JEE ADVANCED-PART TEST-12 (MATHS)

SOLUTION OF MOCK TEST

CONDUCTED ON DOUBTNUT APP

Q-1 - JEE ADVANCED-PART TEST-12 (MATHS)-MATHS

The sum

$$\frac{3}{1!+2!+3!}+\frac{4}{2!+3!+4!}+\;+\frac{2016}{2014!+2015!+2016!} \text{ is}$$

equal to

(A) $\frac{1}{2}-\frac{1}{2014!}$

(B) $\frac{1}{2}-\frac{1}{2016!}$

(C) $\frac{1}{2016!-2018!}$

(D) $\frac{1}{2017!}-\frac{1}{2018!}$

Correct Option : B

SOLUTION

$$S = \sum_{K=1}^{2014} \frac{K+2}{K! + (K+1)! + (K+2)!} = \sum_{K=1}^{2014} \frac{K+2}{K!(K+2)^2} = \sum_{K=}^{201}$$

$$= \sum_{K=1}^{2014} \frac{K+1}{(K+2)} = \sum_{K=1}^{2014} \frac{K+2-1}{(K+2)!} = \sum_{K=1}^{2014} \left[\frac{1}{(K+1)!} - \frac{1}{(K+}\right.$$

Q-2 - JEE ADVANCED-PART TEST-12 (MATHS)-MATHS

Let A,G,H are respectively the A.M., G.M and H.M. between two positive numbers. If $xA = yG = zH$ where x,y,z are non-zero quantities then x,y,z aer in

(A) A.P

(B) G.P

(C) H.P

(D) A.G.P

Correct Option : B

SOLUTION

$$xA = yG \Rightarrow \frac{x}{y} = \frac{G}{A} = \frac{2\sqrt{ab}}{a+b}$$

$$yG = zH \Rightarrow \frac{y}{z} = \frac{2\sqrt{ab}}{a+b} \therefore \frac{x}{y} = \frac{y}{z}$$

Q-3 - JEE ADVANCED-PART TEST-12 (MATHS)-MATHS

The sum of the coefficients of the polynimial obtained by collection of like terms after the expansion of

$\left(1 - 2x + 2x^2\right)^{743}\left(2 + 3x - 4x^2\right)^{744}$ is

(A) 2974

(B) 1487

(C) 1

(D) 0

Correct Option : C

SOLUTION

$$\left(1-2x+2x^2\right)^{743}\left(2+3x-4x^2\right)^{744}=a_0+a_1x+.+a_{2974}x^{2974}$$

put $x=1 \Rightarrow 1=a_0+a_1+,.+a_{2974}$

Q-4 - JEE ADVANCED-PART TEST-12 (MATHS)-MATHS

if $a_i, i=1,2,3,4$ be four real numbers of same sign then the minimum value of $\sum\sum\frac{a_i}{a_j}$ where $I, j \in \{1,2,3,4\}$ and $i \neq j$ is

(A) 6

(B) 8

(C) 12

(D) 24

Correct Option : C

SOLUTION

Let

$$E = \frac{a_1}{a_2} + \frac{a_1}{a_3} + \frac{a_1}{a_4} + \frac{a_2}{a_1} + \frac{a_2}{a_3} + \frac{a_2}{a_4} + \frac{a_3}{a_1} + \frac{a_3}{a_2} + \frac{a_3}{a_4} + \frac{a_4}{a_1} -$$

A.M. $\geq G.\,M \Rightarrow \frac{E}{12} \geq \left(\frac{a_1}{a_2}.\frac{a_1}{a_3}, \ldots.\,\frac{a_4}{a_3}\right)^{1/12} \Rightarrow E \geq 12$

Q-5 - JEE ADVANCED-PART TEST-12 (MATHS)-MATHS

The value of $\left(1 + \frac{1}{3}\right)\left(1 + \frac{1}{3^2}\right)\left(1 + \frac{1}{3^4}\right)\left(1 + \frac{1}{3^8}\right)$.to ∞ is

(A) 3

(B) 43621

(C) 43526

(D) 2

Correct Option : C

SOLUTION

$$\frac{\left(1-\frac{1}{3}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{3^2}\right)\text{...n terms}}{\left(1-\frac{1}{3}\right)} = \frac{3}{2}\left[1-\frac{1}{3^{2^h}}\right] = \frac{3}{2} \text{ as}$$

$n \to \infty$

Q-6 - JEE ADVANCED-PART TEST-12 (MATHS)-MATHS

The remainder when $15^{23} + 23^{23}$ is divided by 38 is

(A) 4

(B) 17

(C) 23

(D) 0

Correct Option : D

SOLUTION

$(19-4)^{23} + (19+4)^{23} = 2\left[.^{23} C_0 10^{23} 4 + . + .^{23} C_{22} 19^1 4^{22}\right]$

Q-7 - JEE ADVANCED-PART TEST-12 (MATHS)-MATHS

The value of $\sum_{r=0}^{20} (r(20-r))\left(.^{20}C_r\right)^2$ is equal to

(A) $400.^{39}C_{20}$

(B) $400.^{40}C_{19}$

(C) $400.^{39}C_{19}$

(D) $400.^{38}C_{20}$

Correct Option : D

SOLUTION

$$\sum_{r=0}^{20} r(20-r).^{20}C_r.^{20}C_r = \sum_{r=0}^{19} r(20-r)^{20}c_r.^{20}C_{20-r}$$

$$= 400\sum_{r=0}^{19} .^{19}C_{r-1}.^{19}C_{19-r} = 400.^{38}C_{18} = 400.^{38}C_{20}$$

The term independent from x in the expansion of

$$\left(1+\sqrt{x}+\frac{1}{\sqrt{x}-1}\right)^{-30} \text{ is}$$

(A) $.^{30}C_{20}$

(B) 0

(C) $.^{30}C_{10}$

(D) $.^{30}C_{5}$

Correct Option : B

SOLUTION

$$\left(1+\sqrt{x}+\frac{1}{\sqrt{x}-1}\right)^{-30} = \left(\frac{\sqrt{x}-1}{x}\right)^{30} = \frac{\left(\sqrt{X}-1\right)^{30}}{X^{30}} \Rightarrow$$

these is no constant term

if $\cos(x-y)$, $\cos y$, $\cos(x+y)$ are H.P. then the value of $\left|\cos y.\sec\left(\frac{x}{2}\right)\right|$ is equal to $(x = 2n\pi)$

(A) 2

(B) 1

(C) $\sqrt{2}$

(D) none of these

Correct Option : C

SOLUTION

if $a, b, c,$ are in H.P then $b = \dfrac{2ac}{a+c}$

If the sum of n terms of an A.P is $cn(n+1)$ where $c \neq 0$ then sum of cubes of these terms is

(A) $c^2n^2(n+1)^2$

(B) $2c^3n^2(n+1)^2$

(C) $\frac{2c^3}{3}n^2(n+1)(2n+1)$

(D) $\frac{2}{3}c^3n^2(n-1)(2n-1)$

Correct Option : B

SOLUTION

$t_n = S_n - S_{n-1}$

Q-11 - JEE ADVANCED-PART TEST-12 (MATHS)-MATHS

Sum of n terms of the series

$\tan\theta \sec 2\theta - \tan 2^{n-1}\theta . \sec 2^2\theta + . + \tan 2^{n-1}\theta . \sec 2^n\theta .$

(A) $\tan 2\theta - \tan 2^{n-1}\theta$

(B) $\tan 2^{n}\theta - \tan\theta$

(C) $\tan\theta - \tan 2^{n}\theta$

(D) $\tan 2^{n-1} - \tan 2\theta$

Correct Option : B

SOLUTION

Consider $\tan 2\theta - \tan\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

$= \tan\theta\left(\dfrac{2}{1-\tan^2\theta} - 1\right) = \tan\theta\left(\dfrac{1+\tan^2\theta}{1-\tan^2\theta}\right)$

$= \tan\theta.\sec 2\theta$

$\therefore S_n = \tan\theta.\sec 2\theta + \tan 2\theta + \tan 2\theta.\sec 2^2\theta + .\ + \tan 2^{n-1}\theta.\mathrm{s}$

$= (\tan 2\theta - \tan\theta) + \left(\tan 2^2\theta - \tan 2\theta\right) + ... + \left(\tan 2^n\theta - \tan 2\right.$

$= \ -\tan\theta + \tan 2^n\theta$

Q-12 - JEE ADVANCED-PART TEST-12 (MATHS)-MATHS

Let $f(n) = \sum_{r=0}^{n} \sum_{k=r}^{n} .^{k} C_{r}$ the total number of divisors of $f(9)$ is

(A) 7

(B) 8

(C) 9

(D) 6

Correct Option : B

SOLUTION

$$\sum_{k=r}^{n} .^{k} C_{r} = .^{r} C_{r} + .^{r+1} C_{r} + .^{r+2} C_{r} + .. + .^{n} C_{r}$$

$$= 1 + .^{r+1} C_{r} + .^{r+2} C_{r} + . + .^{n} C_{r}$$

$$= .^{r+1} C_{0} + .^{r+1} C_{1} + .^{r+2} C_{2} + . + .^{n} C_{n-r}$$

$$= .^{r+2} C_{1} + .^{r+2} C_{2} + . + .^{n} C_{n-r}$$

$$= .^{r+3} C_{2} + ... + .^{n} C_{n-r}$$

$$= .^{n+1} C_{n-r} = .^{n+1} C_{r+1}$$

$$\therefore f(n) = \sum_{r=0}^{n} .^{n+1} C_{r+1} = .^{n+1} C_{1} + .^{n+1} C_{2} + ... + .^{n+1} C_{n+1}$$

$$= 2^{n+1} - 1$$

$$\Rightarrow f(9) = 2^{10} - 1 = 1023 = 3.11.31$$

$$\therefore \text{ number of divisors } = (1+1)(1+1)(1+1) = 8$$

Q-13 - JEE ADVANCED-PART TEST-12 (MATHS)-MATHS

Concentric circles radii 1,2,3,..100 cm are drawn. The interior of the smallest circle is coloured red and the annular regions are coloured alternately green & red such that no two adjacent regions are of the same color. Then the total area of green regions is given by

(A) 1000 π sq. cm

(B) 5050 π sq. cm

(C) 4950 π sq. cm

(D) 5151π sq. cm

Correct Option : B

SOLUTION

Area

$= \pi\left[\left(2^2-1\right)+\left(4^2-3^2\right)+\ +\left(100^2-99^2\right)\right]=\pi[1+2+3+\cdot$

Q-14 - JEE ADVANCED-PART TEST-12 (MATHS)-MATHS

The coefficient of x^n in the expansion of $\left(1-9x+20x^2\right)^{-1}$ is given by

(A) $5^n - 4^n$

(B) $5^{n+1} - 4^{n+1}$

(C) $5^{n+1} - 4^{n-1}$

(D) $5^{n-1} - 4^{n+1}$

Correct Option : B

SOLUTION

$$\frac{1}{(1-5x)}\frac{1}{(1-4x)}$$
$$=\frac{5}{1-5x}-\frac{4}{(1-4x)}=5\left[1+(5x)+(5x)^2.\right]-4\left[1+(x)+(4\right.$$

Q-15 - JEE ADVANCED-PART TEST-12 (MATHS)-MATHS

if ninth term in the expansion of

$$\left(3^{\frac{1}{3}\log_3\left(9^{x-1}+7\right)}+\frac{1}{3^{\frac{1}{8}\log_3\left(3^{x-1}+1\right)}}\right)^{11}$$

is 660. then the value of x is

(A) 4

(B) 1 or 2

(C) 0 or 1

(D) 3

Correct Option : B

SOLUTION

$$\left(3^{\frac{1}{3}\log_3\left(9^{x-1}+7\right)}+\frac{1}{3^{\frac{1}{8}\log_3\left(3^{x-1}+1\right)}}\right)^{11}$$

$$T_g = .^{11}C_8.\left(9^{x-1}+7\right).\frac{1}{\left(3^{x-1}+1\right)} = 660$$

$$\Rightarrow \frac{9^{x-1}+7}{3^{x-1}+1} = 4 \Rightarrow t^2 - 4t + 3 = 0$$

$$\Rightarrow t = 1, 3 \Rightarrow x = 1 \text{ or } 2$$

Q-16 - JEE ADVANCED-PART TEST-12 (MATHS)-MATHS

The value of $\frac{.^{50}C_0}{3} - \frac{.^{50}C_1}{4} + \frac{.^{50}C_2}{5}. + \frac{.^{50}C_{50}}{53}$ is equal to

(A) $\int_0^1 x^3(1-x)^{50}dx$

(B) $\int_0^1 x(1-x)^{50}dx$

(C) $\frac{1}{2652}$

(D) $\frac{1}{70278}$

Correct Option : D

SOLUTION

$$\frac{.^{50}C_0}{3} - \frac{.^{50}C_1}{4} + \frac{.^{50}C_2}{5}. + \frac{.^{50}C_{50}}{53}$$

$$\int_0^1 .^{50}C_0X^2 - .^{50}C_1x^3 + .^{50}C_2x^4 + .^{50}C_{50}x^{52}dx$$

$$= \int_0^1 x^2(1-x)^{50}dx$$

$$= \int_0^1 (1-x)^2 x^{50}dx = \int_0^1 \left(1 - 2x + x^2\right)x^{50}dx$$

$$= \frac{1}{51} - \frac{2}{52} + \frac{1}{53}$$

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Q-17 - JEE ADVANCED-PART TEST-12 (MATHS)-MATHS

The value of

$.^{n}C_0.\cos n\theta + .^{n}C_1.\cos(n-2)\theta + .^{n}C_2.\cos(n-4)\theta + .. + .^{n}C$

is

(A) $2^n \cos^n \theta$

(B) $2^n \sin^n \theta$

(C) $2^{n+1} \cos n\theta$

(D) $2^{n+1} \sin n\theta$

Correct Option : A

SOLUTION

The given series is real part of

$$\sum_{r=0}^{n} .^{n} C_r e^{i(n-2r)\theta} = e^{in\theta} \sum_{r=0}^{n} \left(e^{-i2\theta}\right)^r = e^{in\theta}\left(1 + e^{-2i\theta}\right)^n$$
$$= \left(e^{i\theta} + e^{-i\theta}\right) = (2\cos\theta)^n$$

Q-18 - JEE ADVANCED-PART TEST-12 (MATHS)-MATHS

if t_n denotes the n^{th} term and S_n denotes sum to first n terms of the series $3 + 15 + 35 + 63 + ..$ then

(A) $t_{50} = 50^2 - 1$

(B) $S_{20} = 11460$

(C) $t_{50} = 4.50^2 - 1$

(D) $S_{20} = 11640$

Correct Option : B

SOLUTION

$t_n = an^2 + bn + c$

Q-19 - JEE ADVANCED-PART TEST-12 (MATHS)-MATHS

if $a = \sum_{r=0}^{20} .^{20} C_r, b = \sum_{r=0}^{9} .^{20} C_r. c = \sum_{r=11}^{20} .^{20} C_r$ then

(A) $a = b + c$

(B) $b = 2^{19} - \frac{1}{2}.^{20} C_{10}$

(C) $c = 2^{19} + \frac{1}{2}.^{20} C_{10}$

(D) $a - 2c = \frac{2^{10}(1.3.6 � .19)}{10!}$

Correct Option : B

SOLUTION

$b = .^{20}C_0 + .^{20}C_1 + \; + .^{20}C_9 = .^{20}C_9 = .^{20}C_{20} + \; + .^{20}C_{11} = c$

$\Rightarrow a = b + c + .^{20}C_{10} \Rightarrow a = 2b + .^{20}C_{10}$

$$\Rightarrow a - 2b = \frac{20!}{10!10!} = \frac{(2.4.20)(1.3.519)}{10!10!} = \frac{2^{10}(1.3.5.19)}{10!}$$

Q-20 - JEE ADVANCED-PART TEST-12 (MATHS)-MATHS

Consider the series

$2016. n + 2015. (n-1) + 2014. (n-2) + 2013. (n-3) + .$

where S_n is the sum of first n terns of the series which of the following is/are true

(A) $S_n = \dfrac{n(n+1)(6949-n)}{6}$

(B) $S_n = \dfrac{n(n+1)(3025-n)}{3}$

(C) $S_{20} = 422030$

(D) $S_{20} = 420700$

Correct Option : A

SOLUTION

$T_r = (2017 - r)(n - r + 1)$

$= 2017.(n+1) - (2018 + n)r + r^2$

$\therefore S_n = \sum_{r=1}^{n} T_r = 2017(n+1)\sum_{r=1}^{n} 1 - (2018+n)\sum_{r=1}^{n} r + \sum_{r=1}^{n} r^2$

$S_n = \dfrac{n(n+1)(6949-n)}{6}$

Q-21 - JEE ADVANCED-PART TEST-12 (MATHS)-MATHS

The natural numbers are written as a sequence of digits

123456678910112. Then in the sequence

(A) 190^{th} digit is 1

(B) 201^{st} digit is 3

(C) 2014 th digit is 8

(D) 2013^{th} digit is same as 2014^{th} digit

Correct Option : A

SOLUTION

$1, 1. .9 \Rightarrow 9$

$10, 11.99 \Rightarrow 180$

190^{th} digit 1

201^{st} digit is 3

$100, 101, 102707 \Rightarrow 608 \times 3 = 1824 \Rightarrow 9 + 180 + 1824 = 2013$

so 2014^{th} digit is 7. $(\therefore 708)$

Q-22 - JEE ADVANCED-PART TEST-12 (MATHS)-MATHS

if $N = 7^{2014}$ then

(A) sum of last four digits of N is 23

(B) number of divisors of N are 2014

(C) number of composite divisors of N are 2013

(D) if number of prime divisors of N are p then number of ways to express a non-zero vector coplanar with two given non-collinear vectors as a linear combination of the two vectors is $p+1$

Correct Option : A

SOLUTION

$7^{2014} = 49(1+2400)^{503} = 49\left(1207201 + 10^4_\lambda\right) = 59152849 + 10^4$

divirsors are $7^0, 7^1, 7^2, 7^{2014}$

$\Rightarrow$ No. of divisors are 2015, composite divisors 2013 and prime divisors $1 \Rightarrow p = 1$ also no of ways to express a non-zero vector coplanar with two given non-collinear vectors as a linear combination of the two vectors $= 1$

Q-23 - JEE ADVANCED-PART TEST-12 (MATHS)-MATHS

Consider the sequnce of numbers $\alpha_0, \alpha_1, \ldots, \alpha_n$ where

$\alpha_0 = 17.23.\ \alpha_1 = 33.23$ and $\alpha_{r+2} = \dfrac{\alpha_r + \alpha_{r+1}}{2}$ then

(A) $|\alpha_{10}| = \dfrac{1}{32}$

(B) $\alpha_0 - \alpha_1, \alpha_2, \alpha_3,$ � are in G.P

(C) $\alpha_0 - \alpha_2, 2(\alpha_1 - \alpha_2), \alpha_1 - \alpha_3$ are in H.P

(D) $|\alpha_{10} - \alpha_9| = |\alpha_6 + \alpha_7|$

Correct Option : A

SOLUTION

$2\alpha_{r+2} = \alpha_l + \alpha_{r+1}$

$\Rightarrow 2(\alpha_{r+2} - \alpha_{r+1}) = \alpha_t - \alpha_{r+1} \Rightarrow \alpha_{r+2} - \alpha_{r+1} = -\dfrac{1}{2}(\alpha_{r+1}$

$\Rightarrow \alpha_{10} - \alpha_9 = -\dfrac{1}{2}(\alpha_9 - \alpha_8) = \dfrac{1}{4}(\alpha_8 - \alpha_7) = \ldots. = -\dfrac{1}{2^9}(\alpha_1$

As $\alpha_0 - \alpha_1, \alpha_1 - \alpha_2, \alpha_2 - \alpha_3$ are in G.P.

$\Rightarrow \alpha_0 - \alpha_2, 2(\alpha_1 - \alpha_2), \alpha_1 - \alpha_3$ are in H.P. (adding middle term to all terms)

Q-24 - JEE ADVANCED-PART TEST-12 (MATHS)-MATHS

Give n arithmetic means are inserted between each of the two sets of numbers a, 2b and 2a, b where $a, b, \in, R$. If m^{th} mean of the two sets of numbers is same then

(A) $\dfrac{a}{b} = \dfrac{m}{n - m + 1}$

(B) $\dfrac{a}{b} = \dfrac{n}{n - m + 1}$

(C) $\dfrac{a}{b} < n$

(D) $\dfrac{a}{b} \leq m$

Correct Option : A

SOLUTION

$$A_m = a + m\left(\frac{2b-a}{n+1}\right)$$

$$A'_m = 2a + m\left(\frac{b-2a}{n+1}\right)$$

$$\Rightarrow a(n+1) + m(2b+a) = 2a(n+1) + m(b-2a)$$

$$\Rightarrow bm = a(n-m+1)$$

Now if $\frac{a}{b} < m < n^2 - mn + n$

$\Rightarrow m - n < n(n-m)$ which is false for $n = m$

$\frac{a}{b} \le m \Rightarrow \frac{m}{n-m+1} \le m \Rightarrow 0 \le m(n-m)$ which is true.

Q-25 - JEE ADVANCED-PART TEST-12 (MATHS)-MATHS

if a,b,c, are any three terms of an A.P such that $a = b$ then $\frac{b-c}{a-b}$ may be equal to

(A) 0

(B) $\sqrt{3}$

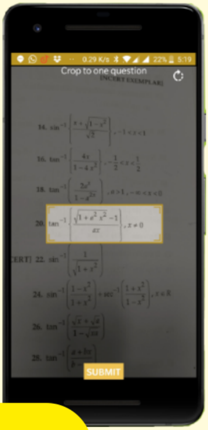
(C) 1

(D) 2

Correct Option : C

SOLUTION

$$\frac{b-c}{a-b}=\frac{[A+(q-1)D]-[A+(r-1)D]}{[A+(p-1)D]-[A+(q-1)D]}=\frac{q-r}{p-q}$$ rational number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