EXERCISE 1.1 - Question No. 2

Show that the relation R in the set R of real numbers, defined as

$R = \{(a, b) : a \leq b^2\}$ is neither reflexive nor symmetric nor transitive.

EXERCISE 1.1 - Question No. 3

Check whether the relation R defined in the set $\{1, 2, 3, 4, 5, 6\}$ as

$R = \{(a, b) : b = a + 1\}$ is reflexive, symmetric or transitive.

EXERCISE 1.1 - Question No. 4

Show that the relation R in R defined as $R = \{(a, b) : a \leq b\}$, is reflexive and transitive but not symmetric.

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EXERCISE 1.1 - Question No. 5

Check whether the relation R in R defined by $R = \{(a, b) : a \leq b^3\}$ is reflexive, symmetric or transitive.

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EXERCISE 1.1 - Question No. 6

Show that the relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 2), (2, 1)\}$ is symmetric but neither reflexive nor transitive.

EXERCISE 1.1 - Question No. 7

Show that the relation R in the set A of all the books in a library of a college, given by R = {(x, y) : x and y have same number of pages} is an equivalence relation.

EXERCISE 1.1 - Question No. 8

Show that the relation R in the set $A = \{1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : |ab| iseven\}$, is an equivalence relation. Show that all the elements of $\{1, 3, 5\}$ are related to each other and all the e

EXERCISE 1.1 - Question No. 9

Show that each of the relation R in the set $A = \{x \in Z : 0 \leq x \leq 12\}$, given by (i) $R = \{(a, b) : |ab| isa\mu ltipleof 4\}$ (ii) $R = \{(a, b) : a = b\}$ is an equivalence relation. Find the set of a

EXERCISE 1.1 - Question No. 10

Give an example of a relation. Which is (i) Symmetric but neither reflexive nor transitive. (ii) Transitive but neither reflexive nor symmetric. (iii) Reflexive and symmetric but not transitive. (iv) Reflexive and transitive but not symmetric. (v) Symmetric and transitive but not reflexive.

EXERCISE 1.1 - Question No. 11

Show that the relation R in the set A of points in a plane given by

$R = \{(P, Q):$ distance of the point P from the origin is same as the distance of the point Q from the origin$\}$, is an equivalence relation. Further, show that the set o

EXERCISE 1.1 - Question No. 12

Show that the relation R defined in the set A of all triangles as

$R = \{(T_1, T_2) : T_1 (is similar to T)_2\}$, is equivalence relation. Consider three right angle triangles T_1 with sides 3, 4, 5, T_2 with sides 5, 12, 13 and T_3 w

EXERCISE 1.1 - Question No. 13

Show that the relation R defined in the set A of all polygons as

$R = \{(P_1, P_2) : P_1(\text{ and } P)_2 havesamenumberofsides\}$, is an equivalence

relation. What is the set of all elements in A related to the right angle triangle

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EXERCISE 1.1 - Question No. 14

Let L be the set of all lines in XY plane and R be the relation in L defined as

$R = \{(L_1, L_2) : L_1 \text{ (is parallel to } L_2)\}$ Show that R is an equivalence relation.

Find the set of all lines related to the line $y - 2x + 4$.

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EXERCISE 1.1 - Question No. 15

Let R be the relation in the set $\{1, 2, 3, 4\}$ given by

$R = \{(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)\}$. Choose the correct

answer. (A) R is reflexive and symmetric but not transitive. (B) R is reflexive

and transitive but not symmetric (C) R is symmetric and transitive butnot

reflexive (D) R is an equivalence relation

EXERCISE 1.1 - Question No. 16

Let R be the relation in the set N given by $R = \{(a, b) : a = b2, b > 6\}$.

Choose the correct answer. (A) $(2, 4) \in R$ (B) $(3, 8) \in R$ (C) $(6, 8) \in R$ (D)

$(8, 7)R$

EXERCISE 1.2 - Question No. 1

Show that the functions $f : R_{.} \rightarrow R_{.}$ defined by $f(x) = \frac{1}{x}$ is one-one and onto. where R* is the set of all non zero real numbers. Is the result true, if the domain R* is replaced by N with co-domain being same as R*.

EXERCISE 1.2 - Question No. 2

Check the injectivity and surjectivity of the following functions: (i) $f : N \rightarrow N$ given by $f(x) = x^2$ (ii) $f : Z \rightarrow Z$ given by $f(x) = x^2$ (iii) $f : R \rightarrow R$ given by $f(x) = x^2$ (iv) $f : N \rightarrow N$ given by $f(x) = x^3$ (v) `f : Z rarr

EXERCISE 1.2 - Question No. 3

Prove that the Greatest Integer Function $f: R \to R,$ given by $f(x) = [x]$, is neither one-one nor onto, where [x] denotes the greatest integer less than or equal to x.

EXERCISE 1.2 - Question No. 4

Show that the Modulus Function $f: R \to R,$ given by $f(x) = |x|$, is neither one-one nor onto, where $|x|$ is x, if x is positive or 0 and $|x|$ is x , if x is negative.

EXERCISE 1.2 - Question No. 5

Show that the Signum Function $f: R \rightarrow R$, given by

$f(x) = \{1, \quad \text{if} \quad x > 00, \quad \text{if} \quad x = 0 - 1, \quad \text{if} \quad x < 0$ is neither one-one nor onto.

EXERCISE 1.2 - Question No. 6

Let $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and let $f = \{(1, 4), (2, 5), (3, 6)\}$ be a function from A to B. Show that f is one-one.

EXERCISE 1.2 - Question No. 7

In each of the following cases, state whether the function is one-one, onto or bijective. Justify your answer. (i) $f: R \rightarrow R$, defined by $f(x) = 34x$ (ii)

$f: R \to R,\ \text{defined by } f(x) = 1 + x^2$

EXERCISE 1.2 - Question No. 8

Let A and B be sets. Show that $f: A \times B \to B \times A$ such that $f(a, b) = (b, a)$

is bijective function.

EXERCISE 1.2 - Question No. 9

Let $f: N -$ and $>$; N be defined by

$f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } \quad nisodd \frac{n}{2}, & \text{if } \quad niseven \end{cases}$ for all $n \in N$. State whether

the function f is bijective. Justify your answer.

EXERCISE 1.2 - Question No. 10

Let $A = R - \{3\}$ and $B = R - \{1\}$. Consider the function $f: A \to B$ defined by $(x) = \left(\frac{x-2}{x-3}\right)$. Is f one-one and onto? Justify your answer.

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EXERCISE 1.2 - Question No. 11

Let $f: R \to R$ be defined as $f(x) = x^4$. Choose the correct answer. (A) f is one-one onto (B) f is many-one onto (C) f is one-one but not onto (D) f is neither one-one nor onto

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EXERCISE 1.2 - Question No. 12

Let $f: R \rightarrow R$ be defined as $f(x) = 3x$. Choose the correct answer. (A) f is one-one onto (B) f is many-one onto (C) f is one-one but not onto (D) f is neither one-one nor onto.

EXERCISE 1.3 - Question No. 1

Let $f: \{1, 3, 4\} \rightarrow \{1, 2, 5\}$ and $g: \{1, 2, 5\} \rightarrow \{1, 3\}$ be given by $f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(1, 3), (2, 3), (5, 1)\}$. Write down gof .

EXERCISE 1.3 - Question No. 2

Let f, g and h be functions from R to R. Show that

$(f + g)oh = foh + goh\left(f\dot{g}\right)oh = (foh)g\dot{o}h$

EXERCISE 1.3 - Question No. 3

Find fog and gof , if (i) () $f(x) = |x|$ and $g(x) = |5x - 2|$ (ii) $f(x) = 8x^3$ and $g(x) = x^{1/3}$

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EXERCISE 1.3 - Question No. 4

If $f(x) = \frac{4x + 3}{(6x - 4)}$,$x \neq \frac{2}{3}$, show that $fof(x) = x$, for all $x \neq \frac{2}{3}$. What is the inverse of f?

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EXERCISE 1.3 - Question No. 5

State with reason whether following functions have inverse (i)

$f: \{1, 2, 3, 4\} \rightarrow \{10\} with f = \{(1, 10), (2, 10), (3, 10), (4, 10)\}$ (ii)

$g : \{5, 6, 7, 8\} \rightarrow \{1, 2, 3, 4\} with g = \{(5, 4), (6, 3), (7, 4), (8, 2)\}$ (iii) `h :

{2,3,4,5} rarr {7,9

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EXERCISE 1.3 - Question No. 6

Show that $f : [-1, 1] \rightarrow R$, given by $f(x) = \frac{x}{(x + 2)}$ is one- one . Find the

inverse of the function $f : [-1, 1]$

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EXERCISE 1.3 - Question No. 7

Consider $f : R \rightarrow R$ given by $f(x) = 4x + 3$. Show that f is invertible. Find

the inverse of f.

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EXERCISE 1.3 - Question No. 8

Consider $f: R_+$ and $> ; [4, \infty)$ given by $f(x) = x^2 + 4$. Show that f is invertible with the inverse f^{-1} of given f by $f^{-1}(y) = \sqrt{y - 4}$ where R_+ is the set of all non-negative real numbers.

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EXERCISE 1.3 - Question No. 9

Consider $f: R_+ \rightarrow [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible with $f^{-1}(y) = \left(\frac{\left(\sqrt{y+6}\right) - 1}{3}\right)$

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EXERCISE 1.3 - Question No. 10

Let $f: X \rightarrow Y$ be an invertible function. Show that f has unique inverse. (Hint: suppose $g_1($ and $g)_2$ are two inverses of f. Then for all $y \in Y, fog_1(y) = I_Y(y) = fog_2(y)$ Use one oneness of f).

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EXERCISE 1.3 - Question No. 11

Consider $f: \{1, 2, 3\} \rightarrow \{a, b, c\}$ given by $f(1) = a$, $f(2) = b$ and $f(3) = c$. Find f^{-1} and show that $(f^{-1})^{-1} = f$.

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EXERCISE 1.3 - Question No. 12

Let $f: X \rightarrow Y$ be an invertible function. Show that the inverse of f^{-1} is f, i.e., $(f^{-1})^{-1} = f$.

EXERCISE 1.3 - Question No. 13

If $f: R \rightarrow R$ be given by $f(x) = \left(3 - x^3\right)^{1/3}$, then $fof(x)$ is (a) $\frac{1}{x^3}$ (b) x^3 (c) x (d) $\left(3 - x^3\right)$

EXERCISE 1.3 - Question No. 14

Let $f: R - \left\{-\frac{4}{3}\right\} \rightarrow R$ be a function as $f(x) = \frac{4x}{3x+4}$. The inverse of f is map, $g: Ran \geq f \rightarrow R - \left\{-\frac{4}{3}\right\}$ given by. (a) $g(y) = \frac{3y}{3-4y}$ (b) $g(y) = \frac{4y}{4-3y}$ (c) $g(y) = \frac{4y}{3-4y}$ (d) $g(y) = \frac{3y}{4-3y}$

EXERCISE 1.4 - Question No. 1

Determine whether or not each of the definition of given below gives a binary operation. In the event that * is not a binary operation, give justification for this. (i) $OnZ^+, def \in e \cdot bya \cdot b = a - b$ (ii) `O n Z^+,

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EXERCISE 1.4 - Question No. 2

For each binary operation * defined below, determine whether * is commutative or associative. (i) $OnZ, def \in ea \cdot b = a - b$ (ii) $OnQ, def \in ea \cdot b = ab + 1$ (iii) $OnQ, def \in ea \cdot b = \frac{ab}{2}$ (iv) `O

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EXERCISE 1.4 - Question No. 3

Consider the binary operation $\wedge$ on the set $\{1, 2, 3, 4, 5\}$ defined by

$a \wedge b = m \in \{a, b\}$. Write the operation table of the operation $\wedge$.

EXERCISE 1.4 - Question No. 4

Consider a binary operation * on the set $\{1, 2, 3, 4, 5\}$ given by the following multiplication table. (i) Compute (2*3)*4 a n d 2*(3*4) (ii) I s * com mu t a t i v e ? (iii) Compute (2*3)

EXERCISE 1.4 - Question No. 5

Let $\cdot\ '$ be the binary operation on the set $\{1, 2, 3, 4, 5\}$ defined by

$a \cdot 'b = H\dot{C}\dot{F}.$ of a and b. Is the operation $\cdot\ '$ same as the operation $\cdot$ defined in Exercise 4 above? Justify your answer.

EXERCISE 1.4 - Question No. 6

Let $\cdot$ be the binary operation on N given by $a \cdot b = L\dot{C}\dot{M}.$ of a and b. Find (i) $5 \cdot 7$, $20 \cdot 16$ (ii) Is $\cdot$ commutative? (iii) Is $\cdot$ associative? (iv) Find the identity of $\cdot$ in N (v) Which elements of N are invert

EXERCISE 1.4 - Question No. 7

Is $\cdot$ defined on the set $\{1, 2, 3, 4, 5\} bya \cdot b = L\dot{C}\dot{M}.$ of a and b a binary operation? Justify your answer.

EXERCISE 1.4 - Question No. 8

Let $*$ be the binary operation on N defined by $a * b = H\dot{C}\dot{F}.$ of a and b. Is $*$ commutative? Is $*$ associative? Does there exist identity for this binary operation on N?

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EXERCISE 1.4 - Question No. 9

Let $\cdot$ be a binary operation on the set Q of rational numbers as follows: (i) $a \cdot b = a - b$ (ii) $a \cdot b = a^2 + b^2$ (iii) $a \cdot b = a + ab$ (iv) $a \cdot b = (a - b)^2$ (v) $a \cdot b = \frac{ab}{4}$ (vi) $a \cdot b = ab^2$ Find which of the binary operations are commutative and which are associative

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EXERCISE 1.4 - Question No. 10

Show that none of the operations given below has identity. (i) $a * b = a - b$

(ii) $a * b = a^2 + b^2$ (iii) $a * b = a + ab$ (iv) $a * b = (a - b)^2$ (v) $a * b = \frac{ab}{4}$

(vi) $a * b = ab^2$

EXERCISE 1.4 - Question No. 11

Let A = N N and ? be the binary operation on A defined by (a , b) ? (c , d) = (a + c , b + d) . Show that ? is commutative and associative. Find the identity element for ? on A, if any.

EXERCISE 1.4 - Question No. 12

State whether the following statements are true or false. Justify. (i) For an arbitrary binary operation on a set N, a *a = a ? a ? N (ii) If . is a commutative binary operation on N, then $a * (b * c) = (c * b) * a$

EXERCISE 1.4 - Question No. 13

Consider a binary operation. on N defined $a.\,b = a^3 + b^3.$ Choose the correct answer. (A) Is . both associative and commutative? (B) Is. commutative but not associative? (C) Is . associative but not commutative? (D) Is

MISCELLANEOUS EXERCISE - Question No. 1

Let $f: R \rightarrow R$ be defined as $f(x) = 10x + 7$. Find the function $g: R \rightarrow R$ such that $gof = fog = 1_R$

MISCELLANEOUS EXERCISE - Question No. 2

Let $f: W \rightarrow W$ be defined as $f(n) = n - 1$, if is odd and $f(n) = n + 1$, if n is even. Show that f is invertible. Find the inverse of f. Here, W is the set of all whole numbers.

MISCELLANEOUS EXERCISE - Question No. 3

If $f: R \rightarrow R$ is defined by $f(x) = x^2 - 3x + 2$, find $f(f(x))$.

MISCELLANEOUS EXERCISE - Question No. 4

Show that function $f: R \rightarrow \{x \in R: -1 < x < 1\}$ defined by

$f(x) = \frac{x}{1+|x|}, x \in R$ is one one and onto function

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MISCELLANEOUS EXERCISE - Question No. 5

Show that the function $f: R \rightarrow R$ given by $f(x) = x^3$ is injective.

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MISCELLANEOUS EXERCISE - Question No. 6

Give examples of two functions $f: N - \quad \text{and} \quad > ; Z and g: Z - \quad \text{and} \quad > ; Z$

such that go f is injective but g is not injective. (Hint: Consider

$f(x) = x and g(x) = |x|$)

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MISCELLANEOUS EXERCISE - Question No. 7

Given examples of two functions

$f : N - \quad \text{and} \quad > ; N and g : N - \quad \text{and} \quad > ; N$ such that g o f is onto but f is

not onto. (Hint: Consider $f(x) = x + 1 and g(x) = |x|$) .

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MISCELLANEOUS EXERCISE - Question No. 8

Given a non-empty set X, consider P(X) which is the set of all subsets of X.

Define the relation R in P(X) as follows: For subsets A, B in P(X), ARB if and

only if $A \subset B$. Is R an equivalence relation on P(X)? Justify you answer

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MISCELLANEOUS EXERCISE - Question No. 9

Given a non-empty set X, consider the binary operation

$\cdot : P(X) \times P(X) \to P(X)$ given by $A \cdot B = A \cap B \forall A, B \in P(X)$ is the power set of X. Show that X is the identity element for this operation and X is the only invertible element i

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MISCELLANEOUS EXERCISE - Question No. 10

Find the number of all onto functions from the set $\{1, 2, 3, , n)$ to itself.

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MISCELLANEOUS EXERCISE - Question No. 11

Let $S = \{a, b, c\} and T = \{1, 2, 3\}$. Find F^{-1} of the following functions F from S to T, if it exists. (i)$F = \{(a, 3), (b, 2), (c, 1)\}$ (ii) $F = \{(a, 2), (b, 1), (c, 1)\}$

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MISCELLANEOUS EXERCISE - Question No. 12

Consider the binary operations $\cdot : R \times R -$ and $>$; R and $o: R \times R -$ and $>$; R defined as $a \cdot b|a - b|$ and $aob = a, \forall a, b \in R$.

Show that * is commutative but not associative, o is associative but not commutative.

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MISCELLANEOUS EXERCISE - Question No. 13

Given a non -empty set X, let $\cdot : P(X) \times P(X) -$ and $> ; P(X)$ be defined as $A \cdot B = (A - B) \cup (B - A), \forall A, B \in P(X)$. Show that the empty set φ is the identity for the operation * and all the elements A of P(A) are invertible withA^{-1}=A

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MISCELLANEOUS EXERCISE - Question No. 14

Define a binary operation *on the set $\{0, 1, 2, 3, 4, 5\}$ as $a \cdot b = \{a + b \quad \text{if} \quad a + b \text{ and } \quad < ; 6 \; a + b - 6, \quad \text{if} \quad a + b \geq 6$ Show that zero is the identity for this operation and each element $a \neq 0$ of the set is invertible with 6 a being t

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MISCELLANEOUS EXERCISE - Question No. 15

Let $A = \{-1, 0, 1, 2\}$, $B = \{-4, -2, 0, 2\}$ and $f, g: A -$ and $>; B$

be functions defined by $f(x) = x^2 - x, x \in A$ and

$g(x) = 2\left|x - \left(\frac{1}{2}\right)\right| - 1, x \in A$. Are f and g equal? Justify your answer

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MISCELLANEOUS EXERCISE - Question No. 16

Let$A = \{1, 2, 3\}$ Then number of relations containing $(1, 2) and (1, 3)$ which are reflexive and symmetric but not transitive is (A) 1 (B) 2 (C) 3 (D) 4

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MISCELLANEOUS EXERCISE - Question No. 17

Let $A = \{1, 2, 3\}$. Then number of equivalence relations containing (1, 2) is (A) 1 (B) 2 (C) 3 (D) 4

MISCELLANEOUS EXERCISE - Question No. 18

Let $f: R \rightarrow R$ be the Signum Function defined as

$f(x) = \{1, x > 00, x = 0 - 1, x < 1$ and $g: R \rightarrow R$ be the Greatest Integer

Function given by $g(x) = [x]$, where [x] is greatest integer less than or equal

to x. Then does fo

MISCELLANEOUS EXERCISE - Question No. 19

Number of binary operations on the set {a, b} are (A) 10 (B) 16 (C) 20 (D) 8

SOLVED EXAMPLES - Question No. 1

Let A be the set of all students of a boys school. Show that the relation R in A given by R = {(a, b) : a is sister of b} is the empty relation and R' = {(a, b) : the difference between heights of a and b is less than 3 meters} is the un

SOLVED EXAMPLES - Question No. 2

Let T be the set of all triangles in a plane with R a relation in T given by $R = \{(T_1, T_2) : T_1 (is congruent to T)_2\}$. Show that R is an equivalence relation.

SOLVED EXAMPLES - Question No. 3

Let L be the set of all lines in a plane and R be the relation in L defined as

$R = \{(L_1, L_2) : L_1(\text{is perpendicular to } L_2\}$ Show that R is symmetric but

neither reflexive nor transitive.

SOLVED EXAMPLES - Question No. 4

Show that the relation R in the set $\{1, 2, 3\}$ given by

$R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}$ is reflexive but neither symmetric

nor transitive.

SOLVED EXAMPLES - Question No. 5

Show that the relation R in the set Z of integers given by R = { (a , b) : 2 d i v i d e s a b } is an equivalence relation.

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SOLVED EXAMPLES - Question No. 6

Let R be the relation defined in the set $A = \{1, 2, 3, 4, 5, 6, 7\}$ by $R = \{(a, b):$ both a and b are either odd or even$\}$. Show that R is an equivalence relation. Further, show that all the elements of the subset `{1, 3

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SOLVED EXAMPLES - Question No. 7

Let A be the set of all 50 students of Class X in a school. Let $f: A \rightarrow N$ be function defined by f (x) = roll number of the student x. Show that f is one-one but not onto.

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SOLVED EXAMPLES - Question No. 8

Show that the function $f: N \rightarrow N$, given by $f(x) = 2x$, is one-one but not onto.

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SOLVED EXAMPLES - Question No. 9

Prove that the function $f: R \rightarrow R$, given by $f(x) = 2x$, is one-one and onto.

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SOLVED EXAMPLES - Question No. 10

Show that the function $f: N \rightarrow N$, given by $f(1) = f(2) = 1$ and $f(x) = x1$

, for every $x > 2$, is onto but not one-one.

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SOLVED EXAMPLES - Question No. 11

Show that the function $f: R \rightarrow R$, defined as $f(x) = x^2$, is neither one-one

nor onto.

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SOLVED EXAMPLES - Question No. 12

Show that $f: N \rightarrow N$, given by

$f(x) = \{x + 1, \quad \text{if} \quad \xi sad\ddot{x} - 1, \quad \text{if} \quad \xi seven_.$ is both one-one and onto.

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SOLVED EXAMPLES - Question No. 13

Show that an onto function $f:\{1, 2, 3\} \rightarrow \{1, 2, 3\}$ is always one-one.

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SOLVED EXAMPLES - Question No. 14

Show that a one-one function $f:\{1, 2, 3\} \rightarrow \{1, 2, 3\}$ must be onto.

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SOLVED EXAMPLES - Question No. 15

Let $f:\{2, 3, 4, 5\} \rightarrow \{3, 4, 5, 9\}$ and $g:\{3, 4, 5, 9\} \rightarrow \{7, 11, 15\}$ be

functions defined as $f(2) = 3$, $f(3) = 4$, $f(4) = f(5) = 5$ and

$g(3) = g(4) = 7$ and `g (5)

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SOLVED EXAMPLES - Question No. 16

Find gof and fog, if $f: R \rightarrow R$ and $g: R \rightarrow R$ are given by $f(x) = \cos x$ and $g(x) = 3x^2$. Show that $gof \neq fog$.

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SOLVED EXAMPLES - Question No. 17

Show that if $f: R - \left\{\frac{7}{5}\right\} -$ and $>; R - \left\{\frac{3}{5}\right\}$ is defined by $f(x) = \frac{3x+4}{5x-7}$ and $g: R - \left\{\frac{3}{5}\right\} -$ and $>; R - \left\{\frac{7}{5}\right\}$ is define by $g(x) = \frac{7x+4}{5x-3}$, then $fog = I_A$ and $gof = I_B$, where $A = R - \left\{\frac{3}{5}\right\}, B = R - \left\{\frac{7}{5}\right\}; I_A(x) = x, \forall x \in A, I_B(x) = x, \forall x \in B$ are called ideal

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SOLVED EXAMPLES - Question No. 18

Show that if $f: A \to B$ and $g: B \to C$ are one-one, then $gof: A \to C$ is also one-one.

SOLVED EXAMPLES - Question No. 19

Show that if $f: A \to B$ and $g: B \to C$ are onto, then $gof: A \to C$ is also onto.

SOLVED EXAMPLES - Question No. 20

Consider functions f and g such that composite gof is defined and is one-one. Are f and g both necessarily one-one.

SOLVED EXAMPLES - Question No. 21

Are f and g both necessarily onto, if gof is onto?

SOLVED EXAMPLES - Question No. 22

Let $f: \{1, 2, 3\} \rightarrow \{a, b, c\}$ be one-one and onto function given by $f(1) = a$,

$f(2) = b$ and $f(3) = c$. Show that there exists a function

$g: \{a, b, c\} \rightarrow \{1, 2, 3\}$ such that $gof = I_x$ and `fog=

SOLVED EXAMPLES - Question No. 23

Let $f: N \rightarrow Y$ be a function defined as $f(x) = 4x + 3$, where,

$Y = \{y \in N: y = 4x + 3$ for some $x \in N$ $\}$. Show that f is invertible. Find the inverse

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SOLVED EXAMPLES - Question No. 24

Let $Y = \{n^2 : n \in N\} \in N$. Consider $f : N \to Y$ as $f(n) = n^2$. Show that f is invertible. Find the inverse of f.

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SOLVED EXAMPLES - Question No. 25

Let $f : N \to R$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f : N \to S$, where, S is the range of f, is invertible. Find the inverse of f.

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SOLVED EXAMPLES - Question No. 26

Consider $f: N \rightarrow N$, $g: N \rightarrow N$ and $h: N \rightarrow R$ defined as $f(x) = 2x$,

$g(y) = 3y + 4$ and $h(z) = s \in z$, ∀ x, y and z in N. Show that ho(gof) =

(hog) of.

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SOLVED EXAMPLES - Question No. 27

Consider $f: \{1, 2, 3\} \rightarrow \{a, b, c\}$ and $g: \{a, b, c\} \rightarrow \{apple, ball, cat\}$

defined as $f(1) = a$, $f(2) = b$, $f(3) = c$, $g(a) = apple$,`g(b) = b a l l a n d

g(c) = c

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SOLVED EXAMPLES - Question No. 28

Let $S = \{1, 2, 3\}$. Determine whether the functions $f: S \rightarrow S$ defined as below have inverses. Find f^{-1} , if it exists. (a) $f = \{(1, 1), (2, 2), (3, 3)\}$ (b) $f = \{(1, 2), (2, 1), (3, 1)\}$ (c) `f=

SOLVED EXAMPLES - Question No. 29

Show that addition, subtraction and multiplication are binary operations on R, but division is not a binary operation on R. Further, show that division is a binary operation on the set R of nonzero real numbers.

SOLVED EXAMPLES - Question No. 30

Show that subtraction and division are not binary operations on N.

SOLVED EXAMPLES - Question No. 31

Show that $\cdot : R \times R \to R$ given by $(a, b) \to a + 4b^2$ is a binary operation.

SOLVED EXAMPLES - Question No. 32

Let P be the set of all subsets of a given set X. Show that $\cup : P \times P \to P$ given by $(A, B) \to A \cup B$ and $\cap : P \times P \to P$ given by $(A, B) \to A \cap B$ are binary operations on the set P.

SOLVED EXAMPLES - Question No. 33

Show that the $\vee : R \to R$ given by $(a, b) \to max\{a, b\}$ and the $\wedge : R \to R$

$\to$ given by $(a, b) \to m \in \{a, b)$ are binary operations.

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SOLVED EXAMPLES - Question No. 34

Show that $+ : R \times R \to R$ and $\times : R \times R \to R$ are commutative binary operations, but $: R \times R \to R$ and $\div : R_{.} \times R_{.} \to R_{.}$ are not commutative.

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SOLVED EXAMPLES - Question No. 35

Show that $\cdot : R \times R \to R$ defined by $a \cdot b = a + 2b$ is not commutative.

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SOLVED EXAMPLES - Question No. 36

Show that addition and multiplication are associative binary operation on R.

But subtraction is not associative on R. Division is not associative on R*.

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SOLVED EXAMPLES - Question No. 37

Show that $\cdot : R \times R \to R$ given by $a \cdot b = a + 2b$ is not associative.

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SOLVED EXAMPLES - Question No. 38

Show that zero is the identity for addition on R and 1 is the identity for

multiplication on R. But there is no identity element for the operations

$- : R \times R -$ and $> ; R$ and $\div : R. \times R. -$ and $> ; R..$

SOLVED EXAMPLES - Question No. 39

Show that a is the inverse of a for the addition operation + on R and $\frac{1}{a}$ is the inverse of $a \neq 0$ for the multiplication operation $\times$ on R.

SOLVED EXAMPLES - Question No. 40

Show that $-a$ is not the inverse of $a \in N$ for the addition operation + on N and $\frac{1}{a}$ is not the inverse of $a \in N$ for multiplication operation $\times$ on N, for $a \neq 1$.

SOLVED EXAMPLES - Question No. 41

If R_1 and R_2 are equivalence relations in a set A, show that $R_1 \cap R_2$ is also an equivalence relation.

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SOLVED EXAMPLES - Question No. 42

Let R be a relation on the set A of ordered pairs of positive integers defined by $(x, y)R(u, v)$ if and only if $xv = yu$. Show that R is an equivalence relation.

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SOLVED EXAMPLES - Question No. 43

Let $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Let R_1 be a relation in X given by $R_1 = \{(x, y) : xy$ is divisible by $3\}$ and R_2 be another relation on X given by

`R_2= {(x , y): {x , y}sub{1, 4, 7}}

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SOLVED EXAMPLES - Question No. 44

Let $f: X \to Y$ be a function. Define a relation R in X given by

$R = \{(a, b): f(a) = f(b)\}$. Examine if R is an equivalence relation.

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SOLVED EXAMPLES - Question No. 45

Determine which of the following binary operations on the set N are associative

and which are commutative. (a) $(b)(c)a \cdot b = 1 \forall a, b \in N(d)$ (e) (b)

$(f)(g)a \cdot b = (h)\left((i)\dfrac{a+b}{j}2(k)(l) \forall a, b \in N(m)\right.$ (n)

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SOLVED EXAMPLES - Question No. 46

Find the number of all one-one functions from set $A = \{1, 2, 3\}$ to itself.

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SOLVED EXAMPLES - Question No. 47

Let $A = \{1, 2, 3\}$. Then show that the number of relations containing $(1, 2)$ and $(2, 3)$ which are reflexive and transitive but not symmetric is four.

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SOLVED EXAMPLES - Question No. 48

Show that the number of equivalence relation in the set $\{1, 2, 3\}$ containing $(1, 2)$ and $(2, 1)$ is two.

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SOLVED EXAMPLES - Question No. 49

Show that the number of binary operations on $\{1, 2\}$ having 1 as identity and having 2 as the inverse of 2 is exactly one.

SOLVED EXAMPLES - Question No. 50

Consider the identity function $I_N : N \rightarrow N$ defined as $I_N(x) = x \, \forall x \in N$. Show that although IN is onto but $I_N + I_N : N \rightarrow N$ defined as $(I_N + I_N)(x) = I_N(x) + I_N(x) = x + x = 2x$ is not onto.