically by the equations given,

without eliminating the parameter, Find $\frac{dy}{dx}$.

$x = a\left(\cos t + \frac{\log \tan t}{2}\right) y = a \sin t$

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EXERCISE 5.6 - Question No. 9

If x and y are connected parametrically by the equations given,

without eliminating the parameter, Find $\frac{dy}{dx}$. $x = a \sec \theta, y = b \tan \theta$

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EXERCISE 5.6 - Question No. 10

If x and y are connected parametrically by the equations given,

without eliminating the parameter, Find $\frac{dy}{dx}$.

$x = a(\cos\theta + \theta\sin\theta), y = a(\sin\theta - \theta\cos\theta)$

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EXERCISE 5.6 - Question No. 11

If $x = \sqrt{a^{\sin\hat{}((-1)t)}}, y = \sqrt{a^{\cos\hat{}((-1)t)}}$, show that

$\frac{dy}{dx} = -\frac{y}{x}$

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EXERCISE 5.7 - Question No. 1

Find the second order derivatives of the functions given $x^2 + 3x + 2$

EXERCISE 5.7 - Question No. 2

Find the second order derivatives of the functions given. x^{20}

EXERCISE 5.7 - Question No. 3

Find the second order derivatives of the functions given. $x\cos x$

EXERCISE 5.7 - Question No. 4

Find the second order derivatives of the functions given. $\log x$

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EXERCISE 5.7 - Question No. 5

Find the second order derivatives of the functions given. $x^3 \log x$

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EXERCISE 5.7 - Question No. 6

Find the second order derivatives of the functions given. $e^x \sin 5x$.

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EXERCISE 5.7 - Question No. 7

Find the second order derivatives of the functions given. $e^{6x} \cos 3x$.

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EXERCISE 5.7 - Question No. 8

Find the second order derivatives of the functions given. $\tan^{-1} x$.

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EXERCISE 5.7 - Question No. 9

Find the second order derivatives of the functions given. $\log(\log x)$

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EXERCISE 5.7 - Question No. 10

Find the second order derivatives of the functions given. sin(log x)

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EXERCISE 5.7 - Question No. 11

If $y = 5\cos x - 3s \in x$, prove that $\frac{d^2y}{dx^2} + y = 0$

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EXERCISE 5.7 - Question No. 12

If $y = \cos^{-1} x$, Find $\frac{d^2y}{dx^2}$ in terms of y alone.

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EXERCISE 5.7 - Question No. 13

If $y = 3\cos(\log x) + 4\sin(\log x)$, show that $x^2y_2 + xy_1 + y = 0$.

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EXERCISE 5.7 - Question No. 14

If $y = Ae^{mx} + Be^{nx}$, show that $\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$

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EXERCISE 5.7 - Question No. 15

If $y = 500e^{7x} + 600e^{-7x}$, show that $\frac{d^2y}{dx^2} = 49y$

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EXERCISE 5.7 - Question No. 16

If $e^y(x+1) = 1$, show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$.

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EXERCISE 5.7 - Question No. 17

If $y = \left(\tan^{-1} x\right)^2$, show that $\left(x^2+1\right)^2 y_2 + 2x\left(x^2+1\right) y_1 = 2$

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EXERCISE 5.8 - Question No. 1

Verify Rolles theorem for the function $f(x) = x^2 + 2x - 8$,

$x \in [-4, 2]$.

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EXERCISE 5.8 - Question No. 2

Examine if Rolles theorem is applicable to any of the following functions. Can you say something about the converse of Rolles theorem from these example? (i) $f(x) = [x]$ for $x \in [5, 9]$ (ii) $f(x) = [x]$ for $x \in [-2, 2]$ (iii) `f(x)=x^

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EXERCISE 5.8 - Question No. 3

If $f: [5, 5]R$ is a differentiable function and if $f'(x)$ does not vanish anywhere, then prove that $f(5)f(5)$.

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EXERCISE 5.8 - Question No. 4

Verify Mean Value Theorem, if $f(x) = x^2 - 4x - 3$ in the interval [a, b], where $a = 1 and b = 4$.

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EXERCISE 5.8 - Question No. 5

Verify Mean Value Theorem, if $f(x) = x^3 - 5x^2 - 3x$ in the interval [a, b], where $a = 1 and b = 3$. Find all $c \in (1, 3)$ for which $f'(c) = 0$.

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EXERCISE 5.8 - Question No. 6

Examine the applicability of Mean Value Theorem for all three functions given in the above exercise 2.

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MISCELLANEOUS EXERCISE - Question No. 1

Differentiate w.r.t. x the function. $\left(3x^2 - 9x + 5\right)^9$

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MISCELLANEOUS EXERCISE - Question No. 2

Differentiate w.r.t. x the function $\sin^3 x + \cos^6 x$

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MISCELLANEOUS EXERCISE - Question No. 3

Differentiate w.r.t. x the function $(5x)^{3\cos 2x}$.

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MISCELLANEOUS EXERCISE - Question No. 4

Differentiate w.r.t. x the function $\sin^{-1}\left(x\sqrt{x}\right), 0 \leq x \leq 1$

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MISCELLANEOUS EXERCISE - Question No. 5

Differentiate w.r.t. x the function $\frac{\cos^{-1}\left(\frac{x}{2}\right)}{\sqrt{2x+7}}$,

-2 and $<;x$ and $<;2$

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MISCELLANEOUS EXERCISE - Question No. 6

Differentiate w.r.t. x the function

$$\frac{\cot^{-1}\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}, 0<x<\frac{\pi}{2}$$

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MISCELLANEOUS EXERCISE - Question No. 7

Differentiate w.r.t. x the function $(\log x)^{\log x}, x>1$

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MISCELLANEOUS EXERCISE - Question No. 8

Differentiate w.r.t. x the function $\cos(a\cos x + b\sin x)$, for some constant a and b.

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MISCELLANEOUS EXERCISE - Question No. 9

Differentiate w.r.t. x the function

$(\sin x - \cos x)^{(\sin x - \cos x)}, \frac{\pi}{4} < x < (3\pi)4$ le b= gt lt /x lt (3pi) gt

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MISCELLANEOUS EXERCISE - Question No. 10

Differentiate w.r.t. x the function $x^x + x^a + a^x + a^a,$ for some fixed $a > 0 and x > 0$.

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MISCELLANEOUS EXERCISE - Question No. 11

Differentiate w.r.t. x the function $x^x \hat{} (2-3) + (x-3)^x \hat{} 2$ for $x > 3$.

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MISCELLANEOUS EXERCISE - Question No. 12

Find $\frac{dy}{dx}$, if $y = 12(1-\cos t), x = 10(t - \sin t), -\frac{\pi}{2} < t < \frac{\pi}{2}$

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MISCELLANEOUS EXERCISE - Question No. 13

Find $\frac{dy}{dx}$, if $y = \sin^{-1} x + \sin^{-1} \sqrt{1 - x^2}$, $-1 \leq x \leq 1$.

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MISCELLANEOUS EXERCISE - Question No. 14

If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, for, $-1 < x < 1$, prove that

$$\frac{dy}{dx} = -\frac{1}{(1+x)^2}.$$

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MISCELLANEOUS EXERCISE - Question No. 15

If $(x-a)^2 + (y-b)^2 = c^2$, for some $c > 0$, prove that

$$\frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$$ is a constant independent of a and b.

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MISCELLANEOUS EXERCISE - Question No. 16

If $\cos y = x\cos(a+y)$, with $\cos a \neq \pm 1$, prove that

$$\frac{dy}{dx} = \left(\frac{\cos^2(a+y)}{\sin a}\right).$$

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MISCELLANEOUS EXERCISE - Question No. 17

If $x = a(\cos t + t\sin t)\ and y = a(\sin tt\cos t)$, find $\frac{d^2y}{dx^2}$.

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MISCELLANEOUS EXERCISE - Question No. 18

If $f(x) = |x|^3$, show that f^x exists for all real x and find it.

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MISCELLANEOUS EXERCISE - Question No. 19

Using mathematical induction prove that $\frac{d}{dx}(x^n) = nx^{n-1}$ for all positive integers n.

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MISCELLANEOUS EXERCISE - Question No. 20

Using the fact that $s \in (A + B) = s \in A \cos B + \cos As \in B$ and the differentiation, obtain the sum formula for cosines.

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MISCELLANEOUS EXERCISE - Question No. 21

Does there exist a function which is continuous everywhere but not differentiable at exactly two points? Justify your answer.

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MISCELLANEOUS EXERCISE - Question No. 22

If $y = |f(x)g(x)h(x)lmnabc|$, prove that

$\frac{dy}{dx} = |f'(x)g'(x)h'(x)lmnabc|$

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MISCELLANEOUS EXERCISE - Question No. 23

If $y = e^{a} \cos^{(-1)x}, \ -1 \leq x \leq 1,$ show that

$$(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} - a^2 y = 0 .$$

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SOLVED EXAMPLES - Question No. 1

Check the continuity of the function f given by

$$f(x) = 2x + 3 at x = 1 .$$

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SOLVED EXAMPLES - Question No. 2

Examine whether the function f given by $f(x) = x^2$ is continuous at

$$x = 0 .$$

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SOLVED EXAMPLES - Question No. 3

Discuss the continuity of the function f given by $f(x) = |x| at x = 0$.

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SOLVED EXAMPLES - Question No. 4

Show that the function f given by

$f(x) = \{x^3 + 3 \quad \text{if} \quad x \neq 01 \quad \text{if} \quad x = 0$ is not continuous at $x = 0$.

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SOLVED EXAMPLES - Question No. 5

Check the points where the constant function $f(x) = k$ is continuous.

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SOLVED EXAMPLES - Question No. 6

Prove that the identity function on real numbers given by $f(x) = x$ is continuous at every real number.

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SOLVED EXAMPLES - Question No. 7

Is the function defined by $f(x) = |x|$, a continuous function?

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SOLVED EXAMPLES - Question No. 8

Discuss the continuity of the function/given by $f(x) = x^3 + x^2 - 1$.

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SOLVED EXAMPLES - Question No. 9

Discuss the continuity of the function f defined by $f(x) = \frac{1}{x}, x \neq 0$.

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SOLVED EXAMPLES - Question No. 10

Discuss the continuity of the function f defined by

$f(x) = \{x + 2 \quad \text{if} \quad x \leq 1 x - 2 \quad \text{if} \quad x > 1$

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SOLVED EXAMPLES - Question No. 11

Find all the points of discontinuity of the function f defined by

$f(x) = \{x + 2, \quad \text{if} \quad x < 10 \quad \text{if} \quad x = 1x - 2 \quad \text{if} \quad x > 1$

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SOLVED EXAMPLES - Question No. 12

Discuss the continuity of the function defined by

$f(x) = \{x + 2, \quad \text{if} \quad x < 0 - x + 2, \quad \text{if} \quad x > 0$

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SOLVED EXAMPLES - Question No. 13

Discuss the continuity of the function f given by

$f(x) = \{x, \quad \text{if} \quad x \geq 0 x^2, \quad \text{if} \quad x < 0$

SOLVED EXAMPLES - Question No. 14

Show that every polynomial function is continuous.

SOLVED EXAMPLES - Question No. 15

Find all the points of discontinuity of the greatest integer function defined by $f(x) = [x]$, where [x] denotes the greatest integer less than or equal to x.

SOLVED EXAMPLES - Question No. 16

Prove that every rational function is continuous.

SOLVED EXAMPLES - Question No. 17

Discuss the continuity of sine function.

SOLVED EXAMPLES - Question No. 18

Prove that the function defined by $f(x) = tanx$ is a continuous function.

SOLVED EXAMPLES - Question No. 19

Show that the function defined by $f(x) = \sin(x^2)$ is a continuous function.

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SOLVED EXAMPLES - Question No. 20

Show that the function f defined by $f(x) = |1 - x + x|$, where x is any real number, is a continuous function.

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SOLVED EXAMPLES - Question No. 21

Find the derivative of the function given $byf(x) = \sin(x^2)$.

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SOLVED EXAMPLES - Question No. 22

Find the derivative of $\tan(2x+3)$.

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SOLVED EXAMPLES - Question No. 23

Differentiate $\sin(\cos(x^2))$ with respect to x.

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SOLVED EXAMPLES - Question No. 24

Find $\frac{dy}{dx}$ if $x-y=\pi$

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SOLVED EXAMPLES - Question No. 25

Find $\frac{dy}{dx}$, if $y + \sin y = \cos x$

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SOLVED EXAMPLES - Question No. 26

Find the derivative of f given by $f(x) = \sin^{-1} x$ assuming it exists.

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SOLVED EXAMPLES - Question No. 27

Find the derivative of / given by $f(x) = \tan^{-1} x$ assuming it exists.

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SOLVED EXAMPLES - Question No. 28

Is it true that $x = e^{\log x}$ for all real

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SOLVED EXAMPLES - Question No. 29

Differentiate the following w.r.t. x: (i) e^{-x} (ii)

$\sin(\log x), x$ and $> ; 0$ (iii) $\cos^{-1}(e^x)$ (iv) $e^{\cos x}$

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SOLVED EXAMPLES - Question No. 30

Differentiate $\sqrt{\frac{(x-3)(x^2+4)}{3x^2+4x+5}}$ w.r.t x.

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SOLVED EXAMPLES - Question No. 31

Differentiate a^x w.r.t. x, where a is a positive constant.

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SOLVED EXAMPLES - Question No. 32

Differentiate $x^{\sin x}$, $x > 0$ w.r.t. x.

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SOLVED EXAMPLES - Question No. 33

Find $\frac{dy}{dx}$, if $y^x + x^y + x^x = a^b$.

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SOLVED EXAMPLES - Question No. 34

Find $\frac{dy}{dx}$, if $x = a\cos\theta, y = a\sin\theta$.

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SOLVED EXAMPLES - Question No. 35

Find $\frac{dy}{dx}$, if $x = at^2, y = 2at$.

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SOLVED EXAMPLES - Question No. 36

Find $\frac{dy}{dx}$, if $x = a(\theta + \sin\theta), y = 1(1 - \cos\theta)$.

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SOLVED EXAMPLES - Question No. 37

Find $\frac{dy}{dx}$, if $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$.

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SOLVED EXAMPLES - Question No. 38

Find $\frac{d^2y}{dx^2}$, if $y = x^3 + \tan x$.

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SOLVED EXAMPLES - Question No. 39

If $y = A\sin x + B\cos x$, then prove that $\frac{d^2y}{dx^2} + y = 0$.

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SOLVED EXAMPLES - Question No. 40

If $y = 3e^{2x} + 2e^{3x}$. Prove that $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$.

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SOLVED EXAMPLES - Question No. 41

If $y = \sin^{-1} x$, show that $\left(1 - x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$.

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SOLVED EXAMPLES - Question No. 42

Verify Rolles theorem for the function $y = x^2 + 2, a = -2$ and $b = 2$.

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SOLVED EXAMPLES - Question No. 43

Verify the Mean Value Theorm for f(x)=x^2 in the interval [2,4].

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SOLVED EXAMPLES - Question No. 44

Differentiate the following w.r.t x. (i) $\sqrt{3x+2} + \left(\frac{1}{\sqrt{2x^2+4}}\right)$ (ii) $e^{\sec^2(x)} + 3\cos^{-1}(x)$ (iii) $\log_7(\log x)$

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SOLVED EXAMPLES - Question No. 45

Find $f'(x)$ if $f(x) = (\sin x)^{\sin x}$ for all $0 < x\pi$

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SOLVED EXAMPLES - Question No. 46

Find df/dx if f(x) = (sin x) ^ sin x for all $o < x < \pi$.

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SOLVED EXAMPLES - Question No. 47

Differentiate $\sin^2 x$ w.r.t $e^{\cos x}$.

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